COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF - 68/17
28 Marzo 1968

V. Silvestrini: A REVIEW OF HIGH ENERGY WORK IN QUANTUM ELECTRODYNAMICS. -

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

Laboratori Nazionali di Frascati del CNEN
Servizio Documentazione

LNF-68/17

Nota Interna: n. 394
28 Marzo 1968

V. Silvestrini: "A REVIEW OF HIGH ENERGY WORK IN QUANTUM E-LECTRODYNAMICS".

(Invited talk at the 3rd Rencontre de Moriond sur les interaction electro magnetic, Lac de Tigres, 9-19 Mars 1968).

INTRODUCTION -

Considerable work has been done during last years in order to test the validity of quantum electrodynamics (QED) at high energy: we know that up to now there is no serious indication of a possible break-down, although very high momentum transfers have been reached in some experiments.

At present we have in Frascati an experiment running at the synchrotron, and many in preparation with Adone, whose aim is to further test the validity of QED.

It is important to understand, in this respect, in which directions and to what extent we can still expect a breakdown of QED, consistent with the available experimental and theoretical information.

The present review is intended as a contribution to this understanding. The theoretical part is extremely simplified, and of course is not addressed to theoreticians.

I - VALIDITY OF QED -

We say that a given set of spin 1/2 fermions (μ, e) is correctly described by QED if:

1) For the free particles the Dirac equation holds

$$(\not{p} + m_o)\psi = 0 \qquad (1)$$

(m_o is a c number).

2) The interaction with an electromagnetic field can be introduced - as in the classical case - through the substitution

$$p \rightarrow p - eA \qquad (2)$$

(minimal interaction) into equation (1).

It is well known that if we want A to represent the external field (and not to include the interaction of each particle with its own field) the first effect of the interaction is to turn the constant m_o into the physical mass m of the particle (mass renormalization).

Equation (1) then turns into:

$$(\not{p} + m - e\not{A})\psi = 0 \qquad (3)$$

or

$$(\not{p} + m)\psi = e\gamma_\mu A^\mu \psi$$

In the absence of an external field, this equation becomes

$$(\not{p} + m)\psi = 0 \qquad (4)$$

(m = physical mass).

We will hereafter refere to this equation, rather than to (1), as to the free equation.

Equation (3) can be derived from the Lagrangian density

$$L = \bar{\psi}(\not{p} + m)\psi + e\bar{\psi}\gamma_\mu \psi A^\mu + F_{\mu\nu}F^{\mu\nu} \qquad (5)$$

$$F_{\mu\nu} = \frac{\partial A_\mu}{\partial x^\nu} - \frac{\partial A_\nu}{\partial x^\mu}$$

or

(6) $$L = L_1 + J_\mu A^\mu + L_{em}$$

In analogy with the classical case, the quantity $J_\mu = \bar{\psi} \gamma_\mu \psi$ is interpreted as the current generated by the particles. It is easily verified that J_μ is conserved

(7) $$\frac{\partial J_\mu}{\partial x_\mu} = 0$$

L is in addition gauge invariant: it is a question of simple algebra to verify that the substitution

$$A_\mu \rightarrow A_\mu - \frac{\partial \phi}{\partial x^\mu}$$

$$\psi \rightarrow \psi e^{-ie\phi}$$

(ϕ is a function whatever of the coordinates x, y, z) does not affect (5). Gauge invariance corresponds to the classical fact that the physical meaning is attached to the fields $\vec{E}$, $\vec{H}$, and not to the potential A.

The solution of equation (3) is usually performed through perturbation, making profit of the fact that the coupling constant e is of the order of 10^{-2}.

In terms of Feynman graphs equation (3) implies that:

a) each fermion propagator S_F is given by

(8) $$S_F = \frac{\not{p} - m}{p^2 + m^2}$$

b) each vertex function Γ_μ is given by

(9) $$\Gamma_\mu = \gamma_\mu$$

We recall also that in the pure electromagnetic scheme (absence of strong interactions) a photon propagator is given by $S_\gamma = 1/q^2$ (q four-momentum of the photon).

2 - MODIFIED QED THEORIES -

We call breakdown of QED any modification of the Lagrangian density (6)

(10) $$L \longrightarrow L'$$

L' must of course satisfy some general requirement: it must be Lorentz invariant, gauge invariant, C-invariant, and the most trivial limit cases must be preserved.

The requirement that L' be gauge invariant implies that the vector

(11) $$K_\mu = \frac{\partial L'}{\partial A^\mu}$$

is conserved

$$\frac{\partial K_\mu}{\partial x_\mu} = 0 \qquad \text{(Noether's theorem}^{(1)}\text{).}$$

As a consequence of this we can write, in analogy with (6):

(12) $$L' = L_1' + J_\mu' A^\mu + L_{em}'$$

with L_1' free lagrangian, L_{em}' Lagrangian of the electromagnetic field (also not dependent explicitly on A_μ) and J_μ' (satisfying the equation

$$J_\mu' = K_\mu - A^\nu \frac{\partial J_\nu'}{\partial A^\mu},$$

as one easily checks by deriving (12) with respect to A^μ) a conserved vector.

The fact that J_μ' is conserved ($\partial J_\mu' / \partial x_\mu = 0$) can be easily shown if J_μ' contains only a linear dependence (or no dependence) on A_μ, making use of (11) and of the Lorentz condition $\partial A_\mu / \partial x_\mu = 0$. In the general case, the demostration requires some more mathematics. J_μ' is of course interpreted as the current generated by the particle.

The general requirements imply severe restrictions on the form of L_1', J_μ', L_{em}'.

Since the calculations of ED processes are performed using the Feynman graphs method, a closer connection with the expected experimental results is obtained if one thinks in terms of vertexes and propagators rather than of Lagrangian. For this reason models of modified QED theories are usually built in terms of modifications of vertexes and/or propagators. In this case it is not easy in general to verify to all orders if the parent Lagrangian satisfies to the general requirements (in particular, gauge invariance and conservation of current). Note that the propagator is strictly related with L_1', and the vertex function with the current J'_μ: if $J'_\mu = \bar{\psi} 0_\mu \psi$, the vertex function Γ_μ is given by $\Gamma_\mu = 0_\mu(A_\mu = 0)$. A very important identity, connecting propagators and vertex functions, has been established by Ward and Takahashi(2): This identity must be satisfied in order that the current be conserved, and reads:

$$q_\mu \Gamma^\mu = S_F^{-1}(p') - S_F^{-1}(p) \tag{13}$$

where: $\Gamma_\mu(p, p', q)$ = vertex function;
S_F = fermion propagator;
p = initial momentum of the fermion;
p' = final momentum of the fermion;
q = $p' - p$.

We see that a propagator modification always requires a modification of the vertex function, in order that current be conserved. However, (13) does not require the vice-versa: a vertex modification $\gamma_\mu \rightarrow \Gamma_\mu$ not affecting the product $q_\mu \Gamma^\mu (q_\mu \Gamma^\mu = q_\mu \gamma^\mu)$ does not modify the propagator. A modification of this kind is called an intrinsic vertex modification, in contrast with the modifications induced by propagator modifications.

The validity of (13) is a condition which is necessary, but not sufficient for the conservation of current: this is easily understood considering that Γ_μ does not take into account a possible part of J'_μ depending on A_μ, and giving rise to terms described by multiphoton vertexes. To guaranty the conservation of current in the general case also the more complicated Chang-Mani(3) identities must be satisfied: they administrate in fact the multiphoton vertexes. The importance of the multiphoton vertexes is in the fact that they are induced (in order that current be conserved) by propagator modifications.

The restrictions imposed by the Ward-Takahashi and Chang-Mani identities to possible modified QED theories have been systematically treated by N. Kroll(4).

Here we will give some simple examples, in order to introduce the general Kroll's conclusions.

3 - PROPAGATOR MODIFICATIONS -

A modification $L_1 \rightarrow L_1'$ involves a propagator modification, and vice-versa. The most general form of L_1 consistent with Lorentz invariance originates the propagator(5)

$$S_F^{-1}(\not{p}, p^2) = A\,(p^2)\not{p} + B\,(p^2)m.$$

We require that $S_F^{-1}(\not{p}, m^2)\,\psi = 0$ represents the equation for the free particle. In order that this reduces to the Dirac equation it is necessary that:

$$\left.\begin{array}{l} A(p^2) \rightarrow 1 \\ B(p^2) \rightarrow 1 \end{array}\right\} \quad \text{as } p^2 \rightarrow m^2$$

Example: There exist two electrons, with masses m and M.

