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G. Bologna: THE 375-MeV FRASCATI LINAC AS A POSSIBLE SOURCE OF A MONOCHROMATIC AND LINEARLY POLARIZED PHOTON BEAM BY COHERENT BREMSSTRAHLUNG IN CRYSTALS. -

(Nota interna: n. 387)

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(Communication to the Meeting on the use of electron linear accelerators for nuclear structure study-Frascati, October 19-21, 1967).

1 - INTRODUCTION -

Up to now two proposal have been presented for the production of a monoenergetic photon beam in the range of 100 MeV to be used in experiments at LEALE (the Frascati laboratory for nuclear structure study).

The first proposal[1] is based on the well known method of positron annihilation in flight. The second one[2] is unconventional, as it should use the compton scattered laser photons in the storage ring ADONE.

The purpose of this note is to propose a third method which is based on the well established coherence effect in bremsstrahlung from crystals.

Coherent bremsstrahlung is a facility of the Frascati 1-GeV electronsynchrotron since 1960; this γ-ray beam has been used in several experiments, mainly because of its relatively high linear polarization, the degree of monochromaticity beeing essentially poor.

However both monochromaticity and polarization could be increased by using a very thin crystals as well as a sharp collimation of the γ-ray beam. Detailed calculations for this case have been performed by the present author; numerical results for the case of a 6-GeV electron beam have been presented[3]. Here numerical results relevant for the Frascati linac are given. It is shown that sufficiently good features can be obtained, despite of the small electron energy (375 MeV). A comparison with the other proposed methods is given in section 4.

The present is a preliminary paper which reports only the expected features of the beam; a detailed project of the experimental set up will be left for another time.

This paper is intended to be a continuation of ref. (3). The same notations are used. Thus, unless otherwise stated, energies are measured in units of the rest mass mc^2 of the electron; angles are measured in units of the reciprocal primary electron energy $1/E_1$. For other definitions see ref. (3).

2 - DISCUSSION OF EXPERIMENTAL CONDITIONS -

The Frascati linac has the following characteristics

- $i_o = 5 \cdot 10^{14}$ electron/sec = maximum current
- $mc^2 E_1 = 375$ MeV = Electron energy at maximum current
- $\delta E_1/E_1 = 1\%$ = Energy spread at half maximum current
- Emittance $\simeq 10^{-3} \pi$ rad mm both vertically and horizontally
- Duty cycle = $8 \cdot 10^{-4}$.

By assuming that the electron angular distribution has a gaussian shape, we choose a root mean square deviation $\omega_o/E_1 = 2 \cdot 10^{-4}$ rad ($\omega_o = 0.15$). As indicated in ref. (3), a round collimator centred in the γ-ray beam will be considered. Its diameter must be as small as possible in order to achieve a good monochromaticity (see eq. (4), ref. (3)). In practice it cannot be smaller than some millimeters. It turns out that the beam size at the radiator must be less than 1 mm (otherwise collimation is not effective). Thus the emittance must be reduced to less than $2 \cdot 10^{-4}$ π rad mm (by using a suitable magnetic device as well as a collimator for the electron beam). In this way the intensity is decreased. We conservatively assumed the reduced intensity as large as 10^{13} electron/sec.

The thickness of the crystal as well as the electron beam angular divergence are chosen in such a way to avoid a large broadening of the desidered line width. As a matter of fact $\sqrt{\omega_o^2 + \omega_M^2}$, ω_M being the root mean square angle of scattering (see eq. (15) and errata in ref. (3)), must be less than the half collimation angle U_o (otherwise the line width is essentially determined by beam divergence and multiple scattering).

Similarly there is no gain in monochromaticity by reducing crystal thickness under a value T such that $\omega_M(T)$ is less than the ultimate beam divergence ω_o.

