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C. Bernardini : HIGH ENERGY EXPERIMENTS ON QED. -

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C. Bernardini: HIGH ENERGY EXPERIMENTS ON QED. -

(Two Lectures at Boulder, Colo.)

I - Many physicists would bet in favour of predictions of QED in it's present form. At the same time there are many among us searching for an experimental basis to this confidence: I will review the efforts recently done to test QED in the high energy region.

Let me first try to say what I mean by "high energy". We are well acquainted with Feynman's language and very often use simple diagrams like the one shown in Fig. 1

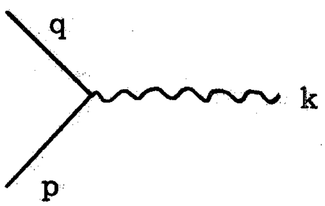


FIG. 1

I am not authorized to neglect "a priori" such possibilities as shown in Fig. 2 (many photon diagrams). But this is what time-honored QED does

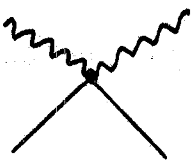
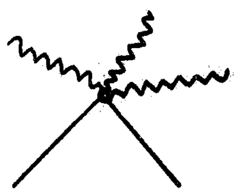


FIG. 2



and nobody found till now good reasons to complicate life and sketches[1]; so, I will continue traditions.

The diagram in Fig. 1 describes a one-photon vertex related to the emission or absorption of a γ ray (4-momentum k) by a charged lepton line. The charged lepton 4-momentum changes from p to q and 4-momentum conservation is required.

Three independent invariants can be formed out of the three 4-momenta p, k, q. The particular choice: p^2, k^2, q^2 allows rapid recognition of particles on the mass shell since in that case the invariants of external (or real) legs will trivially reduce to squared masses. Structure factors like form factors in the vertex or Lorentz invariant propagator modifications will in general be described by functions of p^2, k^2, q^2.

At least one of the three particles involved can be virtual: in every first order process (in the sense of perturbation theory) there is in fact only one (there would be none only in unobserved decays like $\mu \rightarrow e + \gamma$): when the modulus of the squared 4-momentum of a virtual particle is large compared to the squared electron mass I say that we are in the high energy region.

High energy contributions will occur both in the measurement of a static property, (like the g-2 factor of charged leptons) and in dynamical tests of QED (like the determination of cross sections of genuine electromagnetic processes). When measuring static properties, high order processes give the relevant contributions and large virtual momenta appear in loop diagrams only i.e. as a part of an integration range extending from the mass shell to infinity. In dynamical tests it is possible however to select by kinematics sharply defined virtual momenta.

Present techniques allow the measurement of static properties to parts per million accuracy or better; the measurement of cross sections to a few percent. Both kind of experiments are thus relevant[2] but I will review only part of the whole story, namely cross sections.

Even in it's semplicity, QED makes transparent predictions only in the case of first order processes; I mean by this that interference mixing of first order diagrams is reasonably analizable. Thus, my personal preference will go in the following to the examination of two body electromagnetic reactions both because od their semplicity and, even more important, the fact that they have non negligible cross sections.

II - Let me briefly summarize the kinematics of the two-body reactions I will discuss in the following sections. I will use such a metric that $q^2 > 0$ is time-like and $q^2 < 0$ is space-like.

Three distinct kind of diagram can occur depending on the specific nature of particles involved.

I will call a diagram having the structure shown in Fig. 3 (time goes upwards) a scattering diagram. When particles 3 and 4 are identical, the exchange of p_3 and p_4 gives the same physical result to be accounted for by the exchange diagram shown in Fig. 4

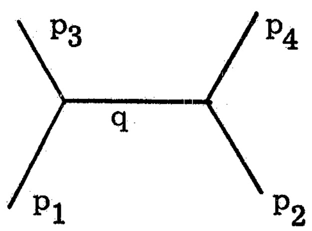


FIG. 3

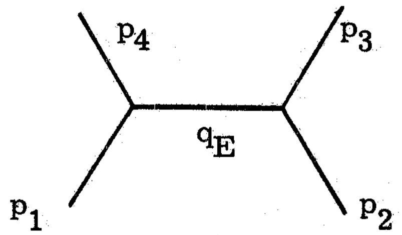


FIG. 4

When particles 1 and 2 can annihilate into a single virtual particle (like $e^+e^- \to$ "γ" or $\gamma e \to$"e") annihilation diagrams occur like the one shown in Fig. 5

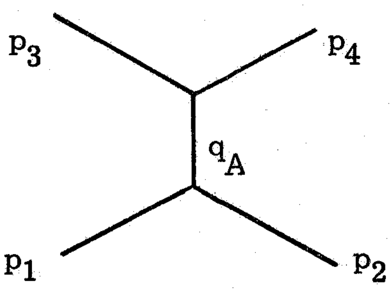


FIG. 5

Now, the distinctive kinematical feature of the 4-momenta q, q_E, q_A is their space-time character. Calling m_i the mass of particle i in a diagram, one has

$$q^2 = (p_3 - p_1)^2 = m_3^2 + m_1^2 - 2\, p_1 \cdot p_3 \leq (m_1 - m_3)^2$$

$$q_E^2 = (p_4 - p_1)^2 = m_4^2 + m_1^2 - 2\, p_1 \cdot p_4 \leq (m_1 - m_4)^2$$

$$p_A^2 = (p_1 + p_2)^2 = m_1^2 + m_2^2 + 2\, p_1 \cdot p_2 \geq (m_1 + m_2)^2$$

We see from the inequalities to the right that q^2 and q_E^2 will usually reach the space-like region whereas q_A^2 is limited to time-like values.

Two important reference systems can be introduced: the center of mass system (C.M.S.; actually met when working with storage rings) and the laboratory system (L.S.; to be used in connection with conventional beams and targets).

C.M.S. is defined by $\vec{p}_1 = -\vec{p}_2$; $\vec{p}_3 = -\vec{p}_4$ then follows. The total energy of incoming particles will be denoted by $W = E_1 + E_2$ and used as parameter. Clearly $q_A^2 = W^2$. Also, calling θ_{ik} the angle between $\vec{p}_i$ and $\vec{p}_k$

$$q^2 = m_1^2 + m_3^2 - 2E_1 E_3 + 2 |\vec{p}_1| \, |\vec{p}_3| \cos\theta_{13}$$

$$q_E^2 = m_1^2 + m_3^2 - 2E_1 E_3 - 2 |\vec{p}_1| \, |\vec{p}_3| \cos\theta_{13}$$

since particles 3 and 4 must be identical to have exchange contributions. q^2 and q_E^2 clearly exchange their role at $\theta_{13} = 90^o$; thus only the range $0 \leq \theta_{13} \leq 90^o$ has a physical meaning.