We can give to the propagators S_F the form(x)

(15) $$S_F^{-1} = \frac{(\not p + M)(\not p + m)}{M + m}$$

working out the product:

(16) $$S_F^{-1} = (\not p + \frac{p^2 + Mm}{M + m})$$

so

$$\left.\begin{array}{l} A(p^2) = 1 \\ B(p^2) = \frac{1}{m}\,\frac{p^2 + Mm}{M + m} \longrightarrow 1 \end{array}\right\} \text{as } p^2 \longrightarrow m^2$$

Now suppose we introduce the EM interaction through the substitution (which authomathically guaranties gauge invariance)

(2) $$p \longrightarrow p - eA$$

into the Lagrangian density

$$L_1' = \bar{\psi}\,\frac{(\not p + m)(\not p + M)}{m + M}\,\psi\,.$$

It is easily seen that the equation of the motion comes out to be:

$$(\not p + \frac{p^2 + Mm}{M+m})\,\psi = eA^{\mu}(\gamma_{\mu} + \frac{2p_{\mu}}{m+M} - e\,\frac{A_{\mu}}{m+M})\psi + \frac{e}{4}\,\sigma_{\mu\nu}\,F^{\mu\nu}\psi$$

The conserved current is:

(x) - This form for the propagator is not consistent with the positive definitiones of the metric. This difficulty can be overcome, within the two masses model by giving to S_F^{-1} a some what more complicated form. Form demonstration purpose, we prefere here to deal with a simple formula like (15), forgetting the above difficulty.

(17) $$J' = e\overline{\psi}\left[\gamma_\mu + \frac{p_\mu + p'_\mu}{m+M} - e\frac{A_\mu}{m+M} - \frac{1}{4}\frac{\sigma_{\mu\nu}q^\nu}{m+M}\right]\psi$$

So we see that a propagator modification has induced a modification of the current: the vertex function is now:

$$\Gamma_\mu = \gamma_\mu + \frac{1}{(m+M)4}\ \sigma_{\mu\nu}q^\nu + \frac{p_\mu + p'_\mu}{m+M}$$

and in addition a two-photon interaction $\left[e^2(A_\mu A^\mu)/(M+m)\right]$ has been introduced. The general statement is that when the inverse propagator S_F^{-1} contains p^n, an n-photon vertex is required to restore current conservation(4). Note that $\sigma_{\mu\nu}q^\nu$ is an intrinsic part since $q^\mu\sigma_{\mu\nu}q^\nu$ is identically zero. Remembering our comments on (13), this means that we could cancel the magnetic-moment part $(e/M+m)\sigma_{\mu\nu}q^\nu$ just by the ad-hoc introduction of a term $(-e/M+m)\sigma_{\mu\nu}q^\nu$ in the vertex function, since this term can be introduced without destroying the self-consistency of the Lagrangian density. This sounds like a combersone procedure. However, it has been shown by N. Kroll that we can define a new kind of minimality (in place of (2)), such that the modifications induced by the propagator into the current do not contain any intrinsic part. The striking result obtained by Kroll is that the effect of a propagator modification cancels with the effect of the induced multiphoton vertexes + induced vertex modifications (not with possible intrinsic vertex modifications) apart from corrections which are outside the possibility of present experimental checking.

So that from the point of view of experimentalists only vertex modifications are effective.

4 - VERTEX MODIFICATIONS -

Let us now see which is the most general form for the vertex function Γ_μ.

The vectors we can use to build up Γ_μ are(6, 7)

(18) $$\gamma_\mu \qquad \sigma_{\mu\nu}q^\nu \qquad q_\mu$$

multiplied in all possible combinations by $\not{p}^i\ \not{p}'^K$. The coefficients will be functions of p^2, p'^2. Since $\not{p}^2 = \not{p}^0 p^2$, we can restrict to $i, K = 0, 1$. Actually it is more convenient to multiply by $(\not{p} + m)^i\ (\not{p}' + m)^K$ (instead of $\not{p}^i\ \not{p}'^K$): in this way the conditions imposed by the free Dirac equation are self evident.

So we can write:

$$(19)\quad \Gamma_\mu = \sum_{i,K}^{0,1} (\not p' + m)^i \left[G_1^{iK}\gamma_\mu + G_2^{iK}\sigma_{\mu\nu}q^\nu + G_3^{iK}q_\mu \right] (\not p + m)^K$$

The G's are functions of p^2, p'^2 and q^2. Invariance under charge conjugation requires

$$G_{1,2}^{iK} = G_{1,2}^{Ki}; \qquad G_3^{iK} = - G_3^{Ki} \qquad \text{so}$$

that (19) involves nine form factors. Note that $q^\mu \Gamma_\mu \neq q^\mu\gamma_\mu$ unless the form factors G_3^{iK} are identically zero. According to (13), $G_3^{iK} \neq 0$ require a modification of the propagator, and multiphoton vertexes must be introduced. However, it is still true that the overall effect is negligible from an experimental point of view.

We can thus put $G_3^{iK} = 0$, and remain with the six form factors which describe all possible intrinsic vertex modifications. Note that if the final (initial) fermion is on the mass-shell, then we can exclude the case i = 1 (K = 1), because $(\not p'+m)\Psi = 0$ $[(\not p+m)\Psi = 0]$: so that we have only 4 form factors. Finally, if both fermions are on the mass-shell, only the form factors G_1^{oo} and G_2^{oo} appear in Γ_μ, and Γ_μ assumes the form which we all know from the example of the proton in e-p scattering(8):

$$(20)\quad \begin{aligned} \Gamma_\mu &= \left[G_1^{oo}(q^2)\gamma_\mu + G_2^{oo}(q^2)\sigma_{\mu\nu}q^\nu \right] \\ G_1^{oo} &= e\,F_1(q^2) \qquad G_2^{oo} = e\,\frac{K}{m}F_2(q^2) \end{aligned}$$

The only possible static properties of spin 1/2 particles are thus charge and magnetic moment (K anomalous magnetic moment).

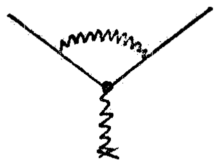
We will now review the experimental results concerning checks of the validity of QED for electrons and muons.

We will separate the following classes of experiments:

- Experiments concerning the static properties;
- High energy experiments;
- Tests of particular models.

5 - MEASUREMENTS OF STATIC PROPERTIES. (Magnetic moment) -

In the case of exact validity of QED, although no magnetic moment term appears in the current ($\Gamma_\mu = \gamma_\mu$), the giromagnetic factor g of electrons and muons is not expected to be exactly 2: $g - 2 \neq 0$. This is due to higher order corrections, from graphs like:



However, the value of g has been computed[9] with a precision of the order of 10^{-8}. Precision experiments of (g-2) have also been performed, providing g with an accuracy 10^{-8} for electrons, and 10^{-6} for muons.

Experiments[10] and theory give results in good agreement.

	$(\frac{g-2}{2})_{exp}$	$(\frac{g-2}{2})_{th}$
e^-	$(11596.22 \pm .27)\times10^{-7}$	$(11596.40 \pm .04)\times10^{-7}$
μ^-	$(1165 \pm 3)\times10^{-6}$	11655.2×10^{-7}
μ^+	$(1162 \pm 5)\times10^{-6}$	11655.2×10^{-7}

This means that $(K/M)F_2(0) \lesssim 10^{-6}$ ($\lesssim 10^{-8}$ for electrons). Remembering the physical interpretation of the form factors, $F_2(0) = 1$ is generally assumed. This corresponds to the fact that every particle is point-like, when we look at it from very far.

The g-2 experiments then tell us that (K/M) is consistent with zero within the above very good accuracy: within the same limits, we can set $G_2^{oo} = 0$ in (19), so that (19) involves five form factors (remember that we have already put $G_3^{iK} = 0$).

When both fermions are on the mass shell $[(\not p' + m)\psi = 0;\ (\not p + m)\psi = 0]$, G_1^{o1} and G_1^{11} disappear, and only G_1^{oo} is present in Γ_μ; when one (or both) is off the mass-shell, G_1^{o1} and G_2^{o1} (G_1^{o1}, G_1^{11}, G_2^{o1} and G_2^{11}) appear. In principle, there could be cancellations, at least in some kinematical regions, among the different G's. In addition the effect of multiphoton vertexes should be kept into account.

Leaving open the possibility of more sophisticated analyses for when (and if) a discrepancy between experiment and QED will be found, we will analyze the experiments from here on in terms of a vertex function $\Gamma_\mu = \gamma_\mu F(q^2, p^2, p'^2)$ involving only one form factor: we will see which are the limits on the quantity ε defined as

$$\varepsilon(q^2, p^2, p'^2) = 1 - F^2(q^2, p'^2, p^2) \tag{21}$$

One can now try to use further the results of the (g-2) experiments: Since the (perturbative) calculation of (g-2) involves propagators and vertex functions, one asks to what extent the agreement with experiment can be used to set a limit on ε. An answer to this question, however, is extremely model dependent[11], since g-2 involves only integrals over the momentum transfers, and very different functions may have the same integral.

To make an analysis of this kind, it is in general assumed <u>without any justification</u>, $F^2(q^2, p^2, p'^2) = [1 + (q^2/\Lambda^2)]^{-2}$. The presence of the term q^2/Λ^2 modifies the value of $(g-2)/2$ th of an amount(11):

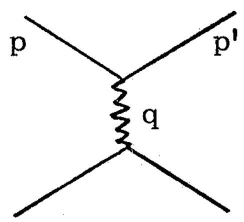
(22) $$\Delta \left(\frac{g-2}{2}\right)_{th} = - 1.6 \times 10^{-3} \left(\frac{m}{\Lambda}\right)^2$$

The experiment on the (g-2) of the muon then sets a limit $\Lambda \gtrsim 1.5$ GeV (two standard deviations).

