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A. Turrin: EXTRACTION OF ELECTRON BEAMS FROM ELECTRON SYNCHROTRONS. -

(Nota interna: n. 367)

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(Review Report presented at the "International Conference on Electromagnetic Interactions at Low and Intermediate Energies, Dubna, February 7-15, 1967")

Introduction

In recent years there has been increasing interest in obtaining from Electron Synchrotrons external electron bursts whose duration could be controlled over a wide range, say from the duration of the order of one particle revolution period up to a few milliseconds.

In this report we shall review very briefly the most advantageous methods in use today for producing these bursts out of the Machines. Moreover, in order to make our survey as complete as possible, we shall also include here the results achieved with the Proton Synchrotrons, considering that all the extraction processes described below seem well applicable to Synchrotrons both for electrons and protons.

We shall distinguish between three ejection schemes:

i) Short burst techniques: the particles are ejected during a single revolution by a suitable pulsed beam deflector.

ii) Linac-like burst techniques: the particles are ejected during a few revolutions by exciting a linear resonance between the radial betatron oscillations and the revolutions.

iii) Long burst techniques: the particles are ejected during several thousand revolutions by exciting a non linear resonance between the radial betatron oscillations and the revolutions.

Anyway, the extraction efficiency can reach very high values (above 90%), and external beams can be produced having practically the same optical properties as the circulating beam.

1. Short burst techniques

The fast scheme, originally proposed for Betatrons, consists basically[1] of removing the beam from the Accelerator by means of a pulsed deflecting magnet ("kicker magnet") acting in a region of small azimuthal width (fig. 1). In the first proposal[1] the beam is made to spiral slowly outwards right up to the edge of the focusing region. After the orbit has been expanded, the kicker magnet is fully energized in a time interval shorter than the revolution period.

The fast rise in time requirement of the magnetic field in the kicker magnet leads to many difficult technological problems; therefore, such a deflecting device is placed at the edge of the focusing region, in order to reduce the deflection angle to a value as small as possible.

This ejection mechanism has been considered by several authors[2-12] and has been increasingly improved in recent years[10-12].

A considerable reduction in size of the deflection angle required is possible by first producing a distortion ("bump") of the equilibrium orbit[12], so that the beam can be deflected more easily into the aperture of a final extraction magnet with septum (fig. 2).

Single bunch ejection has been achieved, and 19 bunches out of the 20 circulating can be extracted[12] from the CERN Proton Synchrotron, leaving the remaining circulating bunches still unaffected, by exciting the kicker magnet with "rectangular" pulses having rise and fall times shorter than the time interval between the passage of two particle bunches.

2. Linac-like burst techniques

The first regenerative deflection system - a narrow peeler - was operating[13] for a Betatron, and essentially the same magnetic discontinuity was first proposed[15] to extract the beam from the Brookhaven Cosmotron, at the time when this Machine was under construction. Meanwhile, the use of a first harmonic field inhomogeneity was proposed[14] for the 300 MeV Electron Synchrotron at the MIT, and the effect of such a perturbation has been studied[19] for application to UCRL's 320 MeV Electron Synchrotron. These two methods appear to be basically different, but it was shown[18] that there is no essential difference between them so far as the resonant condition attained for the radial betatron oscillations is concerned.

To prove this, let us first of all consider briefly what happens in a Synchrocyclotron when the particles are captured into a motion through a regenerative deflecting structure[16,17].

In an unperturbed Synchrocyclotron, the Q_r value (that is, the number of radial betatron oscillations per revolution executed by the particle about the circular synchronous orbit) is slightly less than 1, because of the very small field index "n" value of the guide field. The betatron oscillations of the individual particles are notoriously not coherent, so that all possible phases of oscillation exist within the circulating beam.

The regenerative deflector consists (fig. 3a) of two narrow and strong perturbations of the field gradient, spaced about 90^o in azimuth, called "peeler" (where $n \gg 1$) and "regenerator" (where $n \ll -1$), so that the beam passes alternatively - as in an alternating gradient system - through strong defocusing and focusing lenses, and is deflected alternatively outward and inward.

The whole system is designed to build up the radial oscillations and to deliver them to a magnetic channel, maintaining limited at the same time the vertical oscillation amplitude.