E_i and p_i can be expressed in terms of W:

$$E_{1,3} = \sqrt{m_{1,3}^2 + |\vec{p}_{1,3}|^2} = \frac{W^2 + m_{1,3}^2 - m_{2,4}^2}{2W}$$

$$E_{2,4} = \sqrt{m_{2,4}^2 + |\vec{p}_{2,4}|^2} = W - E_{1,3}$$

where only indices on the same side of the comma must be associated.

L.S. is defined by $p_2 = (m_2, 0, 0, 0)$. The energy of the incoming particle is $E_1 = E$ to be taken as parameter. Then

$$q_A^2 = m_1^2 + m_2^2 + 2Em_2$$

Also,

$$q^2 = m_2^2 + m_4^2 - 2m_2 E_4$$

$$q_E^2 = m_2^2 + m_4^2 - 2m_2 E_3$$

having put $m_3 = m_4$ for exchange. Note that $q^2 + q_E^2 = 2m_4^2 - 2Em_2$. At $E_3 = E_4$, q^2 and q_E^2 exchange their role. Thus, only the range

$$E_3 \leq \frac{1}{2} (E + m_2)$$

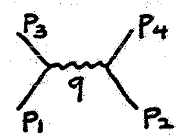
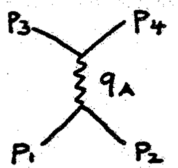
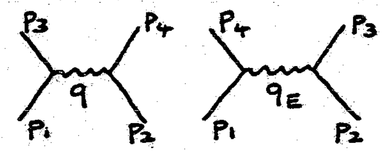
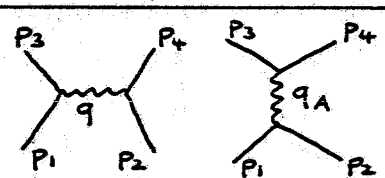
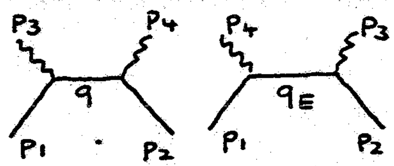
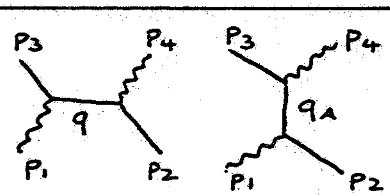
has a physical meaning. This inequality also defines a maximum meaningful scattering angle for reactions allowing exchange as given by

$$\cos^2 \theta_{max} = \frac{E^2 - m_2^2}{(E + m_2)^2 - 4m_4^2}$$

Both in the C.M.S. and the L.S. a threshold condition for the reaction to occur is easily expressed by $q_A^2 \geq (m_3 + m_4)^2$.

III - The complete collection of first order electromagnetic processes is shown in Table I. The fact that electromagnetic transitions changing the nature of a lépton line at a photon vertex have not been observed (compare sect. XII A in ref. (2)) introduces some peculiarities: $e\mu$ scattering and

TABLE I

Reactions	Name	Diagrams	Literature for cross sections
$e\mu \rightarrow e\mu$ (also, bound $e^-\mu^+$ or μ^-e^+)	eμ scattering (muonium)		Use e-p cross section for point protons with no anomalous magnetic moment, substituting muon to proton mass. Ref. (5)
$e^+e^- \rightarrow \mu^+\mu^-$	2 μ annihilation of e - pairs		Ref. (3), formula (21) pag. 115. Delete form factor $F(K^2)$ and note that e-mass is neglected.
$e^-e^- \rightarrow e^-e^-$ $e^+e^+ \rightarrow e^+e^+$ $\mu^+\mu^+ \rightarrow \mu^+\mu^+$ $\mu^-\mu^- \rightarrow \mu^-\mu^-$	Møller scattering		Ref. (4), pag. 252. Both C. M. S. and L. S. cross sections are given.
$e^+e^- \rightarrow e^+e^-$ $\mu^+\mu^- \rightarrow \mu^+\mu^-$ (also, bound e^+e^- or $\mu^+\mu^-$)	Bhabha scattering (positronium)		Ref. (4), pg. 257. Both C. M. S. and L. S. cross sections are given.
$e^+e^- \rightarrow 2\gamma$ $\mu^+\mu^- \rightarrow 2\gamma$	2γ annihilation of pairs		Ref. (3), (4). C. M. S. used in (3) pag. 106, relativistic limit, put F = 1. L. S. used in (4) pg. 263.
$\gamma e^- \rightarrow \gamma e^-$ $\gamma e^+ \rightarrow \gamma e^+$ $\gamma\mu^- \rightarrow \gamma\mu^-$ $\gamma\mu^+ \rightarrow \gamma\mu^+$	Compton scattering		Ref. (4), pg. 229. Explicit formula is given in L. S. only.

$e^+e^- \rightarrow \mu^+\mu^-$ (two muon annihilation of e-pairs) are described by single diagrams, thus defining a single invariant mass for the virtual particle. All other first order reactions involving only one kind of leptons are described by a combination of two diagrams because of either exchange or annihilation contributions.

In the table, electrons and muons are shown by light lines, photons by wavy lines; besides, time goes upwards. Bound states (muonium and positronium) are mentioned but will not be examined in the following.

IV - Kinematics show that the center of mass is a highly privileged system since the whole 4-momentum of reacting particles can be put into the virtual intermediate states without having to pay for center of mass momentum conservation.

