COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF - 68/26
14 Maggio 1968

# G. Pancheri : INFRARED RADIATIVE CORRECTIONS FOR RESONANT PROCESSES. -

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

Laboratori Nazionali di Frascati del CNEN
Servizio Docuementazione

LNF-68/26

Nota Interna: n. 398
14 Maggio 1968

# G. Pancheri: "INFRARED RADIATIVE CORRECTIONS FOR RESONANT PROCESSES".

## 1) - INTRODUCTION -

In view of the importance of radiative corrections for the interpretation of the results of the colliding beam experiments, E. Etim and B. Touschek[1] proposed a division of labour between theorists and experimentalists. This division of labour was intended to enable two experimentalists to compare the results of their experiments directly and to allow the theorist to concentrate in the essential ultraviolet aspects of the process without entering into the details of the experiments.

This proposal was further elaborated by E. Etim, G. Pancheri and B. Touschek[2] and it was shown that the method should give highly accurate results for a certain class of experiments (in which the statistical error was matched to the momentum resolution) and for processes for the cross section calculated in lowest order perturbation theory does not rapidly vary with the energy of the colliding particles. M. Greco and G. Rossi[3] have shown that the proposed procedure can be derived from perturbation theory[4 ÷ 7] by means of a canonical transformation applied to the final state of the reaction.

The case of strongly energy dependent processes has been discussed in a very rough approximation in ref. (1), but it was not included in ref. (2) since the basic assumption of separability of the infrared process from the high energy part of the interaction cannot be expected to hold in this case. From an experimental point of view the radiative corrections are of particular importance just in this case of resonant processes. This is due to the fact that the exploration of a resonance calls for a maximum resolution of the energy of the produced particles: this makes the radiative corrections quite big. U. Amaldi, A. de Gasperis and P. Stein[8] have discussed the behaviour of the $\psi$-resonance assuming an energy resolution of 0.5 MeV and applying perturbation theory in a form suggested by P. Kessler[9]. In these conditions the perturbative radiative correction is about -45%, the treatment suggested in refs. (1) and (2) would lead one to estimate that this correction should be -29%. It is seen that the difference is quite considerable and that there is a case for suspecting the applicability of straightforward perturbation theory.

The breakdown of the separability of the low and high energy aspects of the process has the following reason: the condition for separability is that the high energy part of the process takes place in a time interval, which is very short compared to the inverse of the energy loss in the form of radiation. In a resonant process this will generally only be the case as long as the energy loss due to the radiation field is contained well within the width of the resonance. If one wants informations on the radiative corrections to the form of the resonance curve (and in particular to the form of the "shoulders" of this curve) one has to take explicit account of the finite life time of the intermediate resonant state.

In sect. II we give a semiclassical treatment of this phenomenon and define the Bond factor which characterized it. We propose a generalization of this first approximation result to cover higher powers. The purpose of this generalization is to free the treatment of resonant states from the limitations of perturbation theory.

Sect. III confirms the results of sect. II in lowest order perturbation theory.

In sect. IV the explicit form of the infrared correction factor is given.

## II) - NON-PERTURBATIVE TREATMENT OF THE BREMSSTRAHLUNG EMITTED IN THE FORMATION OF RESONANT STATES -

For non resonant processes the current attributed to the transition from the initial state $|i)$ to the final state $|f)$ can be assumed to be

of the form

$$(1) \qquad j_c(x) = j_f(x)\,\theta(t) + j_c(x)\,\theta(-t)$$

where $j_f$ and $j_i$ are respectively the currents represented by the charged particles of the final and initial states. It is assumed in (1) that the transition from the initial to the final state takes place in an infinitesimal time interval at $t = 0$.

$j_c(x)$ can be understood as the meanvalue of the quantized current in a scattering state $|s)$ defined by $\lim_{t\to\infty} |s) = |f)$.

