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E. Schiavuta and F. Soso: PROPOSAL FOR A NEW ORGANIZATION OF DECISION ELECTRONICS IN MULTICOUNTER EXPERIMENTS. -

(Nota interna: n. 366)

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E. Schiavuta[(x)] and F. Soso: PROPOSAL FOR A NEW ORGANIZATION OF DECISION ELECTRONICS IN MULTICOUNTER EXPERIMENTS. -

Fast, reliable and cheap integrated circuit elements, suitable for building counter experiments electronics, are becoming available. They allow to carry out in short time complex logical functions, and then introduce more involved and sophisticated experimental techniques.

The aim of this paper is to study the possibility of taking advantage of this situation, in the research program for Adone.

1. INTRODUCTION. -

The design of complex systems out of simple functional elements, requires finding the logical and topological organization which guarantees the maximum flexibility and efficiency for the widest possible class of experiments.

There are two ways of facing the problem. One can systematically study some typical experiments, and find which general requirements the electronics must satisfy; the second way, of course, is to analyze the kind of circuits that can be built, and then discuss their utilization

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in experiments.

The class of possible experiments is certainly much wider than we are familiar with, since our skill in figuring out new experiments is conditioned by the instruments at our disposal and our knowledge of them.

We will therefore follow in this paper the second view point, i.e. study the electronic problem; its general connections with the physics are given as an introductory step.

By this choice we will get to a rather unexpected result: a quite new circuit module.

2. STATEMENT OF THE PROBLEM.-

Let us consider a counter experiment and indicate with $x_1, \dots, x_n$ the n binary variables[1] whose actual values define the "answer" of the apparatus to each event, and with $F_\gamma(x_1, \dots, x_n)$ an arbitrary function of these variables; then, according to a well known Boolean theorem we can write.

$$(1) \qquad F_\gamma(x_1, \dots, x_n) = \sum_{1}^{k}{}_{\alpha} \lambda_\alpha^\gamma N_\alpha(x_1, \dots, x_n)$$

where $k = 2^n$ and $N_\alpha(x_1, \dots, x_n)$ for $\alpha = 1, \dots, k$ are the normal products (or minterms) of $x_1, \dots, x_n$, and λ_α^γ are binary numbers.

We define a "word"

$$(2) \qquad w_\alpha = (x_1, \dots, x_n)$$

as the set of values of the variables for which N_α is "true"; we define also an s-word phrase

$$(3) \qquad \Phi_\gamma = (\lambda_1^\gamma, \dots, \lambda_k^\gamma)$$

as the set of k bits that completely specify F_γ, being s the number of non vanishing λ_j^γ in Φ_γ.

A one-word phrase will then identify a simple AND type function, while many-word functions are needed to identify a complex AND-OR type function.

We may distinguish between two main types of experiments.

a) Experiments where many kinds of particles are detected, each kind giving a well defined word as "answer" of the apparatus.

The problem is to change a general decision signal ("master trigger") from one kind of particles to another.

An example is shown in Fig. 1. Three scintillation counters A, B, C, in the focus of an analyzing magnet, divide a particle beam in eight momentum bands, all of which are analyzed in a spark chamber SC.

The behaviour of the SC for different momenta can be systematically studied by changing the SC trigger from function $F_1 = A\overline{BC}$ to $F_2 = A\bar{B}C$, $F_3 = ABC$, etc.

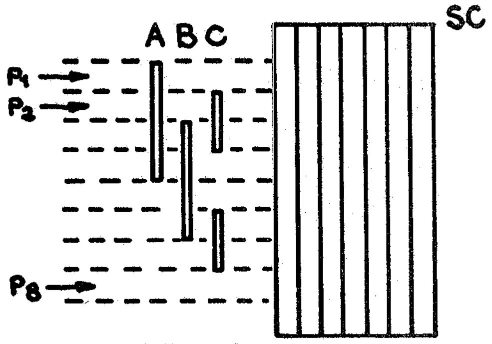

FIG. 1

b) Experiments where different kinds of particles are detected, but each kind can give different words as an auswer. Many-word programs (phrases) are needed to separate different particles, usually by multiparametric correlation analysis.