Storage rings have this good quality to allow working in the center of mass system: but the technique is limited to e^-e^- and e^+e^- collisions. Thus, the reactions to be considered in the realm of storage rings are, at present,

$$e^+e^- \rightarrow \mu^+\mu^-$$

$$e^-e^- \rightarrow e^-e^-$$

$$e^+e^- \rightarrow e^+e^-$$

$$e^+e^- \rightarrow 2\gamma$$

For all the reactions in Table I, the kinematic situation in the laboratory system is very poor. Suppose you want to observe e^-e^- scattering by sending $E = 20$ GeV electrons (at SLAC) on atomic electrons in a target. Then the maximum momentum transfer you get is only

$$k^2 \approx - m_e E \cong - (100 \text{ MeV})^2$$

Performing experiments serious difficulties would come from the fact that relevant events are produced, in the lab system, on a very narrow cone: for instance in the e^-e^- case the cone is restricted to

$$\theta_{max} \approx \sqrt{2} \left(\frac{m_e}{E}\right)^{1/2} \approx 7 \cdot 10^{-3} \text{ rad for SLAC}$$

A way out is to allow for the presence of a massive body taking the role of momentum absorber: such bodies exist (the nucleons or the nuclei) but they have strong interactions besides electromagnetic interactions. As a consequence QED tests involving nuclear targets are somewhat contaminated; nevertheless, having some confidence in the possibility of evaluating the contamination and carefully restricting the kinematical conditions to small-contamination regions a lot of work can and has been done.

Consider for instance Compton scattering, $e\gamma \to e\gamma$. The electron bremsstrahlung on nucleons is a quite similar process

$$ep \to ep\gamma$$

as can be seen from the first order diagrams

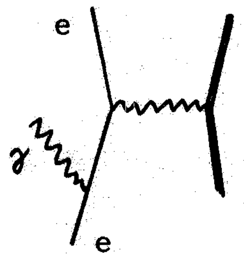
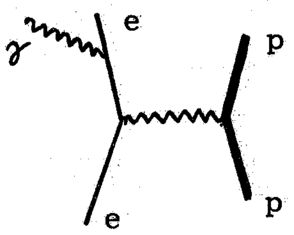


FIG. 6



Compare the diagrams in Fig. 6 to the two diagrams for Compton scattering in Table I: the difference is that one of the photons (being virtual) ends on a proton line. Assuming that the $p\gamma p$ vertex is known from ep scattering one can proceed to calculate the cross section giving enphasis to the virtual lepton, like in the Compton case(6).

Contamination is not yet in. A γ ray in the final state can be produced in a different way than shown in Fig. 6. Give a look to the graph in Fig. 7, called a "virtual-proton-Compton contribution".

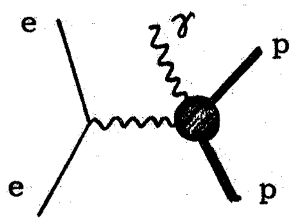


FIG. 7

The bubble on the proton line masks the cussedness of strong interactions, for instance the production of intermediate $N^*(1238)$ and it's successive decay into proton plus gamma.

Analysis of virtual Compton contributions is delicate, depends on models for the strong particles and has been done till now only in special cases.

V - The contamination I just spoke of using the bremsstrahlung as an example is not very serious "in principle". There is a more fundamental one: the γ ray, being a $J^{PC} = 1^{--}$ neutral vector particle can make transitions to other 1^{--} massive neutral vector particles (V^{o}'s) like the ρ^o, ω^o, ϕ^o. Thus, even if we carefully prepare the reacting bodies by eliminating strong interacting particles from the initial states, these can appear and break pure QED just because of a $\gamma \to V^o$ transition. It is true that even a non resonant

$\gamma \to \pi^+\pi^-$ transition would do when the $\pi^+\pi^-$ system is 1^{--}, but this possibility is not so serious as the resonant $\gamma \to V^o$ process.

A convincing evidence for the role of $\gamma \to V^o$ is given for instance by the observation of

$$e^+e^- \to \pi^+\pi^-$$

around the ρ^o mass in storage rings[7].

The branching ratio $(\rho^o \to e^+e^-)/(\rho^o \to \pi^+\pi^-)$ is about 5×10^{-5}, very near to what reasonable models predict[3].

From QED's point of view, the most striking prediction perhaps concerns a structure of the $e^+e^- \to \mu^+\mu^-$ cross section in the region of total center of mass energy comparable to the ϕ^o mass. In this case(not yet observed but likely to be very soon)[8] the intervention of a ϕ^o in the vacuum polarization contribution to 2μ annihilation should give a peculiar wavy structure to the cross section, shown in Fig. 8.

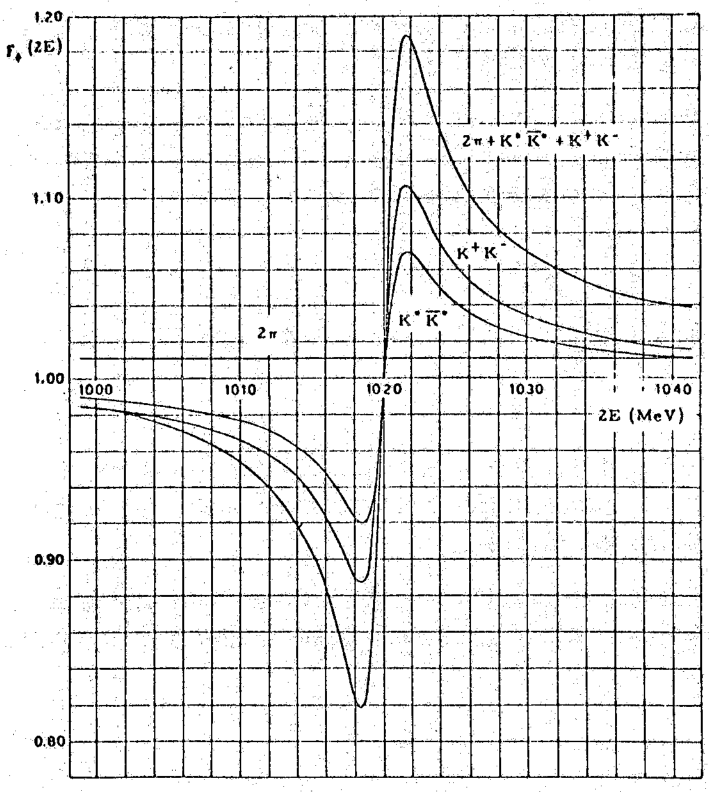


FIG. 8 - $F_\phi(2E)$ is the ratio of cross section for $e^+e^- \to \mu^+\mu^-$ including vacuum polarization effect to cross section without vacuum polarization around the ϕ-mass; separate contributions of 2π (non-resonant) and 2K (resonant) are also shown.

VI - A review of simple possibilites in high energy QED tests is given in Table II. I hope the table is intelligible and make just a few comments on it.