If the transition $|i) \longrightarrow |f)$ is resonant it can no longer be assumed that the process takes place in a negligibly short time (compared to the reciprocal of the frequency of the emitted radiation). One can take account of the finiteness of the time interval between the creation of the final state at time $t = 0$ and the creation of the resonant state at time $-t_0 < 0$ by putting instead of (1)

$$(2) \qquad j_c(x) = j_f(x)\theta(t) + \overline{j_i(x)}$$

where $\overline{j_i(x)}$ is given by

$$(3) \qquad \overline{j_i(x)} = \left(\frac{\Gamma}{2} - i(2E-M)\right)\int_0^\infty dt_o\, j_i(x)\,\theta(-t-t_o)\,e^{-\frac{\Gamma}{2}t_o + i(2E-M)t_o}$$

We have chosen the center of mass system and assumed that the incoming particles (an electron and a positron) have an energy E, that the mass of the resonant state is M and its width is $\Gamma$. Putting

$$j_c(x) = \int j_c(k)\, e^{i(kx)} d^4k$$

one gets

$$(4) \qquad (2\pi)^4 \vec{j}_c(k) = -\sum_f \frac{ie_f \vec{v}_f}{(\vec{k}\vec{v}_f) - \omega} + S\sum_i \frac{ie_i \vec{v}_i}{(\vec{k}\vec{v}_i) - \omega}$$

with

$$(5) \qquad S = \frac{\frac{\Gamma}{2} - i(2E - M)}{\frac{\Gamma}{2} - i(2E - \omega - M)}$$

Here we have only written the space part of $j_c(k)$; $e_f$, $\vec{v}_f$ are the charges and velocities of the final state particle, $e_i$ and $\vec{v}_i$ refer to the initial state and eq. (4) holds for $\omega = |k_0| = |\vec{k}|$. It is seen that for $(2E-M) \gg \Gamma/2$ and $(2E-M) \gg \omega$ one has $S = 1$. Eq. (4) therefore reduces to the Fourier-transform of (1).

The Bond factor $\beta$ introduced in ref. (1) is defined by $dn(\omega) = \beta(d\omega/\omega)$ where $dn(\omega)$ is the average number of photons produced in the transition $|i) \longrightarrow |f)$. Using the method explained in the appendix of ref. (2) we find

$$\beta = \beta_f + |S|^2 \beta_i \tag{6}$$

Because of charge conjugation invariance there is no interference term.

The knowledge of $\beta$ allows one to determine the first approximation to the cross section of the resonant process accompanied by the emission of a single photon. Denoting this cross section by $d^2\sigma(\omega)$ one has

$$d^2\sigma(\omega) = \beta \frac{d\omega}{\omega} d\sigma_o(M) \tag{7}$$

where $d\sigma_o$ is the cross section for the resonant process without the emission of quanta, that is calculated in lowest non vanishing order. If the non relativistic Breit Wigner formula is used for $d\sigma_o$, one finds

$$d\sigma_o(M) \propto \left(\frac{\Gamma^2}{4} + (2E - M)^2\right)^{-1}$$

Within the range of validity of the Breit Wigner formula one can therefore write for (7)

$$d^2\sigma(\omega) = \frac{d\omega}{\omega} (\beta_f d\sigma_o(M) + \beta_i d\sigma_o(M+\omega)) \tag{8}$$

One can easily see that this expression holds also in the relativistic case where

$$S = \frac{M\Gamma - i((2E)^2 - M^2)}{M\Gamma - i((2E)^2(1 - \frac{\omega}{E}) - M^2)} \tag{5'}$$

and

$$d\sigma_o(M) \propto (M\Gamma + ((2E)^2 - M^2)^2)^{-1}$$

It is seen from eq. (8) that the cross section for bremsstrahlung, considered as a function of the machine energy E and for a fixed frequency $\omega$ of the emitted radiation, will show two peaks: one an undisplaced resonance peak at 2E = M corresponding to the emission of photons by the particles of the final state and a displaced peak at $2E=M+\omega$ corresponding to the emission of radiation by the initial state of electrons and positrons.

The asymmetry between the initial and final state is due to the fact that the emission of radiation from the initial state diminishes the $q^2$ of the reaction, while the emission of radiation from the final state does not.

For most reactions to be measured with Adone, the undisplaced peak should be smaller that the displaced peak, since, unless the final particles are also electrons and positrons, we shall have $\beta_f \ll \beta_i$: for the $\rho$-resonance one has $\beta_e = 0.063$ and $\beta_\pi = 0.011$.

The first approximation of eq. (8) cannot however be considered sufficient in view of the very high accuracy in the determination of the energy planned for the experiments on resonant annihilation - this was illustrated numerically in the introduction. A generalization of this result, to cover higher powers of the coupling constant, must satisfy the following two requirements:

a) wherever the cross section can be considered to be not rapidly varying it should be described by the formula which has been justified in ref. (2) and (3), namely

$$N d^2\sigma(\omega) = \beta \frac{d\omega}{\omega} \left(\frac{\omega}{E}\right)^{\beta} d\sigma_E$$

where of course $\beta = \beta_i + \beta_f$ (*).

b) it should allow for a definition of a $d\sigma_E$ which is independent of the experimental resolution.

