

LNF - 68/27
24 Maggio 1968

**A. Majewski and P. Picchi : ELASTIC ELECTRON SCATTERING
FROM LIGHT NUCLEI AND NUCLEON-NUCLEON CORRELATIONS.**

Nota interna: n. 399
24 Maggio 1968

A. Małecki^(x) and P. Picchi: ELASTIC ELECTRON SCATTERING
FROM LIGHT NUCLEI AND NUCLEON-NUCLEON CORRELATIONS.

I. - INTRODUCTION. -

The recent measurements^(1, 2) of the elastic electron scattering form factor for He^4 show definite deviations from the Gaussian model at large momentum transfers $q > 2.5 \text{ F}^{-1}$. The data are characterized by a well pronounced minimum at $q^2 = 10 \text{ F}^{-2}$ which is in drastic disagreement with a Gaussian-shaped alpha-particle. Some attempts have been made to explain the experimental results by introducing the short range nucleon-nucleon correlations in the standard Gaussian wave function^(3, 4, 5).

In ref.⁽³⁾ the correlations were treated with the Jastrow method⁽⁶⁾ (see also Appendix) and only the contributions from the one correlated-pair part of the nuclear wave-function have been kept. The theoretical curve⁽³⁾ fitted well the small momentum transfer region,

(x) - On leave of absence from Instytut Fizyki Jądrowej, Krakow,
Poland.

2.

predicted the minimum in the right place, but was too small by a factor 2.5 on the right-hand side of the minimum (at very large momentum transfers). The single-correlated-pair approximation has been questioned in⁽⁵⁾ where it was shown that using the same correlations as in⁽³⁾ and performing the exact calculations (with six correlated pairs) one gets quite a different result (much worse fit) from that of⁽³⁾. In disagreement with this analysis ref. (7) again suggests the validity of the approximation. The authors of⁽⁵⁾ have also made a best fit to data. Their fit, however, gives the position of the minimum at a smaller q than observed. Moreover, this fit has been obtained with an oscillator potential parameter widely different from that usually assumed.

We shall present a method which avoids the mentioned difficulties, as the summation of the troublesome series in the Jastrow method is now automatically performed. This allows us to perform reliable calculations for nuclei heavier than He^4 . The method is presented in Chapter II where the short range correlations corrections to the elastic form factor are calculated for nuclei with nucleons in the first s- and p-shells.

In Chapter III numerical results for the He^4 nucleus are given; other nuclei will be discussed elsewhere⁽⁸⁾. We obtained a good fit to the experimental data by the following modification of the Gaussian wave function that describes the relative motion of two nucleons in He^4 : we introduced a repulsive core between nucleons at short relative distances and assumed that the internucleon forces at large distances are more attractive than implied by the Gaussian model.

Finally, in Appendix we present an analysis which confirms the results of ref. (5). Introducing similar nucleon-nucleon correlations as in⁽³⁾ and⁽⁵⁾ one gets an analytical expression for the form-factor which is consistent with⁽⁵⁾, but not with⁽³⁾.

II. - GENERAL FORMALISM. -

Let us consider the scattering of an electron with incident energy \mathcal{E} through an angle θ to a final state with energy \mathcal{E}' while the nucleus makes a transition from the ground state $|i\rangle$ to the state $|f\rangle$. The cross-section for this process (we assume that one observes only the final electron) is given, in the first Born approximation, by the following⁽⁹⁾ formula^(x):

(x) - We use a metric such that $a_\mu = (a_0, \vec{a})$ and $a_\mu b_\mu = a_0 b_0 - \vec{a} \cdot \vec{b}$. The magnitude of the three vector is $a = |\vec{a}|$. We also use units $c = \hbar = 1$, $e^2 = 1/137$. Since we consider high-energy electrons the rest mass of the electron is neglected.

$$(1) \quad \frac{d^2\sigma}{d\varepsilon'd\Omega'} = \frac{e^4 \cos^2 \theta}{4\varepsilon^2 \sin^4 \theta/2} \frac{\theta}{2} \sum_{|i\rangle} \sum_{|f\rangle} \delta(\omega - E_f + E_i) \left\{ \frac{q_\mu^4}{q^4} Q_{fi}^x Q_{fi} + \right. \\ \left. + \left(\tan^2 \frac{\theta}{2} - \frac{q_\mu^2}{2q^2} \right) \left[\vec{J}_{fi}^x \cdot \vec{J}_{fi} - \frac{1}{q^2} (\vec{J}_{fi}^+ \cdot \vec{q}) (\vec{J}_{fi}^- \cdot \vec{q}) \right] \right\}_{\text{LAB.}}$$

where $\omega = \varepsilon - \varepsilon'$, $q_\mu^2 = \omega^2 - q^2$; \vec{q} being the three momentum transfer to the nucleus. $Q_{fi} = \langle f | Q(\vec{q}) | i \rangle$, $\vec{J}_{fi} = \langle f | \vec{J}(\vec{q}) | i \rangle$ are the matrix elements between the ground and excited states of the charge and current operators of the target nucleus, respectively. In Eq. (1) all the quantities are to be taken in the laboratory frame.

Let us consider elastic scattering from spin zero nuclei. In this case the formula (1) simplifies considerably. It can be shown⁽¹⁰⁾ that in elastic scattering the contribution coming from the interaction with the transverse components of the nuclear current $\vec{J}(\vec{q})$ (or, in other words, the contribution from the transverse multipoles) vanishes for the spin zero nuclei. From (1) we get:

$$(2) \quad \left(\frac{d\sigma}{d\Omega} \right)_{\text{el}} = \frac{e^4 \cos^2 \theta/2}{4\varepsilon^2 \sin^4 \theta/2} \frac{|Q_{ii}|^2}{1 + \frac{2\varepsilon}{M_T} \sin^2 \theta/2} \left(1 - \frac{q_\mu^2}{2M_T^2} \right) \frac{q_\mu^4}{q^4} = \\ = \frac{Z^2 e^4 \cos^2 \theta/2}{4\varepsilon^2 \sin^4 \theta/2} \frac{|F_{\text{ch}}(q_\mu^2)|^2}{1 + \frac{2\varepsilon}{M_T} \sin^2 \theta/2}$$

where M_T is the mass of the target nucleus. Equation (2) defines the quantity $F_{\text{ch}}(q_\mu^2)$ which is called the charge elastic form factor of the nucleus.