VI/A - Comparison of ep and μp elastic scattering (not mentioned in the table) at the same momentum transfer can give informations on the relative behaviour of e γ e and $\mu\gamma\mu$ vertices. Suppose however that electrons and muons can exchange a boson, other than the photon[9], not very much coupled to nucleons: you would miss this extravagant possibility in indirect tests. In this sense, only direct tests are quoted in the table without denying by this choice the full value of charged lepton-proton scattering experiments as sources for QED[10].

VI/B - For some reactions in the table, a direct and an inverse process can be defined. The one indicated is a reaction to be considered actually feasible as compared to it's inverse. This is trivially obvious for $\gamma\gamma \rightarrow e^+ e^-$ for instance. In the case of contaminated processes, inverse reactions have the handicap of three-body encounters in the initial state; in the interesting case of one-photon annihilation of e^+ on bound electrons, $e^+ H \rightarrow p\gamma$, (the inverse of wide angle e-pairs), the cross section is strongly depressed by atomic form factors in the high-energy region.

VI/C - Higher order possibilities of testing QED have often been examined. A well known example is trident production $eN \rightarrow eNe^+e^-$[11]. In the case of tridents, eight diagrams occur (4 basic diagrams plus 4 of the exchange type). In the limit of minimum nuclear recoil, the off-mass shell photon producing the pair plays the only relevant role. This photon will appear in the cross section at four different 4-momenta both time - and space-like : nevertheless conclusions can be drawn concerning the behaviour of photon propagators and vertices by accurate choice of the kinematics.

VII - Before going to experimental results I need make two remarks, namely:

VII/A - Radiative corrections.

VII/B - Currently used methods of analysis of the data.

VII/A - Radiative corrections constitute a serious headache when working with light charged particles. It is true that they do not pose any fundamental problem and can, in principle, be evaluated with a high confidence level given the present accuracy of experiments. But to get involved into specific radiative correction calculations looks very disappointing to most of the people I know: the attitude against radiative corrections has several roots ranging from the very unelegant aspect of formulae to the subordination to dirty experimental details.

TABLE II

Process	Equivalent contaminated process	Main contamination channel	Present observation techniques available	Space like lepton	Time like lepton	Space like photon	Time like photon
$e\mu \rightarrow e\mu$			μ's on e^- at rest (also bound states			yes	
$e^+e^- \rightarrow \mu^+\mu^-$		$e^+e^- \rightarrow \rho^0$ $\hookrightarrow \mu^+\mu^-$	storage rings				yes
$e^-e^- \rightarrow e^-e^-$ $e^+e^+ \rightarrow e^+e^+$			storage rings			yes	
$e^+e^- \rightarrow e^+e^-$			storage rings (also bound states)			yes	yes
$e^+e^- \rightarrow 2\gamma$	$\gamma N \rightarrow e^+e^-N$	$\gamma N \rightarrow \rho^0 N$ $\hookrightarrow e^+e^-$	storage rings or wide angle electron pairs	yes			
$\gamma e \rightarrow \gamma e$	$eN \rightarrow e\gamma N$	$eN \rightarrow eN^*$ $\hookrightarrow N+\gamma$	wide angle electron bremsstrahlung	yes	yes		
$\mu^-\mu^- \rightarrow \mu^-\mu^-$ $\mu^+\mu^+ \rightarrow \mu^+\mu^+$			none			yes	
$\mu^+\mu^- \rightarrow \mu^+\mu^-$			none			yes	yes
$\mu^+\mu^- \rightarrow 2\gamma$	$\gamma N \rightarrow \mu^+\mu^-N$	$\gamma N \rightarrow \rho^0 N$ $\hookrightarrow \mu^+\mu^-$	wide angle muon pairs	yes			
$\gamma\mu \rightarrow \gamma\mu$	$\mu N \rightarrow \mu N\gamma$	$\mu N \rightarrow \mu N^*$ $\hookrightarrow N+\gamma$	wide angle muon bremsstrahlung	yes	yes		

An effort to use computers from the very beginning (that is, from graphs!) would generate enthousiasm and good results(12).

At present, there are some simple techniques an experimentalist can use to compute by himself at least part of the correction, the part depending on the resolution of the apparatus. This part, in which infrared divergence problems have been eliminated by accurate analysis of the cancellations to any electromagnetic order, can be understood on semiclassical grounds(13).

Also, the so-called "peaking approximation" is very useful in a spiral approach to accurate figures(14).

However, a "genuine" radiative correction (the convergent part of loops, see ref. (13) pag. 284) is left in any case to specialists in Dirac matrices: the task of computing it, though cleaned of experimental details, is still unpalatable.

In describing the experiments I will not quote radiative corrections unless they are a relevant or controversial point in the interpretation of results.

VII/B - Fits of the experimental data are often presented in terms of single-pole structure factors in analogy to ep scattering. That is, the contribution of a diagram having one off-mass shell particle (squared 4-momentum q^2) is multiplied by

$$F(q^2) = (1 - \frac{q^2}{\Lambda^2})^{-1}$$

where Λ^2 is a parameter on which the experiment puts a lower limit, (or "could" give a definite value). People frequently say that QED has been tested to a distance $1/\Lambda$ when the experimental results require that Λ be larger than some value fixed by a confidence criterion on the fit. Also, sometime the length r defined by

$$r^2 = 6/\Lambda^2$$

is introduced as root mean square radius of the involved charged particle.

The parameters Λ obtained in the study of different reactions are hard to compare since, even assuming adequacy of the fitting formula, they usually refer to quite different explored structures. The more, on the time-like side no adequacy can be presumed for single-pole factors diverging at $q^2 = \Lambda^2$.

In my opinion, it is more convincing to present experimental data and say how well QED predictions fit within the quoted errors; to judge how deep an experiment goes into structures consideration of the kinematics (values of the invariants) is far enough.

VIII - EXPERIMENTAL RESULTS. -

VIII/A - Space-like photons - The only actually performed experiment in Table 2 is at present e-e scattering at Stanford by the Princeton-Stanford group using a colliding-beam machine[15].

A look at the cross section formula for Møller scattering[4] shows that QED predicts a rapid decrease of the angular distribution going from small to large scattering angles. The role of the two invariants (compare Table I and section II) is complementary since at fixed total energy their sum $q^2 + q_E^2 = -4E^2$ is constant and the cross section is dominated by the smallest. In this experiment the angular distribution has been measured and no absolute normalization is given; thus, it is sensitive to QED predictions as far as the angular distribution is.

Results at 2 x 300 MeV total center of mass energy are shown in Fig. 9. QED fits pretty well the data: no need for structure factors is recognized (by eye).