6 - HIGH ENERGY DYNAMICAL TESTS -

We will classify the dynamical tests according to which graph is relevant in the first order description of the phenomenon.

<u>Both fermions on the mass shell</u>: $p^2 = p'^2 = m^2$; $q^2 \neq 0$

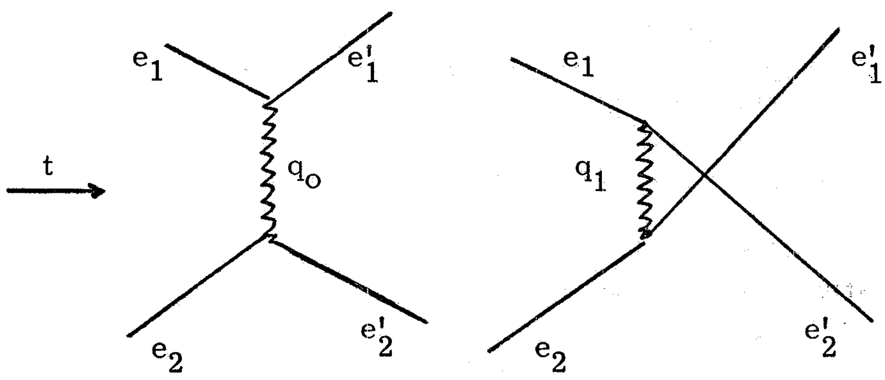


(e.g. scattering of electrons or muons; $e^+ + e^-$ ($\mu^+ + \mu^-$) annihilation).

<u>Let us first consider electrons:</u> The experiments performed up to now are e - e scattering ($q^2 < 0$, space like photons) and fine structure of positronium ($q^2 > 0$). The scattering of electrons on nucleons is of course not useful to test QED, since we already know that there are form factors to describe the nucleon-photon vertex.

The e-e scattering experiment(12) has been performed at Stanford by a Princeton-Stanford group, using a colliding beam machine. The energy of the electrons was 300 MeV. An angular distribution has been measured (<u>not</u> the absolute cross section).

The results are shown in fig. 1, and compared with theory. Since the process is described at lowest order by the two graphs



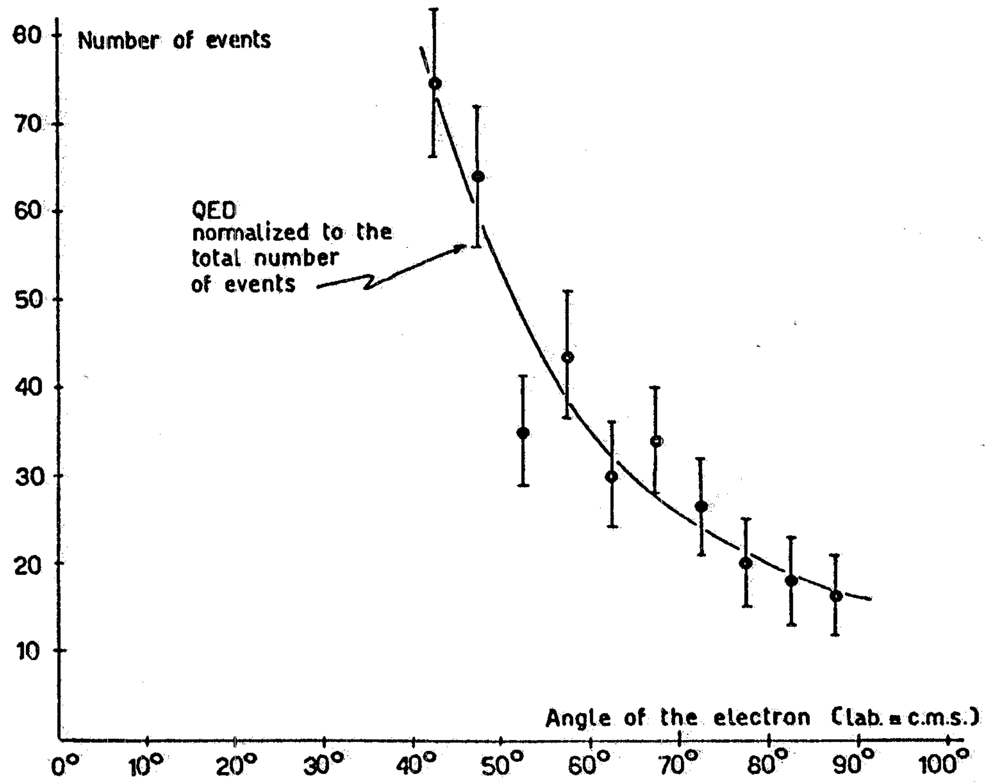


FIG. 1

the angle θ of emission of the electron does not determine uniquely the four-momentum square q^2 of the photon. In the two graphs, q^2 is given respectively by

(23) $$q_o^2 = -4E^2 \sin^2 \theta/2 \qquad q_1^2 = -4E^2 \cos^2 \theta/2$$

E: energy of the incident electrons.

According to QED, the cross section for the process is in first order approximation(13) (radiative corrections are kept into account separately):

(24) $$\frac{d\sigma}{d\Omega} = \frac{r_o^2}{8}\left(\frac{m}{E}\right)^2 \left[\frac{S^4+q_o^4}{q_1^4} + \frac{2S^4}{q_1^2\, q_o^2} + \frac{S^4+q_1^4}{q_o^2}\right]$$

$$S^2 = 4E^2$$

Now we could have a vertex modification ($\gamma_\mu \rightarrow \gamma_\mu F(q^2)$), and a modification of the photon propagator(x)

(x) - Also the function $M(q^2)$ must satisfy general requirements in order that fundamental laws of physics hold (see ref. 4). These requirements or in general not satisfied by simple models currently used in the analysis of experiments.

$$\left(\frac{1}{q^2} \to \frac{M(q^2)}{q^2}\right).$$

In this case the cross section would become:

$$\frac{d\sigma}{d\Omega} = \frac{r_o^2}{8}\left(\frac{m}{E}\right)^2 \left[\frac{S^4 + q_o^2}{q_1^4} G^2(q_1^2) + \frac{2\,S^4}{q_1^2 q_o^2} G(q_1^2)G(q_o^2) + + \frac{S^4 + q_1^2}{q_o^4} G^2(q_o^2) \right] \tag{25}$$

$$G(q^2) = F^2(q^2)M(q^2)$$

(the square of $F(q^2)$ appears, since the photon attaches to an electron on both sides).

The first point to be noted is that the effect of a form factor of the electron is not distinguishable from a modification of the photon propagator. By the way, we recall that modifications of the photon propagator are expected, due to the coupling of the photon with the vector bosons (ρ, ω, φ): these modifications are particularly important in the time-like region, as q^2 approaches the masses of the bosons.

The second point is that we cannot put a limit on $1 - G(q^2)$ unless we make a specific model.

The authors put $G(q^2) = (1 - q^2/K^2)^{-1}$ and then the experimental results say that $K > .76$ GeV with 95% confidence level.

One could have put equally well $G(q^2) = (1 - q^2/K^2)^{-2}$ [for instance $M(q^2) = 1$; $F(q^2) = (1 - q^2/K^2)^{-1}$], giving then the limit $K > 1.5$ GeV. Personally, we would prefere to say, as a comment to Fig. 1: the slope of the cross section $d\sigma/d\Omega(q_o^2 q_1^2)$ has been measured; the values of q^2 involved are between $(\sim .2\ \text{GeV})^2$ and $(\sim .5\ \text{GeV})^2$; the agreement with QED is good enough to exclude (95% confidence level) a smooth variation of $G(q^2)$, from one edge to the other of the above region, of a factor bigger than ~ 2.

In conclusion, for $q^2 < 0$, only one relative measurement is available, exploring the region between $\sim .2\ \text{GeV} < |q| < \sim .5\ \text{GeV}$.

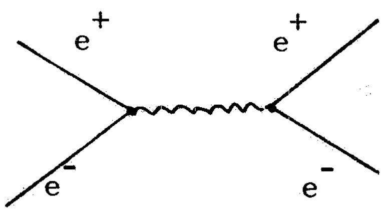
As far as we know, $F^2(q^2)$ could go for instance from 1 to .5 when $|q|$ goes from 0 to 200 MeV, and then remain fairly constant up to 500 MeV, without contradicting any experiment.

An absolute measurement of the cross section, involving momentum transfers in whatever region, would thus provide very interesting information.

Also new measurements at different energies would complete the available information, allowing to extract $G(q^2)$ from (26) in a model independent way.

A new measurement, with $E \simeq 550$ MeV, is being performed by the same Princeton-Stanford Group.