These requirements can be fulfilled by the following expression:

$$N d^2\sigma(\omega) = \frac{d\omega}{\omega} \left(\frac{\omega}{E}\right)^{\beta} (\beta_f d\sigma_E(M) + \beta_i d\sigma_E(M+\omega)) \tag{9}$$

In the above equation, $d\sigma_E(M)$ is defined by postulating no emission of photons by the initial state, i.e. by putting $\beta_i = 0$. Since the radiation

---

(*) - The condition of small variability of the cross section is satisfied for $\omega \ll \Gamma$ as well as for $(2E-M) \gg \Gamma$ and $\omega \ll (2E-M)$. It is not satisfied on the shoulders of the resonance curve.

losses from the final state do not affect the $q^2$ of the reaction, this case would coincide with the one which has been treated in ref. (2) and therefore we can write

$$d\sigma_E(M) = d\sigma_o(M)(1+\lambda) \tag{10}$$

where $d\sigma_o(M)$ is the differential cross section calculated at the lowest perturbative order and $\lambda$ represents the finite contribution of the ultraviolet radiative corrections.

Eq. (9) takes then the following form:

$$N d^2\sigma(\omega) = \frac{d\omega}{\omega}\left(\frac{\omega}{E}\right)^{\beta}(\beta_f + |S|^2\beta_i)\, d\sigma_E(M) \tag{11}$$

where S is given by eq. (5) or (5'). It is immediately seen that the expression (11) satisfies both requirements (a) and (b) with $d\sigma_E(M)$ given by eq. (10). It has to be noticed that in the case of $\beta_f = 0$ this expression coincides with the case which has been treated in ref (1).

## III - COMPARISON WITH PERTURBATION THEORY -

We confirm the result (11) to first order in $\beta$ by actually computing the radiative corrections at the first perturbative order to a specific process and by comparing the relative expressions.

We have taken as an example the process

$$e^+ + e^- \longrightarrow \rho^o \longrightarrow \pi^+ + \pi^-$$

which is described, with its radiative corrections, through the Feynman diagrams of fig. 1.

Assuming that the energy of the real emitted photon is much smaller than the energy of the emitting particle, the differential cross section in the center of mass system of the colliding particles can be written as

$$d\sigma_1 = (1+\delta)\, d\sigma_o(M)$$

where $d\sigma_o(M)$ is the cross section at the lowest perturbative order and

$$\delta = \frac{\alpha}{\pi}\left[4\left(1-\ln\frac{2E}{m}\right) - \ln\left(\frac{2E}{\mu_\pi}\right)\ln\frac{E}{\Delta E} + \right.$$

