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Elastic electron scattering is usually analyzed in terms of the elastic charge form factor F_{ch} defined as the Fourier transform of the ground state charge density. Using the nuclear charge operator as given in (1) we have the form factor:

$$F_{ch} = (G_{Ep} + G_{En})(1 + \frac{q_\mu^2}{8M^2})\, e^{\frac{q^2}{4A\alpha^2}} \left\langle \Psi_{SM} \right| \frac{1}{A} \sum_{j=1}^{A} e^{i\vec{q}\cdot\vec{r}_\gamma} \left| \Psi_{SM} \right\rangle \qquad (1)$$

where G_E is the electric form factor(2) of the nucleon; M, $\vec{q}$, q_μ^2 are the nucleon mass, three momentum transfer and four-momentum squared, respectively. The third term in (1) is the correction for the center-of-mass motion of the target(3) evaluated in the shell model with oscillator potential for the nucleus. We shall use this model henceforth; α is the parameter of the oscillator well. The last term in (1) is called the shell model elastic form factor of the nucleus F_{SM}; Ψ_{SM} being the completely antisymmetrised shell model ground state wave function. In (1) the same number of protons and neutrons Z = 1/2 A is assumed.

Recently the influence of the short range nucleon-nucleon correlations on the elastic form factor has been discussed extensively(4)-(9). In refs. (5)-(8) the correlations were introduced by using a Jastrow type nuclear density of the form(10):

$$\left|\tilde{\Psi}(\vec{r}_1 \dots \vec{r}_A)\right|^2 = \left|\Psi_{SM}(\vec{r}_1 \dots r_A)\right|^2 \prod_{j<k}^{A} \left[1 - h(s_{jk})\right] = \qquad (2)$$
$$= \left|\Psi_{SM}\right|^2 \left[1 - \sum_{j<k} h(s_{jk}) + \sum_{j<k,\, l<m} h(s_{jk}) h(s_{lm}) - \dots\right]$$

(x) - Instytut Fizyki Jadrowej, Krakow (Poland).

The function $h(s_{jk})$ introduces correlations of the (j, k) pair; in order to simulate at small relative distances $s_{jk} = |\vec{r}_j - \vec{r}_k|$ the hard-core repulsion between nucleons the gaussian for of $k(s_{jk}) = ecx\,(-(1/2)\,\Lambda^2 s^2_{jk})$ was assumed.

Because of the complexity of (2) the expression for the correlated form factor (with $\tilde{\Psi}$ instead of Ψ_{SM}) can be evaluated exactly only for simple systems like He^4. In fact, in refs. (5)-(7) only the contributions from the one correlated pair part of $\tilde{\Psi}$, i.e. terms in the series (2) with powers, of $h(s_{jk})$ smaller than two were retained. The single correlated pair approximation (s.c.p.a.) has, however, been questioned in ref. (8) where it was shown that the exact calculations for He^4 give quite a different result from that obtained in (5) with the help of this approximation.

Does the s.c.p.a. fail also for heavier nuclei? In order to answer this question we perform the correlation calculations for Li^6, C^{12} and O^{16}. As it would be almost impossible to perform the exact Jastrow type calculations for these nuclei we use a method which has been described in our previous paper(9).

In(9) the shell model form factor F_{SM} was expressed in terms of the matrix elements between the two-particle states. In the case of harmonic oscillator wave functions one can go over from the motion of two particles about a common center to a description of the relative and c.m. motion of the two particles. The nucleon-nucleon correlations were intruduced in (9) by modifying the radial wave function $R_{n\ell}(r)$ of the relative two-nucleon motion:

$$(3) \qquad |\widetilde{n\ell m}\rangle = \frac{g(r)}{\sqrt{N_{n\ell}}} R_{n\ell}\, Y_{\ell m}, \qquad N_{n\ell} = \int_0^\infty dr\, r^2 R^2_{n\ell}\, g^2(r)$$

where g(r) is a certain function.