In order to evaluate F_{ch} we have to make some assumption about the nuclear charge operator. Following McVoy and Van Hove⁽¹¹⁾ we use the nonrelativistic form of $Q(\vec{q})$ including terms to the order $1/M^2$ (M -nucleon mass):

$$(3) \quad Q(\vec{q}) = \sum_{j=1}^A \left\{ \left[F_{ij} + \frac{q_\mu^2}{8M^2} (F_{ij} + 2K_j F_{2j}) \right] e^{i\vec{q} \cdot \vec{r}_j} \right\}$$

where one sums over all the nucleons in the target nucleus and \vec{r}_j, K_j are the position operator and the anomalous magnetic moment (in nuclear magnetons) for the j -th nucleon, respectively. $F_1(q_\mu^2)$ and $F_2(q_\mu^2)$ are the Dirac and Pauli electromagnetic form factors of the

4.

nucleon, normalized to $F_{1p}(0) = F_{2p}(0) = 1$ for the proton and $F_{1n}(0) = 0$, $F_{2n}(0) = 1$ for the neutron. Using (2) and (3) one gets:

$$\begin{aligned}
 F_{ch}(q_\mu^2) &= \frac{1}{Z} \left[(F_{1p} + F_{1n})(1 + q_\mu^2/8M^2) + \frac{q_\mu^2}{4M^2} (K_p F_{2p} + K_n F_{2n}) \right] \\
 (4) \quad &\cdot \langle i \left| \sum_{j=1}^Z e^{i\vec{q} \cdot \vec{r}_j} \right| i \rangle = \\
 &= \frac{1}{Z} (G_{Ep} + G_{En}) \left(1 + \frac{q_\mu^2}{8M^2}\right) \langle i \left| \sum_{j=1}^A e_j e^{i\vec{q} \cdot \vec{r}_j} \right| i \rangle
 \end{aligned}$$

where we have introduced the electric form-factors of the nucleon⁽¹²⁾

$$(5) \quad G_E(q_\mu^2) = F_1(q_\mu^2) + \frac{q_\mu^2}{4M^2} K F_2(q_\mu^2)$$

In Eq. (4) the same number of protons and neutrons $Z = \frac{A}{2}$ was assumed.

We intend to use the shell-model for the description of the ground state of the nucleus. The shell-model wave-functions are functions of the $3A$ spatial coordinates of the nucleons. In fact there are only $3(A - 1)$ independent internal coordinates because three of the nucleon coordinates merely give the position of the centre of mass of the nucleus. Thus, in the usual shell-model treatment one neglects the effect of the c. m. motion. The correction due to this effect could be evaluated in the shell-model with the harmonic oscillator potential well. In this case⁽¹³⁾ the shell-model wave function ψ_{SM} can be related easily to the c. m. wave function ϕ_{CM} and to the actual wave function of internal coordinates. Using the fact⁽¹⁴⁾ that

$$(6) \quad \langle \phi_{CM} \left| e^{i\vec{q} \cdot \frac{1}{A} \sum_{j=1}^A \vec{r}_j} \right| \phi_{CM} \rangle = e^{-\frac{q^2}{4A\alpha^2}}$$

we get

$$\begin{aligned}
 (7) \quad \langle i \left| \sum_{j=1}^A e_j e^{i\vec{q} \cdot \vec{r}_j} \right| i \rangle &= e^{q^2/4A\alpha^2} \langle \psi_{SM} \left| \sum_{j=1}^A e_j e^{i\vec{q} \cdot \vec{r}_j} \right| \psi_{SM} \rangle = \\
 &= Z e^{q^2/4A\alpha^2} F_{SM}(q_\mu^2)
 \end{aligned}$$

where α is the oscillator potential parameter (the oscillator spacing equals α^2/M). Equation (7) defines the shell-model elastic form-factor of the target - FSM.

We shall use henceforth the harmonic oscillator potential. This potential (as besides any shell-model potential well) has a certain drawback. It does not account for possible dynamical nucleon-nucleon correlations. Let us consider short-range correlations as an example. It is known that the nucleon-nucleon interaction contains a hard-core, that is, the interaction becomes strongly repulsive at short distances in the relative coordinate of two nucleons. The short range correlations induced by such a hard-core are, however, neglect in the usual shell-model treatment.

In our calculations we will modify the relative two nucleon wave function. We shall use the method which was recently applied for calculations of short-range correlations corrections to the inelastic electron scattering sum rule⁽¹⁵⁾. For sake of completeness let us briefly describe this method. We introduce the correlations into the completely antisymmetrized shell model wave function Ψ_{SM} employing a unitary operator U:

$$(8) \quad |\tilde{\Psi}\rangle = U |\Psi_{SM}\rangle, \quad U^+ = U^{-1}$$

The unitarity of U guarantees us the correct normalization of the modified function. Let us consider the matrix element $\langle \Psi_{SM} | \theta | \Psi_{SM} \rangle$, where θ is a two-particle operator i. e.

$$\theta = \sum_{j \neq k}^A \theta(j, k).$$

Taking into account only the two-nucleon correlations we can write the correlated matrix element as follows⁽¹⁵⁾:

$$(9) \quad \langle \tilde{\Psi} | \theta | \tilde{\Psi} \rangle = \langle \Psi_{SM} | U^{-1} \sum_{j \neq k}^A \theta(j, k) U | \Psi_{SM} \rangle = \\ = \sum_{\alpha, \beta} \left[\langle \alpha(1)\beta(2) | \theta(1, 2) | \alpha(1)\beta(2) \rangle - \langle \alpha(1)\beta(2) | \theta(1, 2) | \beta(1)\alpha(2) \rangle \right]$$

where α, β are complete sets of quantum numbers of single-particle states and the summation extends over all occupied states. In (9) we have introduced the correlated two-particle wave functions:

$$(10) \quad |\tilde{\alpha}\tilde{\beta}\rangle = U |\alpha\beta\rangle$$

The indices α, β contain the spatial, spin and isospin quantum num-

6.

bers. The single particle spatial quantum numbers we denote a, b ; the one-particle orbital state is $|a\rangle = |n_a l_a m_a\rangle$. The summation over the spin and isospin quantum numbers can be easily performed as our simple oscillator model does not introduce any spin- or isospin dependent couplings.