As a particle passes through the regenerative system, a phase change is introduced in the radial oscillations (fig. 3a), so that the frequency of all oscillations becomes closer and closer to value 1, and the field gradient pattern in these asymptotic conditions repeats itself identically in each turn. Another consequence of the peeler-regenerator action consists in an increase in amplitude of the radial oscillations; this action, repeated at the same phase of the oscillatory motion in successive revolutions, results in a rapid resonant build-up of amplitude.

If non linearities of the field are everywhere negli gible, the simple matrix formalism[17] permits one to study the regenerative process: It is found that after h revolutions, the radial oscillation $x(\Theta)$ in the unperturbed azimuthal gaps behaves, in first approximation, as follows:

1) $$x(\Theta) = A \exp(h\lambda) \sin(Q_r\Theta+\Phi)+B \exp(-h\lambda)\sin(Q_r\Theta - \Phi)$$

$$(Q_r = \sqrt{1-n} \quad , \quad 0 \leqslant \Theta \leqslant 2\pi),$$

where A and B depend on the initial conditions (λ and Φ are functions of the system parameters only).

In addition, it has been shown[18] that this resonance in Synchrocyclotrons is driven essentially by the second harmonic component of such an azimuthally variable field index.

Fig. 3b shows the distributed action of this Fourier component: the periodicity of the oscillations shrinks to the value 2π, while the amplitude grows continuously. Under these circumstances, the first approximation solution of the radial motion equation is

2) $$x(\Theta) = A \exp(\alpha\Theta) \sin(\Theta+\varphi) + B \exp(-\alpha\Theta) \sin(\Theta - \varphi)$$

$$(0 \leqslant \Theta \leqslant \infty),$$

where again α and φ are functions of the system parameters only. Eq. 2) is equivalent to Eq. 1) because $Q_r \cong 1$.

Similar conclusions hold also for Synchrotrons: if the field index n value is near to 3/4, the Q_r value is close to 1/2; for this case the behaviour of the radial oscillations in the presence of a strong and narrow peeler[21] is shown in fig. 4a; equation 1) holds. The harmonic component of such an azimuthally variable field index responsible for the resonant build up of the amplitude is the first one[18] (fig. 4b). Under these half--integer resonance conditions, the first approximation solution of the radial motion equation is

3) $$x(\Theta) = A \exp(\alpha\Theta) \sin(\frac{1}{2}\Theta + \varphi) + B \exp(-\alpha\Theta) \sin(\frac{1}{2}\Theta - \varphi),$$

$$(0 \leqslant \Theta \leqslant \infty)$$

equivalent to eq. 1) for $Q_r \cong 1/2$.

Finally, it has been pointed out[19] that a first harmonic variation in the guide field in Synchrotrons causes not only the addition of a forcing term (far from the resonance) in the equation of motion: when $Q_r \cong 1/2$ build-up of amplitudes can also occur, thanks to the presence of a first harmonic variation in the linear restoring term. Omitting the forcing term which cannot contribute to amplitude growth, the resulting (Mathieu type) equation becomes

4) $$\frac{d^2x}{d\Theta^2} + \left[Q_r^2 - \varepsilon \cos\Theta\right] x = 0,$$

whose first order solution, when $Q_r \cong 1/2$, is just eq. 3).

The use of a first harmonic component in the guide field was initially proposed[19] with a view to obtaining short bursts from UCRL's 320 MeV Electron Synchrotron, and a first harmonic perturbation of the field gradient was successfully employed[20,22] for resonant fast ejection from the 70 MeV Iowa State College Electron Synchrotron. A similar system is used[24] for the Synchrophasotron at Dubna.

The extension[21,23] of this analytical treatment to cover the cases of both the Racetracks and Alternating Gradient Machines is quite simple, if the smooth approximation technique is adopted. Suppose the Q_r value approaches m/2 , with m integer, either odd or even. A linear half-integer resonance occurs when a perturbation in the field gradients lattice is introduced, periodic in azimuth, having the form $\varepsilon \cos m\Theta$. The equation of the radial betatron oscillations is

$$5) \qquad \frac{d^2x}{d\Theta^2} + \left[Q_r^2 - \varepsilon \cos m\Theta \right] x = 0 \qquad (Q_r \cong m/2),$$

and the first order solution of this Mathieu equation is

$$6) \qquad x(\Theta) = A \exp(\alpha\Theta) \sin(\frac{m}{2}\Theta + \varphi) + B \exp(-\alpha\Theta) \sin(\frac{m}{2}\Theta - \varphi).$$

Cases m = 1 and m = 2 belong to the weak focusing Synchrotrons and to the Synchrocyclotrons, respectively.