Each particle is identified by applying indipendent selection criteria to all or some of the binary variables x_i. As these variables are associated to basically stochastic processes, the answer to each type of event will show statistical fluctuations.

Let $p_\beta^{(i)}$ be the probability that a particle of kind β give an answer x_i = "1" (true), then the probability for the value of x_i associated to particles β, be it "zero" or "one", is(x)

(4)
$$p_\beta (x_i) = x_i \, p_\beta^{(i)} + \bar{x}_i \, (1 - p_\beta^{(i)})$$

and the probability that the answer of the apparatus to a particle β be the word w_α is

(5)
$$p_\beta (w_\alpha) = \prod_i p_\beta (x_i^\alpha) = \prod_i \left[x_i^\alpha \, p_\beta^{(i)} + (1 - p_\beta^{(i)}) \bar{x}_i^\alpha \right]$$

Note that, by definition, minterms N_α, when $\alpha = 1, \dots, k$, are the set of all possible answers, therefore

(6)
$$\sum_{1}^{k} {}_\alpha \, p_\beta (w_\alpha) = 1$$

Finally, the probability of having answer $F_\gamma = 1$ for a function F_γ whose phrase is Φ_γ, from a particle β, is

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(x) - x_i, $\bar{x}_i$ are here ordinary numbers 1, 0, rather than boolean variables.

(7) $$w(\beta, \gamma) = \Sigma_\alpha \, p_\beta (w_\alpha) \lambda_\alpha^\gamma$$

The electronic problem, if we want to select particles of kind β, is that of building a logic function F_γ of the variables $x_1, \ldots, x_n$, such that

(8) $$\varepsilon_\gamma = w(\beta, \gamma) \simeq 1 \qquad \text{if} \quad \beta = \gamma$$

and

$$C_\gamma = \sum_{\beta \neq \gamma} w(\beta, \gamma) \ll 1$$

Here ε_γ is the detecting efficiency of function F_γ for γ particles, while C_γ is a contamination factor from background.

Clearly, optimization of efficiency and rejection, often contradictory goals, may require a very complex F_γ function.

Changing from one kind of particles to another, or even modifying the selection criteria for the same kind, involves manipulation of the phrase Φ_γ pertaining to the decision function F_γ ; this should be done easily, during the experimental live time.

We can consider, as an example, a telescope of N counters, in which pions are to be separated from k-mesons through correlated amplitude analysis (see Fig. 2.)

We assume, for simplicity, that the probability p_i that a k give "1" in the i^{th} discriminator equals the probability that a π give "0".

If we put $q_i = 1 - p_i$ (see Fig. 3) the probability that the answer to a k be the word w_α is (5)