We are interested in the shell-model form factor of the target- F_{SM} . We can write it as follows:

$$(11) \quad F_{SM} = \frac{1}{2Z(A-1)} \langle \psi_{SM} | \sum_{j \neq k}^A (e_j e^{iqZ_j} + e_k e^{iqZ_k}) | \psi_{SM} \rangle,$$

where we chose \vec{q} along the z-axis. As F_{SM} depends only on the absolute value of \vec{q} we can choose its direction as we please. Performing the summations over the spin and isospin quantum numbers we get from (11):

$$(12) \quad F_{SM} = \frac{1}{Z(2Z-1)} \left[4 \sum_{ab} \langle a(1)b(2) | e^{iqZ_1} + e^{iqZ_2} | a(1)b(2) \rangle - \sum_a \langle a(1)a(2) | e^{iqZ_1} + e^{iqZ_2} | a(1)a(2) \rangle \right]$$

Let us now consider the two-particle state^(x)

$$(13) \quad \begin{aligned} |ab\rangle &= |n_a l_a m_a\rangle |n_b l_b m_b\rangle = \\ &= \sum_{\lambda\mu} \langle l_a m_a l_b m_b | l_a l_b \lambda\mu \rangle |n_a l_a, n_b l_b, \lambda\mu\rangle. \end{aligned}$$

In the case of harmonic oscillator wave functions, it is possible to define a transformation that takes us from motion of two particles about a common center to a description of the relative and center-of-mass of the two particles. Following Moshinsky⁽¹⁷⁾ this transformation may be written:

$$(14) \quad |n_1 l_1, n_2 l_2, \lambda\mu\rangle = \sum_{n_1 N L} \left\{ n_1 l_1, n_2 l_2, \lambda \right\} |n_1 N L, \lambda\mu\rangle,$$

(x) - We use Edmonds⁽¹⁶⁾ conventions for the Clebsh-Gordan coefficients and spherical harmonics.

where (nl) are the quantum numbers of relative motion and (N, L) are the quantum numbers of the c. m. motion. The relative and the c. m. coordinates are defined as follows⁽¹⁷⁾

$$(15) \quad \vec{r} = \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2) \quad , \quad \vec{R} = \frac{1}{\sqrt{2}} (\vec{r}_1 + \vec{r}_2)$$

As

$$(16) \quad |nl, NL, \lambda\mu\rangle = \sum_{mM} \langle 1m LM | 1L \lambda\mu \rangle |nlm\rangle |NLM\rangle$$

we have from (13), (14) and (16)

$$(17) \quad |ab\rangle = \sum_{\lambda\mu mM} \sum_{n_1 N_1 L_1} \langle 1_{a m_a} 1_{b m_b} | 1_a 1_b \lambda\mu \rangle \langle 1m LM | 1L \lambda\mu \rangle \\ \left\{ nl, NL, \lambda \left| n_a 1_a, n_b 1_b, \lambda \right. \right\} |nlm\rangle |NLM\rangle$$

We introduce the nucleon-nucleon correlations in (17) modifying the radial functions of the relative motion

$$(18) \quad |\widetilde{nlm}\rangle = \widetilde{R}_{nl}(r) Y_{lm}(\theta, \psi), \quad \widetilde{R}_{nl} = \frac{g(r)}{\sqrt{N_{nl}}} R_{nl}(r)$$

$$(19) \quad N_{nl} = \int_0^{\infty} dr r^2 R_{nl}^2 g^2(r)$$

where the function $g(r)$ which modifies the standard radial oscillator function $R_{nl}(r)$ has the following properties:

$$(20) \quad \begin{aligned} g(r) &= 0 & \text{for } r \leq \frac{1}{\sqrt{2}} r_c, \text{ and} \\ g(r) &= \sqrt{N_{nl}} & \text{for } r \approx \frac{1}{\sqrt{2}} r_h; \end{aligned}$$

r_c is here the radius of the hard core, and r_h is the so-called healing distance. We must now check whether the transformation (18), (19) is a unitary one, i. e. whether the correlated wave functions $|\widetilde{nlm}\rangle$ satisfy the same orthonormality conditions as $|nlm\rangle$. This we shall do for the case of light nuclei with nucleons in the first s- and p-shells of the shell model ($4 \leq A \leq 16$). In this case we have $n_a = n_b = 0$, and $l_a, l_b = 0, 1$ (here we have used Moshinsky's⁽¹⁷⁾ notation for the radial quantum number). These states lead to the following $|nlm\rangle$ states of the relative two-nucleon motion: $|000\rangle, |01m\rangle, |02m\rangle, |100\rangle$.

8.

After introducing the modifications defined by (18), (19) we get four normalized states $\widetilde{|nlm\rangle}$ which are orthogonal to each other except for $\langle 000 | 000 \rangle \neq 0$. This situation can be remedied as follows. Instead of (18), (19) we apply for the $|100\rangle$ state a slightly different modification:

$$(18a) \quad \widetilde{R}_{10}(r) = \sqrt{6} \pi^{-1/4} \alpha^{3/2} \frac{g(r)}{\sqrt{N_{10}}} (1 + \delta - 2/3 \alpha^2 r^2) e^{-\frac{\alpha^2 r^2}{2}}$$

$$(19a) \quad \left\{ \begin{array}{l} N_{10} = 6 \pi^{-1/2} \alpha^3 \int_0^\infty dr r^2 e^{-\alpha^2 r^2} (1 + \delta - 2/3 \alpha^2 r^2)^2 g^2(r) = \\ = -3/2 \frac{N_{01}^2}{N_{00}} + \frac{5}{2} N_{02} \\ \delta = \frac{N_{01}}{N_{00}} - 1 \end{array} \right.$$

After this modification we recover orthornormality of all the states involved in calculations, hence the unitarity of the transformation. Let us note that δ should be very small compared to unity.