Let us now consider the time-like region ($q^2 > 0$): there is only one-though very precise-measurement, coming from the fine structure of positronium. In the fundamental S state of positronium, the spins of the particles can be parallel (3S state) on antiparallel (1S state). There are many reasons[14, 15] why the two states have a different binding energy: one of the must important, contributing for $\sim 3/7$ of the total energy splitting, is that the 3S state lives part of the time as a photon



A form factor $F(q^2)$ in the vertex function ($q^2 = (1 \text{ MeV})^2$), and a photon propagator modification

$$\frac{1}{q^2} \rightarrow \frac{M(q^2)}{q^2} ,$$

would affect the energy splitting due to this phenomenon of a factor $F^2(q^2)M(q^2)$.

Experimentally, the frequency of the line is

$$\Delta W = 203.389 \pm 27 \text{ mc} \qquad \text{Ref. (16)}$$

$$\Delta W = 203.403 \pm 12 \text{ mc} \qquad \text{Ref. (17)}$$

The theoretical value is[15]

$$\Delta W_{th} = 203.373 \quad \text{mc} \qquad \text{using } \alpha_{DL}$$

$$\Delta W_{th} = 203.381 \quad \text{mc} \qquad \text{using } \alpha_j$$

according to which value for α is used[18] (α_{DL} is obtained from the deuteron fine structure and the Lamb shift; α_j from the Josephson effect).

This results in a value of $F^2(q^2)M(q^2)$:

$$M(q^2)F^2(q^2) = (1 + 2.5 \times 10^{-4}) \pm 1.2 \times 10^{-4} \qquad \text{at } q^2 = 1 \text{ MeV}$$

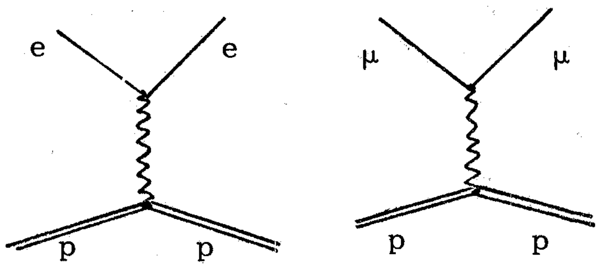
In conclusion, all what we know about the electron-photon vertex (the electron being an the mass shell) is:

$$M(q^2)F^2(q^2) = 1+(2.5)\times 10^{-4} \pm (1.2)\times 10^{-4} \quad |q| = (1\text{ MeV}) \quad (q^2 > 0)$$

$M(q^2)F^2(q^2)$ varies of less than a factor ~ 2 for $|q|$ between .2 and .5 GeV ($q^2 < 0$).

Now we consider muons. -

Here we have no measurement of a possible form factor in the muon-photon vertex, but only a comparison between muons and electrons. For $q^2 < 0$ (space-like photons), the scattering of muons on protons is compared with the scattering of electrons.



If the muon and/or the electron have a structure, the ratio of the cross sections would be given by

(26) $$\frac{d\sigma_\mu}{d\sigma_e} = \frac{F_\mu^2(q^2)}{F_e^2(q^2)}$$

the effect of the proton structure and of a possible photon propagator modification being cancelled in the ratio.

There is one experiment(19) covering a range of q^2

$$.62\text{ BeV} < |q| < 1.1\text{ BeV}.$$

The results for the ratio (26) are shown in fig. 2 (Black points).

A second experiment(20) covers the region $.45\text{ GeV} < |q| < .85\text{ GeV}$ with larger errors (open points o, in fig. 2).

Note that:

1) The cross section $d\sigma_\mu/d\Omega$ varies of ~ 2 orders of magnitude in the range of $|q|$ involved in fig. 2.

2) The data on electron scattering necessary for the comparison come from different experiments.

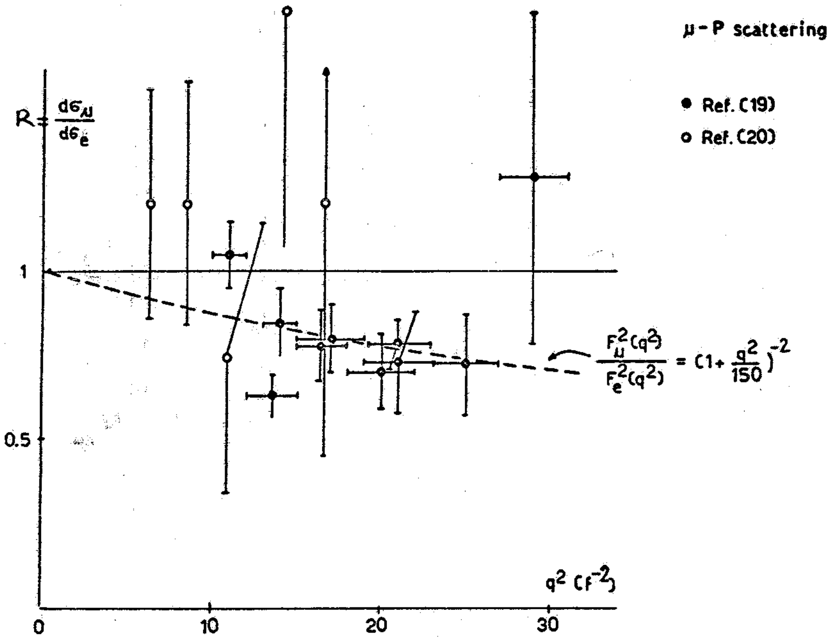


FIG. 2

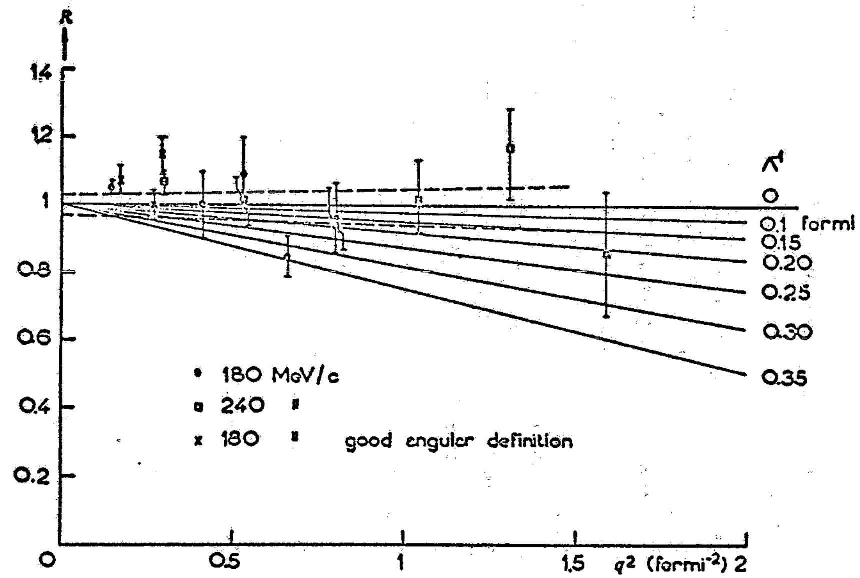


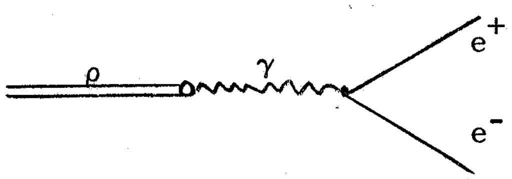
FIG. 3 - Ratio R of observed to calculated scattering rates as a function of the square of momentum transfer. The dashed lines on either side of the axis R = 1 correspond to the estimated standard systematic error. The error bars represent statistical errors. Lines labelled with values of Λ^{-1} replace the line R = 1 if the calculated rates are based on a structure of the μ vertex described by this cut-off.

3) An estimated normalization uncertainty (4-10%) is included in the error bars of ref. (19).

In addition to the above μ-p scattering experiments, there are experiments of μ-scattering on nuclea. These results are summarized in Fig. 3[21] and 4[22] (scattering on carbon) and Fig. 5 (scattering on nuclear emulsions)[23, 24, 25]. Taking into account the possibility of systematic effects, all the above results are usually considered consistent with 1, allowing to conclude that for $0 < -q^2 < (1.1\ \text{GeV})^2$ a possible form factor squared in the muon vertex does not differ of more than $\sim$ 20% from the form factor squared of the electron.