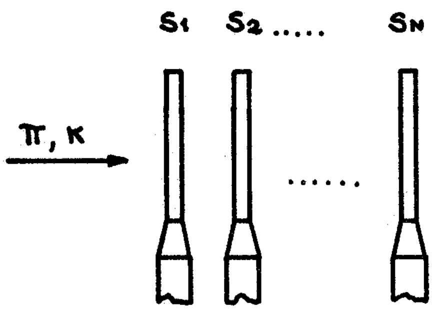


FIG. 2

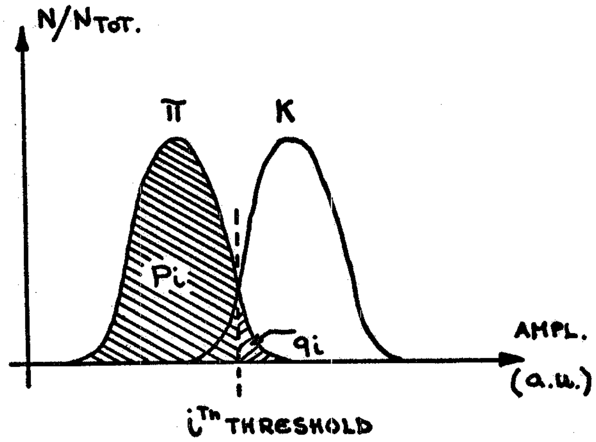


FIG. 3

(9a) $$p_k(w_\alpha) = \prod_i p_k(x_i^\alpha) = \prod_i (x_i^\alpha p_i + \bar{x}_i^\alpha q_i)$$

while for a π

(9b) $$p_\pi(w_\alpha) = \prod_i p_\pi(x_i^\alpha) = \prod_i (x_i^\alpha q_i + \bar{x}_i^\alpha p_i) \, .$$

We can now perform step by step the following systematic procedure:

1) Evaluate the probabilities $p_\pi(w_\alpha)$ and $p_k(w_\alpha)$ of all possible answers.

2) Order all words w_α according to decreasing values of $\rho_\alpha = p_\pi(w_\alpha)/p_k(w_\alpha)$, for function p_π, and of ρ_α^{-1} for p_k.

3) Take $\lambda_\alpha^\pi = 1$ in (7) up to an $\alpha = \alpha^*$ such that

$$\sum_{1}^{\alpha^*}{}_\alpha \, p_\pi(w_\alpha) \, \lambda_\alpha^\pi \geqslant \varepsilon_\pi$$

where ε_π is an assigned efficiency for π's detection. The procedure for k's is analogous.

Note that these steps for optimization of F_π or F_k can easily be accomplished by a small computer; if the computer is on-line, it can also perform real time measurements of integral efficiencies needed to evaluate the terms $p(w_\alpha)$, or modify F_π or F_k as some counter efficiency drifts, etc.

We examine two numerical cases

I[st] example. $N = 3$, $p_1 = p_2 = 0.9$, $p_3 = 0.8$. The detection efficiencies for words w_α are tabulated (see Table I).

TABLE I

α	w_α	$p_\pi(w_\alpha)$	$p_k(w_\alpha)$
1	$\bar{S}_1\bar{S}_2\bar{S}_3$	.648	.002
2	$\bar{S}_1\bar{S}_2S_3$	.162	.008
3	$\bar{S}_1S_2\bar{S}_3$	.072	.018
4	$S_1\bar{S}_2\bar{S}_3$	.072	.018
5	$\bar{S}_1S_2S_3$	.018	.072
6	$S_1\bar{S}_2S_3$	.018	.072
7	$S_1S_2\bar{S}_3$	.008	.162
8	$S_1S_2S_3$	.002	.648

Overall efficiencies for different choices of S^* are (see Table II)

TABLE II

S^*	ε_π	ε_k	$R_k = \varepsilon_\pi / \varepsilon_k$
3	0.882	0.028	31.5
4	0.954	0.046	20.7
5	0.972	0.112	8.68

One can have a low efficiency, high rejection function ($S^*=3$)

$$F_\pi^{(1)} = \bar{S}_1\bar{S}_2\bar{S}_3 + \bar{S}_1\bar{S}_2 S_3 + \bar{S}_1 S_2 \bar{S}_3 = \bar{S}_1\bar{S}_3 + \bar{S}_1\bar{S}_2$$

or a high-efficiency, low rejection one ($S^*=5$)

$$F^{(2)} = \bar{S}_1\bar{S}_3 + \bar{S}_1\bar{S}_2 + S_1\bar{S}_2\bar{S}_3 + \bar{S}_1 S_2 S_3 = \bar{S}_1 + \bar{S}_2\bar{S}_3$$

Both of them may be useful, and are advantageous when compared with the undimensional efficiency ($\varepsilon_\pi = 90\%$) and rejection ($R_k = 9$).