Let us now turn to the shell-model form factor F_{SM} . Using (12) and (17) we get after some tedious though straightforward algebra (for more details of calculations of this type see ref. (15)) the following short range correlations correction:

$$(21) \quad \Delta F_{SM} = \frac{e^{-t}}{Z(2Z-1)} \left\{ \left[7Z - 8 - \frac{20}{3} (Z-2)t + (Z-2)t^2 \right] \cdot \right. \\ \left. \cdot \Delta \langle 000 | e^{iqZ/\sqrt{2}} | 000 \rangle + 1/2 (Z-2) \Delta \langle 100 | e^{iqZ/\sqrt{2}} | 100 \rangle - \right. \\ \left. - \frac{\sqrt{2}}{3} (Z-2)t \Delta \langle 100 | e^{iqZ/\sqrt{2}} | 000 \rangle \right\},$$

where $t = q^2/8\alpha^2$ and $\Delta(\dots)$ denotes the difference between correlated and uncorrelated magnitudes. The formula (21) is valid for nuclei with two protons in the s-shell and $Z-2$ protons in the p-shell. In proving (21) we employ the following simplification. We neglect the difference $\Delta(\dots)$ for all the terms whose integrands behave like $r^k (k > 4)$ for small r . This amounts to keeping the short-range correlations in the relative s-states only. In such a way we extract the leading terms of the short range correlation corrections. This

procedure simplifies the computations enormously.

Concluding this chapter we give the final expression for the charge elastic form factor of the target F_{ch} the quantity which can be directly measured in the elastic scattering experiment. From (4) and (7) we get:

$$(22) \quad F_{ch} = (G_{Ep} + G_{En}) \left(1 + \frac{q^2}{8M^2}\right) e^{q^2/8Z\alpha^2} \left[\left(1 - \frac{Z-2}{6Z} \frac{q^2}{\alpha^2}\right) e^{-q^2/4\alpha^2} + \Delta F_{SM} \right]$$

The electric nucleon form factors are known quite accurately over the wide range of our momentum transfers, e. g. Ref. (18) gives:

$$(23) \quad G_{Ep} + G_{En} = \frac{2.50}{1 - \frac{q^2}{15.7}} - \frac{1.60}{1 - \frac{q^2}{26.7}} + 0.10$$

where momentum transfer is expressed in F^{-1} . The formulae (22), (23) together with (18), (19) and (21) allow us, after a proper choice of the modifying function $g(r)$, to perform numerical calculations.

III. - ELASTIC ELECTRON SCATTERING FROM He^4 . -

The He^4 nucleus represents the very simple case where only one single-particle orbital state ($n_a = l_a = 0$) is occupied. This leads, as can be seen from (17), to the only one state of the relative motion $|000\rangle_{rel}$ of two nucleons, and to the only one state of their c. m. motion $|000\rangle_{CM}$. From (12), (15) and (17) we get for the He^4 target:

$$(24) \quad F_{SM} = \langle 000_{CM} | e^{iqZ/\sqrt{2}} | 000_{CM} \rangle \langle 000_{rel} | e^{iqZ/\sqrt{2}} | 000_{rel} \rangle$$

Modifying the relative state $|000\rangle_{rel}$ according to (18) and (19) we have the correlated shell-model form factor:

$$(25) \quad \tilde{F}_{SM} = \frac{2}{q} e^{-q^2/8\alpha^2} \frac{\int_0^\infty ds s e^{-1/2\alpha^2 s^2} \sin(1/2 qs) g^2(s)}{\int_0^\infty ds s^2 e^{-1/2\alpha^2 s^2} g^2(s)}$$

where $s = |\vec{r}_1 - \vec{r}_2|$ is the real distance between two nucleons.

The results of our analysis for He^4 are presented in Fig. 1 where the four theoretical expressions for the charge elastic form-factor are compared with the recent experimental data^(1,2).

The curve 1 represents the uncorrelated charge form factor corresponding to the standard, shell-model form-factor

$$(26) \quad F_{\text{SM}} = e^{-q^2/4\alpha^2}$$

The experimental data are consistent with the Gaussian form-factor (26) at small momentum transfers. The curve 1 has been calculated with the oscillator parameter $\alpha = 148.3$ MeV. This value gives the r-m-s radius of the charge distributions as determined in a recent low- q^2 experiment⁽¹⁹⁾. For momentum transfers $q^2 > 6F^{-2}$ the experimental results show definite deviations from (26); first of all, the Gaussian form factor does not account for a minimum is observed at $q^2 = 10F^{-2}$.

The curve 2 in Fig. 1 represents the charge form factor corrected, according to (25), for the short range nucleon-nucleon correlations. We have modified the Gaussian wave function of the relative two-nucleon motion only at small distances, introducing a hard core, but we did not change it at large distances between nucleons. This means that the modifying function $g(s)$ has the following properties:

$$(27) \quad g(s) = \begin{cases} 0 & 0 \leq s \leq r_c \\ h(s) & r_c \leq s \leq r_h \\ N_{00} & s > r_h \end{cases}$$

where r_c is the radius of the hard core and r_h is the so-called healing distance.