Furthermore beam divergence is more effective in line broodening than multiple scattering is. This is, be cause, for a given $\sqrt{\omega_o^2 + \omega_M^2(T)}$, the electron distribution $G(\omega)$ has approximately a gaussian shape for $\omega_o > \omega_M(T)$ while it has approximately an exponential integral shape for $\omega_o < \omega_M(T)$.

We have seen in ref. (3) that diamond and beryllium are the "best" crystals as far as coherent bremsstrahlung is concerning.

In the case of a beryllium crystal we have chosen $T = 9.25 \cdot 10^{-3}$ gr/cm^2 = 0.05 mm. This is a very small thickness indeed. It is questionable wether such a crystal can be obtained without defects or not. But such thickness is required, as $\omega_M(T) = 0.24$; as a consequence, the lower limit of line width (for a collimation $U_o = 0$) is, at the value X = 0.2 of the fractional photon energy

$$\frac{\Delta X}{X} = (1 - 0.2)\,(0.15^2 + 0.24^2) = 6\%.$$

For diamond the condition is even more severe, as $\omega_M = 0.26$ is obtained for a thickness $T = 7 \cdot 10^{-3}$ gr/cm^2 = 0.02 mm. There are no many chances to obtain so thin diamonds; thus we presented in ref. (3) results for beryllium only. Here we present also results for diamond, just to show what could be obtained with the "best" crystal.

The indetermination in the position of a line at X = 0.2, due to the energy spread of the electron beam is given by

$$\frac{\delta X}{X} = (1 - X)\,\frac{\delta E_1}{E_1} = (1 - 0.2) \cdot 10^{-2} = 0.8\%,$$

which is negligible as compared to the spread $\Delta X/X = 6\%$ of the line, due to electron divergence and multiple scattering. Thus we do not consider this effect.

A serions cause of line broodening could be the mosaic spread of the crystal. In order to prevent this broodening the r.m.s. deviation $\delta\theta$ of a crystal axis must be controlled with a precision

$$\delta\theta/\theta < \omega_o^2 + \omega_M^2(T)$$

where θ/E_1 is the angle between a crystal axis and the electron momentum.

As far as the electron energy loss in the crystal is concerned, an estimation for beryllium 0.05 mm thick gives a power absorption of $2 \cdot 10^{-2}$ Watt for a beam of 10^{13} electron/Sec; this can be easily dissipated by heat conduction, so that there is no danger of a temperatu-

re rise in the crystal. The situation is even more favourable for diamond, which is one of the best heat conductor existing in nature. Thus we will assume the crystals in thermal equilibrium at room temperature.

3 - NUMERICAL RESULTS -

I) Beryllium crystal. -

For beryllium crystal the orientation chosen in ref. (3), which gives a "single point" spectrum, is no longer suitable in the present case, for the ratio of the coherent line to the incoherent background is too small for the electron energy $mc^2E_1 = 375$ MeV.

We will present numerical results for three different values of the angle $\alpha = \sphericalangle(\vec{b}_1, \vec{b}_2)(\vec{b}_1, \vec{p}_1)$, where $\vec{p}_1$ is the electron momentum, and $\vec{b}_1, \vec{b}_2$ are two orthogonal reference axes of the crystal ($\vec{b}_1$ almost parallel to $\vec{p}_1$). In figs. 1, 2, 3 the structure of the reciprocal lattice