For those familiar with veitch diagrams, the situation can be represented in a more functional way:

$p_\pi(w_\alpha)$: (S_1 above, S_2 left, S_3 below)

S_2	.008	.002	.018	.072
	.072	.018	.162	.648

$p_k(w_\alpha)$: (S_1 above, S_2 left, S_3 below)

S_2	.162	.648	.072	.018
	.018	.072	.008	.002

and for the functions $F_\pi^{(1)}$, $F_\pi^{(2)}$

$F_\pi^{(1)} =$ (S_1 above, S_2 left, S_3 below)

S_2				1
			1	1

$F_\pi^{(2)} =$ (S_1 above, S_2 left, S_3 below)

S_2			1	1
	1		1	1

from which their simplified expressions can be easily deduced.

2^{nd} example. $N = 4$, $p_1 = p_2 = 0.9$, $p_3 = p_4 = 0.8$ (see Table III)

TABLE III

α	w_α	$p_\pi(w_\alpha)$	$p_k(w_\alpha)$
1	$\bar{S}_1\bar{S}_2\bar{S}_3\bar{S}_4$	.5184	.0004
2	$\bar{S}_1\bar{S}_2\bar{S}_3S_4$	.1296	.0016
3	$\bar{S}_1\bar{S}_2S_3\bar{S}_4$	.1296	.0016
4	$\bar{S}_1S_2\bar{S}_3\bar{S}_4$	.0576	.0036
5	$S_1\bar{S}_2\bar{S}_3\bar{S}_4$	.0576	.0036
6	$\bar{S}_1\bar{S}_2S_3S_4$	.0324	.0064
7	$\bar{S}_1S_2\bar{S}_3S_4$	.0144	.0144
8	$\bar{S}_1S_2S_3\bar{S}_4$	.0144	.0144
9	$S_1\bar{S}_2\bar{S}_3S_4$	.0144	.0144
10	$S_1\bar{S}_2S_3\bar{S}_4$	.0144	.0144
11	$S_1S_2\bar{S}_3\bar{S}_4$	.0064	.0324
12	$\bar{S}_1S_2S_3S_4$	.0036	.0576
13	$S_1\bar{S}_2S_3S_4$	.0036	.0576
14	$S_1S_2\bar{S}_3S_4$	.0016	.1296
15	$S_1S_2S_3\bar{S}_4$	.0016	.1296
16	$S_1S_2S_3S_4$	.0004	.5184
		1.0000	1.0000

The various choices for S^*, that give (see Table IV):

TABLE IV

S^*	ε_π	ε_K	$R_k = \varepsilon_\pi / \varepsilon_k$	F_π (minimized)
2	.648	.0020	324	$F_2=\bar{S}_1\bar{S}_2\bar{S}_3$
3	.7776	.0036	216	$F_3=F_2+\bar{S}_1\bar{S}_2\bar{S}_4$
4	.8352	.0072	116	$F_4=F_3+\bar{S}_1\bar{S}_3\bar{S}_4$
5	.8928	.0108	82.7	$F_5=F_4+\bar{S}_2\bar{S}_3\bar{S}_4$
6	.9252	.0172	53.8	$F_6=\bar{S}_1\bar{S}_2+\bar{S}_1\bar{S}_3\bar{S}_4+\bar{S}_2\bar{S}_3\bar{S}_4$
7	.9396	.0316	29.7	$F_7=\bar{S}_1\bar{S}_2+\bar{S}_1\bar{S}_3+\bar{S}_2\bar{S}_3\bar{S}_4$
8	.9540	.0460	20.7	$F_8=F_7+\bar{S}_1\bar{S}_4$
9	.9684	.0604	16.0	$F_9=F_8+\bar{S}_2\bar{S}_3$
10	.9828	.0748	13.1	$F_{10}=\bar{S}_1\bar{S}_2+\bar{S}_1\bar{S}_3+\bar{S}_1\bar{S}_4+$ $+\bar{S}_2\bar{S}_3+\bar{S}_2\bar{S}_4+\bar{S}_3\bar{S}_4$

can all be interesting, from $S^*=2$ which gives low efficiency and extremely high rejection, to $S^*=10$ which gives almost 100% efficiency and still 13:1 rejection.