Employing the Moshinsky technique(11) we obtain after some algebra the following correlation correction:

$$\begin{aligned}
\Delta F_{SM} = \frac{1}{Z(2Z-1)} \Big\{ & 6\, e^{-t_s} \Delta(000,000;s) + (Z-2)\, e^{-t_p} \Big[(3 - 4\,t_p + t_p^2) \times \\
& \times \Delta(000,000;p) + \tfrac{1}{2} \Delta(100,100;p) - \sqrt{\tfrac{2}{3}}\, t_p\, \Delta(100,000;p) + \\
(4) \qquad & + \Delta \sum_m (01m,01m;p) - 2\,t_p\, \Delta(011,011;p) + \tfrac{1}{2} \Delta \sum_m (02m,02m;p) + \\
& + \frac{2}{\sqrt{3}}\, t_p\, \Delta(020,000;p) \Big] + 4(Z-2)\, e^{-t_{sp}} \Big[(1 - \tfrac{2}{3} t_{sp})\, \Delta(000,000,s_p) + \\
& + \tfrac{1}{3} \Delta \sum_m (01m,01m;s_p) \Big] \Big\}
\end{aligned}$$

where $t_c = q^2/(8\alpha_c^2)$; c = s, p, sp, $(n\,l\,m,\ n'\,l'\,m';\ c)$ = $= \langle (n\,l\,m)_c | \exp[(i/\sqrt{2})qz] | (n'\,l'\,m')_c \rangle$ and $\Delta(\ldots)$ denotes the difference between correlated and uncorrelated magnitudes(12). The formula (4) is valid for nuclei with two protons in the s-shell and Z-2 protons in the p-shell; the oscillator parameters for the two shells are assumed to be different. The index c denotes the source of the correlation correction: two correlated nucleons from the s/p/-shell yield the terms with c = s/p/ while the correlation between two nucleons from the different shells introduces the terms with c=sp(14).

We use the following form of g(s) function (s being the distance between two nucleons);

$$g(s) = \sqrt{1 - e^{-1/2\,\Lambda^2 s^2}} \tag{5}$$

Thus we are able to compare our results with those(6) obtained with help of the Jastrow method and s.c.p.a. The results of our analysis for the Li^6, C^{12} and O^{16} nuclei are presented in Figs. 1, 2 and 3.

Our conclusions and comments are the following :

a) In the case of Li^6 the elastic electron scattering turns out to be sensitive to the nucleon-nucleon correlations only at large momentum transfers. The low momentum transfer data (up to $q = 2\ fm^{-1}$) for this nucleus are well explained in the oscillator shell model provided one assumes that the s- and p-shell protons move in different wells. At large momentum transfers there is, however, a deviation from the uncorrelated shell model. Introducing the correlations one improves the situation. We have obtained a good fit to the experimental data over the wide range of momentum transfer (see Fig. 1). Our calculations predict a diffraction minimum for Li^6 at $q = 3.7\ fm^{-1}$.

b) In the elastic electron scattering from C^{12} and O^{16} the short range correlations are less important. The experimental results for these nuclei are fairly well explained in the oscillator shell model without the correlations.

c) Our calculations are in a drastic disagreement with the results(6) which were based on the Jastrow method with the s.c.p.a. Using the oscillator and correlation parameters as given in ref. (6) we have evaluated the dotted lines in Figs. 1, 2 and 3. These curves are inconsistent with the experimental results while in ref. (6) applying the same parameters, a good accord with experiment was obtained. This comparison allows us to state that the single correlated pair approximation is wrong. In our opinion, this approximation considerably overestimates the effect of the short range correlations.

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T. Janssens, R. Hofstadter, E.B. Hughes and M.R. Yearian, Phys. Rev. 142, 922 (1966).
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If one assumes for the s-p nuclei the oscillator model in which the s - and p - nucleons move in different wells ($\alpha_s \neq \alpha_p$) the parameter in the c.m. correction is given by:

$$A\alpha^2 \rightarrow 4\,\alpha_s^2 + 2(Z-2)\,\alpha_p^2 \; .$$

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(9) - A. Małecki and P. Picchi, Frascati report LNF-68/27 (1968); Phys. Rev. Letters, to be published.
(10) - R.J. Jastrow, Phys. Rev. 98, 1479 (1955).
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(12) - We use Moshinsky's notation for the radial quantum number n. The more usual n' can be obtained by adding unity to Moshinsky's values. In order to obtain orthonormality of all the correlated states involved in calculations for the $|100\rangle$ state a slightly different from (3) radial dependence should be introduced(13):

$$\tilde{R}_{10} = 6^{1/2}\,\pi^{-1/4}\,\alpha^{3/2}\,\frac{g(r)}{\sqrt{N_{10}}}\,(1 + \delta - 2/3\,\alpha^2 r^2)\,e^{-1/2\,\alpha^2 r^2}$$

where

$$N_{10} = -3/2\,\frac{N_{01}^2}{N_{00}} + \frac{5}{2}N_{02} \quad \text{and} \quad \delta = \frac{N_{01}}{N_{00}} - 1 \; .$$

(13) - W. Czyz, L. Lesniak and A. Małecki, Ann. Phys. 42, 119 (1967).
(14) - The average oscillator parameter α_{sp} was assumed to be:

$$\alpha_{sp} = \sqrt{5Z-4}\,\left[6\,\alpha_s^{-2} + 5(Z-2)\,\alpha_p^{-2}\right]^{-1/2} \; .$$

(15) - L.R. Suelzle, M.R. Yearian and H. Crannel, Phys. Rev. 162, 992 (1967).