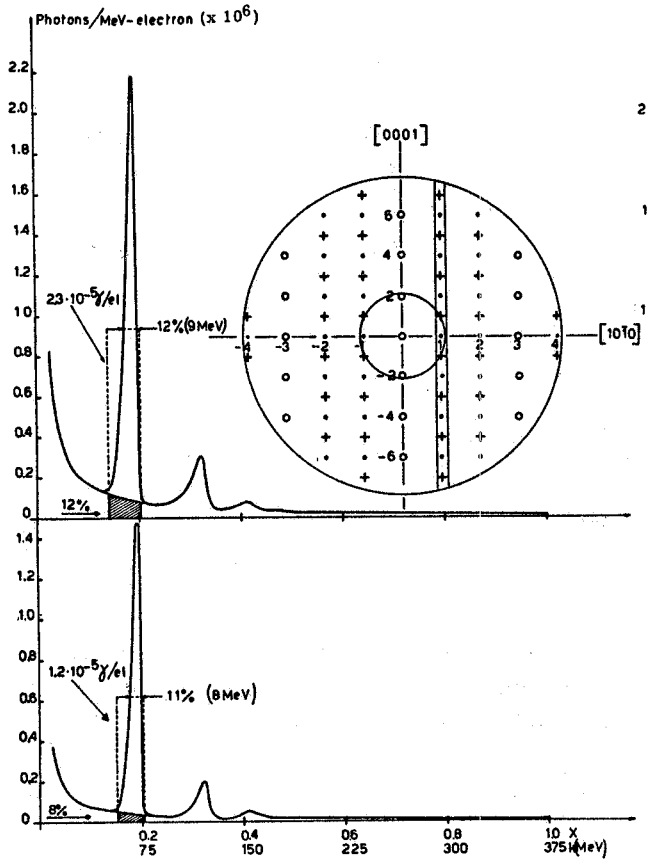

Fig. 1

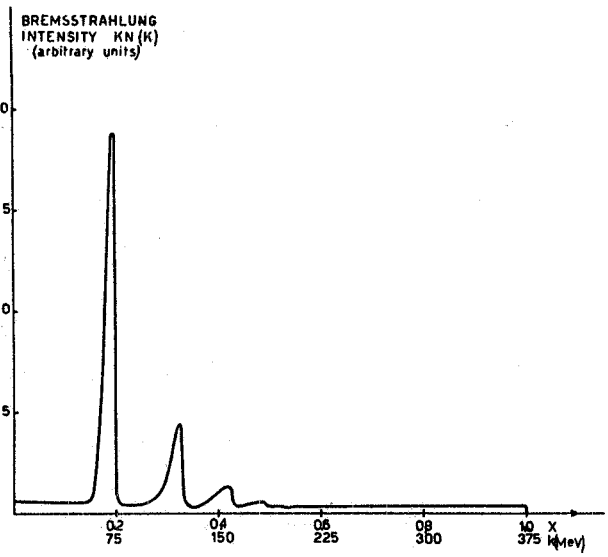

Fig. 1a

Figs. 1, 1a - Beryllium crystal $\alpha = 0$; $\theta/E_1 = 13.8$ mrad. Abscissas are photon energies both in MeV and in units of electron energy.
Fig. 1 Lower curve $U_o = 0.2$; Upper curve $U_o = 0.3$.
Fig. 1a $U_o = 0.2$.

plane through the origin is given. Crosses, dots, small circles refer to different structure factor values. The inner circle passes through the reciprocal lattice points for which the function $\phi(g^2)$ of ref. (3) is a maximum. The outer circle delimits the region of lattice plane considered

