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G. De Franceschi, F. Guerra, V. Silvestrini and F. Vanoli:
THE WIDTH $\Gamma_{X_0 \to 2\gamma}$ AS A TEST OF THE MASS FORMULA FOR BOSON NONETS. -

(Nota interna: n. 376)

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G. De Franceschi, F. Guerra[x], V. Silvestrini and F. Vanoli[x]:
THE WIDTH $\Gamma_{X_0 \to 2\gamma}$ AS A TEST OF THE MASS FORMULA FOR BOSON NONETS. -

According to a recent experiment[1], the width of the decay $\eta \to 2\gamma$ is $\Gamma_{\eta \to 2\gamma} = (1.21 \pm 0.26)$ KeV.

We will make some simple consideration on this result. We put[2]

$$(1)\qquad \begin{aligned} X_o &= \eta_1 \cos\alpha + \eta_8 \sin\alpha \\ \eta &= -\eta_1 \sin\alpha + \eta_8 \cos\alpha . \end{aligned}$$

SU_3 then predicts[3]:

$$(2)\qquad \Gamma_{\eta \to 2\gamma} = \frac{1}{3}\left(\frac{m_\eta}{m_{\pi^o}}\right)^3 \left(\cos\alpha - \frac{A'}{M'}\sin\alpha\right)^2 \Gamma_{\pi^o \to 2\gamma}$$

$$(3)\qquad \Gamma_{X_o \to 2\gamma} = \frac{1}{3}\left(\frac{m_{X_o}}{m_{\pi^o}}\right)^3 \left(\sin\alpha + \frac{A'}{M'}\cos\alpha\right)^2 \Gamma_{\pi^o \to 2\gamma}$$

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where A' is the amplitude $\eta_1 \to 2\gamma$ and M' the amplitude $\eta_8 \to 2\gamma$.

The mixing angle α can be deduced[2,4,5] from the known masses of the pseudoscalar nonet. Both hypotheses will be left open, that a quadratic or a linear mass formula holds for the boson nonet.

We get :

$$\alpha_{sq} = 10^{\circ}.4 \qquad\qquad \alpha_{lin} = 23^{\circ}.4 \ .$$

We now use relation (2) to evaluate the A'/M' ratio :

$$\left.\begin{array}{lll} (4) & A'/M' = 22.1 \pm 2.6 \\ (5) & A'/M' = -11.2 \pm 1.3 \end{array}\right\} \text{ using } \alpha_{sq} = 10^{\circ}.4$$

and

$$\left.\begin{array}{lll} (6) & A'/M' = 9.9 \pm 1.2 \\ (7) & A'/M' = -5.3 \pm 0.6 \end{array}\right\} \text{ using } \alpha_{lin} = 23^{\circ}.4$$

We have used $\Gamma_{\pi^0 \to 2\gamma} = (6 \pm 0.6)$ eV[6].

The values (4) to (7) can be used as input to (3) to get the width $\Gamma_{X_0 \to 2\gamma}$.

We obtain :

$$\begin{array}{ll} (8) & \\ (9) & \\ (10) & \\ (11) & \end{array} \Gamma_{X_0 \to \gamma\gamma} = \begin{cases} (343 \pm 90) \text{ keV} & (A'/M' = 22.1 ; \ \alpha_{sq} = 10^{\circ}.4) \\ (84 \pm 22) \text{ keV} & (A'/M' = -11.2 ; \ \alpha_{sq} = 10^{\circ}.4) \\ (64 \pm 16) \text{ keV} & (A'/M' = 9.9 ; \ \alpha_{lin} = 23^{\circ}.4) \\ (14 \pm 4) \text{ keV} & (A'/M' = -5.3 ; \ \alpha_{lin} = 23^{\circ}.4) . \end{cases}$$

Experimentally, one has not much information to compare with.

We know that :

$$\Gamma_{X_0 \to \text{all modes}} < 4 \text{ MeV}^{(7)}$$

$$\Gamma_{X_0 \to 2\gamma} < 0.15 (\Gamma_{X_0 \to \text{all modes}})^{(8)} < 600 \text{ keV}$$

(12) $$\Gamma_{X_0 \to \rho\gamma} < 1 \text{ MeV}^{(7)}$$

(13) $$\Gamma_{X_o \to \text{neutrals}} = (0.26 \pm 0.04)\,\Gamma_{X_o \to \text{all modes}}{}^{(9)}$$

(14) $$\Gamma_{X_o \to \pi^+\pi^-\eta} = (0.48 \pm 0.05)\,\Gamma_{X_o \to \text{all modes}}{}^{(9)} .$$

The upper experimental limit $\Gamma_{X_o \to 2\gamma} < 600$ keV is consistent with all the solutions (8) to (11).

A somewhat lower figure can be obtained using isotopic spin invariance and $(\Gamma_{\eta \to \text{neutrals}})/(\Gamma_{\eta \to \text{all modes}}) = 0.73$.

From (14) one predicts

$$\Gamma_{X_o \to \pi^o\pi^o\eta\,(\eta \to \text{neutrals})} = 0.18\,\Gamma_{X_o \to \text{all modes}} .$$

Comparing with (13) we see that about 8 % of the total width is allowed for the $X_o \to 2\gamma$ decay mode.