At this point it is of interest to note the essential features of the above described half-integer resonance ejection scheme:

a) The oscillations tend toward a constant phase value determined only by the physical parameters inherent in the system.

b) The oscillations grow exponentially, whatever the initial amplitude value. This means that, as soon as the perturbation is introduced, all oscillations become unstable simultaneously.

c) If m is odd, there is a unique stationary behaviour of oscillations, represented by the first term of equation 6) (see also figs. 4a, 4b). If m is even, we see that there are two distinct stationary oscillations opposite in phase, depending on whether the arbitrary constant A is positive or negative.

A single quadrupole lens[21], placed in a straight section of a weak focusing or strong focusing Machine, may be used to bring the oscillations to this linear half-integer resonance: the zero-th Fourier harmonic of the quadrupole strength acts in changing the Q_r value up to approaching m/2, while the perturbation $\varepsilon \cos m\Theta$ is created by their m th harmonic.

3. Long burst techniques

Long burst ejection is possible by setting up a controlled non linear resonance condition[25,26].

For the sake of simplicity and clearness we shall confine ourselves only to the radial motion.

Non linear resonances[27-33] occur when the perturbations introduced into the field gradients lattice are not only periodic in azimuth, but also depend on the displacement from the equilibrium orbit; the resulting motion equation is therefore a non linear one.

In the smooth approximation, where sinusoidal betatron oscillations are assumed, the equations of motion describing such a resonant situation take the forms

7) $$\frac{d^2x}{d\Theta^2} + Q_r^2 x = \eta_\ell x^\ell \cos(m\Theta),$$

where $Q_r \cong m/p$ (m and p integers)

$$2 \leqslant \ell \begin{cases} \text{even, if p is odd} \\ \text{odd, if p is even} \end{cases}.$$

For investigating the behaviour of the oscillations one may adopt the well-known Krylov and Bogoliubov linearization procedure. The results of such an analysis are the following:

a) If the amplitude is sufficiently small, stability of motion is maintained.

b) If the amplitude exceeds a limiting value (determined by the distance of Q_r from the resonance m/p value and by the perturbation strength η_ℓ) the motion becomes unstable. Once an oscillation becomes unstable, growth rates become larger than may be achieved by making use of a linear resonance, and the phases of each oscillation tend towards a common asymptotic behaviour.

Slow spill operation can therefore be accomplished as follows: after the beam has reached the desired energy, the magnetic perturbation is introduced, but all the particle oscillations remain limited since their amplitudes are sufficiently small.

The critical amplitude value (the separatrix between stable and unstable solutions) can then be caused to decrease slowly to zero by gradually approaching the Q_r value to m/p, so that the particles are squeezed out sequentially and the spill-out can be spread over a long time.

Let us now first consider how the problem of obtaining a slow external beam has been solved at the CERN PS[34] (fig. 5). For such a Synchrotron the Q_r value is shifted near the 6 resonance (m = 6 and p = 1 in eq. 7) by means of a quadrupole lens, and the perturbation is produced by six of the existing sextupole lenses distributed around the ring, and excited to give the required sixth harmonic (m = 6) azimuthal variation of the sextupole moment (l = 2 in eq. 7). Besides, the sextupole perturbation introduces a variation of Q_r with radius across the gap. Thus, at the end of the acceleration cycle, with the quadrupole and sextupole perturbations turned on and the R. F. off, the beam spirals smoothly outwards ($\dot{B} < 0$) towards the resonance radius ($Q_r \rightarrow 6$). It should be emphasized here that this non-linear integer (p = 1) resonance may be generated only by a single lens containing both a quadrupole and a sextupole field component: such a magnetic discontinuity contains all the azimuthal harmonics, so that by tuning the Q_r value to 6, the resonance condition is fulfilled by the term $\eta_2 x^2 \cos(6\Theta)$.