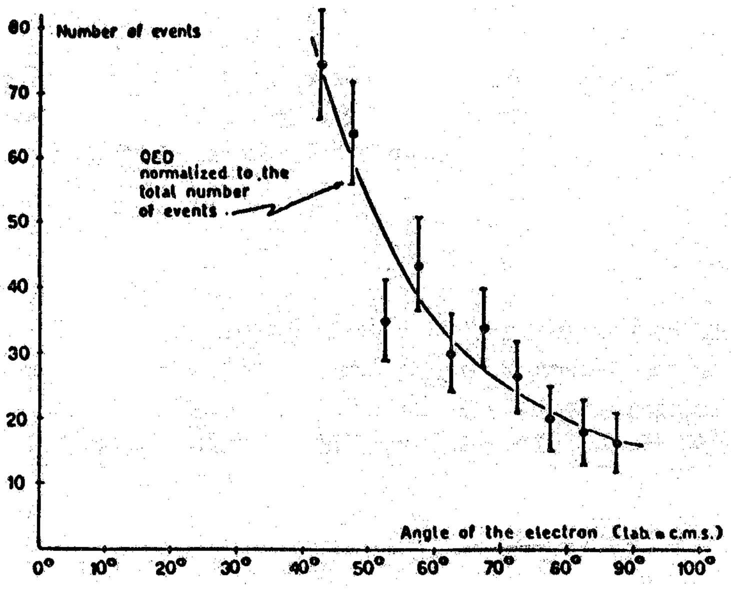


FIG. 9

A new set of data has already been taken at 2 x 550 MeV total center of mass energy and will soon be presented[16].

One can, for his own curiosity, assume that the photon behaves honestly, the electron has no intrinsic anomalous magnetic moment and use a charge form factor $F(q^2)$ in the vertices in analogy to the e-p scattering case.

The Taylor series $F(q^2) = 1 + (q^2/\Lambda^2) + \ldots$, to the first order in q^2, does not depend a lot on specific models and allows putting an upper limit to the electron r.m.s. charge radius

$$r_e^2 = \frac{6}{\Lambda^2} \leq (0.33 \text{ fermis})^2$$

to 95% confidence-level. Remember that the proton radius, defined in the same way for point electrons, is much larger than this limit: $r_p \approx 0.8$ fermis, so that reinterpretation of e-p scattering data at low momentum transfer would only give a slight change in the proton size $((0.73\ f)^2 < r_p^2 \leq (0.8\ f)^2$.

Comparison of the $e\gamma e$ and $\mu\gamma\mu$ vertices at space-like momentum transfers can be obtained from e-p and μ-p scattering (see however section

VI/A). Measurements of μp scattering cross section are still very poor as compared to the ep case. Fig. 10 shows recent results(17) going up to $q^2 = -(1.1 \text{ GeV})^2$.

As seen by the special formula chosen to fit $R(q^2)$ an e-μ difference as measured by $1/D < 0.09$ f is still possible.

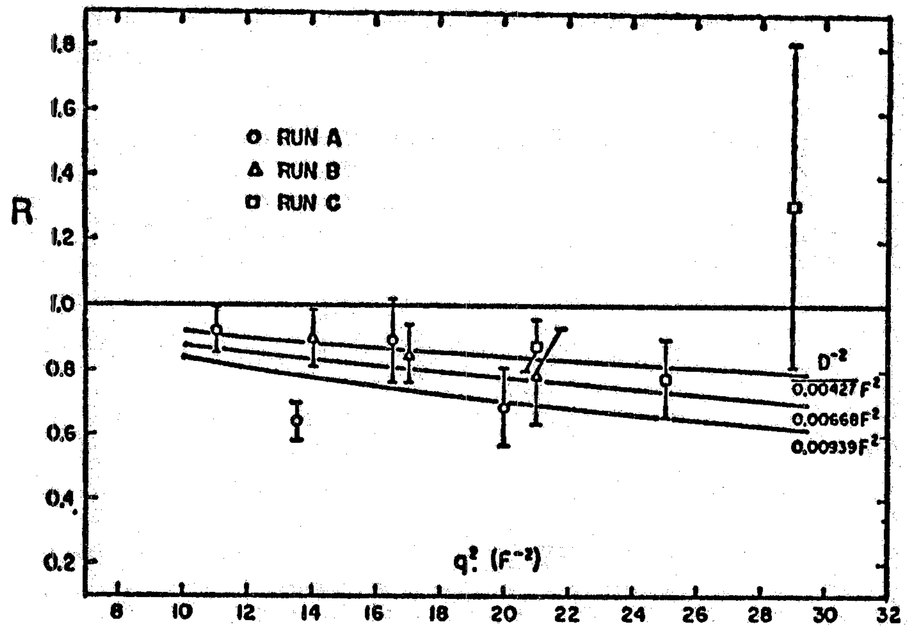


FIG. 10 - Ratio of measured to predicted yields for μ scattering. The predicted yield is calculated by using the dipole form factors for the proton:

$$G_E = \frac{G_M}{\mu} = \left(1 - \frac{q^2}{0.71 \text{ GeV}^2}\right)^{-2}.$$

Also, fits of the form

$$R(q^2) = \frac{1}{(1-q^2/D^2)}$$

are shown in the figure (best fit and 95% confidence levels).

VIII/B - Time-like photons - Apart from positronium(2) neither 2μ annihilation nor Bahabha scattering have yet been investigated; colliding beam experiments of this kind are in preparation at Frascati(8).

A trident production experiment(10) has been performed but it is not very sensitive to accidents of QED. An interesting, though very limited, result concerns the comparison of the eγe and $\mu\gamma\mu$ vertices in the leptonic decays of ρ^0. The average value of the branching ratio is(7)(18)

$$R = \frac{\rho^0 \to e^+ e^-}{\rho^0 \to \mu^+ \mu^-} = 0.97 \pm 0.17$$

The γ ray at the lepton vertex has $q^2 = (.75\,\mathrm{GeV})^2$, a quite high value. Thus the μ and e form factors are equal (at $q^2 = m_\rho^2$) to within 9% (one standard deviation; R is very little affected by the μ-e mass difference).

Also, a difficult experiment on electroproduction of μ-pairs on carbon is under way at CEA by a Northeastem University group(19). Results did not yet appear in the literature, but more than thousand events have been collected in the region where time-like photons contribute significantly.

