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In the past few years a great deal of experimental effort has been devoted to study accurately the elastic scattering of strongly interacting particles, at high energy and large momentum transfers. Several experiments(1-5) have measured particularly the proton-proton scattering, where a very high accuracy has been achieved because of the high intensities of proton beams. A considerable amount of very precise data, with a close spacing of the data points, are now available. Any reliable theory must be able to reproduce these results. Many speculations concerning high-energy processes have been made(6), and a wide range of models and ideas, more or less accepted, have been suggested. Many attempts(3, 7-9) have been also done to search for a suitable universal function which would represent all the large momentum transfer data.

In the present note we wish to make the following observation. Let us assume the nucleon as a complex object having an internal structure. At high energies and as the momentum transfer becomes higher, many different channels are open, so that the probability that the nucleon does not break up will decrease more and more. This picture, together with the Bolzmann's statistical definition for the entropy of a thermodynamical state, namely

(1) $$S = k \lg W$$

in terms of the probability W for the occurrence of the state, has led us to the following assumptions:

(i) For hard collisions each process can be described in terms of the entropy which grows up, with the different channels that are open, to a maximal value which depends on the specific process.

(ii) The number of channels open, and therefore the entropy, are an increasing function of the momentum transfer t, for fixed energy in the center of mass system (C.M.S.).

By making these hypothesis, we find the following formula for the differential cross section for the elastic nucleon-nucleon scattering:

(2) $$\frac{d\sigma}{dt} = \frac{A}{p^4} e^{-\frac{1}{T}(\sqrt{t}+\sqrt{s})}$$

where t is the momentum transfer, s is the square of the total energy in C.M.S., p is the nucleon momentum in C.M.S., A and T are two constants.

The formula (2) is valid in asymptotic conditions, and shows the following features:

a) It is relativistic invariant.
b) It is consistent with the upper limits which have been set on the scattering amplitudes by Cerulus and Martin[10].
c) It is in a very good agreement with the experimental results, when it is compared to the asymptotic proton-proton scattering data.

More explicity let us now define $P_{NB}(t)$ as the probability that a nucleon does not break up when the momentum transfer takes the value t. The assumptions (i) and (ii) suggest the following equation:

(3). $$\frac{dP_{NB}(t)}{P_{NB}(t)} = -\frac{dW}{W} = -dS$$

where $dP_{NB}(t)$ is the variation of $P_{NB}(t)$ when t is increased by an amount dt, and W, which is proportional to the channels open at t, is defined in (1). We assume for semplicity k = 1. By integrating (3) we obtain:

(4) $$P_{NB}(t) = P_o e^{-\int_{t=o}^{t} dS} = P_o e^{-[S(t) - S(0)]}$$

The expression (4) must be associated to each nucleon which, emerging from the interaction region, will not emit pions, kaons, giving in this way an elastic event.

We want to calculate now the probability that, for a process of elastic scattering, the final state will be caracterized by a value of the momentum transfer between t and t + dt. In order to achieve this, let us recall that the system, during the interaction, will proceed to states of greater probability, or maximum entropy. This state, for fixed energy, will be realized at the maximum value to of the momentum transfer, according to the assumptions (i) and (ii). We can however observe some fluctuations. In particular let S(t) be the entropy of the state characterized by the value t of the momentum transfer, and $S(t_o)$ the maximum value of the entropy, attained at $t = t_o$. Then thermodynamics tells us that the probability dP of the parameter t to have a value lying inside the infinitesimal interval dt is given by the equation:

(5) $$dP = f\, e^{[S(t) - S(t_o)]}\, dt$$

where the function f is a distribution function f(t) whose size must be chosen so that the integral of dP over all physicallt realizable values of t amounts to unity.

We are now able to write down an expression for the overall probability $d\pi$ for an elastic scattering process with the momentum transfer lying between t and t + dt:

(6) $$d\pi = \int f^2\, e^{[S(t)-S(t_o)]}\, P_{NB}(t)\, P_{NB}(t')\, dt dt'\, \delta(t-t')$$

where $P_{NB}(t)$ and $P_{NB}(t')$ refer to the two particles present in the final state, and the δ-function has been used to impose a correlation between them. The term $e^{[S(t)-S(t_o)]}$, moreover, has been counted only once, because the fluctuation concerns the system as a whole.