In fact, a single non linear element (a Robinson current strip[37]) has been employed to eject the electron beam from the CEA[35], and the same non linear perturbation is used for the DESY[36]. The resonance chosen is the half-integer one at $Q_r = 6+1/2$ (m = 13 and p = 2 in eq. 7). The field of an octupole lens (l = 3 in eq. 7) may be able to create the resonance situation (fig. 6). However, both the shift of the betatron oscillation frequency Q_r to the appropriate value 13/2 and the 13 th harmonic variation of the octupole moment requirements are simultaneously satisfied by the magnetic field shape of the current strip (fig. 7).

The strip, placed inside the equilibrium orbit in a straight section, acts as a regenerative (i. e. focusing) element. Because of the strong non linearity created by the strip, deflection of a particle is considerable only if its displacement from the equilibrium orbit is inward.

As long as the equilibrium orbit is close to the central orbit, Q_r is far from the resonance value and the betatron oscillation amplitudes are sufficiently small. Therefore, the perturbing element is unable to bring the betatron oscillations into resonance. When the electrons are caused to fall out of synchronism with the R.F. system (by slowly reducing the R.F. peak voltage) their equilibrium orbits contract, so that the strip action becomes increasingly more powerful: when the equilibrium orbit contracts due to energy loss, Q_r tends toward the resonance value, and the "o c t u p o l e" perturbation causes the betatron oscillations to grow rapidly, as soon as instability conditions are reached.

Another way of accomplishing the slow approach to the strip field in the CEA consists in producing a distortion (bump) of the equilibrium orbit[38]. It should be noted that there exist relationships which make possible the calculation of the distorted orbit as a function of the guide field disturbance also for the 4-sector weak focusing Synchrotrons[39].

Third integer non linear resonances (p = 3 in eq. 7) are employed for slow ejection from the weak focusing Frascati Electron Synchrotron[25,40,41] and the weak focusing PPA[42-44] (fig. 8), as well as from the Brookhaven AGS[45] (fig. 9).

At the Frascati Electron Synchrotron a second harmonic (m = 2 in eq. 7) in the radial gradient of the field index (l = 2 in eq. 7) is introduced to excite the oscillations. Since the radius where Q_r approach 2/3 is inside, the electrons are extracted by allowing the beam to spiral inward.

The same $Q_r = 2/3$ resonance was successfully[44] employed for ejection from the PPA. Both the tuning of Q_r to this resonance and the production of a second harmonic pattern of the sextupole moment, which is required to excite the oscillations, are accomplished with four current-sheet magnets(modified sextupole magnets), spaced 90° apart, and placed in short straight sections around the Accelerator. The two combined functions (i.e. those of a quadrupole perturbation and of a sextupole perturbation periodic in azimuth) are simultaneously carried out by a suitable current-flow configuration in these four current-sheet magnets. Moreover, the amplitude of unstable betatron oscillations is caused to grow in an exponential manner[25] by softening the sextupole field component to a constant gradient field at the edges (fig. 10). Thus, once the oscillations amplitude is large, the resonance is driven by the nonlinear term $\eta |x| \cos 2\theta$

For such a Synchrotron the procedure for slow extraction consist in turning on the "sextupole" field component, then in adjusting the quadrupole field component to gradually approach the resonance.

The ejection system recently proposed for the Brookhaven AGS employs the 8+2/3 resonance (m = 26 in eq. 7). Four sextupoles (l = 2 in eq. 7), placed symmetrically in four straight sections around the Machine are energized with alternate polarities to provide the 26 th azimuthal harmonic in the sextupole perturbation (fig. 9). Since at all energies the radius where Q_r is equal to 8+2/3 remains close to the centre of the aperture, no quadrupole adjustments are required.

4. Conclusion

We have seen that the few slow ejection systems existing today are based on integer, half-integer, or third-integer non linear resonances. At present it would be very difficult to compare the merits of each scheme, since they are subject to rapid variations and improvements[46]. However, the systems mentioned above may facilitate the choice of the most appropriate set of linear and non-linear elements for every Synchrotron. In practice, when designing the proper non-linear perturbations, there may exist difficulties on account of the small number and short length of the straight sections available, or even owing to the limited current-carrying capacity of the existing pole face windings on the Synchrotron magnet, or, else, due to problems connected with the stability of the vertical betatron motion.