From (25) and (27) we get the correlated form factor of the shell model:

$$(28) \quad \tilde{F}_{\text{SM}} = e^{-q^2/4\alpha^2} \left[1 - \frac{4}{\sqrt{2\pi}} \frac{\alpha^3}{q} e^{q^2/8\alpha^2} \left(h_1 - \frac{h_2 h_3}{h_4} \right) \right]$$

with

$$h_1 = \int_0^{r_h} ds s e^{-1/2 \alpha^2 s^2} \sin(1/2 q s)$$

$$h_2 = \int_0^{r_h} ds s e^{-1/2 \alpha^2 s^2} \sin(1/2 q s) h^2(s)$$

$$h_3 = \int_0^{r_h} ds s^2 e^{-1/2 \alpha^2 s^2}$$

$$h_4 = \int_{r_c}^{r_h} ds s^2 e^{-1/2 \alpha^2 s^2} h^2(s)$$

The function $h(s)$ which "heals" the relative wave function at medium internucleon distances has been chosen to be

$$(29) \quad h(s) = \alpha^2 (s - r_c)^2 e^{-\gamma \alpha^2 (s - r_c)^2}$$

The curve 2 has been calculated with the following parameters: $\alpha = 148.3$ MeV, $r_c = 0.56F$, $\gamma = 1.0$; the healing distance corresponding to these parameters equals $r_h = 2.37 F$. The curve fits well the experimental data down to the minimum at $q^2 = 10 F^{-2}$; also the position of this minimum is very well accounted for the value of r_c we have used is very close^(*) to that suggested in ref. (4) and not too different from the usually accepted⁽²⁰⁾ radius ($0.5F$) of the hard-core. Comparison of the curves 1 and 2 shows that the effect of the repulsive core in He^4 is particularly important; elastic electron scattering on the He^4 at large momentum transfers turns out to be very sensitive to the short range nucleon-nucleon correlations.

For very large momentum transfers, $q^2 > 10F^{-2}$, the curve 2 is no longer consistent with the experimental data. It gives the values of $|F_{ch}|$ much smaller than the experimental ones. This suggests that one should modify the Gaussian relative wave function not only at small internucleon distances, but also at large distances. This suggestion is confirmed by an analysis done in ref. (2) which yielded the conclusion that the surface of the He^4 charge distribution is less diffuse than that indicated by the Gaussian model. In order to account for this effect we have used the following $g(s)$ function:

$$(30) \quad g(s) = \begin{cases} 0 & 0 \leq s \leq r_c \\ \alpha^2 (s - r_c)^2 e^{-\gamma \alpha^2 (s - r_c)^2} & r_c \leq s \leq \infty \end{cases}$$

(*) - Strictly speaking the experimental data down to the minimum could be fairly well explained with various sets of r_c and γ . From our analysis follows that $0.5 \leq r_c \leq 0.6F$. The value $r_c = 0.56F$ has been chosen as it gives the best fits in the case of curves 3 and 4.

The modification of the relative two-nucleon motion, according to (30) means that the internucleon forces at large distances between nucleons are more attractive than those implied by the harmonic-oscillator model; at short distances the interaction contains a repulsive core, as before. Consequently, the modification (30) will make the surface of the nucleus less diffuse. This is evident from Fig. 2 where we have presented the squared wave functions of the relative two-nucleon motion. The relative wave function modified according to (30) is represented by the curve 3. For a comparison we have also shown in Fig. 2 the Gaussian wave function (curve 1) and the relative wave function, modified according to (27) and (29) - curve 2.

The charge form-factor resulting from (30) and (25) is represented in Fig. 1 by the curve 3. We have used $\gamma = 0.74$ and the same values of α and r_c , as for the curve 2. From Fig. 1 one can see that the modification (30) of the relative motion at large distances between nucleons corrects the curve 2 in the right direction: it lifts much higher the shoulder of the curve on the right hand side of the minimum. The curve 3 is consistent with the experimental data up to $q^2 = 15 F^{-2}$. Unfortunately the curve falls too rapidly with increasing momentum transfer. This feature of the theoretical seem to be the consequence of the Born approximation which gives zeros instead of shallow minima. One can say that the curve 3 "feels" too strongly the presence of the second minimum at very large q .

We have also tried to fit the experimental data using the $g(s)$ functions somewhat different from (30). We found that combining various powers of s before the exponential and in the exponent one can obtain fits very similar to curve 3. On the other hand, making very crude assumptions about the relative wave-function, as a rectangular or parabolic form, we could not fit the data at all.

One could obtain a better fit than that of curve 3 by a simultaneous modification of both the relative and the center-of-mass motion of two nucleons. The latter modification could be performed, for instance, by changing the oscillator parameter in the c. m. wave function. Putting

$$(31) \quad \alpha_{CM}^2 = (1 + \Gamma) \alpha^2$$

with $\Gamma > 0$ one makes the surface of the nucleus still less diffuse than was obtained by the modification of the relative motion only. Using $\Gamma = 0.1$ and the remaining parameters the same as for curve 3, we obtain curve 4 which reproduces the data very well. We do not want, however, to stress the importance of this fit, since the last modification introduces an additional parameter.

APPENDIX. -

In this appendix we make a comparison between the Jastrow⁽⁶⁾ method, used in⁽³⁾ and⁽⁵⁾, and ours. In⁽³⁾ and⁽⁵⁾ the nucleon correlations were introduced by applying a Jastrow type wave function for He⁴:

$$(A1) \quad \tilde{\psi}_{SM}(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = N e^{-1/2 \alpha^2 (r_1^2 + r_2^2 + r_3^2 + r_4^2)} \cdot \left[\prod_{j < k}^4 (1 - h_{jk}) \right]^{1/2}$$

where N is a normalization factor. The product

$$\prod_{j < k}^4 (1 - h_{jk})$$

could be expanded in powers of h_{jk} giving thus a series of contributions of different numbers of correlated pairs (no correlated pairs, one correlated pair, two correlated pairs etc.). In⁽³⁾ terms with more than one correlated pair were neglected while in⁽⁵⁾ the exact calculations, including even six correlated pairs, have been performed. In both analyses a gaussian form of h_{jk}

$$(A2) \quad h_{jk} = e^{-1/2 \alpha^2 (\vec{r}_j - \vec{r}_k)^2}$$

which simulate to some extent a hard core repulsion between nucleon, was assumed.

We would like to stress that although

$$\prod_{j < k} [1 - h(r_{jk})]$$

in the Jastrow method is constructed from two-nucleon functions and apparently includes only pair correlations, effects involving ≥ 3 particles are also present: for example, the 3-particle group (j, k, l) is connected by the factor $h(r_{jk}) h(r_{jl}) h(r_{kl})$. On the contrary, in our calculations only the two-nucleon correlations are taken into account - see Eq. (9). For this reason only a qualitative comparison between the two methods is allowed.