By approximating $f \sim (const/p^2)$, we obtain from (4) and (6) the following expression for the differential cross section $d\sigma$, which must be proportional to $d\pi$:

(7) $$\frac{d\sigma}{dt} = \frac{C}{P^4}\, e^{[-S(t)-S(t_o)+2S(0)]}$$

where we have grouped all together the constants in C.

In the C.M.S., if θ is the scattering angle, we have $t = 2p^2(1-\cos\theta)$ and $t_o = 4p^2$.

We want now to calculate the explicit expression of S(t). Let us write:

(8)
$$\frac{\partial S}{\partial (2E)} = \frac{1}{2} \frac{\partial S}{\partial t} \frac{\partial t}{\partial E}$$

where 2E is the total energy in C.M.S. We define T, the "temperature" of the system, as follows:

(9)
$$\frac{1}{T} = \frac{\partial S}{\partial (2E)} \Bigg|_{t=t_o} .$$

We assume that in asymptotic conditions T will attain a constant value(11). Then by integrating the equation (8) at $t=t_o$, S(t) being a function of t alone, we finally obtain:

(10)
$$S(t) = \frac{\sqrt{t}}{\beta T}$$

where $\beta = p/E$. At high energies we shall put $\beta = 1$.

Consequently the exponential part of the right-hand side of (7) can be written:

(11)
$$e^{-S(t) - S(t_o) + 2S(0)} = e^{-\frac{1}{\beta T}(\sqrt{t} + \sqrt{t_o})}$$

The equation (10), together with (11) allows to express the differential cross section in terms of a well defined function of alone.

Finally, taking to $= 4p^2 \simeq s$, we arrive to the equation (2). This formula presents a very interesting feature, that is the momentum transfer t and the square of the C.M. total energy s play a completely symmetric role.

For proton-proton scattering, however, because of the two identical particles present in the final state, the experimental data must be compared with the quantity:

(12)
$$\left(\frac{d\sigma}{d\Omega}\right)_{p-p} = \frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma}{d\Omega}(\pi - \theta)$$

which can be written as

(13)
$$\left(\frac{d\sigma}{d\Omega}\right)_{p-p} = \frac{B}{p^2} e^{-\frac{\sqrt{s}}{T}} \left[e^{-\frac{\sqrt{t}}{T}} + e^{-\frac{\sqrt{u}}{T}} \right]$$

where u is defined as: $u = 2p^2(1 + \cos\theta)$.

In table I are listed all the experimental data for $t \gtrsim 8\ (GeV/c)^2$,

TABLE I

Differential cross-section $d\sigma/d\Omega$, in units of 10^{-30} cm^2/sr, as function of the incoming lab. momentum p_o, and the C.M. scattering angle θ. $(d\sigma/d\Omega)_{th}$, in the same units, is calculated with the formula (13).

p_o (GeV/c)	Ref.	$\theta_{C.M.}$ (deg)	t $(GeV/c)^2$	$(d\sigma/d\Omega)_{exp}$ (μb/ster)	$(d\sigma/d\Omega)_{th}$ (μb/ster)
9.2	5	68.	4.87	0.264±(3-4)%	0.200
9.2	5	71.	5.26	0.200±(3-4)%	0.174
9.2	5	75.	5.78	0.148±(3-4)%	0.148
9.2	5	80.	6.44	0.111±(3-4)%	0.125
9.2	5	90.	7.79	0.105±(3-4)%	0.109
10.1	5	65.	4.99	0.169±(3-4)%	0.122
10.1	5	68.	5.40	0.128±(3-4)%	0.104
10.1	5	71.	5.82	0.934×10^{-1}±(3-4)%	0.905×10^{-1}
10.1	5	75.	6.40	0.731×10^{-1}±(3-4)%	0.755×10^{-1}
10.1	5	80.	7.13	0.600×10^{-1}±(3-4)%	0.635×10^{-1}
10.1	5	90.	8.63	0.516×10^{-1}±(3-4)%	0.548×10^{-1}
10.2	4	90.	8.73	0.441×10^{-1}±4.8%	0.505×10^{-1}
10.4	4	90.	8.91	0.386×10^{-1}±4.7%	0.435×10^{-1}
10.6	4	90.	9.10	0.356×10^{-1}±4.8%	0.376×10^{-1}
10.8	4	90.	9.29	0.303×10^{-1}±4.9%	0.326×10^{-1}
11.0	4	90.	9.47	0.284×10^{-1}±5.5%	0.282×10^{-1}
11.1	5	68.	5.98	0.572×10^{-1}±(3-4)%	0.525×10^{-1}
11.1	5	71.	6.45	0.465×10^{-1}±(3-4)%	0.440×10^{-1}
11.1	5	75.	7.09	0.384×10^{-1}±(3-4)%	0.374×10^{-1}
11.1	5	80.	7.9	0.335×10^{-1}±(3-4)%	0.312×10^{-1}
11.1	5	90.	9.57	0.304×10^{-1}±(3-4)%	0.270×10^{-1}
11.2	4	90.	9.66	0.255×10^{-1}±5.4%	0.248×10^{-1}
11.4	4	90.	9.85	0.202×10^{-1}±5.4%	0.212×10^{-1}
11.6	4	90.	10.04	0.190×10^{-1}±5.2%	0.185×10^{-1}
11.8	4	90.	10.22	0.153×10^{-1}±5.4%	0.160×10^{-1}

