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P. Di Vecchia, F. Drago and M. L. Paciello: CONSPIRACY AND FACTORIZATION IN VECTOR MESON PHOTOPRODUCTION. -

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ABSTRACT. -

We deduce the conspiracy relations for reactions of the type

$$\gamma + N \rightarrow V + N$$

In order to study the factorization of the Regge pole residues at $t=0$ we introduce a reasonable hypothesis, suggested by the work of Freedman and Wang in the spin-less case, which allows to continue the Regge representation at $t=0$ for the scattering of particles with spin and unequal masses.

We then show that the residue factorization cannot give any information about the conspiracy or the evasion: we show in fact that the factorization requirements give rise to relations which are not self-consistent.

We suggest that the factorization may fail where the residues are singular.

(x) - Istituto di Fisica dell'Università, Roma.

I. INTRODUCTION. -

Recently the importance of conspiracy relations among scattering amplitudes at $t=0$ has been discussed[1]. The first example of conspiracy relations has been discovered by Goldberger, Grisaru, Mac Dowell and Wong[2] in their classical paper on nucleon-nucleon scattering; the same example has been extensively discussed by Gribov and Volkov[3]. In these early papers constraints between the helicity amplitudes were found by writing down the relations which connect the invariant scalar amplitudes to the helicity amplitudes: in order that spurious singularities are not introduced in the invariant amplitudes, some suitable combinations of kinematic singularity-free helicity amplitudes must vanish in particular kinematical configurations.

In recent papers[4, 5, 6] the problem of conspiracy relations has been discussed in a more general framework starting from the crossing relations[7, 8] between the helicity amplitudes. However a uniform discussion, able to cover in a satisfactory way all the possible kinematical situations, has not yet been given. In fact in the equal mass case the deduction of conspiracy relations at $t=0$ does not give rise to any difficulty and their physical meaning is very clear being linked in a very simple way to the forward angular momentum conservation in the crossed channel. On the contrary the situation is not so clear in the unequal mass case.

In the present paper we discuss some aspects of conspiracy, Regge behaviour and factorization for reactions of the type $\gamma+N\rightarrow V+N$. However we believe that some of our conclusions are of more general validity.

In Sec. II we derive the conspiracy relations at $t=0$ following essentially the method due to Cohen-Tannoudji et al.[6] (see however also ref. (5)). In this way conspiracy relations have been derived for the reactions $\gamma+N\rightarrow\pi+N$[9] and $\pi+N\rightarrow V+N$[10].

In Sec. III we study the Regge representation (i.e. asymptotic behaviour of the Sommerfeld Watson transformed partial wave expansion) for our reactions. The different mass kinematics plays here a crucial role: indeed the variable transformation from t and s to t and $\cos\theta_t$ being singular in this case at $t=0$, any representation of a scattering amplitude in the form $A(t,s)=F(t,\cos\theta_t)$ appears suspicious at $t=0$[11]. This problem has been studied by Freedman and Wang[11] in the spinless case and solved for the Regge representation introducing the "daughter trajectories". We propose, without proof, a reasonable way to extend the Regge representation at $t=0$ in analogy with the Freedman and Wang work.

In Sec. IV we study the consequences of the factorization of the residues at $t=0$ on the sets of reactions $N+\bar{N}\rightarrow\gamma+V$; $\bar{N}+N\rightarrow\bar{N}+N$; $\gamma+V\rightarrow\gamma+V$. We show how the factorization hypothesis leads to internal contradiction at $t = 0$. We give a possible explanation of the factorization breakdown.

In Appendix A we give some kinematical factors involved in the crossing matrix.

II. CONSPIRACY RELATIONS IN $\gamma + N \rightarrow V + N$. -

We shall use the customary notation $f^t_{cd;\,ab}$ to denote a helicity amplitude[12] for the t-channel reaction $a + b \rightarrow c + d$. Helicity amplitudes free from kinematical singularities in s and t must be used in the derivation of conspiracy relations. The first step is to define amplitudes[13, 14] free from kinematical singularities in s:

$$\text{(II. 1)} \qquad \bar{f}^t_{cd\,;\,ab} = \left(\operatorname{sen}\frac{\theta_t}{2}\right)^{-|\lambda-\mu|} \left(\cos\frac{\theta_t}{2}\right)^{-|\lambda+\mu|} f^t_{cd\,;\,ab}$$

where $\lambda = a - b$, $\mu = c - d$.