For $q^2 > 0$, the available information comes from a comparison of the decay widths

$$\Gamma(\rho \to e^+ + e^-) \qquad \Gamma(\rho \to \mu^+ + \mu^-)$$



The available data are:

$$A = \frac{\Gamma(\rho \to \mu^+ + \mu^-)}{\Gamma(\rho \to \pi^+ + \pi^-)} = \begin{cases} (3.3^{+1.6}_{-.7})\times 10^{-5} & \text{ref. (26)} \\ (9.7 \pm 3.2)\times 10^{-5} & \text{ref. (27)} \\ (5.1 \pm 1.2)\times 10^{-5} & \text{ref. (28)} \\ (7.4 \pm 2)\times 10^{-5} & \text{ref. (29)} \end{cases}$$

$$B = \frac{\Gamma(\rho \to e^+ + e^-)}{\Gamma(\rho \to \pi^+ + \pi^-)} = \begin{cases} (6.5 \pm 1.4)\times 10^{-5} & \text{ref. (30)} \\ (6.5^{+1.1}_{-.5})\times 10^{-5} & \text{ref. (31)} \\ (3.9 \pm 1.2)\times 10^{-5} & \text{ref. (32)} \\ (4.9 \pm .8)\times 10^{-5} & \text{ref. (33)} \\ (6.2 \pm 1.1)\times 10^{-5} & \text{ref. (34)} \end{cases}$$

The average value of A is $\bar{A} = 5.75 \pm .90$; the average value of B is $\bar{B} = 5.60 \pm .45$. So that

$$R = \frac{\Gamma(\rho \to e^+ + e^-)}{\Gamma(\rho \to \mu^+ + \mu^-)} = \frac{\bar{B}}{\bar{A}} = .97 \pm .17$$

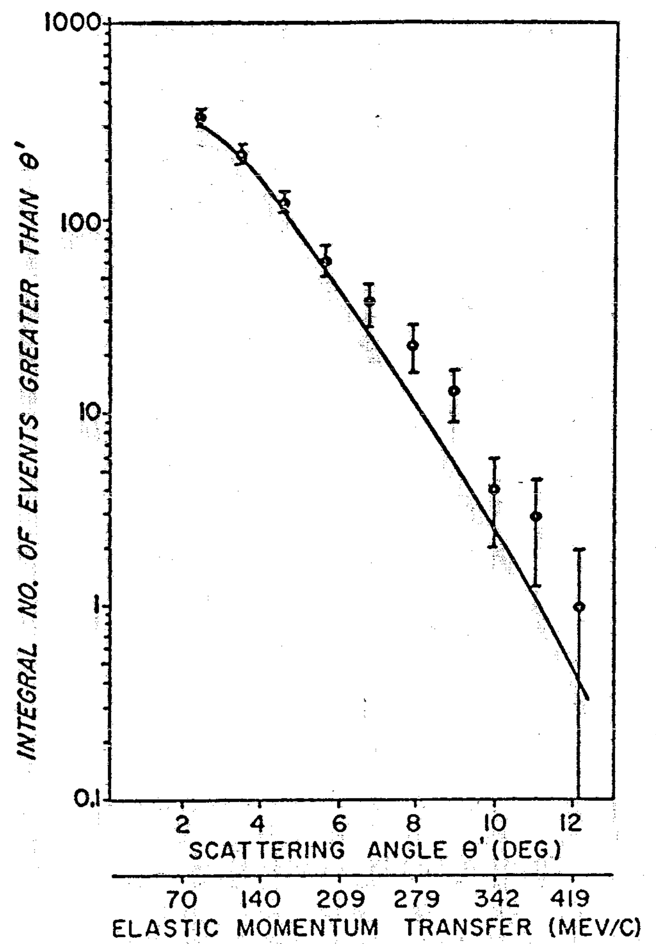


FIG. 4 - Curves showing the theoretical and experimental integral angular distributions of 2 BeV/c muons scattered in 27.0 g/cm^2 of carbon. Target-out subtractions have be- en made to the experimental points. The errors shown a- re statistical only. The theoretical curve was obtained by integrating the cross-section over all angle greater than θ'.

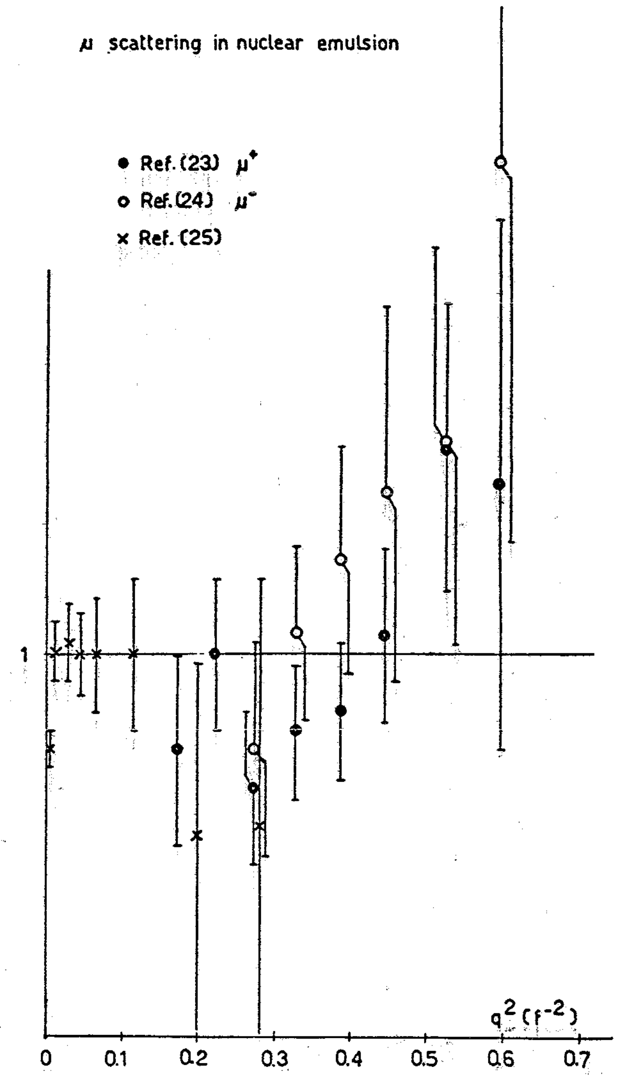


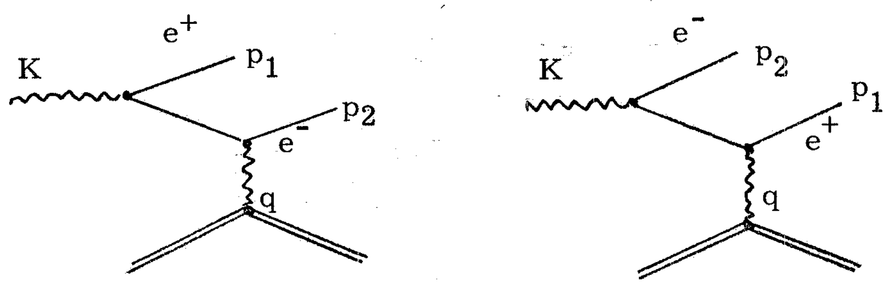
FIG. 5

QED forsees $R \approx 1$ (the phase space correction is of the order of the per cent).

So that for $q^2 \approx (.75\ \mathrm{GeV})^2$, $F_\mu^2(q^2)$ differs for less than 17% (1 Standard deviation) from $F_e^2(q^2)$.

One fermion off the mass-shell; photon off the mass-shell.

We first consider lepton pair production



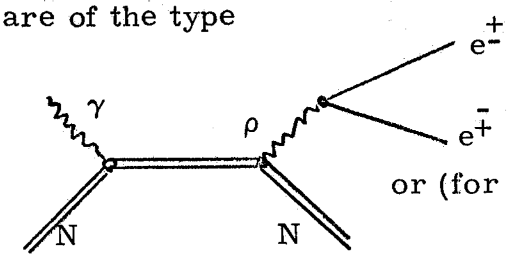
The important parameters in the description are:

p' : four-momentum of the virtual lepton. One in fact looks for the possible presence of an off-mass-shell form factor in the vertex. In the above graphs $p'^2 < 0$ (space-like leptons). $p'^2 = (K-p_{1,2})^2 \simeq -2|K|\cdot|p_{1,2}|(1-\cos\theta_{1,2})$. So that p'^2 is uniquely determined by the apparatus only for symmetric pairs. In this case $p'^2 \simeq -K^2\theta^2/2$.

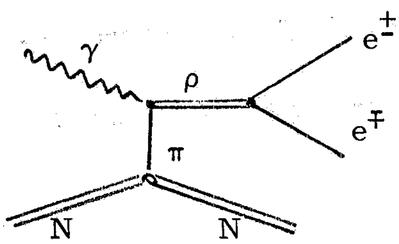
q^2 : four momentum of the virtual photon. For symmetric pairs $q^2 \simeq \simeq -K^2\theta^4/4$(35). Note that the photon attaches on the nuclear target so that the knowledge of the form factor of the target nucleus is important to calibrate-out the E. M. part. All the experiments have been performed up to now on nuclea (C, Al). One usually tries to keep q^2 as low as possible, in order to avoid the possible contribution of inelastic processes (with excitation of the nucleus) which are very little known(36), and whose integral contribution is likely to increase rather drastically with q^2.

$M^2 = (p_1+p_2)^2$, invariant mass of the two-leptons system; for symmetric pairs $M^2 \simeq K^2\theta^2 \simeq 2p'^2$.

The cross section coming from the shown graphs is usually referred to as to the Bethe-Heitler cross section (σ_{BH}). However, there are other graphs contributing to lepton pair production and involving strong interactions. These graphs (usually referred to as to the Compton graphs) are of the type



or (for instance):



Another advantage of the symmetric situation is that the interference term of the Bethe-Heitler and Compton graphs cancels, so that the Compton contribution reduces to of the order of the per cent;

This is not true as M approaches the mass of a vector boson: in this cases the Compton contribution can even dominate over the B. H. process(18, 30, 35).