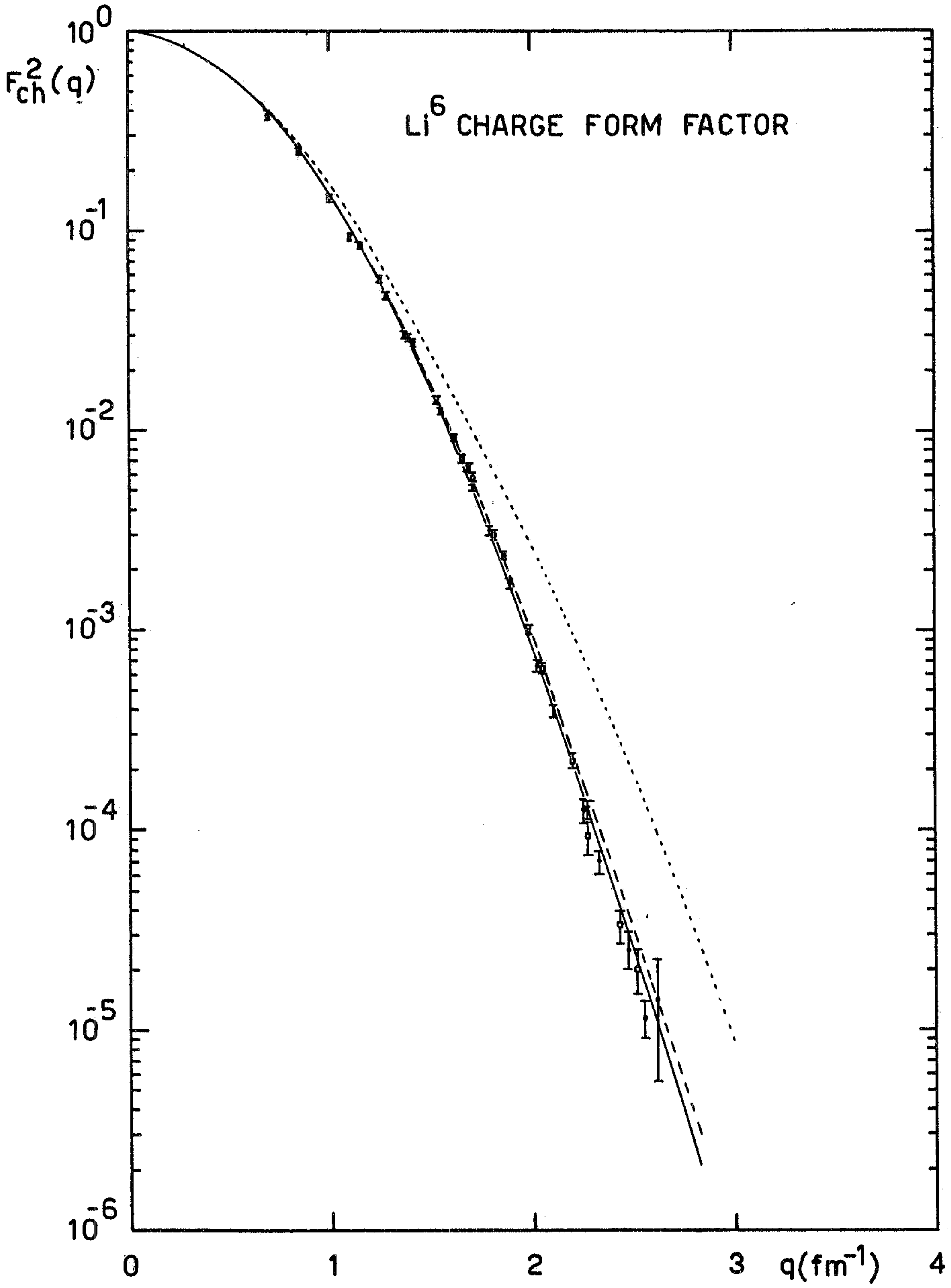

FIG. 1 - Elastic form factor of Li^6. The experimental points are from ref. (15). The dashed line represents the uncorrelated form factor with α_s = 118 MeV, α_p = 106 MeV. The full line was obtained with the same α's and with the correlation parameter Λ = 2 fm^{-1}. The dotted line, calculated with α_s = 127.5 MeV, α_p = 100.6 MeV and Λ = 1.878 fm^{-1}, is given for a comparison with ref. (6).