What should be inferred from this example is that, even with $N=4$ counters, the selection function can be so involved as to be hardly realizable with conventional electronics.

It follows from the examples and discussion above that considerable improvement in counter-experiments data handling can be brought in by a new circuit module.

Besides giving any of the logic functions $F(x_1, \ldots, x_n)$ of a set of n binary variables (the outputs of counter discriminators and other selection circuits), this module should be easily switchable, either manually or by computer control, from one function to the other.

We will see that, for reasonable devices, the maximum number n of handled variables cannot by very large, even by use of integrated circuitry.

However it is worth to keep a general point of view, as long as possible.

3. BASIC DECISION CIRCUITS: GENERAL PRINCIPLES AND BLOCK DIAGRAMS. -

Formula (1) suggests a straightforward way of obtaining the function F_β. It is sufficient to obtain all possible minterms N_α of variables $x_1, \dots, x_n$ and then add them after a line of AND gates controlled by a register, in which the phrase Φ_β is written.

This principles is illustrated in Fig. 4, for $n = 4$.

Some general criteria to evaluate a block scheme may be given.

Both cost and complexity are approximately proportional to the number of gate inputs ($G_n = 128$ in Fig. 4)(x).

The propagation delay t'_ρ of the circuit is important too; it is obviously given by the propagation delay of each gate, t_ρ, times the number S of cascaded gates (S = 3 in Fig. 4). Available high speed micrologic elements have $t_\rho = 2 \div 6$ nsec, so the total delay t'_ρ is in the few nanoseconds range(+).

Undimensional there is no general criterion to evaluate how difficult the wiring of a given block diagram is.

We will call P-type (parallel processing) circuits like the one of Fig. 4. G_n is given below for $n = 2 \div 8$ variables.

n (variables)	2	3	4	5	6	7	8
G_n (gate-inputs number)	20	56	128	288	576	1308	3072

; $G_n \simeq 2^n(n+4)$

Other ways of obtaining the general boolean function of n variables may be found.

Dividing the set of variables $x_1, \dots, x_n$ in two sets

(10) $$Y = (x_1, \dots, x_m) \qquad Z = (x_{m+1}, \dots, x_m)$$

(x) - One should be aware that 2-3-4-8- input gates are generally available, but not 5-6-7- input ones.

(+) - The propagation delay should not be identified with the resolving time, althrough both depend on the speed of the logic elements.