Let us introduce the two-nucleon correlations employing the $g(s)$ function as suggested by (A2):

14.

$$(A3) \quad g(s) = (1 - e^{-1/2 \lambda \alpha^2 s^2})^{1/2} \quad 0 < s < \infty$$

From (25) and (A3) we obtain immediately the correlated elastic form factor of the shell-model

$$(A4) \quad \tilde{F}_{SM} = e^{-q^2/8\alpha^2} \frac{e^{-q^2/8\alpha^2} - (1+\lambda)^{3/2} e^{-\frac{q^2}{8(1+\lambda)\alpha^2}}}{1 - (1+\lambda)^{-3/2}}$$

Applying (A4) we have calculated the charge elastic form-factor of He^4 for the two sets of parameters:

1. $\alpha = 157.9 \text{ MeV}$ $\lambda = 6.80 - \text{see}^{(3)} \text{ and}^{(5)}$
2. $\alpha = 179.4 \text{ MeV}$ $\lambda = 1.45 - \text{see}^{(5)}$

In these calculations the electromagnetic structure of nucleons has been taken into account by using the nucleon form-factor as predicted by the exponential charge distribution of the proton - see ref. (5). We assumed the r. m. s. radius of the proton to be $a = 0.813 \text{ F}^{(21)}$.

Our results are presented in Fig. 3 (solid curves denoted MP) where we compare them with the curves from ref. (3) (CL) and from ref. (5) (SV). It is evident from Fig. 3 that our result is inconsistent with the prediction of ref. (3). Our curves do not agree with the results of (5) either; however, in this case, the difference is much smaller.

The difference between the curves SV and ours could be interpreted as follows. In the calculations (5) based on the Jastrow method, besides the two-nucleon correlation terms one includes also terms due to clusters with more than two correlated particles; all the types of correlations become mixed in this method. The difference between the result of (5) and ours (calculated with only two-body correlations) may suggest that the effect of higher correlations in He^4 is rather important. We do not stress, however, the importance of this conclusion since the two methods are not clearly related to each other. The problem of three-body interactions is very complicated and deserves an extended and thorough investigation.

On the other hand, one may suppose that the curves SV could be fitted by the expression (A4) with proper parameters α and λ . The parameter λ is related to the radius of the hard-core repulsion: the smaller value of λ the stronger repulsion between nucleons. This suggests that, in order to fit the curves SV with our formula (A4), the values of λ should be taken smaller than those in (5), as our calculations do not include the higher correlations. This indeed comes true as is shown in Fig. 3. The dashed curves were calculated with the aid of (A4) and the values of λ as indicated; the va-

lues of α are the same as for the other curves in Fig. 3, respectively.

We have also tried, using A(4), to fit the curve CL in Fig. 3. It turned out, however, to be quite impossible. This curve is by no means consistent with the formula (A4). This result confirms the conclusion following from ref. (5): in the case of the He^4 nucleus the one-correlated-pair approximation is wrong. Is the same true for heavier nuclei? A forth-coming comparison⁽⁸⁾ of the formula (21) with calculations⁽²²⁾ based on the approximated Jastrow method should answer this question.

REFERENCES. -

- (1) - J. P. Repellin, P. Lehmann, J. Lefrancois and D. B. Isabelle, Phys. Letters 16, 169 (1965).
- (2) - R. F. Frosch, J. S. Mc. Carthy, R. E. Rand and M. R. Yearian, Phys. Rev. 160, 874 (1967).
- (3) - W. Czyz and L. Lesniak, Phys. Letters 25B, 319 (1967).
- (4) - Y. C. Tand and R. C. Herndon, Phys. Letters 25B, 307 (1967).
- (5) - T. Stovall and D. Vinciguerra, Orsay preprint L. L. 1191, May 1968.
- (6) - R. J. Jastrow, Phys. Rev. 98, 1479 (1955).
- (7) - F. C. Khanna, Phys. Rev. Letters 20, 871 (1968).
- (8) - A. Malecki and P. Picchi - in preparation.
- (9) - A. Malecki, Frascati report 67/34 (1967).
- (10) - T. de Forest Jr., and J. D. Walecka, Advances in Physics 15, 1 (1966).
- (11) - K. W. McVoy and L. Van Hove, Phys. Rev. 125, 1034 (1962).
- (12) - L. N. Hand, D. G. Miller and R. Wilson, Revs. Modern Phys. 35, 335 (1963).
- (13) - J. P. Elliott and T. H. R. Skyrme, Proc. Roy. Soc. (London) A232, 561 (1965).
- (14) - L. J. Tassie and F. C. Barker, Phys. Rev. 111, 940 (1958).
- (15) - W. Czyz, L. Lesniak and A. Malecki, Ann. Phys. 42, 119 (1967).
- (16) - A. R. Edmonds, Angular Momentum in Quantum Mechanics, Princeton University Press 1957.
- (17) - M. Mosinsky, Nuclear Phys. 13, 104 (1959).
- (18) - T. Janssens, R. Hofstadter, E. B. Hughes and M. R. Yearian, Phys. Rev. 142, 922 (1966).
- (19) - H. Frank, D. Haas and H. Prange, Phys. Letters 19, 391 (1965); 19, 719 (1965).
- (20) - M. A. Preston, Physics of the Nucleus, Addison - Wesley Publishing Company 1962.
- (21) - S. D. Drell, Proceedings of International Symposium on Electron and Photon Interactions, Stanford (1967).
- (22) - C. Ciofi degli Atti, Sanità preprints, ISS-67/49, ISS-68/10.