12.0	4	90.	10.41	$0.143\times10^{-1}\pm5.4\%$	0.140×10^{-1}
12.1	5	65.	6.06	$0.357\times10^{-1}\pm(5-6)\%$	0.318×10^{-1}
12.1	5	68.	6.57	$0.298\times10^{-1}\pm(5-6)\%$	0.274×10^{-1}
12.1	5	71.	7.08	$0.252\times10^{-1}\pm(5-6)\%$	0.232×10^{-1}
12.1	5	75.	7.79	$0.192\times10^{-1}\pm(5-6)\%$	0.190×10^{-1}
12.1	5	80.	8.68	$0.173\times10^{-1}\pm(5-6)\%$	0.157×10^{-1}
12.1	5	90.	10.50	$0.166\times10^{-1}\pm(5-6)\%$	0.131×10^{-1}
12.2	4	90.	10.60	$0.118\times10^{-1}\pm5.3\%$	0.122×10^{-1}
12.4	4	90.	10.78	$0.116\times10^{-1}\pm5.4\%$	0.107×10^{-1}
12.6	4	90.	10.97	$0.953\times10^{-2}\pm6.3\%$	0.937×10^{-2}
12.8	4	90.	11.16	$0.867\times10^{-2}\pm5.7\%$	0.820×10^{-2}
13.0	4	90.	11.35	$0.739\times10^{-2}\pm5.9\%$	0.720×10^{-2}
13.2	4	90.	11.53	$0.722\times10^{-2}\pm7.1\%$	0.630×10^{-2}
13.4	4	90.	11.72	$0.525\times10^{-2}\pm5.7\%$	0.550×10^{-2}
14.25	3	67.	7.63	$(7.58\pm0.23)\times10^{-3}$	7.85×10^{-3}
14.25	3	71.	8.44	$(5.86\pm0.19)\times10^{-3}$	6.15×10^{-3}
14.25	3	77.	9.70	$(4.36\pm0.14)\times10^{-3}$	4.54×10^{-3}
14.25	3	90.	12.52	$(3.31\pm0.09)\times10^{-3}$	3.24×10^{-3}
16.9[15]	2	67.	9.14	$(1.83\pm0.04)\times10^{-3}$	1.83×10^{-3}
16.9	2	70.	9.87	$(1.44\pm0.06)\times10^{-3}$	1.48×10^{-3}
16.9	2	72.	10.36	$(1.30\pm0.05)\times10^{-3}$	1.30×10^{-3}
16.9	2	75.	11.12	$(1.10\pm0.04)\times10^{-3}$	1.08×10^{-3}
16.9	2	77.	11.62	$(0.94\pm0.03)\times10^{-3}$	0.972×10^{-3}
16.9	2	80.	12.39	$(0.84\pm0.04)\times10^{-3}$	0.837×10^{-3}
16.9	2	82.	12.91	$(0.78\pm0.03)\times10^{-3}$	0.775×10^{-3}
16.9	2	85.	13.69	$(0.74\pm0.04)\times10^{-3}$	0.708×10^{-3}
16.9	2	90.	15.00	$(0.68\pm0.05)\times10^{-3}$	0.665×10^{-3}
19.3	3	64.	9.69	$(6.82\pm0.29)\times10^{-4}$	6.80×10^{-4}
19.3	3	69.	11.06	$(4.19\pm0.26)\times10^{-4}$	4.61×10^{-4}
19.3	3	75.	12.78	$(2.70\pm0.20)\times10^{-4}$	3.02×10^{-4}
19.3	3	90.	17.24	$(1.88\pm0.17)\times10^{-4}$	1.78×10^{-4}