The works by Hara[15] and Wang[16] show how one can then remove the t kinematical singularities from the amplitudes formed into "parity conserving" combinations. The Wang result can be written

$$\text{(II. 2)} \qquad \bar{f}^t_{cd;\,ab} \pm \bar{f}^t_{-c-d;\,ab} = K^{\pm}_{cd;\,ab}(t)\ \bar{\bar{f}}^t_{cd;\,ab}$$

where $K^{\pm}$ is a known factor containing the kinematical singularities: the behaviour at $t = 0$ of the relevant $K^{\pm}$ is listed in Table I.

The amplitudes $\bar{\bar{f}}_{cd;ab}(s, t)$ being free from kinematical singularities in s and t contain only the dynamic and can be reggeized[13].

The conspiracy relations provide additional kinematic zeros at $t = 0$ in certain linear combinations of the parity-conserving amplitudes.

The derivation of the conspiracy relations is given in some detail in Appendix B. The relations obtained at $t = 0$ are:

$$\text{(II. 3)} \qquad t^{1/2}\left[\bar{f}^t_{-11;\frac{1}{2}\frac{1}{2}} - \bar{f}^t_{1-1;\frac{1}{2}\frac{1}{2}}\right] = -it^{1/2}\left[\bar{f}^t_{-11;\frac{1}{2}-\frac{1}{2}} - \bar{f}^t_{1-1;\frac{1}{2}-\frac{1}{2}}\right]$$

$$\text{(II. 4)} \qquad t^{1/2}\left[\bar{f}^t_{01;\frac{1}{2}\frac{1}{2}} + \bar{f}^t_{0-1;\frac{1}{2}\frac{1}{2}}\right] = -it^{1/2}\left[\bar{f}^t_{01;\frac{1}{2}-\frac{1}{2}} + \bar{f}^t_{0-1;\frac{1}{2}-\frac{1}{2}}\right]$$

$$\text{(II. 5)} \quad t^{1/2}\left[\bar{f}^{t}_{11;\frac{1}{2}\frac{1}{2}} - \bar{f}^{t}_{-1-1;\frac{1}{2}\frac{1}{2}}\right] = -\frac{i}{2}\,t^{1/2}\left[\bar{f}^{t}_{11;\frac{1}{2}\frac{1}{2}} - \bar{f}^{t}_{-1-1;\frac{1}{2}\frac{1}{2}}\right]$$

For all the parity-conserving amplitudes involved here, the Wang kinematical factor K allows a $t^{-1/2}$ behaviour at $t=0$ (see Table I).