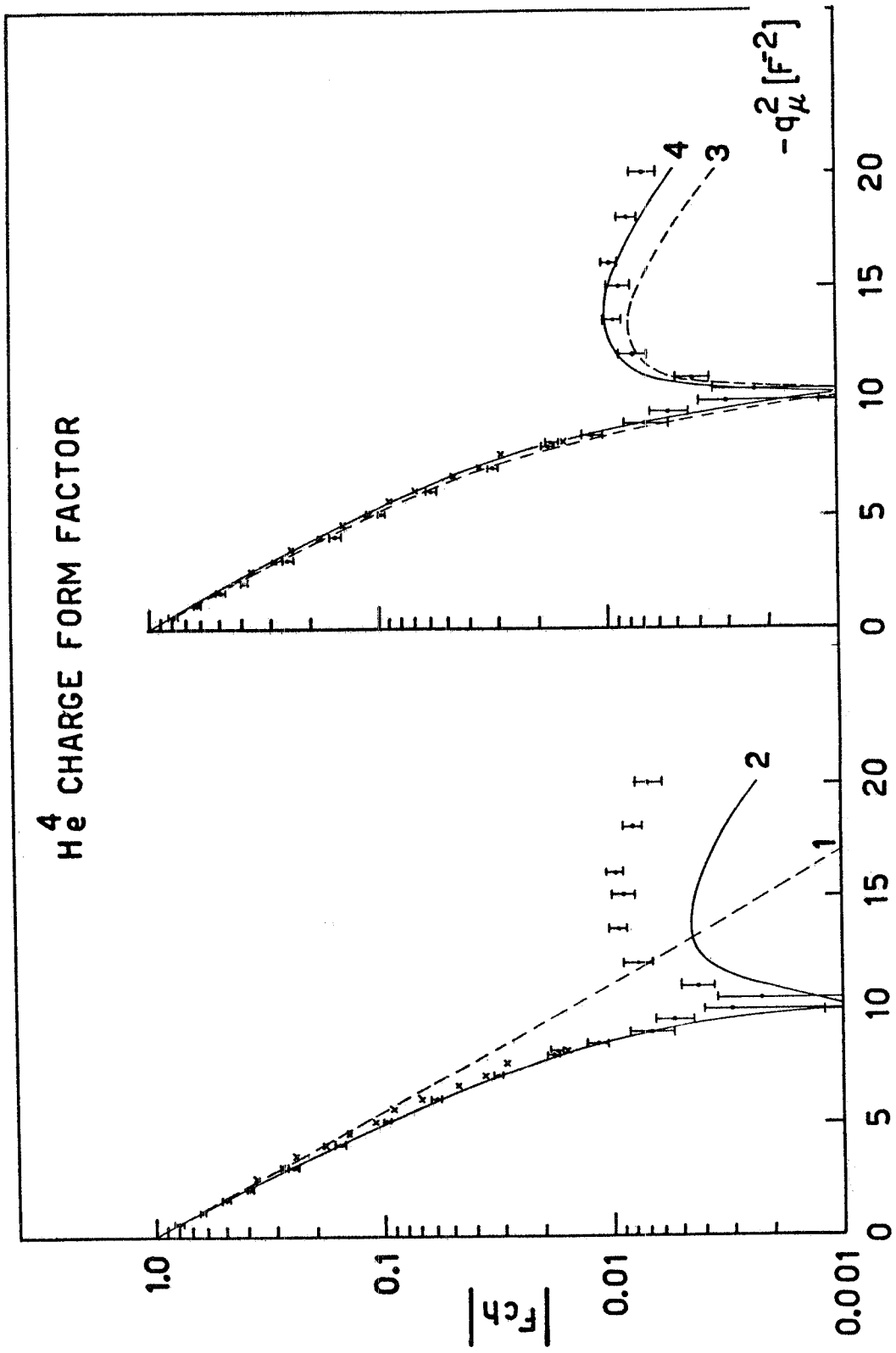


FIG. 1 - Charge form-factors of He^4 . Experimental points marked x one taken from ref. (1) and those marked ϕ from ref. (2). Curve 1 represents the Gaussian form-factor. Curve 2 gives the charge form-factor corrected for the short range nucleon-nucleon correlations. Curves 3 and 4 have been calculated using the wave function of the relative two-nucleon motion obtained by modification of the Gaussian wave function at short and large internucleon distances. In addition, for curve 4 the c. m. motion of two nucleons has been modified.