The available results are summarized in Table I. Only published results are quoted: a summary including also unpublished results can be found for instance in ref. (18). The results of ref. (35) are also not included, since it has been recognized that they are affected by large systematic errors.

Under columns 6, 7, 8, 9 corrections are quoted, as evaluated by each author: correction in parentheses means evaluated but not applied to the results. If no figure is quoted, then the corresponding correction has not been evaluated by the authors (or it has been implicitely considered as negligible). Apart from the measurement of ref. (37), all the experiments have been performed in a symmetric (or nearly symmetric) situation, so that if a form factor $F(p'^2, q^2)$ affects the lepton-photon vertex, F^2 can be factorized from the cross section, R can thus be considered as a measurement of F^2 at the corresponding values of p'^2 and q^2.

The results on electron pairs are plotted against $|p'|$ in Fig. 6, those on muon pairs in Fig. 7.

The electron results are well consistent with no slope as a function of $|p'|$, although the average normalization is $\sim .95$. As for the muons, the Frascati point agrees with QED; while the results of the CEA experiments are not consistent with zero slope [apart from an overal normalization effect of (10 - 15)%] : the contribution of the ρ diagrams has been evaluated by the authors(42) using the results of measurements performed in different kinematical situations(26). It is seen what these contributions are important only for $|p'| \gtrsim 400$ MeV (dashed line). Below this energy, the data are well fitted by a curve of the type (full-line):

$$R = A \left[1 - \frac{p'^{\,2}}{\Lambda^2} \right]$$

$$A = 1.34 \pm .14$$

$$\Lambda^2 = .8 \pm .24 \ \mathrm{GeV}^2$$

The errors are essentially an estimate of systematic effect (statistical errors would be much smaller).

Recently, the hypothesis has been made by one of the authors(18) that the overal effect may be instrumental.

TABLE I

Experiment	Lepton	|p'| (MeV)	|q| (MeV)	|M| (MeV)	Inelastic	Radiative	Compton	Systematic errors	Target	$R = \frac{\sigma_{exp}}{\sigma_{B.H.}}$
Richter '58 (Ref. 37)	e	~115							H_2	.96±.14
Frascati '62 (Ref. 38)	μ	135-185	~20	240-290	3%	(< 1.5%)		(< 2%)	C	1.00±.05
CEA '64 (Ref. 39)	μ	230	~20	320	~0			1%	C	1.09±.13
		275	↓	385	↓			↓		1.22±.07
		315		445						1.20±.06
		355		500						1.14±.06
		395		550						.96±.08
		435		610						1.23±.12
		475		665						1.07±.15
		520		725						2.33±.60
		560	~120	785	8%			10%		1.60±.52
Cornell '67 (Ref. 40)	e	33	~5	47		3-4%	~0	(±5%)	C	.87±.03
		43	↓	61				absolute		.94±.03
		50		72				±1.4%		.94±.05
		63		90				for com-		.98±.04
		89		126				parison		.96±.03
		100		142				among		.96±.04
		109		154				the		1.00±.04
		128		182				points		1.01±.05
		142		202						1.10±.06
		170		241						.99±.03
		180	~25	254						1.00±.08
DESY-Columbia '67 (Ref. 41)	e	114	~6	162		3±1%		negligible	C	.91±.04
		144	~9	204		↓		for com-		1.01±.05
		155	~8	220				parison		.98±.03
		172	~13	244				among		.96±.03
		179	~9	254				different		.92±.03
		195	~12	276				points		.95±.05
		200	~17	284						.89±.06
		221	~11	313						.93±.02
		226	~14	320						.92±.045
		234	~17	331						.85±.06
		271	~20	384						.92±.045
		272	~23	385						.97±.055
		278	~17	394						1.04±.045
		314	~27	445						.85±.045
		333	~25	472						.97±.045
		388	~33	548						.92±.05
CEA '66 (Ref. 42)	μ	248	0	350	0	(< 1%)	0	(~9%	C	1.195±.057
		268	↓	378	↓		↓	normali-		1.222±.038
		298		420				zation)		1.165±.029
		327		461						1.137±.033
		353		498						1.108±.027
		380		535						1.092±.038
		406		572			0			1.065±.060
		435		613			5%			1.074±.089
		465		655			15%			1.089±.137
		491		691			25%			1.207±.169
		524	80	740	~8%		40%			1.394±.336

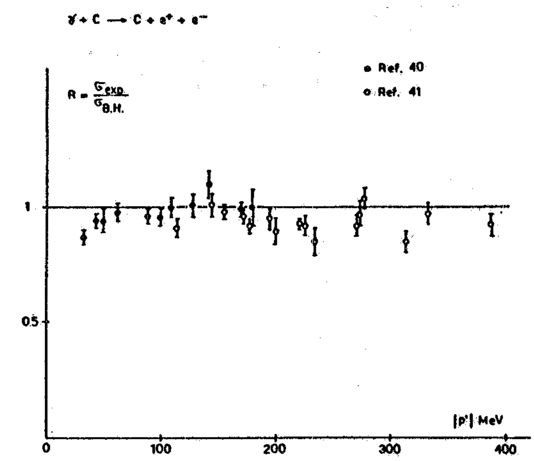


FIG. 6

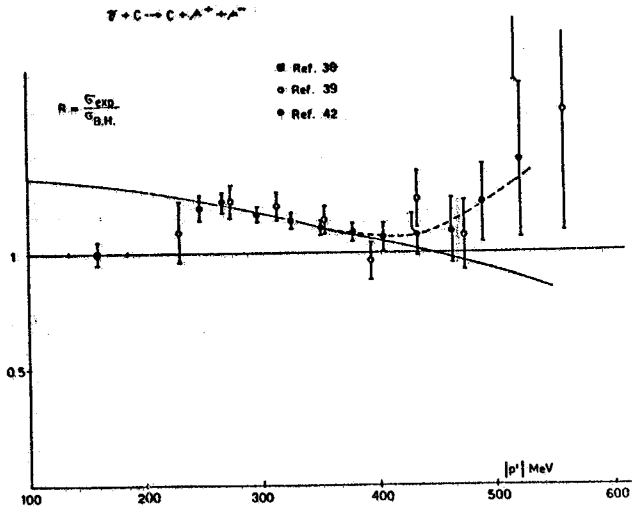


FIG. 7

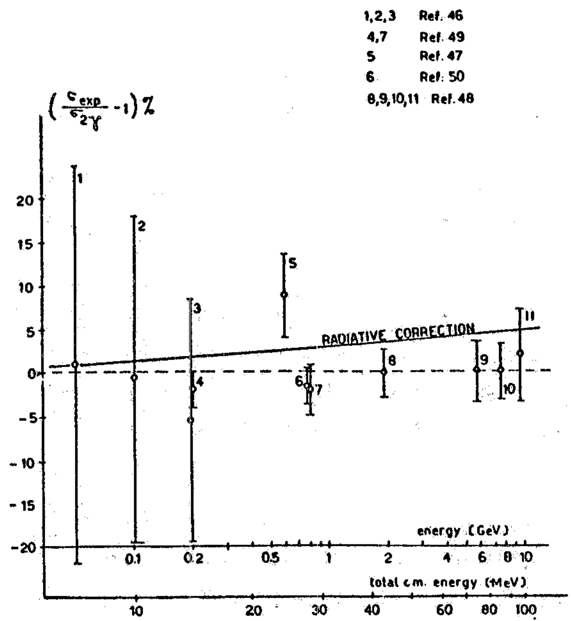
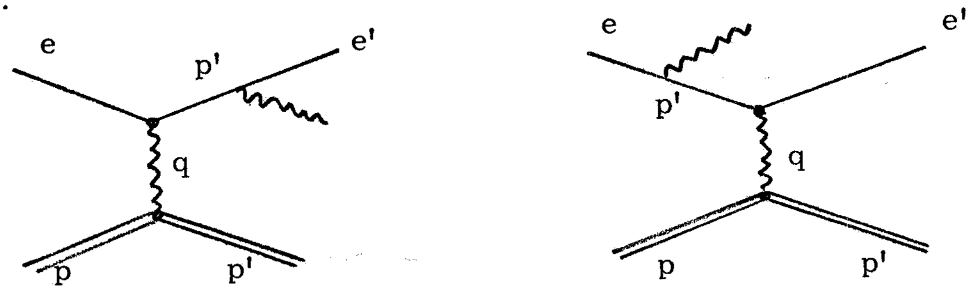


FIG. 8 - Results on positron annihilation in flight.

The lepton pair production has thus been in last years a very important tool to probe QED for space-like momentum transfers to the virtual fermion.

An experiment is in progress in Frascati to explore regions of time-like momentum transfers to the electron. It is an experiment of wide angle bremsstrahlung of electrons on hydrogen. The useful graphs are:



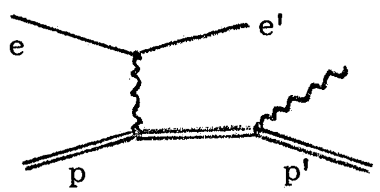
In the first graph the virtual electron is time-like ($t^2 = p'^2 > 0$), in the second one space-like ($S^2 = p'^2 < 0$).