TABLE I

	Amplitude	$\lvert\lambda\rvert$	$\lvert\mu\rvert$	behaviour near t = 0	
				$K^{\pm}(t)$	$G_{\lambda\mu}(t)$
a	$f^{t}_{11;\frac{1}{2}\frac{1}{2}} + f^{t}_{-1-1;\frac{1}{2}\frac{1}{2}}$	0	0	1	1
b	$f^{t}_{11;\frac{1}{2}\frac{1}{2}} - f^{t}_{-1-1;\frac{1}{2}\frac{1}{2}}$	0	0	$t^{-1/2}$	1
c	$f^{t}_{11;\frac{1}{2}-\frac{1}{2}} + f^{t}_{-1-1;\frac{1}{2}-\frac{1}{2}}$	1	0	1	$t^{1/2}$
d	$f^{t}_{11;\frac{1}{2}-\frac{1}{2}} - f^{t}_{-1-1;\frac{1}{2}-\frac{1}{2}}$	1	0	$t^{-1/2}$	$t^{1/2}$
e	$f^{t}_{01;\frac{1}{2}+\frac{1}{2}} + f^{t}_{0-1;\frac{1}{2}\frac{1}{2}}$	0	1	$t^{-1/2}$	$t^{1/2}$
f	$f^{t}_{01;\frac{1}{2}+\frac{1}{2}} - f^{t}_{0-1;\frac{1}{2}\frac{1}{2}}$	0	1	1	$t^{1/2}$
g	$f^{t}_{01;\frac{1}{2}-\frac{1}{2}} + f^{t}_{0-1;\frac{1}{2}-\frac{1}{2}}$	1	1	$t^{-1/2}$	$t^{1/2}$
h	$f^{t}_{01;\frac{1}{2}-\frac{1}{2}} - f^{t}_{01;\frac{1}{2}-\frac{1}{2}}$	1	1	1	$t^{1/2}$
i	$f^{t}_{-11;\frac{1}{2}\frac{1}{2}} + f^{t}_{1-1;\frac{1}{2}\frac{1}{2}}$	0	2	1	t
l	$f^{t}_{-11;\frac{1}{2}\frac{1}{2}} - f^{t}_{1-1;\frac{1}{2}\frac{1}{2}}$	0	2	$t^{-1/2}$	t
m	$f^{t}_{-11;\frac{1}{2}-\frac{1}{2}} + f^{t}_{1-1;\frac{1}{2}-\frac{1}{2}}$	1	2	1	t
n	$f^{t}_{-11;\frac{1}{2}-\frac{1}{2}} - f^{t}_{1-1;\frac{1}{2}-\frac{1}{2}}$	1	2	$t^{-1/2}$	t

If only Regge poles contribute to the scattering amplitudes, these relations can be satisfied in two different ways:

1) All the residues of the $\bar{\bar{f}}^t$ involved contain a factor of t and therefore they vanish individually at t = 0 (evasion).

2) Every amplitude $\bar{f}^t$ retains its singular behaviour at t = 0: in this case both sides of the equation must approach to the same constant (conspiracy).

Similar possibilities are open for the contribution of other singularities in the angular momentum complex plane (cuts and fixed poles). The two possibilities are experimentally distinguishable because in the first case (evasion) all the helicity flip amplitudes are suppressed, in the s-channel, for small t near the forward direction, while in the second case (conspiracy) only those amplitudes which do not conserve the angular momentum are suppressed for small t. This feature is not peculiar of the reactions $\gamma + N \rightarrow V + N$, but is also present in other cases(9, 10) and can be discussed with the method presented in Sec. III of ref. (10).

III. THE REGGE REPRESENTATION AT t = 0. -

It is well known(11) that serious difficulties arise for the Regge asymptotic expansion if the masses of the colliding particles are different. Let us quickly examine the problem in the case in which the external particles have spin. The partial wave expansion for the parity conserving amplitudes, free from s kinematical singularities, is given by(13):

(III.1)
$$\bar{f}^t_{cd;ab} \pm \eta_C \eta_D (-1)^{\lambda + M + S_C + S_D - v} \bar{f}^t_{-c-d;ab} = \sum_J 2J+1 \left[e^{J+}_{\lambda\mu}(\cos\theta_t) F^{J\pm}_{cd;ab}(t) + e^{J-}_{\lambda\mu}(\cos\theta_t) F^{J\mp}_{cd;ab}(t) \right]$$

here η means intrinsic parity, S spin, $M = \max(|\lambda|, |\mu|)$, v is 1/2 for half-integral S_C+S_D and 0 for integral S_C+S_D and the $e^{J\pm}(\cos\theta_t)$ functions are defined in ref. (13). We first extract the Wang kinematical factor K(t), then we perform the Sommerfeld-Watson transformation on the previous expansion (III. 1) and we find that, if $|\cos\theta_t| \gg 1$, the contribution of a single Regge pole $\alpha^{\pm}(t)$ to the (III.1) is given by:

(III. 2)
$$\pi \frac{2\alpha^{\pm}(t)+1}{\mathrm{sen}\,\pi\alpha^{\pm}(t)} E^{\alpha\pm}_{\lambda\mu}(\cos\theta_t) \bar{K}^{\pm}_{cd;ab}(t) \beta^{\pm}_{cd;ab}(t) \sim \sim \bar{K}^{\pm}_{cd;ab}(t)\, \gamma^{\pm}_{cd;ab}(t) \left(\frac{S}{S_o}\right)^{\alpha^{\pm} - M}$$