Anyway, two sextupoles, placed at the opposite end of a diameter of the ring, and excited with appropriate polarities to produce the right Fourier harmonics, would in principle be sufficient for ejecting the beam from all the existing strong and weak focusing Synchrotrons working in the neighbourhood of a third-integer resonance.

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replace $\mathfrak{F}_1(\theta) = \ldots + \ldots$ by $\mathfrak{F}_1(\theta) = \ldots\ldots - \ldots\ldots$
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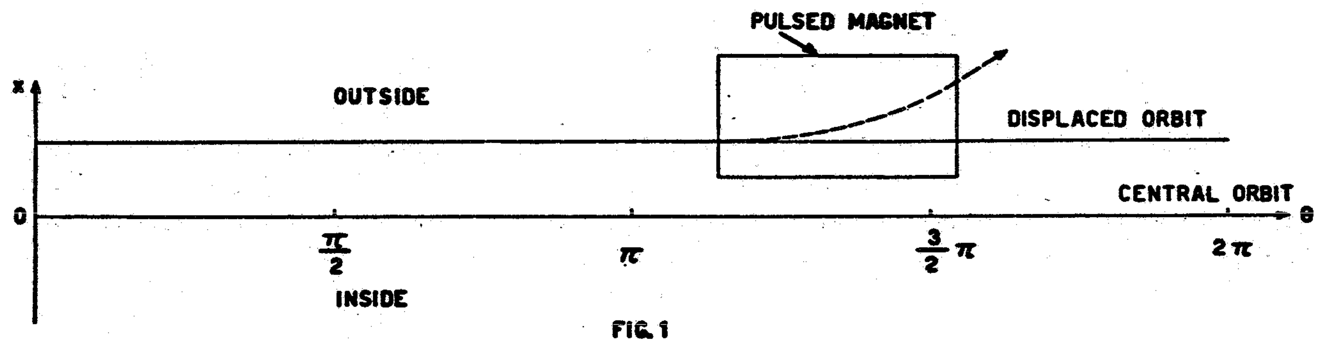

FIG. 1

FIG. 1 - Sketch of early fast ejection systems.

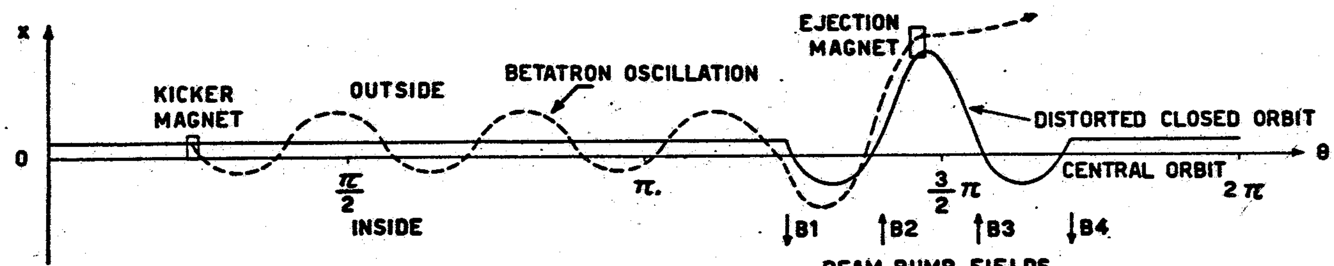

FIG. 2

FIG. 2 - Fast ejection scheme for the CPS (B. Kuiper and G. Plass[12]).

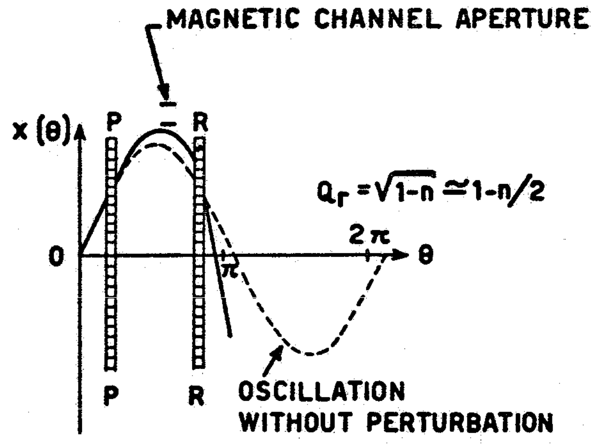

FIG. 3a - Basic ejection scheme from Synchrocyclotrons using regenerative deflection.