The three kinematical situations explored in the experiment are such that

	I situation	II situation	III situation
$\lvert q \rvert$ (MeV)	~ 380	~ 380	~ 380
$\lvert S \rvert$ (MeV)	~ 260	~ 260	~ 260
$\lvert t \rvert$ (MeV)	~ 70	~ 100	~ 140

The use of an H_2 target allows to reach values of $\lvert q \rvert$ as high as ~ 400 MeV, without the danger of introducing large systematic effects due to inelastic contributions.

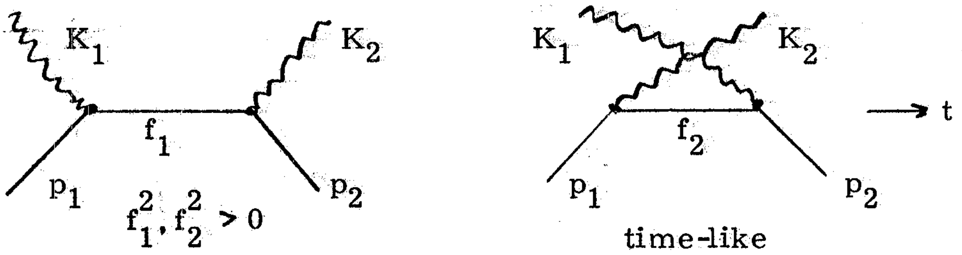
On the contrary, the interference term with the Compton graphs does not vanish. In our kinematical situations, however, this contribution (evaluated according to ref. (43)) does not exceed a few percent.



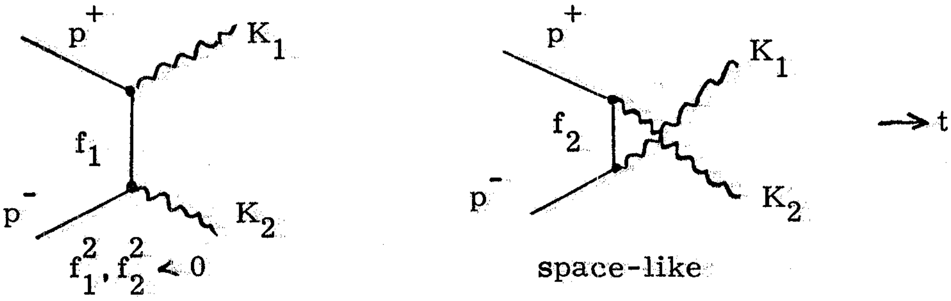
At the moment, data have been collected at t = 70 and 100 MeV. The point at 70 MeV has been completely analyzed (apart from radiative corrections) and the result is consistent with QED within the 6% error. The complete set of data is expected to be available within a few months.

<u>One fermion off the mass-shell; photon on the mass-shell.</u>

Experiments of this kind are Compton scattering



and two photon annihilation



Note that this is the only case in which we have not to do with photon propagators, nor with any kind of strong-interaction contamination.

We will consider the annihilation process, which has been experimentally investigated at high energy, and will be the object of further investigation using colliding beam machines[44].

The differential cross section for two-photon annihilation can be written in the c. m. s. [45], as a function of invariants:

$$d\sigma = -\frac{r_o^2 m^2}{8 p_o \varepsilon_o} \left\{ 4\left(\frac{1}{x_1} + \frac{1}{x_2}\right)^2 - 4\left(\frac{1}{x_1} + \frac{1}{x_2}\right) - \left(\frac{x_1}{x_2} + \frac{x_1}{x_1}\right) \right\} d\Omega$$

where r_o classical radius of the electron
m mass of the electron
$p = (\varepsilon_o, \vec{p}_o) = p_+ = - p_-$ four momentum of the incident positron
$p_o = |\vec{p}_o|$).
$m^2 x_{1,2} = 2\,\varepsilon_o\,(\varepsilon_o \pm p_o \cos\theta) = f_{1,2}^2 + m^2$
θ angle of emission of K_1.

or, as a function of the angle θ:

$$d\sigma = \frac{1}{4}\,\frac{r_o^2 m^2}{p_o \varepsilon_o}\left(\frac{\varepsilon_o^2 + p_o^2 + p_o^2 \sin^2\theta}{\varepsilon_o^2 - p_o^2 \cos^2\theta} - \frac{2 p_o^4 \sin^4\theta}{(\varepsilon_o^2 - p_o^2 \cos^2\theta)^2}\right)\sin\theta\, d\theta\, d\varphi$$

Only total cross section measurements have been performed up to now.

Once $\mathcal{E}$ is fixed, the momentum transfers f^2 range from $2\mathcal{E}_o(\mathcal{E}_o-p_o)\simeq$ $\simeq 0$ to $2\mathcal{E}_o(\mathcal{E}_o+p_o)\simeq 4\mathcal{E}_o^2$ as θ goes from 0^o to 180^o. However, the differential cross section weights heavily the low values of the momentum transfers. The average value of the momentum transfer can be evaluated

$$\overline{f}_1^2 = \overline{f}_2^2 = \frac{\int (M^2 x_1 - m^2) d\sigma}{\int d\sigma} = 2\,\mathcal{E}_o^2 - 2\,\mathcal{E}_o p_o \frac{\int \cos\theta\, d\sigma}{\int d\sigma}$$

and even at 10 GeV incident positrons turns ont to be of the order of $(30\ \mathrm{MeV})^2$.

The available experimental results are shown in Table II and Fig. 8.

We see that the agreement with QED would be perfect if one neglects radiative corrections. However, taking into account radiative corrections there is a discrepancy of the order of 3 ÷ 5% (= 1 ÷ 1.5 standard deviation on each of the 6 points). We are now waiting for the colliding beam experiments. It is worth noticing that the momentum transfers f^2 involved in experiments of this kind(44) will be of the order of some $(\mathrm{GeV})^2$.

7 - TESTS OF A PARTICULAR MODEL -

We have seen a naive example of a modified QED involving an excited electron. A specific model of QED breackdown in terms of an heavy electron has actually been proposed by F. Low(51). The excited electron e' of mass m^* could couple with electron in the form

$$\frac{\lambda_e}{m^*}\ \overline{\psi}_{e'}\,\sigma_{\mu\nu}\,\psi_e\,F_{\mu\nu} + \text{h.c.}$$

λ was proposed to be of the order of 1.

Two experiments(52, 53) have been performed searching for the heavy electron in reaction

$$e + p \rightarrow e' + p$$
$$(\hookrightarrow e + \gamma)$$

The range of mass explored is $\sim 100\ \mathrm{MeV} < m^* < 1\ \mathrm{GeV}$. No evidence for the presence of this reaction was found. The results are expressed in terms of upper limits on λ, and are summarized in Table III.

I thank prof. C. Bernardini for a number of clarifying discussions.

TABLE II

Experiment	Positron energy MeV	$2\mathcal{E}_o$ c. m. total energy MeV	Average momentum transfer MeV	$(\frac{\sigma_{exp}}{\sigma_{2\gamma}} - 1)\%$	Radiative corrections %
Berkeley (Ref. 46)	50 100 200	7 10 14	~ 3 ~ 4 ~ 6	+1.8 ± 23 - .8 ± 19 -6.4 ± 15	+ few percent
Lausanne (Ref. 47)	600	24	~ 9	+7.5 ± 5	+2.2
CERN (Ref. 48)	1940 5800 7710 9640	45 77 89 100	~15 ~22 ~25 ~27	- .6 ± 2.7 - .5 ± 3.5 0 ± 3.2 1.5 ± 5.5	+3.3 +4.1 +4.35 +4.55
Stanford (Ref. 49)	200 811	14 28	~ 6 ~10	-2.1 ± 2.1 -2.1 ± 2.9	+1.8 +2.5
Pisa (Ref. 50)	790	28	~10	-1.9 ± 2.1	+2.5

TABLE III

m^* (MeV)	$\lambda_{max} \times 10^2$	
120	4.1	Ref. 52
150	3.4	
180	3.9	
210	4.1	
240	4.25	
240	1.4	
255	2	
278	2.8	
304	3.8	
330	5	
359	6.9	
391	9.5	
424	11.2	
456	13.5	
493	15	
533	15	
571	15	
500	1.4	Ref. 53
600	1.4	
700	1.5	
800	1.7	
900	1.7	
1000	2.5	