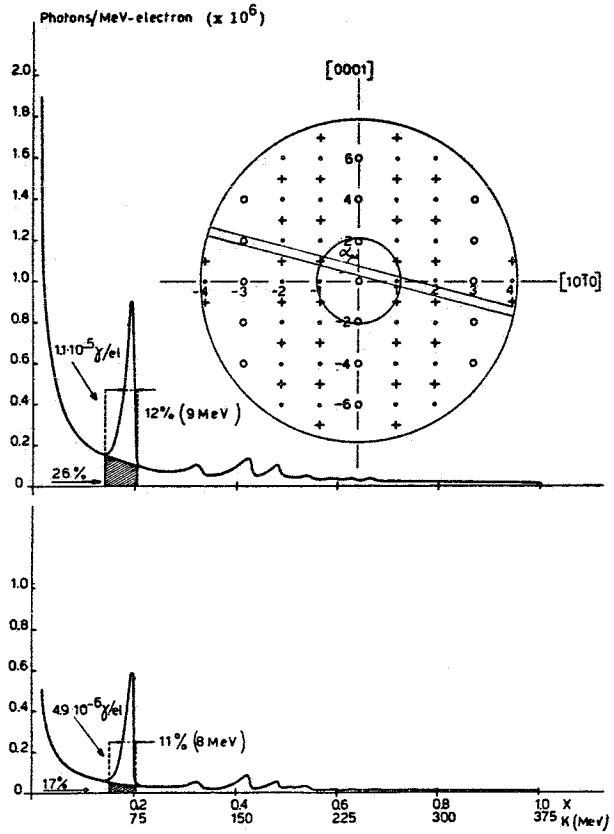

Fig. 2

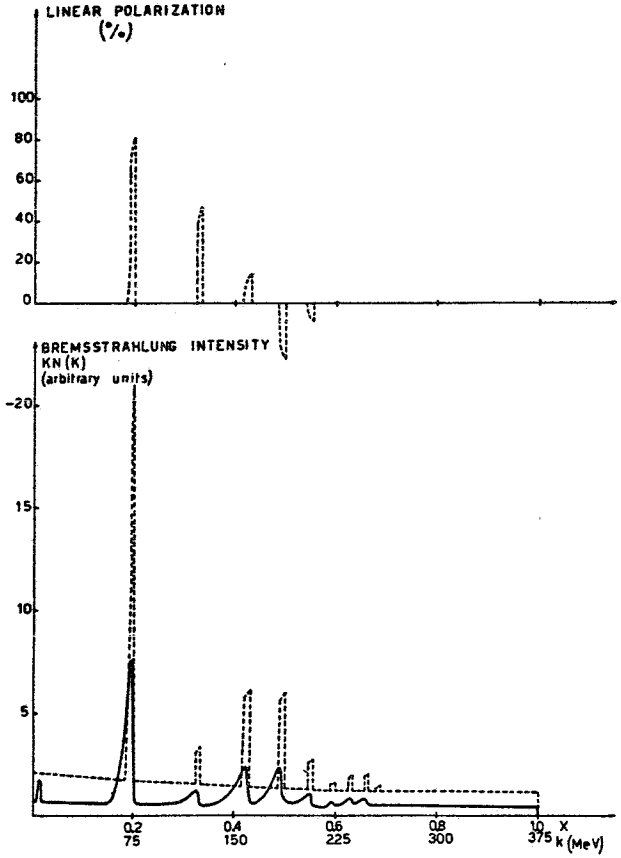

Fig. 2a

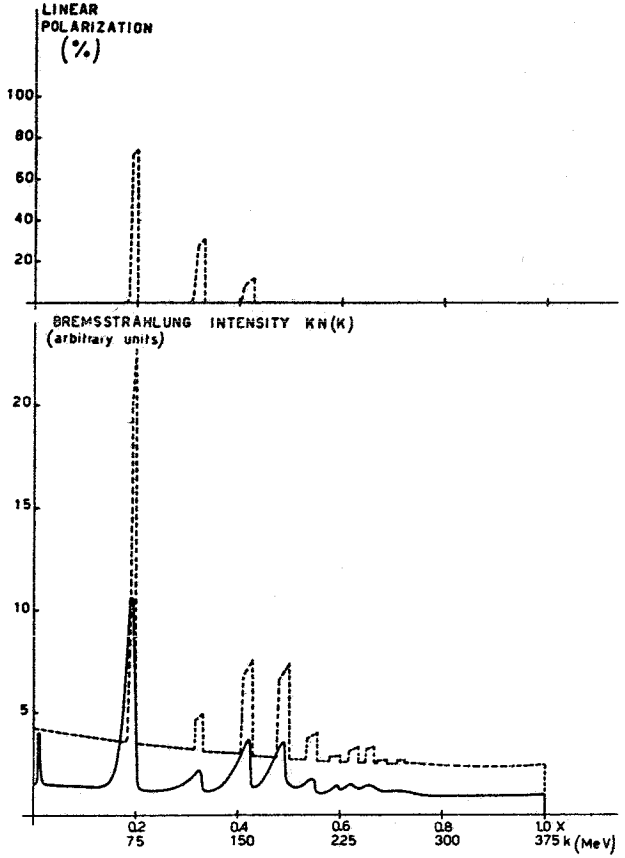

Fig. 2b

Figs. 2, 2a, 2b - Beryllium crystal $\alpha = 74.56^{o}$; $\theta/E_1 = 52.1$ mrad. Fig. 2 Lower curve $U_o = 0.2$, Upper curve $U_o = 0.3$; Fig. 2a $U_o = 0.2$; Fig. 2b $U_o = 0.3$.
Figs. 2a, 2b ———— with electron divergence and multiple scattering; -------- no electron divergence, no multiple scattering.

in calculation (outer points give negligible contributions). The intersection of the "pancake" with the plane is represented by the two parallel straight lines.

Fig. 1 refers to $\alpha = 0$. A high density reciprocal lattice row is contained within the pancake, so that the line height is a maximum, while linear polarization is negligible.

Fig. 2 refers to $\alpha = 74.56^{o}$; in this case only three reciprocal lattice points are contained. Thus a low line height as well as a high polarization is expected.

