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DETERMINATION OF THE PION AND CONSPIRATOR RESIDUES AND TRAJECTORIES, IN  $\pi^+$  PHOTOPRODUCTION FROM CONTINUOUS MOMENT SUM RULES. -

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ABSTRACT. -

Using continuous moment sum rules we obtain a clear evidence for the pion conspiracy. We find the pion and conspirator residues and trajectories, which reproduce with accuracy the forward peak in the high-energy differential cross section.

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Recently much interest has been devoted to the forward high-energy  $\pi^+$  photoproduction. In the framework of the Regge pole model a quite good fit to the experimental data was obtained<sup>(1)</sup> under the assumption that the pion Regge trajectory conspires in a family with  $M=1$ . The hypothesis of the pion conspiracy has been further supported by the study of the imaginary part of the photoproduction amplitudes by means of standard Finite-Energy Sum Rules<sup>(2)</sup>. More details about this problem may be found, for instance, in Ref. (2), to which we also refer for the notations.

In this note we study the conspiracy problem using the much more powerful technique of the Continuous Moment Sum Rules<sup>(3, 4, 5)</sup>. We clearly establish the existence of a conspiracy between two Regge poles (the pion and the "conspirator"); and that the possible contribution of cuts<sup>(6)</sup> or background<sup>(7)</sup> is completely negligible around  $t=0$ . This is possible because the trajectories and the residues of the pion and the conspirator are determined in this way from the low-energy data with much more precision than that of the previous method<sup>(2)</sup>. The high-energy differential cross sections predicted by the Regge parameters found in this manner are in good agreement with the experimental data at small  $t$  (see Fig. 2)<sup>(x)</sup>.

The reader should refer to (5) for a detailed treatment and discussion of the continuous moment sum rules.

We are here interested in the following relations:

$$\begin{aligned}
 \phi_{\pi}(\gamma) &= -(\nu_{\max})^{-\gamma} \left\{ \frac{\mu}{\pi} \int_{\nu_0}^{\nu_{\max}} [\nu^2 - \nu_0^2]^{\gamma/2} \operatorname{Im} \left[ e^{-i \frac{\pi}{2} \gamma} F_2^{(-)}(\nu, t) \right] d\nu + \right. \\
 (1. a) \quad & \left. + \frac{1}{2} \operatorname{ef}(t + \mu^2) [\nu_0^2 - \nu_B^2]^{\gamma/2} \right\} = \\
 &= \frac{\mu}{2\pi M} \sum_k \frac{\alpha_k(t) \beta_k(t)}{\sin[\pi \alpha_k(t)/2]} \cdot \frac{\sin \frac{\pi}{2} [\alpha_k(t) + \gamma]}{\alpha_k(t) + \gamma} \cdot \left( \frac{2M\nu_{\max}}{S_0} \right)^{\alpha_k(t)}
 \end{aligned}$$

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(x) - The  $\chi^2$  (with no free parameters) found for the 13 data points of Boyarski et al.<sup>(9)</sup> at 8, 11, 16 GeV in the peak region ( $\sqrt{-t} \leq 0.12$  GeV) is 13.6.

$$\begin{aligned}
\phi_c(\gamma) = & -(\nu_{\max})^{-\gamma} \left\{ \frac{\mu}{\pi} \int_{\nu_0}^{\nu_{\max}} [\nu^2 - \nu_0^2]^{\gamma/2} I_m \left[ e^{-i\frac{\pi}{2}\gamma} F_3^{(-)}(\nu, t) \right] d\nu + \right. \\
& \left. + \left[ \frac{ef}{4M} (t-4M^2) - \frac{tef}{4M} (1+\mu_p - \mu_n) \right] \cdot [\nu_0^2 - \nu_B^2]^{\gamma/2} \right\} = \\
(1. b) \quad & = \frac{\mu}{2\pi M} \sum_k \frac{\alpha_k(t) \beta_k(t)}{\sin[\pi\alpha_k(t)/2]} \cdot \frac{\sin\frac{\pi}{2} [\alpha_k(t) + \gamma]}{\alpha_k(t) + \gamma} \cdot \left( \frac{2M\nu_{\max}}{S_0} \right)^{\alpha_k(t)}
\end{aligned}$$

For the evaluation of the integrals, the low-energy fit done by R. L. Walker has been used. It covers the energy range from threshold up to  $W = \sqrt{S} = 1.8$  GeV. The essential features of this fit have been discussed elsewhere<sup>(8)</sup>. The above sum rules have been considered for  $-0.25 \leq t \leq 0.30$  GeV<sup>2</sup>, which is the important interval to study the conspiracy problem and for  $\gamma$  ranging from 0 to 5. Both  $\phi_r(\gamma)$  and  $\phi_c(\gamma)$  are very well fitted within 3-4% by one-pole curves for all values of  $t$  in the range considered.