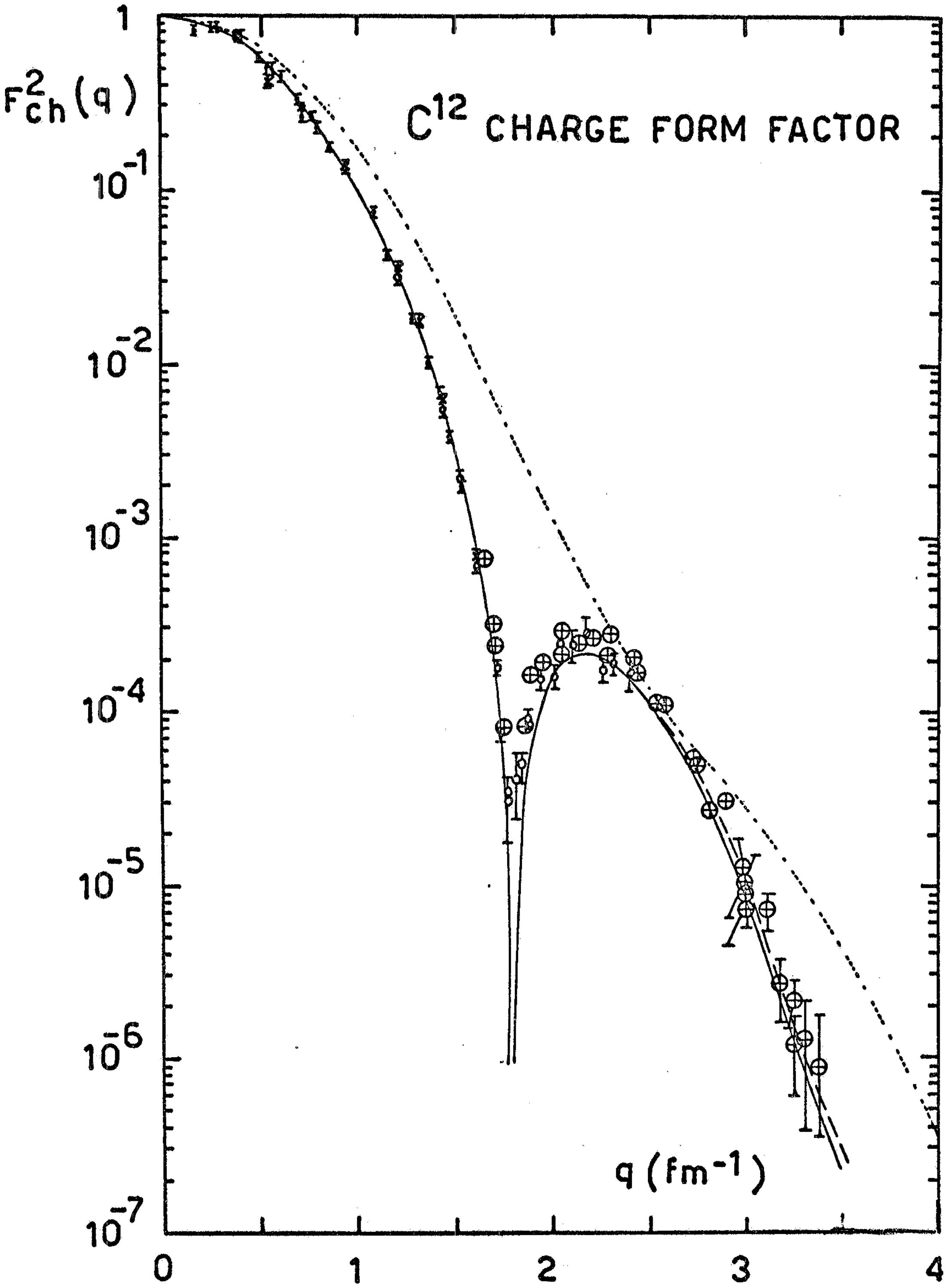

FIG. 2 - Elastic form factor of C^{12}. The experimental data are those reported in ref. (6). The dashed line represents the uncorrelated form factor with $\alpha_s = \alpha_p = \alpha \doteq 119.5$ MeV. The full line was obtained with the same α and with the correlation parameter $\Lambda = 2\ fm^{-1}$. The dotted line, calculated with $\alpha_s = 168.2$ MeV, $\alpha_p = 126.2$ MeV and $\Lambda = 2.156\ fm^{-1}$, is given for the comparison with the result of ref. (6).