VIII/C - Space-like charged leptons. - The annihilation of pairs into 2 photons would provide a clean test at space-like lepton 4-momenta. An experiment of this kind is in preparation at the storage ring Adone(8) and will, much like the case of Møller scattering, study the angular distribution of the produced γ rays.

Most of the work on space-like leptons has been done by the use of pair-photoproduction on nuclei (mainly carbon): both wide angle e-pairs (WAEP) and μ-pairs (WAMP) have been investigated(20).

Symmetric pairs are usually investigated, with one exception (Quinn, Ritson). The symmetric arrangement allows some important considerations: first, charge-conjugation invariance requires the interference between Bethe-Heitler and virtual Compton diagrams to vanish(21). A virtual Compton diagram is shown in Fig. 11 together with the two Bethe-Heitler contributions.

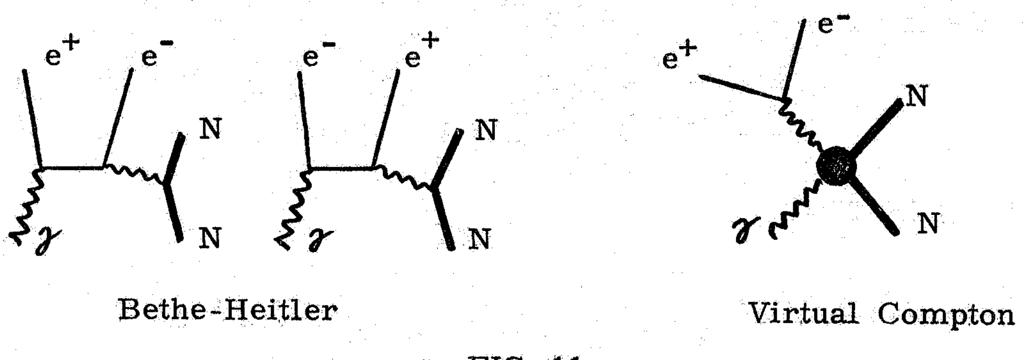


FIG. 11

Second, a low nuclear recoil is produced by symmetric pairs and thus the corrections for form factors are minimized.

The Bethe-Heitler cross section including form factors is well known(22) and shows the characteristic dip at the symmetric condition. This dip can be shown(23) to occur whenever in an electromagnetic process on nuclei the nuclear recoil momentum is colinear with the momentum of the incoming particle. It causes troubles in the evaluation of the radiative corrections because the nonradiating process has a rapidly changing cross section around the dip.

Both virtual-Compton contributions and the inelastic nuclear form factors[24] introduce uncertainties of some percent in the interpretation of wide angle pairs as pure QED processes.

A serious contamination can be simply recognized by examining the invariant mass M of a produced lepton pair: when M is near the ρ^o mass, photoproduction of ρ^o's and their subsequent decay into e^+e^- or $\mu^+\mu^-$ can significantly contribute to the measured yield. This actually is a way to measure the ρ^o branching ratio[7] into lepton pairs to pion pairs. Since however the dependence on the pair aperture angle is rather different in the Bethe-Heitler and ρ^o contribution, a separation can be done to some extent.

Authors frequently use the M value as a parameter for the plot of their results; this is simply translated into the value of the virtual lepton squared 4-momentum for symmetric pairs since in that case

$$(p_+ + p_-)^2 = M^2 = -2(k-p_-)^2 = -2(k-p_+)^2 .$$

When however the pair is asymmetric and only one outgoing particle is observed (Quinn, Ritson[20]), M is not well defined while the determination of the relevant virtual 4-momentum is still possible. Consideration of this case whas suggested by Drell[25] as particularly suited for Linac experiments where coincidences are not permissible. Enphasis should be given to the fact that the Quinn, Ritson experiment has been done using a proton target rather than carbon.

A summary of the available data is given in Table III.

Unpublished work is not shown in this table but has been reported at Stanford by R. Weinstein[19]; new data refer both to WAMP in the q-range from 340 to 830 MeV (symmetric pairs on carbon) and to WAEP in the q-range from 200 to 500 MeV (symmetric pairs on carbon).

New and old data appear to be in substantial agreement and do not show relevant inadequacies of QED.

Also, I did not quote the early work by the Harvard group[20] indicating a large disagreement between experiment and predictions since in my opinion it must be considered as a first effort to perform a difficult experiment; this effort stimulated subsequent work giving warnings on precautions to be taken.

The published experimental results for symmetric pairs are shown in Fig. 12 for WAEP and Fig. 13 for WAMP. The ratio R of measured yields to predicted yields is plotted versus the 4-momentum q of the virtual lepton.

In Fig. 14, the results of Quinn and Ritson for asymmetric μ-pairs are shown. Interference of the Bethe-Heitler and the virtual Compton contri-

TABLE III

Experiment	Tested particle	Observed particles	Target	Range of g (MeV)	Range of M (MeV)
Stanford 1958 P. R. L. 1, 114	e	e^+	H_2	115	
Cornell 1967 P. R. L. 18, 425	e	e^+e^- symmetric	C	33 ÷ 180	47 ÷ 254
Columbia-Desy 1967 P. R. 161, 1344	e	e^+e^- symmetric	C	114 ÷ 388	162 ÷ 548
Frascati 1962 P. R. L. 9, 226	μ	$\mu^+\mu^-$ symmetric	C	135 ÷ 185	240 ÷ 290
Northeastern-MIT 1964 P. R. L. 12, 739	μ	$\mu^+\mu^-$ symmetric	C	230 ÷ 475	320 ÷ 670
Northeastern-MIT 1966 P. R. L. 17, 767	μ	$\mu^+\mu^-$ symmetric	C	230 ÷ 560	320 ÷ 785
Stanford 1968 P. R. L. 20, 890	μ	μ^-	H_2	425	