The trajectories  $\alpha_\pi(t)$  and  $\alpha_c(t)$  and the residues  $\beta_\pi(t)$ ,  $\beta_c(t)$  resulting from the fits to  $\phi_\pi(\gamma)$  and  $\phi_c(\gamma)$  (performed independently at each value of  $t$ ) are plotted in Fig. 1a and 1b respectively.

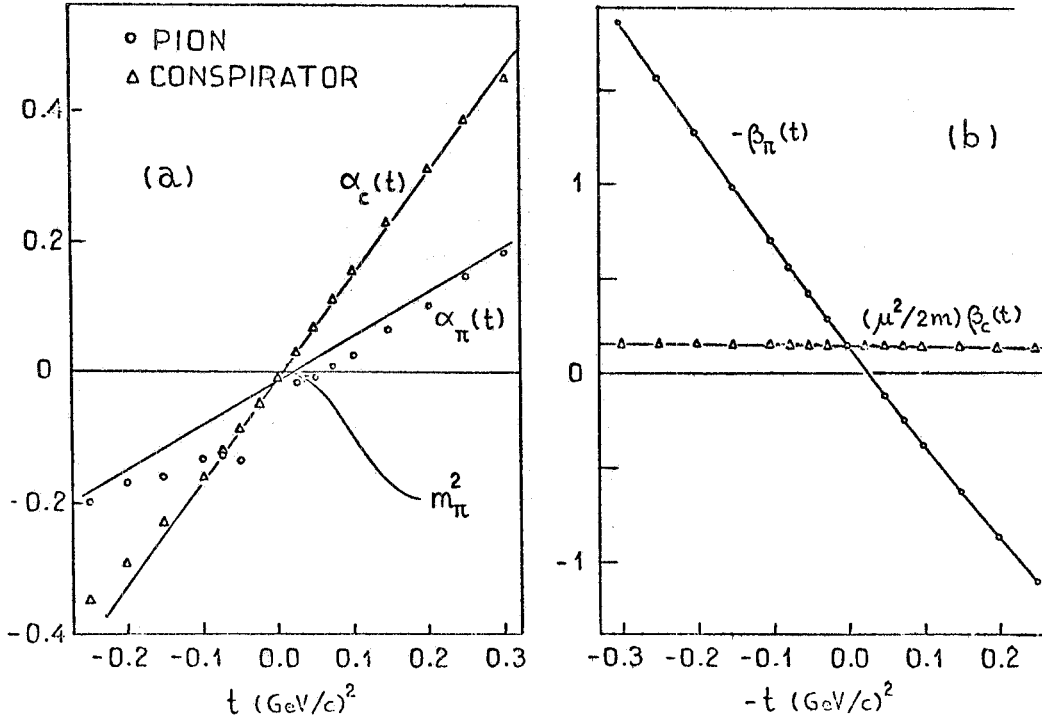


FIG. 1 - Trajectory and residues of the pion and the conspirator as result from the one-pole best fits to  $\phi_\pi(\gamma)$  and  $\phi_c(\gamma)$  (discrete points). Solid curves correspond to Eq. 2.