REFERENCES -

(1) - E. Noether, Nachr. Akad. Wiss. Göttingen, II, Math. Physik, Kl. 1918, p. 235.
(2) - J.C. Ward, Phys. Rev. 78, 182 (1950); Y. Takahashi, Nuovo Cimento 6, 371 (1957).
(3) - N.P. Chang and H.S. Mani, Phys. Rev. 134, B896 (1964).
(4) - N.M. Kroll, Nuovo Cimento 45, A65 (1966).
(5) - H. Lehmann, Nuovo Cimento 11, 342 (1954).
(6) - J.A. McClure and S.D. Drell, Nuovo Cimento 37, 1638 (1965).
(7) - A.M. Bincer, Phys. Rev. 118, 855 (1960).
(8) - See, for instance, S.D. Drell and F. Zachariasen, The electromagnetic structure of nucleons (Clarendon Press, Oxford).
(9) - S.D. Drell and H.R. Pagels, Phys. Rev. 140, B 397 (1965); H.H. Elend, Phys. Letters 20, 682 (E 21, 720), (1966).
(10) - D.T. Wilkinson and H.R. Crane, Phys. Rev. 130, 852 (1962); W. H. Lousille, R.W. Pidd and H.R. Crane, Phys. Rev. 94, 7 (1954); G. Charpak, F.J.M. Farley, R.L. Garwin, T. Muller, J.C. Sens and A. Zichichi, Nuovo Cimento 37, 1241 (1965); F.J.M. Farley, J. Bailey, R.C.A. Brown, M. Giesh, H. Jöstlein, S. Van der Meer, E. Picasso and M. Tannenhaum, Nuovo Cimento 45, 281 (1966).
(11) - See, for instance, K. Okamoto, Nuclear Phys. B4, 226 (1967).
(12) - W.C. Barber, B. Gittelman, G.K. O'Neill and B. Richter, Phys. Rev. Letters 16, 1127 (1966).
(13) - C. Möller, Ann. Phys. 14, 531 (1932).
(14) - S. De Benedetti and H.C. Corben, Ann. Rev. of Nucl. Sci. 4, 191 (1954).
(15) - R. Karplus and A. Klein, Phys. Rev. 87, 848 (1952).
(16) - M. Deutsch and C.S. Brown, Phys. Rev. 85, 1047 (1952); r. Weinstein, M. Deutsch and S. Brown, Phys. Rev. 94, 758 (1954); 98, 223 (1955).
(17) - E.D. Theriot Jr., R.H. Beers and V.W. Huges, Phys. Rev. Letters 18, 767 (1967).
(18) - R. Weinstein, A Review of recent Work in QED, Presented at the Intern. Symp. on Electron and Photon Interactions at High Energies, Stanford (1967).
(19) - R. Cool, A. Mashke, L.M. Lederman, M. Tannenbaum, R. Ellsworth, A. Melissinos, J.H. Tinlot and T. Yamanouchi, Phys. Rev. Letters 14, 724 (1965).
(20) - G.E. Masek, L.D. Heggie, Y.B. Kim and R.W. Williams, Phys. Rev. 122, 937 (1961).
(21) - A. Citron, C. Delorme, D. Fries, L. Goldzahl, J. Heintze, E.G. Michaelis, C. Richard and H. Øveras, Phys. Letters 1, 175 (1962).
(22) - G.E. Masek, L.D. Heggie, Y.B. Kim and R.W. Williams, Phys. Rev. 122, 937 (1961).

(23) - P. L. Connolly, J. G. McEwen and J. Orear, Phys. Rev. Letters 6, 554 (1961).
(24) - D. Kotelchuck, J. G. McEwen and J. Orear, Phys. Rev. 129, 876 (1961).
(25) - C. Y. Kim, S. Kaneko, Y. B. Kim, G. E. Masek and R. W. Williams, Phys. Rev. 122, 1641 (1961).
(26) - J. K. De Pagter, J. I. Friedman, G. Glass, R. C. Chase, M. Gettner, E. VonGoeter, R. Weinstein and A. M. Boyarski, Phys. Rev. Letters 16, 35 (1966).
(27) - B. D. Hyams, W. Koch, D. Pellett, D. Potter, L. VonLindern, E. Lorenz, G. Lütjens, V. Stierlin and P. Weilhammer, Phys. Letters 24B, 634 (1967).
(28) - A. Wehmann, E. Engels Jr., C. M. Hoffman, P. G. Innocenti, R. Wilson, W. A. Blampied, D. J. Drickey, L. N. Hand and G. D. Stairs, Phys. Rev. Letters 18, 929 (1967).
(29) - P. L. Rothwell, A. Boyarski, R. C. Chase, J. DePagter, J. I. Friedman, M. Gettner, G. Glass, E. VonGoeler and R. Weinstein, Intern. Symp. on Electron and Photon Interactions at High Energies, Stanford (1967).
(30) - J. G. Ashbury, U. Becker, W. K. Bertram, P. Joos, M. Rohde, A. J. S. Smith, C. L. Jordan and S. C. C. Ting, Phys. Rev. Letters 19, 869 (1967).
(31) - S. S. Hertzbach, R. W. Kraemer, L. Madansky, R. A. Zdanis and R. Strand, Phys. Rev. 155, 1461 (1967).
(32) - M. N. Khachaturyan, M. A. Azimov, A. M. Baldin, A. S. Belousov, I. V. Chuvilo, R. Firkowski, J. Hladky, M. S. Khvastunov, J. Manca, A. T. Mathyushin, V. H. Mathyushin, G. A. Ososkov, L. N. Shtarkov and L. I. Zhuravleva, Phys. Letters 24B, 349 (1967).
(33) - V. L. Auslander, G. I. Budker, Ju. N. Pestov, V. A. Sidorov, A. N. Skrinskij and A. G. Khabakhpashev, Phys. Letters 25B, 433 (1967).
(34) - J. E. Augustin, J. C. Bisot, J. Buon, J. Haissinski, D. Lalanne, P. C. Marin, J. Perez-y-Jorba, F. Rumpf, E. Silva and S. Tavernier, Orsay report LAL-1181 (1967).
(35) - R. B. Blumenthal, D. C. Ehn, W. L. Faissler, P. M. Joseph, L. J. Lanzerotti, F. M. Pipkin and D. G. Stairs, Phys. Rev. 144, 1199 (1966).
(36) - C. Bernardini, B. Grossetete, B. Jean-Marie, P. Lehmann, V. Silvestrini and R. Tchapoutian, Laboratori Nazionali di Frascati, Internal report LNF-66/33 (1966) (unpublished); See also B. Jean-Marie, Thesis Orsay (unpublished).
(37) - B. Richter, Phys. Rev. Letters 1, 114 (1958).
(38) - A. Alberigi-Quaranta, M. DePretis, G. Marini, A. Odian, G. Stoppini and L. Tau, Phys. Rev. Letters 9, 226 (1962).
(39) - J. K. DePagter, A. Boyarski, G. Glass, J. I. Friedman, H. W. Kendall, M. Gettner, J. F. Larraber and R. Weinstein, Phys. Rev. Letters 12, 739 (1964).

(40) - E. Eislander, J. Feigenbaum, N. Mistry, P. Mostek, D. Rust, A. Silverman, C. Sinclair and R. Talman, Phys. Rev. Letters 18, 425 (1967).
(41) - J.G. Ashbury, W.K. Bertram, U. Becker, P. Joos, M. Rhode, A. J.S. Smith, S. Friedlander, C.L. Jordan and C.C. Ting, Phys. Rev. Letters 18, 65 (1967).
(42) - J.K. DePagter, J.I. Friedman, G. Glass, R.C. Chase, M. Gettner, E. VonGoeler, R. Weinstein and A.M. Boyarski, Phys. Rev. Letters 17, 767 (1966).
(43) - R.A. Berg and C.N. Lindner, Nuclear Phys. 26, 259 (1961).
(44) - See, for instance, R. Baldini-Celio, G. Capon, C. Mencuccini, G. P. Murtas, C. Pellegrini, A. Reale, M. Spinetti, C. Bacci, G. Penso and G. Salvini, Laboratori Nazionali di Frascati internal report LNF-67/73 (1967).
(45) - A.I. Akhiezer and V.B. Berestetskii, Quantum electrodynamics. (Interscience Publ., 1965) pag. 457.
(46) - S.A. Colgate and F.C. Gilbert, Phys. Rev. 89, 790 (1953).
(47) - E. Malamud and R. Weill, Nuovo Cimento 27, 418 (1963).
(48) - F. Fabiani, M. Fidecaro, G. Finocchiaro, G. Giacomelli, D. Harting, N.H. Lipman and G. Torelli, Nuovo Cimento 25, 635 (1962).
(49) - A. Brown and J. Pine, Nuovo Cimento 27, 850 (1963).
(50) - P.L. Braccini, I.X. Ion, A. Stefanini, G. Torelli and R. Torelli Tosi, Nuovo Cimento 29, 1215 (1963).
(51) - F.E. Low, Phys. Rev. Letters 14, 238 (1965); F. Gutbrbt and D. Shildknecht, Zeitsch. f. Physik 192, 271 (1966).
(52) - C. Betournè, H. Nguyen Ngoc, J. Perez-y-Jorba and J. Tran Thanh Van, Phys. Letters 17, 70 (1965).
(53) - H.J. Beherend, F.W. Brasse, J. Engler, E. Ganssauge, H. Hultschig, S. Galster, G. Hartwig and H. Shopper, Phys. Rev. Letters, 15, 900 (1965).