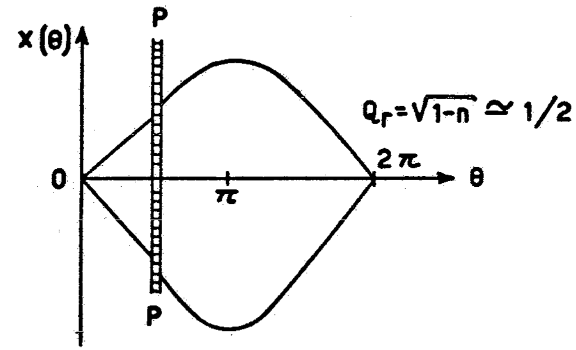

FIG. 4a) - Regenerative deflection in Circular Synchrotrons. Particles go alternately to large and small radii on successive revolutions.

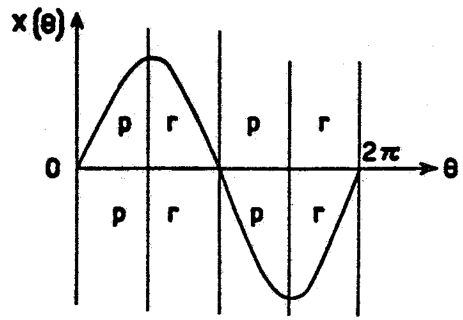

FIG. 3b - Increase of radial betatron oscillation amplitude in Synchrocyclotrons due to the presence of a second harmonic component in the field gradient inhomogeneity as represented in fig. 3a.

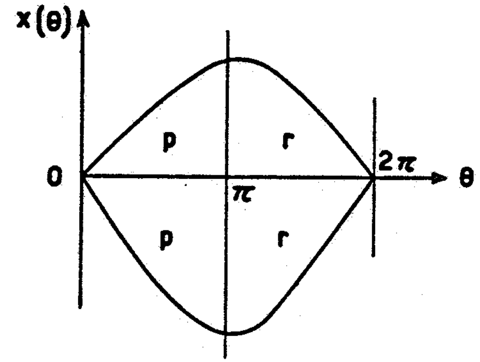

FIG. 4b - Increase of radial betatron oscillation amplitude in Synchrotrons due to the presence of a first harmonic component in the field gradient inhomogeneity as represented in fig. 4a.

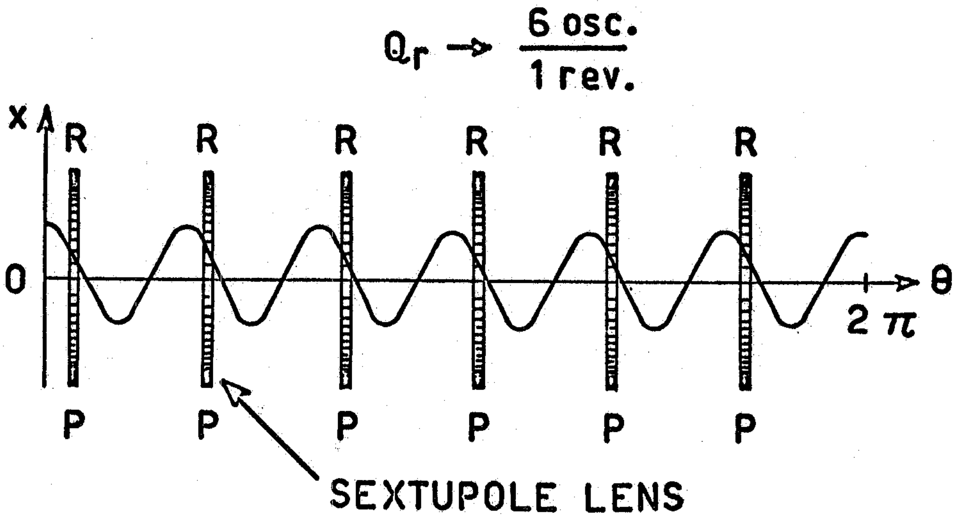

FIG. 5 - A particle oscillation in CPS during slow ejection (schematic, magnification of amplitude not represented) under the influence of an azimuthal sixth-harmonic in the sextupole perturbation.