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The solid curves represent the fits to these points given by:

$$(2.a) \quad \alpha_{\pi}(t) = -0.013 + 0.65 t$$

$$(2.b) \quad \alpha_c(t) = -0.013 + 1.56 t$$

$$(2.c) \quad \beta_{\pi}(t) = -0.14 \left[ (1 + 76.3) \cdot \exp(0.49t) - 76.3 \right]$$

$$(2.d) \quad \beta_c(t) = 13.7 (1 + 0.45t) \text{ GeV}^{-1}$$

$\beta_{\pi}(t)$  may be equivalently parametrized for not too large  $|t|$  by:

$$(2.e) \quad \beta_{\pi}(t) = -0.25 \left( 1 + 0.42 \frac{t - \mu^2}{\mu^2} \right)$$

Near the region where the pion residue vanishes ( $\beta_{\pi}(t) = 0$  for  $t = -0.027$ ) there are some obvious discrepancies between the fit (2.a) to  $\alpha_{\pi}(t)$  and the points obtained from the sum rule (1.a). The expression (2.e) is to be compared with the high-energy fit performed by Ball et al.<sup>(1)</sup>. The form used by these authors is  $\beta_{\pi}(t) = -\pi \text{ eg } \mu^2 \left[ 1 + \lambda (t - \mu^2) / \mu^2 \right]$  and the value they found for  $\lambda$  is  $\lambda = 0.4$  which is to be compared with the  $\lambda = 0.42$  of equation (2.e).

From the expression 2.a,b,c,d) and neglecting all other contributions we have evaluated the differential cross section<sup>(x)</sup>:

$$(3) \quad \frac{d\sigma}{dt} = \frac{1}{16\pi} \left[ \left| A_1^{(-)} + t A_2^{(-)} \right|^2 + \left| A_1^{(-)} \right|^2 - t \left( \left| A_3^{(-)} \right|^2 + \left| A_4^{(-)} \right|^2 \right) \right]$$

near  $t = 0$ .

In Figs. 2a,b,c the calculated cross sections are compared with the experimental data of Boyarski et al.<sup>(5)</sup> at 8, 11 and 16 GeV. (We obviously have no adjustable parameters).

It is difficult to discuss the amount of the errors present in the fit given in Eqs. 2.a, b, c, d). However, the fact that all the curves which are studied are of the pure one-pole kind, (see for instance in Fig. 3  $\phi_{\pi}(\gamma)$  and  $\phi_c(\gamma)$  at  $t = 0$ , looks like a confirmation of the goodness of Walker's fit; and this allows one to hope that the errors are rather small. Also the good fit to the differential cross section provided at small  $t$  by the pion and the conspirator contributions leads one to the same conclusion. Anyway the study of the other amplitudes is in progress, and it will be possible to recover, with the same technique, Regge pole parameters already known in some other way. The comparison will

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(x) - For the corresponding expression quoted in Ref. (2) see the Erratum in Physics Letters.

give clearer indications about the real amount of the errors.

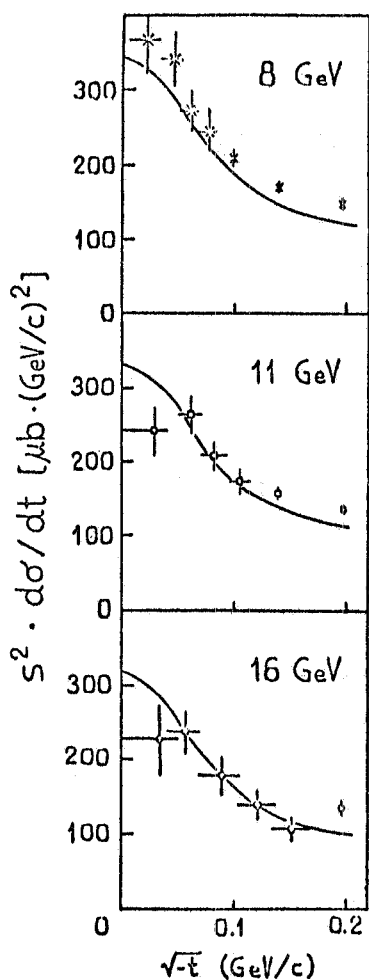


FIG. 2 - Comparison of the predicted differential cross sections with the experimental data of Boyarski et al.<sup>(9)</sup>.

In conclusion, extracting information from the low-energy data we have been able to predict all the Regge pole parameters relevant to the near forward  $\pi^+$  photoproduction at high energy. The obtained parameters quite well reproduce the data of Boyarski et al.<sup>(9)</sup> at small  $t$ . The pion satisfies the kinematical constraint at  $t=0$  by a conspiracy with a pole of opposite parity, strongly supporting its assignment to a Toller family with  $M=1$ . No evidence was found for a sizeable contribution of the  $A_2$  pole in the amplitude  $F_3^{(-)}$  in the range of  $t$  considered.