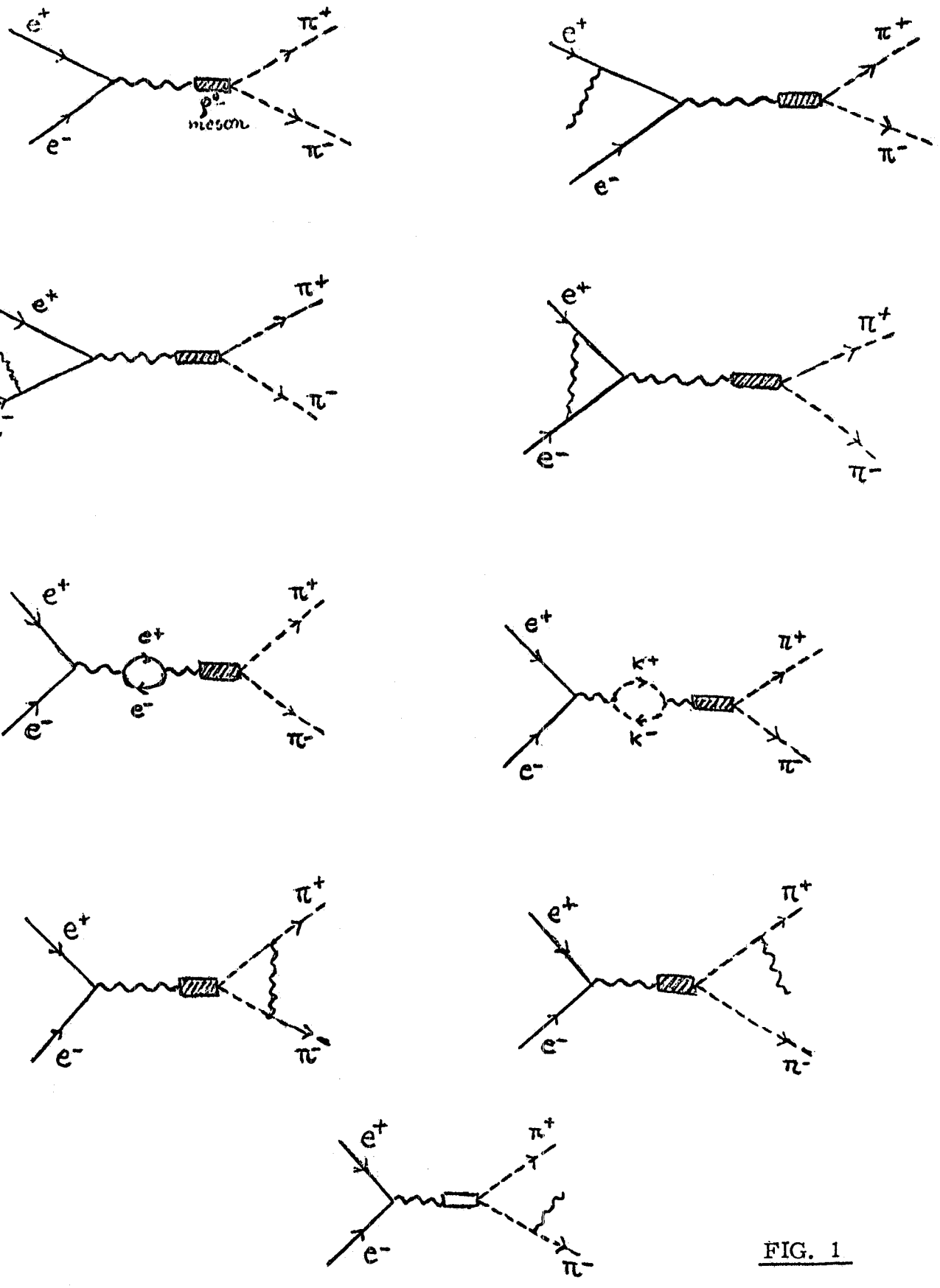


FIG. 1

$$+(1-2\ln(\frac{2E}{m}))\ln(\frac{(4E^2(1-\Delta E/E)-M^2)^2+M^2\Gamma^2}{(4E^2-M^2)^2+M^2\Gamma^2} -$$

$$-2(\frac{4E^2-M^2}{M\Gamma})(1-2\ln(\frac{2E}{m}))\operatorname{arctg}(\frac{M\Gamma}{4E^2-M^2-(\frac{(4E^2-M^2)+M^2\Gamma^2}{4E\,\Delta E})}) +$$

$$+\frac{13}{3}\ln(\frac{2E}{m})+4\ln(\frac{2E}{\mu_\pi})+\frac{1}{3}\ln(\frac{2E}{\mu_k})-\frac{50}{9}+\frac{2\pi^2}{3}\Big].$$

In the above expression E is the energy of each colliding beam, $\Delta E$ the resolution in energy, M, m, $\mu_\pi$ and $\mu_k$ the mass of the $\rho$-meson, of the electron, of the $\pi$ and of the k-meson respectively. $\Gamma$ is the width of the resonance.

To the first order in $\beta$ the integration of equation (11) up to an energy resolution $\Delta E$, gives

$$d\sigma_{exp} \sim d\sigma_o(M)(1-(\beta_e+\beta_\pi)\ln\frac{E}{\Delta E}-\frac{\beta_e}{2}\ln(\frac{(4E^2(1-\frac{\Delta E}{E})-M^2)^2+M^2\Gamma^2}{(4E^2-M^2)^2+M^2\Gamma^2})-$$

$$-\beta_e\frac{4E^2-M^2}{M\Gamma}\operatorname{arctg}(\frac{M\Gamma}{4E^2-M^2-(\frac{(4E^2-M^2)^2+M^2\Gamma^2}{4E\,\Delta E})})+\ )$$

where the Bond factor $\beta_k$ can be written as

$$\beta_k=\frac{4\alpha}{\pi}(\ln(\frac{2E_k}{m_k})-\frac{1}{2})$$

with $E_k$ and $m_k$ to represent the energy and the mass of the emitting particles. One therefore sees that to first order in $\beta$ the non-perturbative treatment gives the same result than the perturbative one, provided one puts

$$\lambda=\frac{d}{\pi}(\frac{13}{3}\ln(\frac{2E}{m})+4\ln(\frac{2E}{\mu_\pi})+\frac{1}{3}\ln(\frac{2E}{\mu_k})-\frac{50}{9}+\frac{2\pi^2}{3})$$