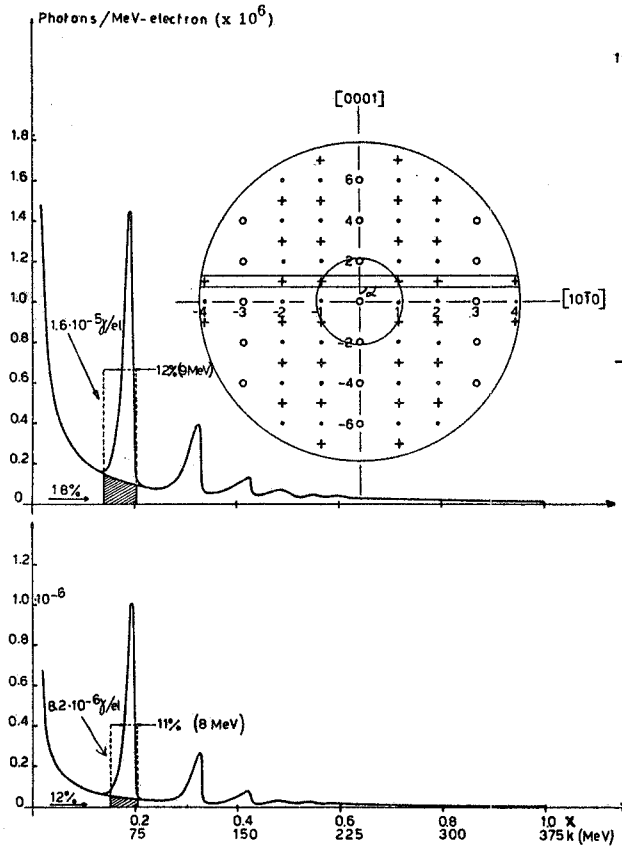

Fig. 3

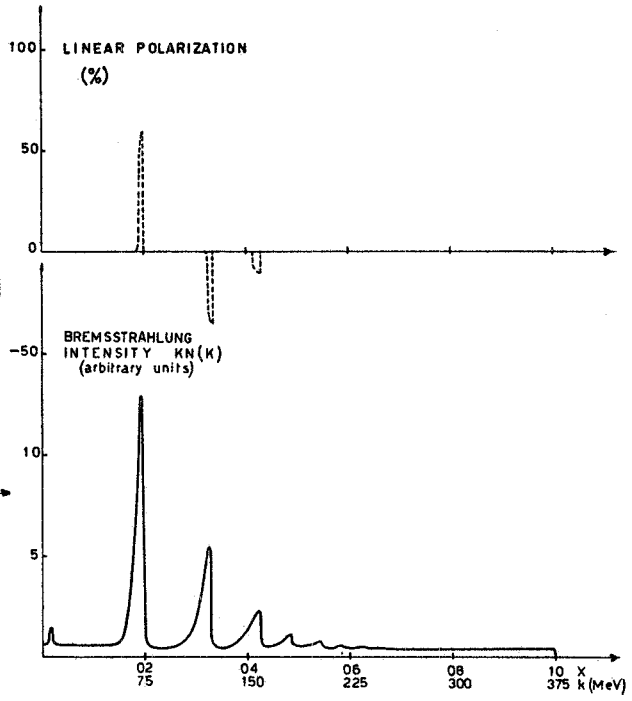

Fig. 3a

Figs. 3, 3a - Beryllium crystal $\alpha = 90^{o}$; $\theta/E_1 = 25.1$ mrad.
Fig. 3 Lower curve $U_o = 0.2$; Upper curve $U_o = 0.3$.
Fig. 3a $U_o = 0.2$.

The situation of fig. 3, where $\alpha = 90^{o}$, is a compromise between the preceding cases.

Let us now examine the numerical results in detail. The following conditions were assumed

- Electron energy $mc^2E_1 = 375$ MeV.
- Nominal position of the right edge of the dominant line $X_p = k_p/mc^2E_1 = 0.2$ ($k_p = 75$ MeV).
- Beryllium crystal reference axes $\vec{b}_1 \equiv [1\bar{2}10]$. $\vec{b}_2 \equiv [10\bar{1}0]$; $\vec{b}_3 \equiv$

[0001] . (Other crystal data are given in ref. (3)).
- Crystal thickness $T = 9.25 \cdot 10^{-3}$ gr/cm^2 = 0.05 mm. Thus the R. M. S. angle of scattering $\omega_M(T)/E_1 = 0.33$ mrad ($\omega_M(T) = 0.24$)
- R. M. S. electron deviation $\omega_o/E_1 = 0.2$ mrad ($\omega_o = 0.15$)
- Round collimator centred in the γ-ray beam (U_o = Half aperture).

Fig. 1 shows the number of photons/MeV electron as a function of $X = k/mc^2 E_1$ as well as of photon energy k, while fig. 1a) shows bremsstrahlung intensity kN(k) (arbitrary units), where N(k)dk is the number of photons between photon energies k and k + dk. Abscissa is again X or k.