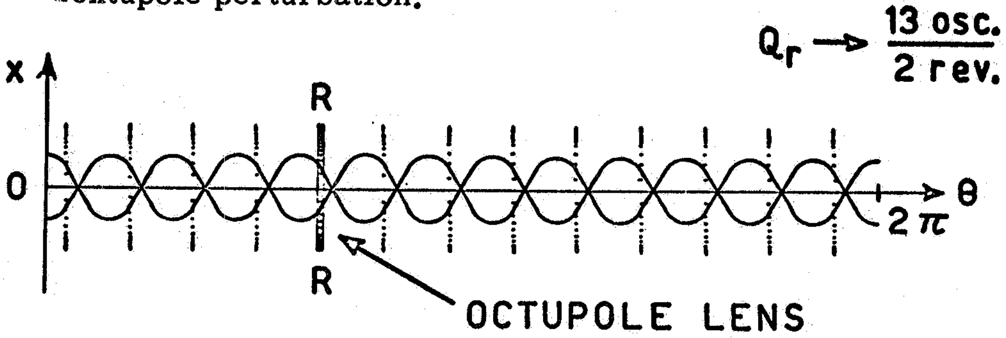

FIG. 6 - Qualitative behaviour of the radial betatron oscillation in the half-integer non linear resonance ejection used at the CEA. An octupole lens may be used to excite the oscillations. The 13th harmonic component of such a perturbation is also represented.

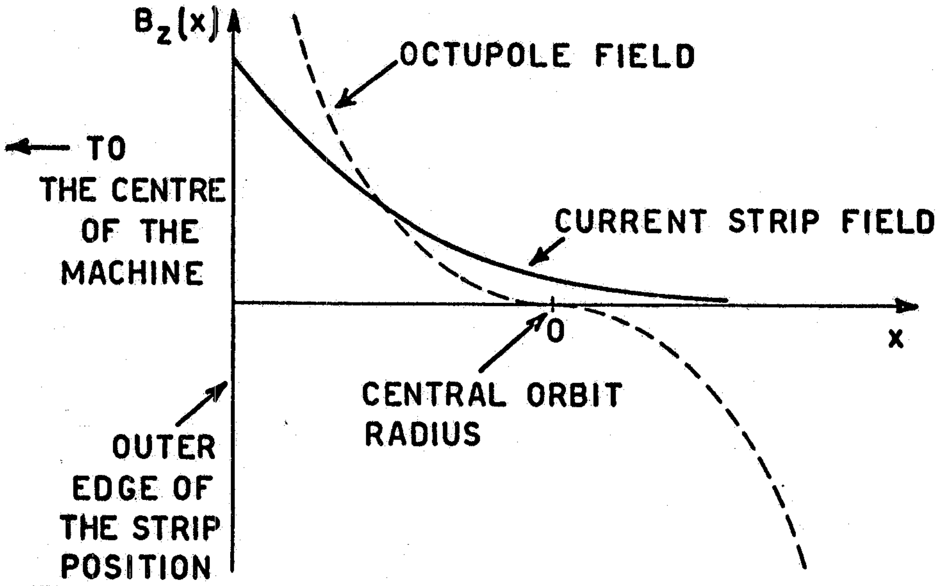

FIG. 7 - Shape of the magnetic field (on the median plane) generated by a current strip. Dotted curve: octupole field.