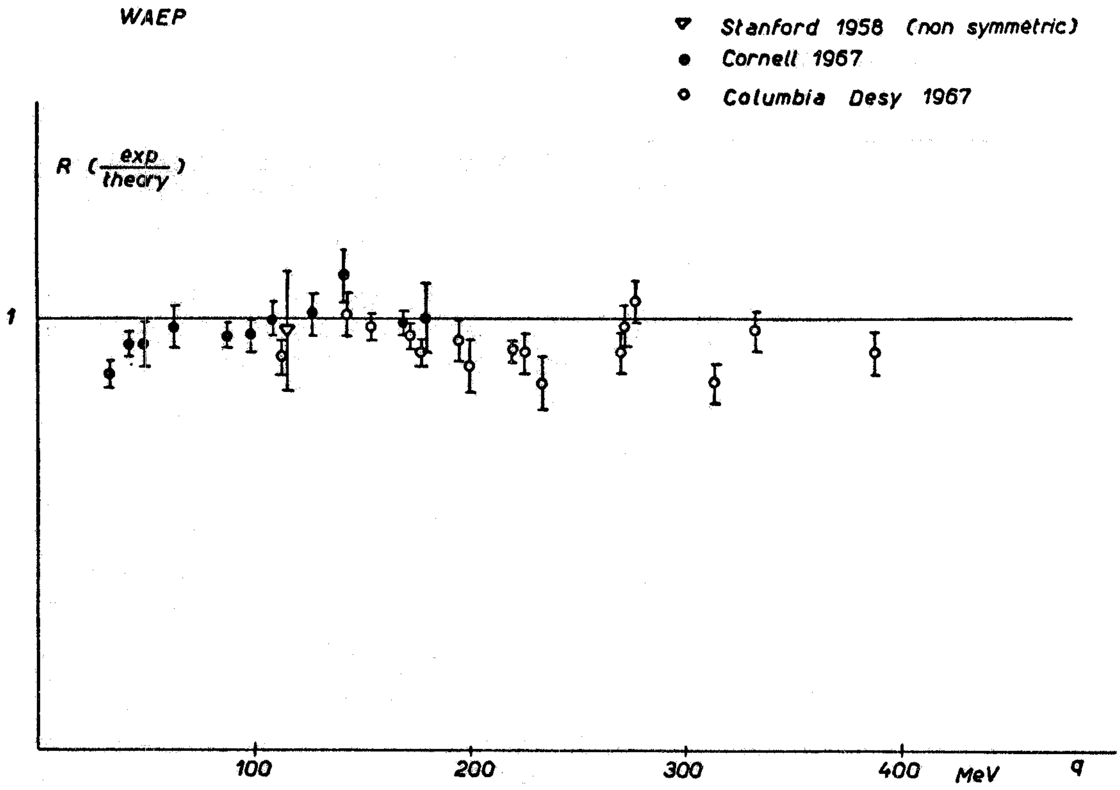

WAEP
Stanford 1958 (non symmetric)
Cornell 1967
Columbia Desy 1967
R (exp/theory)
1
100
200
300
400
MeV
q

FIG. 12

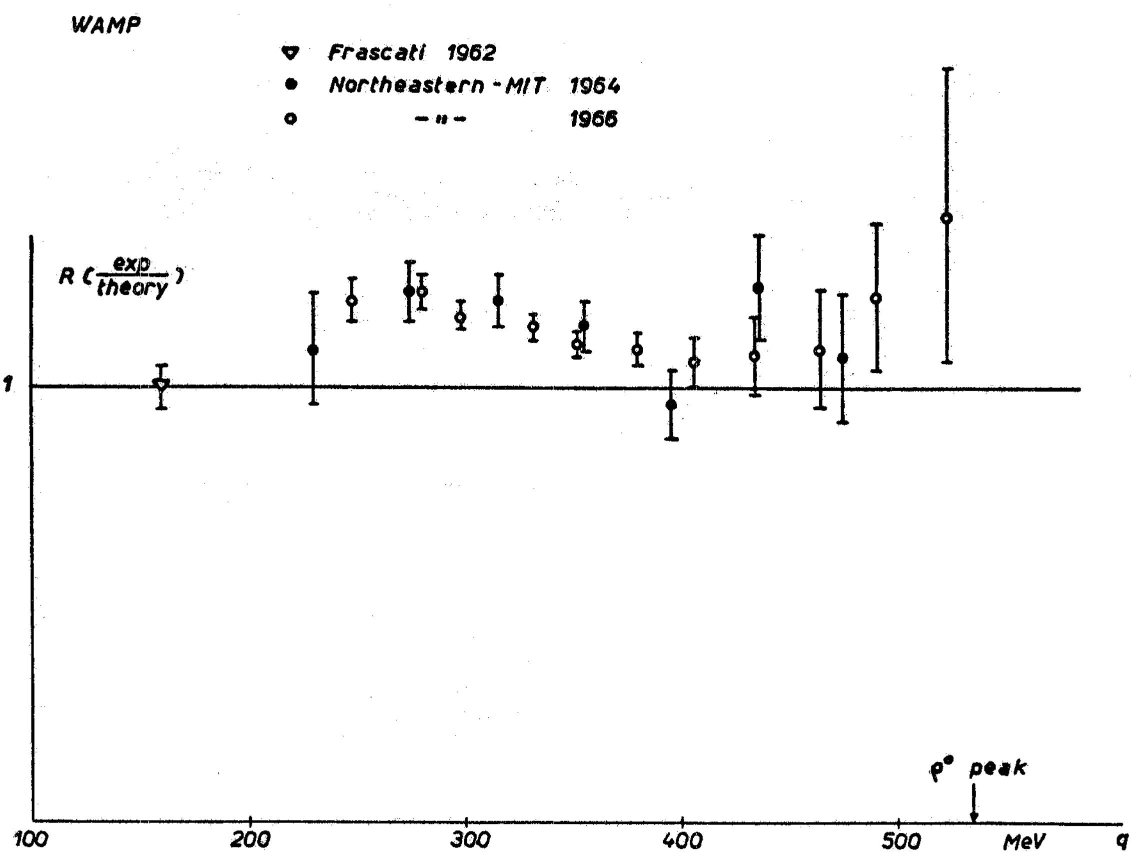

WAMP
Frascati 1962
Northeastern - MIT 1964
- " - 1966
R (exp/theory)
1
ρ° peak
100
200
300
400
500
MeV
q

FIG. 13

butions is now present but it is estimated to be no more than 2%.