Both figures refer to the case $\alpha = 0$ (high line-to-background ratio, small polarization). The line is obtained at Xp = 0.2 for an angle $\theta/E_1 = 13.8$ mrad between electron momentum $\vec{p_1}$ and crystal axis $\vec{b_1}$. The curve in the lower part of fig. 1 was obtained for a half collimation angle $U_o = 0.2$ ($U_o/E_1 = 0.27$ mrad), while the curve in the upper part was obtained for $U_o = 0.3$ ($U_o/E_1 = 0.41$ mrad). Fig. 1a) is relative to $U_o = 0.2$ only ($U_o = 0.3$ giving similar results). Linear polarization is not represented, as it is very small (10% maximum).

In fig. 1 the dashed rectangle at the position of the dominant line represents a number of γ/el. equal to the total number of γ/el. within the line. The intersection of the upper base of the rectangle with the line is drawn as an indication of the line width. The shaded area at the bottom of the rectangle represents the contribution of the incoherent background. The ratio N_B/N_γ of the background photons to the total photons in the rectangle as well as the number of photons per electron in the rectangle is indicated.

Figs. 2, 2a, 2b are relative to $\alpha = 74.56^o$ (high polarization, small line-to-background ratio), $\theta/E_1 = 52.1$ mrad. Description of fig. 2 is the same as that of fig. 1.