Hence $\Gamma_{X_o \to 2\gamma} \lesssim 400$ keV, which again is consistent with all values (8) - (11).

Up to now we used only SU_3 and/or experimental data.

To be able to make a choice between our different solutions (4) to (7), we try to use the pole dominance model of Gell-Mann, Sharp and Wagner (GSW)[10].

In this model, the following relations hold[3, 11]:

(15) $$\frac{\Gamma_{X_o \to 2\gamma}}{\Gamma_{X_o \to \rho\gamma}} \approx 0.1 \left(\frac{1 + \frac{M'}{A'} \operatorname{tg}\alpha}{1 + \frac{M}{A} \operatorname{tg}\alpha} \right)^2$$

(16) $$\frac{\Gamma_{\eta \to \pi^+\pi^-\gamma}}{\Gamma_{\eta \to 2\gamma}} = 0.233 \left(\frac{1 - \frac{A}{M} \operatorname{tg}\alpha}{1 - \frac{A'}{M'} \operatorname{tg}\alpha} \right)^2$$

(17) $$\Gamma_{\eta \to 2\gamma} = 0.67 \times 10^{-3} \left(\cos\alpha - \frac{A'}{M'} \sin\alpha\right)^2 \Gamma_{\omega \to \pi^o\gamma}$$

Here A is the amplitude for $\eta_1 \to \rho\gamma$ and M that for $\eta_8 \to \rho\gamma$. As a consequence of the model $2(A/M) = (A'/M')^{(3)}$.

Using (12) and (15), we get $\Gamma_{X_o \to 2\gamma} \lesssim 100$ keV, quite indipendently of the values inserted for α and A'/M'. On this bases, solution (8) should be rejected.

From (16), with the different solutions (4) to (7), we find :

$$\frac{\Gamma_{\eta\to\pi^+\pi^-\gamma}}{\Gamma_{\eta\to 2\gamma}} = \begin{cases} 0.026 & \text{using (4)} \\ 0.10 & \text{using (5)} \\ 0.028 & \text{using (6)} \\ 0.10 & \text{using (7)} \end{cases}$$

A comparison with the experimental value[7]

$$\frac{\Gamma_{\eta\to\pi^+\pi^-\gamma}}{\Gamma_{\eta\to 2\gamma}} = 0.15 \pm 0.03 ,$$

shows that solutions (4) and (6) do not fit in the model.

However, on the basis of the (GSW) model one can also obtain the prediction

$$\frac{\Gamma_{\omega\to\pi\gamma}}{\Gamma_{\pi^0\to 2\gamma}} = 3.3 \times 10^4 ,$$

indipendently of $\Gamma_{\eta\to 2\gamma}$, A'/M' and α (e. g. from the formulas (2) and (17), reported above).

Considering that the experimental value is

$$\frac{\Gamma_{\omega\to\pi\gamma}}{\Gamma_{\pi^0\to 2\gamma}} = (19.2 \pm 3.5) \times 10^4 ,$$

one must conclude that the (GSW) model does not work satisfactorily in this case. This can cast some doubts on the reliability of our previous results based on (GSW). Some other evidence that solution (4) should be rejected can be obtained if one believes in the current aestimates[3, 12] $\Gamma_{X_0 (\text{total})} < 1$ MeV. In fact, using our previous result

$$\Gamma_{X_0\to 2\gamma} / \Gamma_{X_0\to \text{all modes}} < 8 - 10\%,$$

one would obtain $\Gamma_{X_0\to 2\gamma} < 100$ keV.

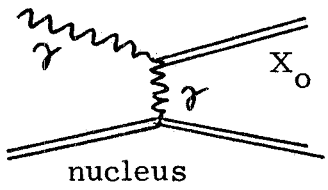
To summarize :

- solution (4) is probably to be rejected;
- the width $\Gamma_{X_0\to 2\gamma}$ is in the range 10 - 100 keV;
- there is some reason to exclude solution (6);

- there is not enough information to discriminate between a quadratic and a linear mass formula for the boson nonets.

In view of this it seems to us that a measurement of the width $\Gamma_{X_o \to \text{total}}$, or better $\Gamma_{X_o \to 2\gamma}$, is highly desirable, since an upper bound like $\Gamma_{X_o \to 2\gamma} \lesssim 20$ keV, would provide definite evidence against the use of the quadratic mass formula.

With the same experimental method used to determine the $\Gamma_{\eta \to 2\gamma}$ width, the "Primakoff effect"[1, 13], an upper limit of 20 keV for $\Gamma_{X_o \to 2\gamma}$ should not be out of experimental possibilities. For $\Gamma_{X_o \to 2\gamma} = 20$ keV, the production cross-section according to



would be ~ 3 time larger than for η production.

The other experimentally well known nonet is the 1^-; for this nonet, the use of a linear rather than quadratic mass formula would change the mixing angle of $\sim 2^o$; an effect which is up to now outside the possibility of an experimental check.

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