Here and in following we omit the signature factor not essential for our considerations. Introducing suitable powers of cos $(\theta_t/2)$ and sen $(\theta_t/2)$ we get the parity conserving helicity amplitudes:

(III. 3)
$$\bar{f}^{t}_{cd;ab} \pm \eta_C \eta_D (-1)^{\lambda + M + S_C + S_D - v} \bar{f}^{t}_{-c-d;ab} \sim$$
$$\sim \gamma^{\pm}_{cd;ab}(t)\, \bar{K}^{\pm}_{cd;ab}(t)\, G_{\lambda\mu}(t) \left(\frac{S}{S_o}\right)^{\alpha^{\pm}(t)}$$

where the factor $G_{\lambda\mu}(t)$ comes out from the asymptotic expansion, for large s and $t \neq 0$, of the powers of cos $(\theta_t/2)$ and sen $(\theta_t/2)$:

(III. 4)
$$\left(\text{sen}\,\frac{\theta_t}{2}\right)^{|\lambda-\mu|} \left(\cos\frac{\theta_t}{2}\right)^{|\lambda+\mu|} \xrightarrow[s \to \infty]{} G_{\lambda\mu}(t)\, S^M$$

and its behaviour near $t = 0$ is given in Table I for our process.

It is now clear why the Regge representation breaks down at $t = 0$: in fact at $t = 0$ $\cos\theta_t = 0$ (See Appendix A) for any s no matter how large and so the expansion (III. 3) is not valid in the limit $t = 0$. The deep reason of this breakdown must probably be found in the singularity at $t = 0$ of the transformation from the t, s to the t, $\cos\theta_t$ variables[11], rendering suspect the description of the scattering amplitude in terms of the latter variables at $t = 0$.

This difficulty was overcome by Freedman and Wang[11] by introducing the daughter trajectories. Using the Khuri representation these authors showed, in the spin-less case, that the $S^{\alpha(t)}$ behaviour survives at $t = 0$ provided that one introduces for any trajectory $\alpha(t) = \alpha_o(t)$ a family of trajectories of alternating signature satisfying

(III. 5)
$$\alpha_n(o) = \alpha_o(o) - n$$

The extension of the Freedman and Wang work to the case of the scattering of particles with spin, described by the helicity formalism, is not trivial and should be investigated in detail.

We shall however assume without proof that a mechanism similar to that proposed by Freedman and Wang works also in our case so that the asymptotic expansion (III. 3) is correct for any t including $t = 0$. As a consequence of this assumption all the helicity amplitudes have the same s power behaviour also at $t = 0$.

The discussion on the factorization in the next section is based on the representation (III. 3).

IV. REGGE RESIDUES FACTORIZATION. -

We shall suppose in this section, that only moving poles are present in the complex angular momentum.

The reason of studying the factorization requirements is that, as shown by several authors(10, 19, 20), factorization definitely implies additional t-dependence in some cases, which could give some informations about conspiracy or evasion.

First of all let us clearly state what we mean by factorization(17, 18): we assume, as expression of the factorization, that, for the contribution of a single Regge pole, the following relation holds all along the trajectory, except at most some isolated points:

$$\text{(IV.1)} \qquad \left[f^{t(a+b\to c+d)}_{cd;ab} \right]^2 = \left[f^{t(cd\to cd)}_{cd;cd} \right] \left[f^{t(a+b\to a+b)}_{ab;ab} \right]$$

Moreover we shall factorize the parity amplitudes since, for s ∞ in the hypothesis of parity conservation, a given pole will only contribute to one of the two parity conserving amplitudes.

Hence if reaction 1 is $a+b\to a+b$, reaction 2 is $a+b\to c+d$ and reaction 3 is $c+d\to c+d$, the factorization condition can be stated as:

$$\text{(IV.2)} \qquad \left[\gamma_2(t) K_2(t) G_2(t) \right]^2 = \left[\gamma_1(t) K_1(t) G_1(t) \right] \left[\gamma_3(t) K_3(t) G_3(t) \right]$$

The behaviour near $t \sim 0$ of both sides of equation (IV. 2) is given in Table II.