21.3	3	66.	11.34	$(2.20 \pm 0.18)\times10^{-4}$	2.24×10^{-4}
21.3	3	70.	12.58	$(1.35 \pm 0.11)\times10^{-4}$	1.62×10^{-4}
21.3	3	75.	14.17	$(1.01 \pm 0.08)\times10^{-4}$	1.11×10^{-4}
21.3	3	87.	18.12	$(0.61 \pm 0.09)\times10^{-4}$	0.645×10^{-4}
12.9	1	72.1	7.80	$9.83\times10^{-3}(^{+25}_{-20})\%$	1.32×10^{-2}
18.2	1	58.8	7.80	2.52×10^{-3} "	1.77×10^{-2}
25.0	1	49.1	7.80	9.78×10^{-4} "	2.30×10^{-4}
11.4	1	90.	9.90	$2.24\times10^{-2}(^{+25}_{-20})$	2.14×10^{-2}
14.2	1	78.4	10.00	5.10×10^{-3} "	4.35×10^{-3}
20.9	1	62.1	10.00	4.84×10^{-4} "	3.75×10^{-4}
28.7	1	52.0	10.00	$1.47\times10^{-4}(^{+30}_{-25})\%$	0.42×10^{-4}
30.7	1	53.7	11.12	4.47×10^{-5} "	1.62×10^{-5}
19.6	1	70.2	11.56	2.82×10^{-4} "	3.62×10^{-4}
16.0	1	81.4	12.01	$1.54\times10^{-3}(^{+25}_{-20})\%$	1.32×10^{-3}
23.8	1	65.2	12.46	$8.41\times10^{-5}(^{+30}_{-25})\%$	7.85×10^{-5}
21.9	1	73.1	13.94	6.90×10^{-5} "	9.61×10^{-5}
18.0	1	86.0	14.50	$3.65\times10^{-4}(^{+25}_{-20})\%$	3.74×10^{-4}
26.6	1	68.1	15.06	$1.46\times10^{-5}(^{+30}_{-25})\%$	1.83×10^{-5}
26.2	1	77.9	18.77	$5.18\times10^{-6}(^{+35}_{-30})\%$	9.60×10^{-6}
21.9	1	90.0	19.65	$5.15\times10^{-5}(^{+30}_{-25})\%$	4.67×10^{-5}
31.8	1	72.8	20.38	$9.20\times10^{-7}(^{+100}_{-50})\%$	1.49×10^{-6}
30.9	1	82.4	24.39	1.10×10^{-6} "	1.01×10^{-6}

and, as an indication, some data for $t < 8\,(\mathrm{GeV}/c)^2$. The experimental cross sections are compared with (13), which is calculated for $B = 3.86 \times 10^5$ μb/sr and $T = .544$ GeV. In Fig. 1 the same points are plotted as a function of t. For semplicity we have reported only a part of the data of table I.

By inspection of the data we can deduce a value of $t, t_{th} \gtrsim 8\,(\mathrm{GeV}/c)^2$, so that, for $t \gtrsim t_{th}$ we note a very good agreement between the theory and the experiments. We can say, therefore, that for $t \gtrsim t_{th}$ we are in asymptotic conditions.

The agreement gets worse for $t \lesssim 8$ (GeV/c) and fails completely for $t \ll t_{th}$. This can occur at fixed energy, by decreasing θ, or alternatively at fixed θ, for instance at $\theta = 90^o$, by decreasing the energy.

The discontinuities, or breaks, in the differential cross section, in particular, discovered by Akerlof et al.[4] and afterwards studied by Allaby et al.[5], are exactly in this region. The above asymptotic considerations cannot, therefore, be applied here. As a final remark, we want to emphasize that the formula (13), in the momentum transfer region defined above, provides the most accurate fit to the p-p elastic scattering data.

I am greatly indebted to Prof. N. Cabibbo for his interest in this work and the many suggestions he made for its improvement.