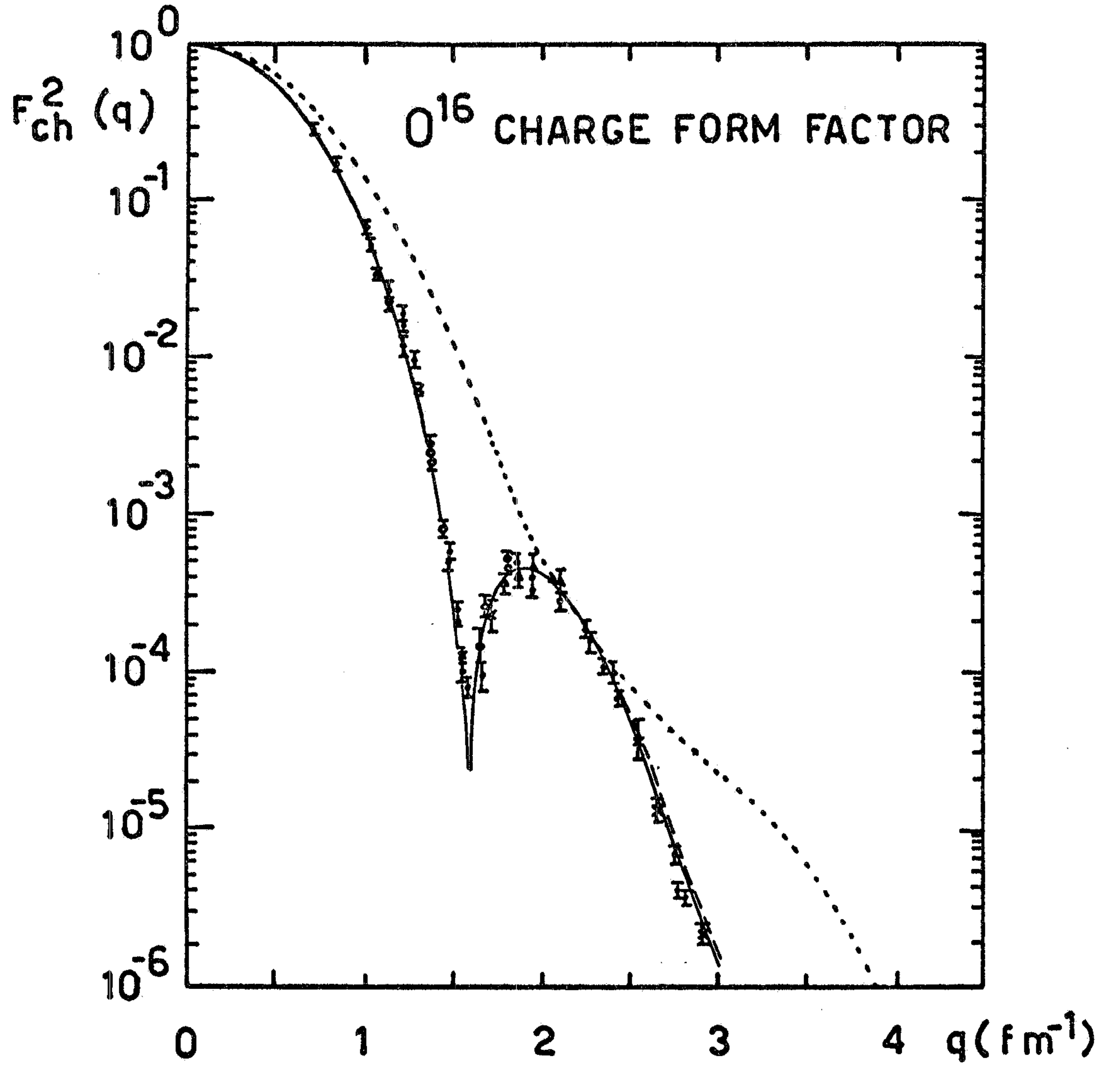

FIG. 3 - Elastic form factor of O^{16}. The experimental data are those reported in ref. (6). The dashed line represents the uncorrelated form factor with $\alpha_s = \alpha_p = \alpha = 10.9$ MeV. The full line was obtained with the same α and with the correlation parameter $\Lambda = 2$ fm^{-1}. The dotted line, calculated with $\alpha_s = 175.1$ MeV, $\alpha_p = 122.3$ MeV and $\Lambda = 2.426$ fm^{-1}, is given for the comparison with the result of ref. (6).