We see that the factorization seems satisfied for the amplitudes a), d), f), h), i), m), while the introduction of extra t factors seems necessary in order to satisfy the factorization requirements for the amplitudes b), c), e), g), 1), n).

However one can prove quite in general that it is impossible, in this case, to satisfy simultaneously all the constraints imposed by factorization introducing an extra integer power of t.

That implies breakdown of the factorization at $t=0$. A possible reason of this breakdown can be the following(x): it is known that in

(x) - We are indebted to Prof. Y. Scrivastava for a discussion about this.

potential scattering the proof of the factorization theorem fails in the point where two trajectories cross one another and there is a double pole[(8)]: but one can look at a double pole as a simple pole with singular residue. Now at $t = 0$ some of our residues are singular and this may suggest that also in the relativistic case the residue factorization holds all along the trajectory, except at most at some isolated points where the residues are singular.

Therefore it is impossible, in our case to derive from factorization any indication about the conspiracy or the evasion.

TABLE II

	Behavior near $t = 0$ $\left[\gamma_2(t)\, K_2(t)\, G_2(t)\right]^2$	Behavior near $t = 0$ $\left[\gamma_1(t)\, K_1(t)\, G_1(t)\right] \left[\gamma_3(t)\, K_3(t)\, G_3(t)\right]$	Factorization satisfied?
a	1	1	Yes
b	t^{-1}	1	No
c	t	1	No
d	1	1	Yes
e	1	t	No
f	t	t	Yes
g	1	t	No
h	t	t	Yes
i	t^2	t^2	Yes
l	t	t^2	No
m	t^2	t^2	Yes
n	t	t^2	No

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APPENDIX A. -

We collect in this appendix some useful kinematical formulas. We consider the t-channel reaction $\bar{p}+p \rightarrow V+\gamma$ $(a+b \rightarrow c+d)$ and the s-channel reaction $\gamma+p \rightarrow V+p$ $(D+b \rightarrow c+A)$; the crossing matrix [7,8] is given by

$$f^s_{cA;Db} = \sum_{a'b'c'd'} d^{J_a}_{a'A}(\chi_a)\, d^{J_b}_{b'b}(\chi_b)\, d^{J_c}_{c'c}(\chi_c)\, d^{J_d}_{d'D}\, f^t_{c'd';a'b'} \tag{A.1}$$

In our case $J_a=J_b=1/2$; $J_c=J_d=1$. Moreover, owing to the zero mass of the photon, it may easily be shown[21] that $\cos\chi_d=-1$, so that $d^1_{d'D}(\pi)=\delta_{d',-D}$ and the crossing matrix reduces to

$$f^s_{cA,Db} = \sum_{a'b'c'} d^{1/2}_{a'A}(\chi_a)\, d^{1/2}_{b'b}(\chi_b)\, d^1_{c'c}(\chi_c)\, f^t_{c'-D;a'b'} \tag{A.2}$$

The crossing angles are explicitely given by

$$\cos\chi_a = \frac{-t(s+M^2-\mu^2)-2\mu^2M^2}{\sqrt{\left[s-(M-\mu)^2\right]\left[s-(M+\mu)^2\right]}\sqrt{t(t-4M^2)}} \tag{A.3}$$

$$\cos\chi_b = \frac{t(s+M^2)-2\mu^2M^2}{(s-M^2)\sqrt{t(t-4M^2)}} \tag{A.4}$$

$$\cos\chi_c = \frac{(t+\mu^2)(s+\mu^2-M^2)-2\mu^4}{\sqrt{\left[s-(M-\mu)^2\right]\left[s-(M+\mu)^2\right]}\,(t-\mu^2)} \tag{A.5}$$

where M = nucleon mass, μ = vector meson mass.