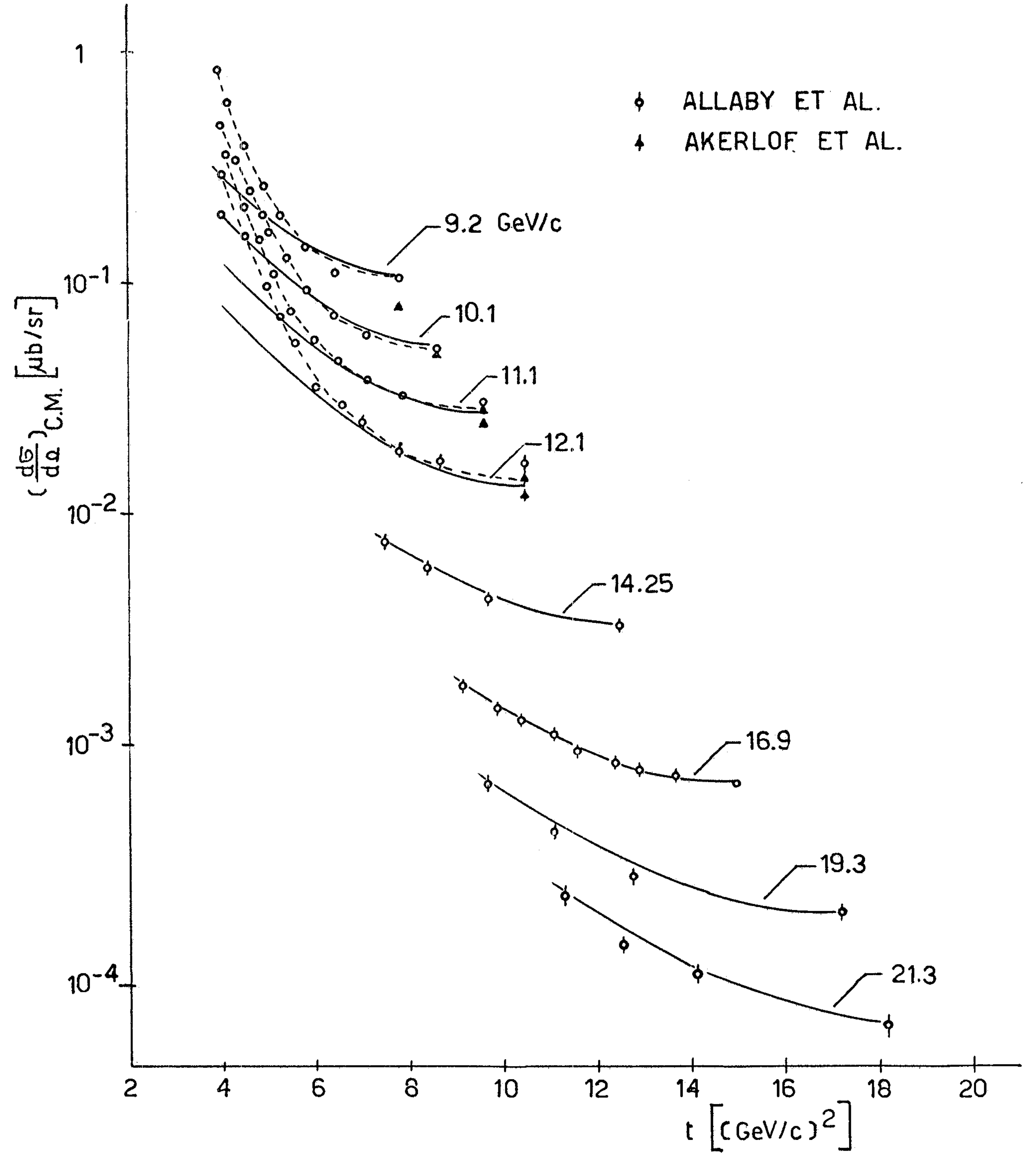

FIG. 1 - Differential cross section ($d\sigma/d\Omega$); in units of $10^{-30}\,cm^2/sr$, as function of the squared four momentum transfer t. Dashed lines connect the experimental points with the same value of p_o. The full lines result from the equation (13).

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(11) - See for more details reference (12) and (13). This assumption does not mean that we put ourselves in the frame work of the usual statistical models. Previous statistical theories(13) do not explain quantitatively the elastic scattering. The $e^{-p_\perp}$ distribution for instance, which has been used previously to fit the large angle data, cannot be applied rigorously, to a two-particle decay of a "fireball", which is quite isotropic. The fact, besides, that experiments(2) have found no Ericson fluctuations(14), does not exclude the existence of the fluctuations, we have considered by deriving the formula (2).

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