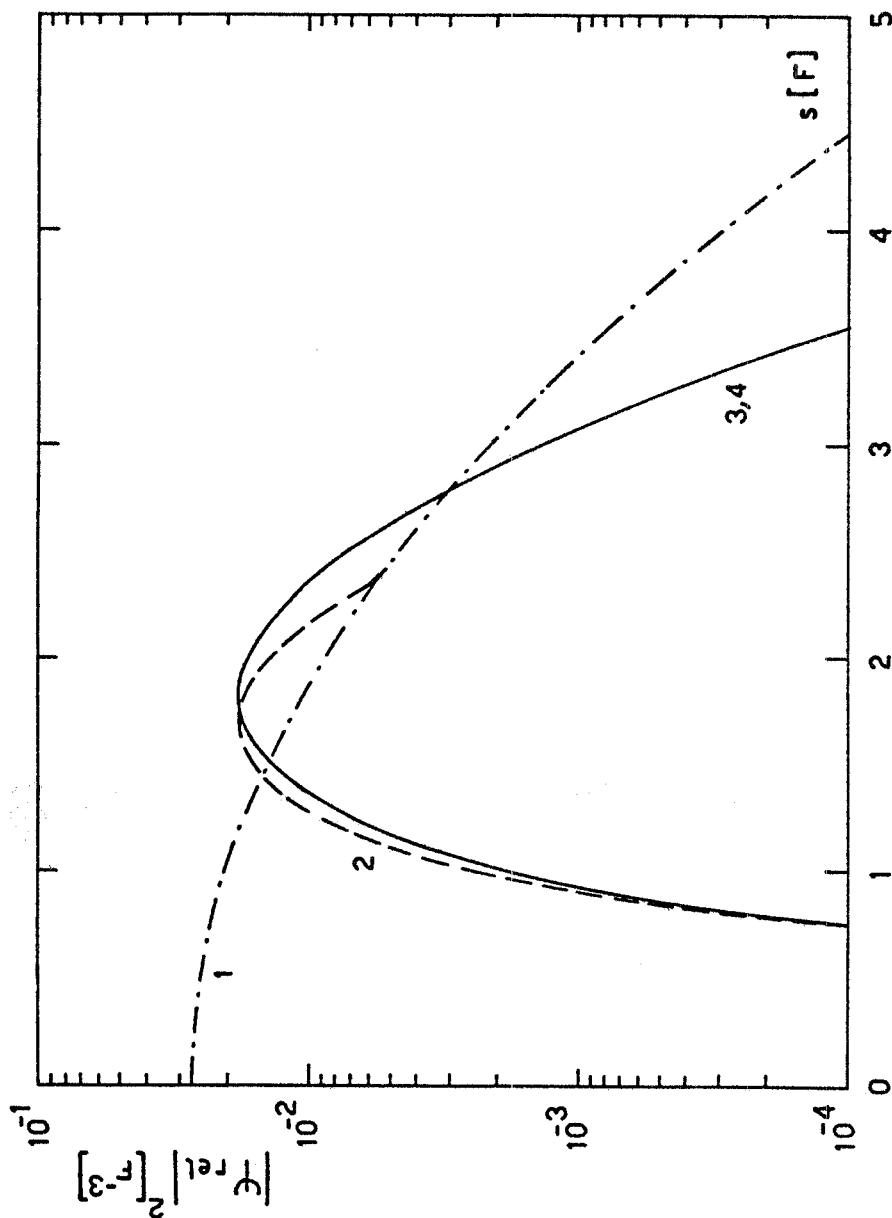


FIG. 2 - Squared wavefunctions of the relative two-nucleon motion. Curve 1 is the Gaussian wave function. The relative wave function 2 describes a hard-core repulsion between nucleons at short distances. It is identical to the Gaussian wave function at large internucleon distances. The relative wavefunction 3, 4 has been obtained by modification of the Gaussian wavefunction at short as well as large internucleon distances.

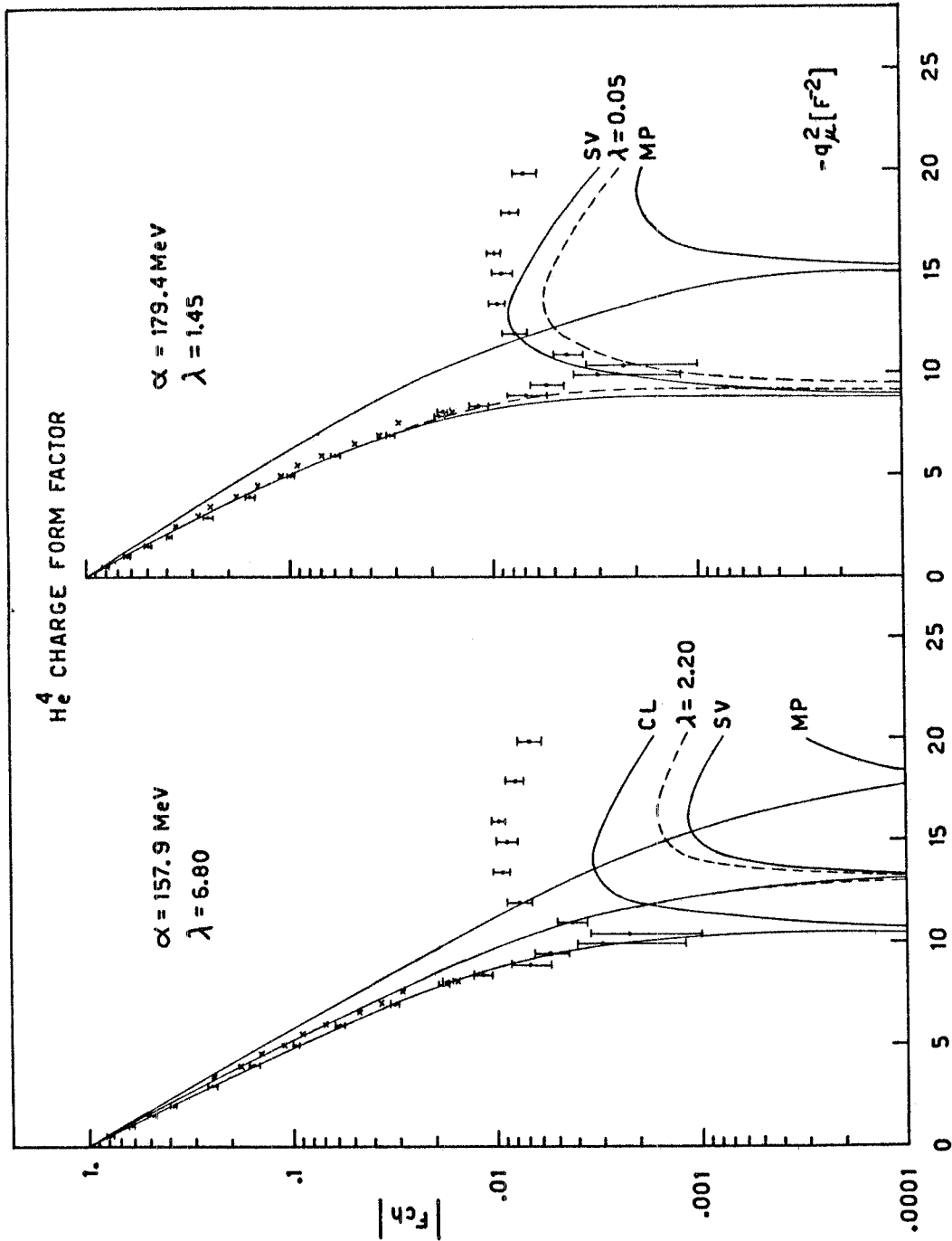


FIG. 3 - Charge form factors of He^4 . Our form factors (solid curves denoted MP, and dashed curves) have been calculated with the help of (A4) and are compared with the results of ref. (3) (solid curve CL and ref. (5) (solid curves SV).