Another useful formula is the following:

$$\cos\theta_t = \frac{2st+t^2-t(2M^2+\mu^2)}{(t-\mu^2)\sqrt{t(t-4M^2)}} \tag{A.6}$$

APPENDIX B. -

The method used for deriving the conspiracy relations given in Sec. II is due to Cohen-Tannoudji, Morel and Navelet[6]. If $\bar{\bar{f}}^t$ is a parity conserving helicity amplitude, free from s and t kinematical singularities, the crossing matrix may be written in the form

$$\bar{f}_i^s = \sum_j \bar{\bar{M}}_{ij} \bar{\bar{f}}_j^t \tag{B.1}$$

where $\bar{f}^s$ is free from t-kinematical singularities.

If all the masses involved are not unequal, near $t=0$ the matrix elements of $\bar{\bar{M}}$ have the form:

$$\bar{\bar{M}}_{ij} = \frac{C_{ij}}{(t)^x} + \quad \text{(terms regular at } t=0) \tag{B.2}$$

Thus the relations

$$\sum_j C_{ij} \bar{\bar{f}}_j^t = 0 \tag{B.3}$$

must generally hold at $t=0$, since neither the $\bar{f}_i^s$ nor the $\bar{\bar{f}}_j^t$ have kinematical singularities at $t=0$. The Eqs. (B.3) are the desired conspiracy relations.

In the case of the reaction $\gamma + N \rightarrow V + N$ the singularity of the elements of the crossing matrix $\bar{\bar{M}}$ at $t=0$ is of the type of $1/t$: $1/t^{1/2}$ comes from the $d^{1/2}_{aA}(\chi_a)$ and $d^{1/2}_{bB}(\chi_b)$ (see Appendix A) and $1/t^{1/2}$ comes from the kinematical K factor (see Table I). Writing down explicitely the relations (B.3) we find the following equations:

$$\begin{aligned}
&\frac{1}{4}\left(\cos\frac{\chi_c}{2}\right)^2 A + \frac{\operatorname{sen}\chi_c}{2\sqrt{2}} B + \left(\operatorname{sen}\frac{\chi_c}{2}\right)^2 C = 0 \\
&\frac{1}{4}\left(\operatorname{sen}\frac{\chi_c}{2}\right)^2 A - \frac{\operatorname{sen}\chi_c}{2\sqrt{2}} B + \left(\cos\frac{\chi_c}{2}\right)^2 C = 0 \\
&-\frac{1}{8}\operatorname{sen}\chi_c A + \frac{1}{2}\cos\chi_c B + \frac{\operatorname{sen}\chi_c}{\sqrt{2}} C = 0 \\
&-\frac{1}{4\sqrt{2}}\operatorname{sen}\chi_c A + \frac{1}{2}\cos\chi_c B + \frac{1}{2}\operatorname{sen}\chi_c C = 0
\end{aligned} \tag{B.4}$$

where:

$$A = \left[-\bar{f}^{t}_{1-1;\frac{1}{2}\frac{1}{2}} - i\bar{f}^{t}_{1-1;\frac{1}{2}-\frac{1}{2}} - i\bar{f}^{t}_{1-1;-\frac{1}{2}\frac{1}{2}} + \bar{f}^{t}_{1-1;-\frac{1}{2}-\frac{1}{2}} \right] t^{1/2}$$

(B. 5) $$B = \left[-\bar{f}^{t}_{0-1;\frac{1}{2}\frac{1}{2}} + i\bar{f}^{t}_{0-1;-\frac{1}{2}\frac{1}{2}} + i\bar{f}^{t}_{0-1;-\frac{1}{2}\frac{1}{2}} - \bar{f}^{t}_{0-1;-\frac{1}{2}-\frac{1}{2}} \right] t^{1/2}$$

$$C = \left[\frac{i}{2}\bar{f}^{t}_{11;-\frac{1}{2}\frac{1}{2}} + \frac{i}{2}\bar{f}^{t}_{11;\frac{1}{2}-\frac{1}{2}} - \bar{f}^{t}_{11;-\frac{1}{2}-\frac{1}{2}} + \bar{f}^{t}_{11;\frac{1}{2}\frac{1}{2}} \right] t^{1/2}$$

One can easily show that the unique solution of the system (B. 4) is

(B. 6) $$A = O \quad ; \quad B = O \quad ; \quad C = O$$

and these are precisely the conspiracy relations given in Sec. II.

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