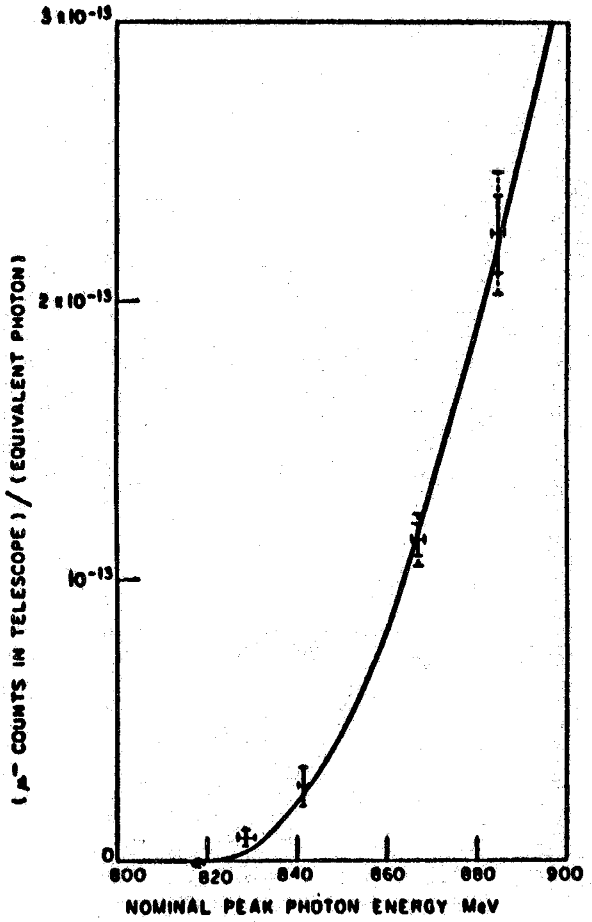


FIG. 14 - μ^- yields versus peak incident photon energy. Solid line is QED prediction.

A measurement of the total cross section of $e^+ e^- \rightarrow 2\gamma$ has also been done using fast positrons on atomic electrons at rest[26].

In spite of being a total cross section measurement it is somewhat sensitive to high virtual momenta.

The reason for the sensitivity is that dominance of small angle events is not so strong thanks to the mild variation of the electron propagator[4] (compare Møller scattering to contrast with the behaviour of photon propagators).

The pity is that the maximum allowed virtual momentum in the laboratory system improves slowly with the positron energy and is only 100 MeV at $E_+ = 10$ GeV. Results are shown in figure 15; the maximum allowed virtual momentum q (actually space-like) is given on the abscissa, the ratio of experimental values to predictions on the ordinate.

Radiative corrections to the points have not been applied; when included, the agreement is slightly worse.

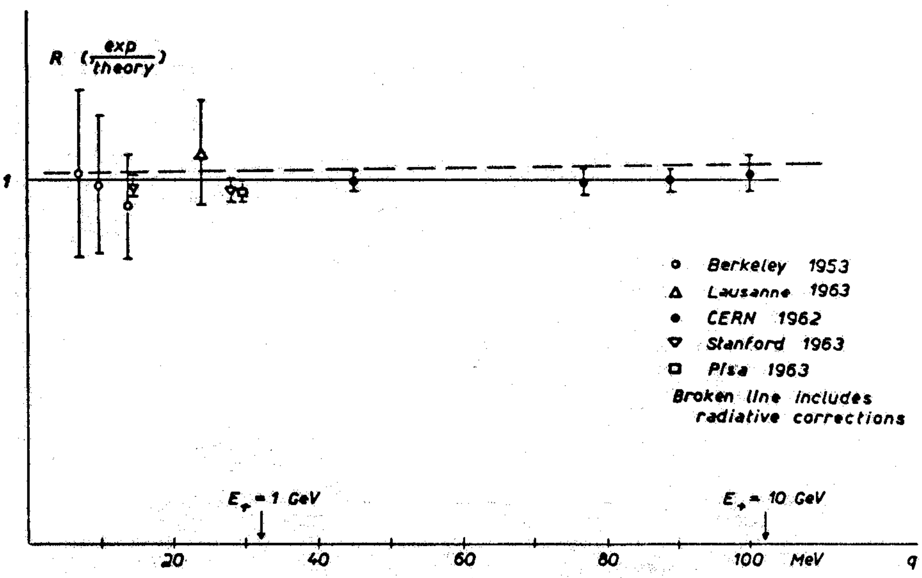


FIG. 15

VIII/D - Time like leptons. - Compton effect, the best reaction in principle, is unlikely to provide data in a near future. Wide angle bremsstrahlung (WAB) must be used to reach significant time like momenta.

Following a suggestion by F. Low[9] a simple type of breakdown of QED could be due to the existence of heavy electrons, e^x, rapidly decaying into $e + \gamma$. Should e^x exist, then a second peak besides the elastic peak would be found, in ep scattering, in the recoil proton energy spectrum at fixed angle corresponding to

$$e + p \rightarrow e^x + p$$

Extensive searches for e^x have been done at various laboratories[27]. These experiments are simplified versions of WAB relying on the strong enhancement of the WAB cross section that should accompany the production of a real though unstable particle. Electromagnetic coupling of e to e^x via their charge is not allowed by current conservation, thus an anomalous magnetic coupling must be introduced in order to permit an $e\,\gamma\,e^x$ vertex.

No e^x has been found till now having a mass less than 1, 3 GeV and a cross section (in ep production) larger than $\sim 10^{-33}$ $cm^2/ster$.

Model-independent WAB experiments have been undertaken in some laboratories. The only one to my knowledge presently completed[28] has provided an absolute measurement of the WAB cross section at space-like momenta of $\sim$300 MeV and time-like momenta of up to 100 MeV (compare Fig. 6) using a proton target and detecting the three outgoing particles (e, p, γ). The Bethe-Heitler cross section[6] is known and the virtual Compton contribution[29] has been reasonably evaluated; radiative corrections have not yet been computed. Preliminary results (not including radiative corrections) give agreement with QED to within 5% errors at the two time like values of 70 and 100 MeV.

IX - Conclusions - Give a look again to Table 2.

A lot of work has yet to be done to support confidence in QED, expecially on the time-like side. Electron data at the space-like virtual momenta appear to be good up to $\sim$700 MeV for virtual photons, 500 MeV for virtual electrons. Muon data are somewhat worse, still go up to $\sim$800 MeV (virtual muons).

As for the time-like momenta, data are poorer or limited to lower values (400$\div$800 MeV for photons from $eC \rightarrow eC\mu^+\mu^-$, 100 MeV for electrons from WAB). Storage rings look promising but time-like virtual leptons will stay in the dark for some time. We can nevertheless say that QED is in better shape than ten years ago[30] and the borderline of skepticism has been removed some hundred MeV from the mass-shell.

I am greatly indebted to Vittorio Silvestrini for stimulating discussions and for the wide use I did of his unpublished recollection of data[31].

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