## IV - DETERMINATION OF THE INFRARED CORRECTION FACTOR -

By the use of eq. (11), we get for the experimental cross-section of resonant intermediate state processes the expression

$$d\sigma_{exp} = C(\rho)(1+\lambda)\, d\sigma_o(M)$$

with

$$NC(\rho) = \int \frac{d\omega}{\omega}\left(\frac{\omega}{E}\right)^{\beta} \rho(\omega)(\beta_f + |S|^2 \beta_i)$$

Let us assume for $\rho(\omega)$, the resolution function discussed in ref. (2), the form

$$\rho(\omega) = e^{-(\omega^2/2\Delta E^2)}$$

Using eq. (5'), we get

$$C(\rho) = N^{-1}\left(\frac{\sqrt{2}\,\Delta E}{E}\right)^{\beta}\left(\Gamma(1+\frac{\beta}{2}) - \frac{\beta_i}{\beta} I_R\right)$$

with

$$I_R = \beta\, 8\sqrt{2} E\,\Delta E \int_o^{\infty} \frac{\left[2\sqrt{2}E\,\Delta E x^2 - (4E^2 - M^2)x\right] x^{\beta-1} e^{-x^2} dx}{\left(4E^2\left(1 - \frac{\sqrt{2}\,\Delta E}{E} x\right) - M^2\right)^2 + M^2\Gamma^2}$$

In order to evaluate $I_R$, it is convenient to distinguish the three following possibilities:

1) $\Gamma \gg E$ or $q^2 \gg M^2$ resonance.

This is the case already seen in ref. (2). We have

$$I_R \simeq 0$$

and

$$C(\rho) = N^{-1}\left(\frac{\sqrt{2}\,\Delta E}{E}\right)^{\beta} \Gamma(1+\frac{\beta}{2})$$

2) $\Gamma \ll \Delta E$ and $q^2$ within the resonance region.

This case was treated in ref. (1) and it is valid in $\varphi$-meson decay

and also in $\omega$ -meson decay. We have

$$I_R \simeq \Gamma(1+\frac{\beta}{2})$$

and we are therefore left only with the correction due to the photon emission from the final state, i. e.

$$C(\rho)=N^{-1}(\frac{\sqrt{2}\,\Delta E}{E})^{\beta}\,\frac{\beta_f}{\beta}\,\Gamma(1+\frac{\beta}{2})$$

3) $\Gamma \simeq \Delta E$ and $q^2$ within the resonance region.

In this case, which is met when measuring the $\rho$-meson decay, no simplification is allowed (but the obvious ones due to $\Delta E \ll E$) and no closed form is easily attainable for the integral $I_R$. However, due to the rapid convergence of the integrand, there should be no difficulty in computing the correction on a machine.

REFERENCES -

(1) - E. Etim and B. Touschek, LNF-66/10 (1966).
(2) - E. Etim, G. Pancheri and B. Touschek, Nuovo Cimento 51B, 276 (1967).
(3) - M. Greco and G. Rossi, Nuovo Cimento 50A, 168 (1967).
(4) - J. Schwinger, Phys. Rev. 76, 790 (1949).
(5) - D.R. Yennie and H. Suura, Phys. Rev. 105, 1378 (1957); D.R. Yennie, S.C. Frautschi and H. Suura, Ann. Phys. 13, 379 (1961).
(6) - K.E. Erikson, Nuovo Cimento 19, 1010 (1961).
(7) - E.L. Lomon, Nuclear Phys. 1, 101 (1956); E.L. Lomon, Phys. Rev. 113, 726 (1959).
(8) - U. Amaldi, A. de Gasperis and P. Stein, Istituto Superiore di Sanità, ISS 65/48 (1965).
(9) - P. Kessler, Nuovo Cimento 17, 809 (1960).