Related work in pion photoproduction is in progress.

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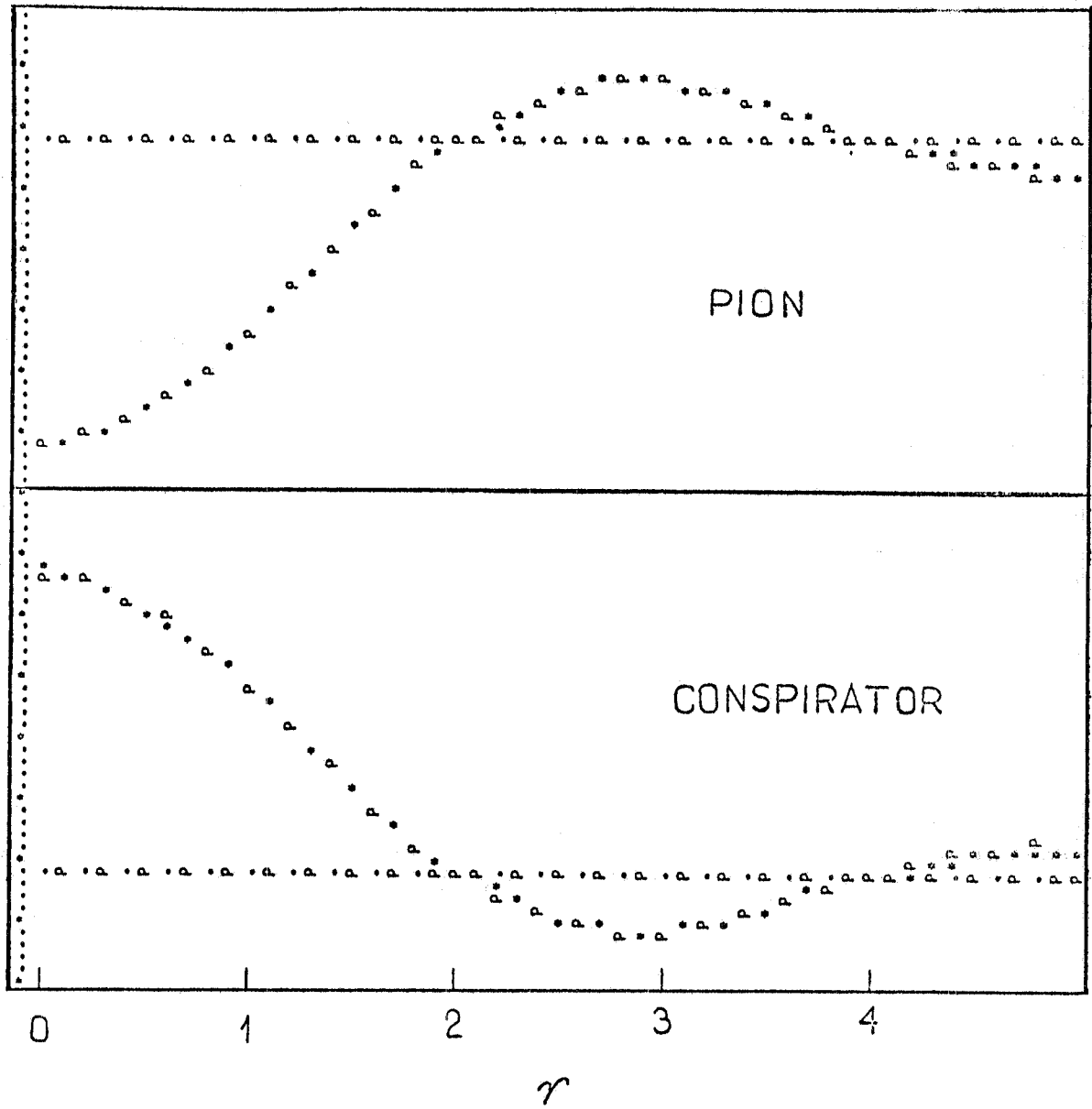


FIG. 3 - Plot of  $\phi_{\pi}(\gamma)$  and  $\phi_c(\gamma)$  at  $t=0$  (x) and their one-pole best fit (P, at alternate values of  $\gamma$ ).

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