As in fig. 1a, the continuous curve in figs. 2a and 2b gives bremsstrahlung intensity for $U_o = 0.2$ and $U_o = 0.3$ respectively. The superimposed dashed curve was obtained for no electron divergence and no multiple scattering; it is given for comparison. Furthermore, the dashed curve in the upper part of the figures represent linear polarization, as referred to the plane ($\vec{b_1}$, $\vec{p_1}$). Polarization is at present computed for no electron divergence and no multiple scattering only. However its reduction, due to the actual experimental conditions is expected to be small.

Figs. 3, 3a present the case $\alpha = 90^o$, $\theta/E_1 = 25.1$ mrad, with features which are intermediate between the previous ones. Description is the same of figs. 1, 1a, with the only addition that the upper part of fig. 3a presents the plot of linear polarization (computed without electron divergence and multiple scattering and referred to

$\vec{b}_1$, $\vec{p}_1$ again). In this particular case the contribution of the first lattice row (which is contained between the parallel straight lines of fig. 3) is about equal to the contribution of the second one. Thus, by shifting the dominant line to the position of the second one (this is accomplished simply by assuming $\theta/E_1 = 50.2$ mrad, other conditions being unchanged), a spectrum is obtained which does not have the line at $X = 0.2$, and coincides with the previous one within $5 \cdot 10^{-8}$ photons/MeV·electron in the remaining part. This could be useful in performing experiments by using the photon difference method. Care must taken as far as the polarization is concerning: in the new spectrum the polarization of the dominant line does not coincide with the corresponding one of the old spectrum. However a compensation can be obtained by suitably changing the angle of the plane ($\vec{b}_1$, $\vec{p}_1$) with respect to a fixed plane.

II) Diamond. -

The best choice of the reference axes for diamond is

$$\vec{b}_1 \equiv [110] \; ; \quad \vec{b}_2 \equiv [001] \; ; \quad \vec{b}_3 \equiv [1\bar{1}2].$$

As already said in section 2, diamond thickness is chosen as large as $T = 7 \cdot 10^{-3}$ gr/cm^2 = 0.02 mm ($\omega_M(T) = 0.26$; $\omega_M(T)/E_1 = 0.36$ mrad).

We made computations for $\alpha = 13.26^o$, which, at our electron energy, is the best compromise in order to obtain a good line-to-background ratio with a high polarization. For $X_p = 0.2$, we have $\theta/E_1 = 38.5$ mrad. The other conditions are the same as for case I .

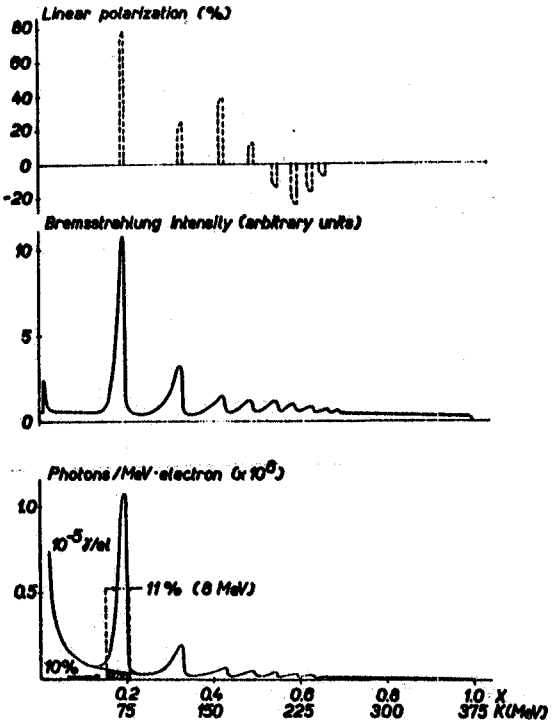

Fig. 4 represents the number of photons/MeV electron, as well as the bremsstrahlung intensity and the linear polarization for $U_o = 0.2$ versus X and k.