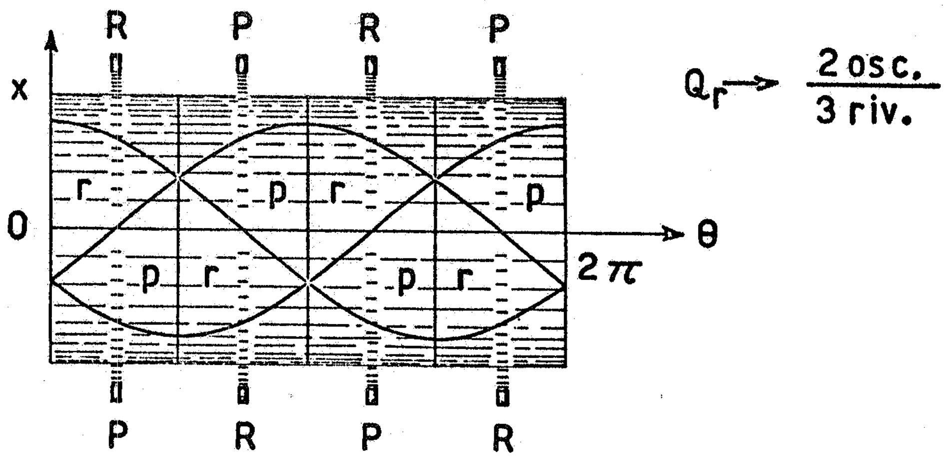

FIG. 8 - Orbit of a circulating particle during the 2/3 resonance ejection. Sextupole lenses (P and R) refer to the PPA. $\pi/2$ wide sextupole fields (p and r) refer to the Frascati Electron-Synchrotron.

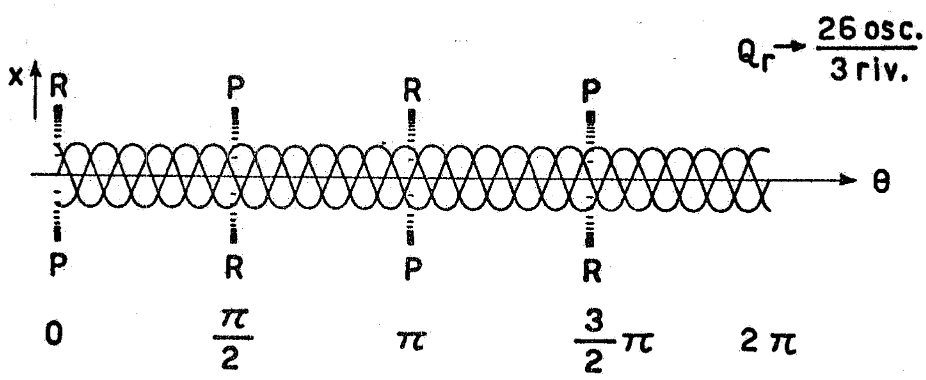

FIG. 9 - Orbit of a circulating particle during the 8+2/3 resonance ejection.

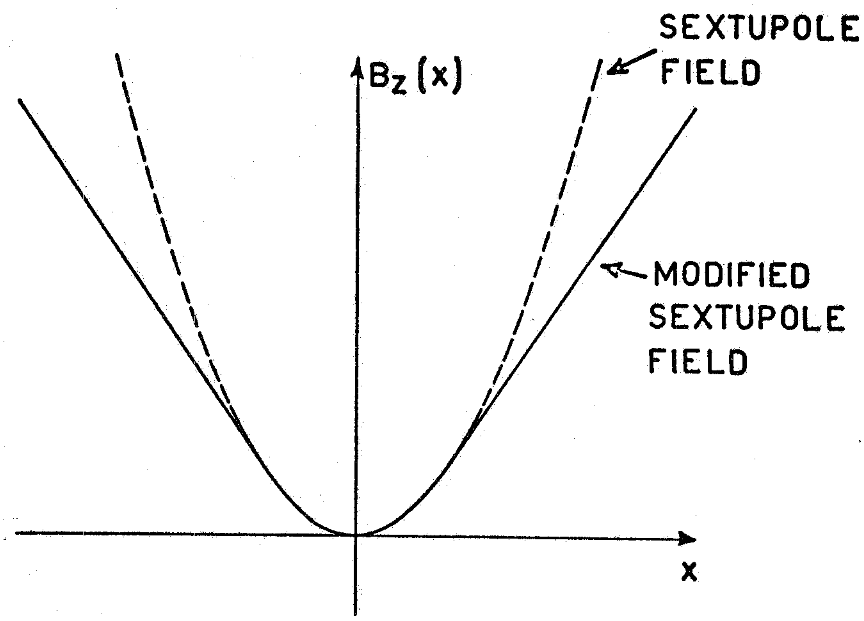

FIG. 10 - Sextupole field and "modified" sextupole field on the median plane.