FIG. 4 - Diamond crystal $\alpha = 13.26^o$ $\theta/E_1 = 38.5$ mrad $U_o = 0.2$.

4 - DISCUSSION OF RESULTS -

The obtained results are summarized in Table 1. Columns 1 and 2 individuate the type of crystal and its orientation, respectively.

TABLE I - Summary of results.

Crystal	α	U_o	Δk(MeV)	$N_\gamma(sec^{-1})$	Polarization	N_B/N_γ	I_B/I_γ
Beryllium	0°	0.2	8	1.2×10^8	--	0.08	1.6
"	0°	0.3	9	2.3×10^8	--	0.12	
"	90°	0.2	8	8.2×10^7	0.59	0.12	2.8
"	90°	0.3	9	1.6×10^8	0.56	0.18	
"	74.56°	0.2	8	4.9×10^7	0.80	0.17	4.2
"	74.56°	0.3	9	1.1×10^8	0.74	0.26	4.2
Diamond	13.26°	0.2	8	1.0×10^8	0.79	0.10	2.2

U_o gives the half collimation angle (in $1/E_1$ units). Δk in MeV gives the width of the dominant line (at k = 75 MeV), as indicated in figs. 1 through 4. $N\gamma$ gives the number of photons per second obtained in the dominant line for 10^{13} electron/sec as well as for a crystal thickness of 0.05 and 0.02 mm for beryllium and diamond, respectively. Next comes the maximum linear polarization. N_B/N_γ represents the ratio of the background photons (shaded areas in figs 1 through 4) to the total photons in the line. Finally, in the ratio I_B/I_γ, I_γ represents the integral of the bremsstrahlung intensity in the dominant line, while I_B is the integral in the remaining part. It gives the ratio of the power radiated in the unwelcome part of the spectrum to the power radiated in the required energy region of the line.

The two ratios N_B/N_γ and I_B/I_γ are significant for determining the influence of the background in two extreme situations.

The following remarks can be made:

1) By examining Table I it results that, if a diamond 0.02 mm thick could be obtained, its spectrum has characteristics which are intermediate between the cases of Beryllium with α = 0 and α = 90°, but with a large polarization (79%) in addition.

The case of Beryllium with α = 74.56° has about the same polarization (80%), but smaller N_γ and larger backgrounds.

2) By comparing the present results with those of ref. (1) for positron annihilation, it results that in the latter case energy resolution can be essentially better than the resolution of our case, intensity of the γ-ray beam being in effect essentially the same. On the other hand coherent bremsstrahlung could have the advantage of a large polarization, this

being zero for positron annihilation. Background are smaller for positron annihilation. The two methods are complementary in the sense that coherent bremsstrahlung works for $k_p < 100$, while positron annihilation works for $k_p > 100$ MeV, the upper limit given by the maximum linac energy.

3) By comparing the present results with those of ref. (2) for Compton scattered laser photons in ADONE, we can say that energy resolution and linear polarization are about the same in the two cases, while backgrounds are much smaller in the latter case (the ratios analogous to N_B/N_γ and I_B/I_γ have the values 10^{-4} and $3 \cdot 10^{-2}$ respectively[4]. Unfortunately the laser power cannot larger than 4 Watt, in order to avoid reduction of the beam life-time in ADONE. Consequently the γ-ray beam has an upper limit of 10^6 photon/sec[4], to be compared with $\sim 10^8$ photon/sec for coherent bremsstrahlung. On the other hand duty cycle for ADONE is essentially better ($1.3 \cdot 10^{-2}$) than the linac duty cycle ($8 \cdot 10^{-4}$).

ACKNOWLEDGMENTS. -

I wish to thank A. Turrin for his collaboration in mathematical considerations.

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(3) - G. Bologna, Nuovo Cimento 49, 756(1967); Errata 50, 756 (1967).
(4) - C. Schaerf, private communication.