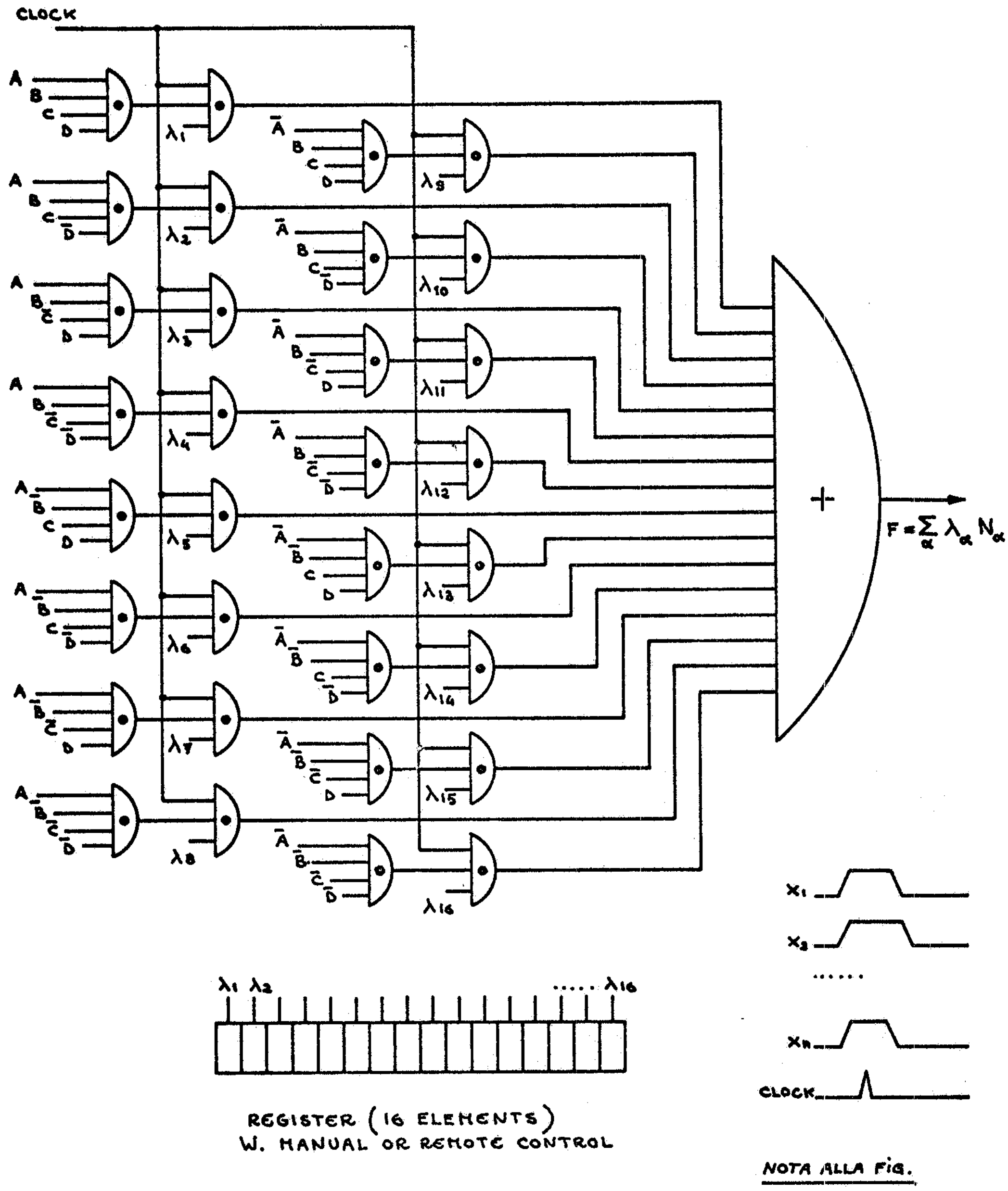
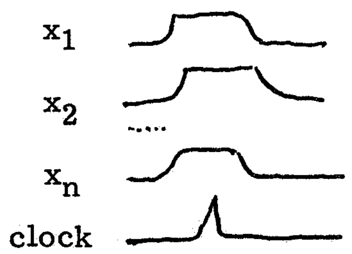


FIG. 4

(x) - The clock signal is essential to the apparatus performance, Its time relation with all other signals, sketched aside, allows each signal to be used either in coincidence or in anticoincidence without changing its width or timing.



The clock can be a machine characteristic, as for Adone, or it can be obtained from a coincidence between low-threshold counter discriminators.

(1) can be written

$$F(x_1, \dots, x_n) = \sum_{r,s} \lambda_{rs} N_r(Y) N_s(Z) \tag{11}$$

where $N_r(Y)$, $N_s(Z)$ are minterms of the sets $(x_1, \dots, x_m)$, $(x_{m+1}, \dots, x_n)$ respectively.

Taking $m = n/2$ (n even), or $m = (n-1)/2$ (n odd), diagrams like that of Fig. 5 are obtained, PS type (parallel-series processing).

These schemes are considerably simpler than P-ones, as can be seen from G_n values below.

n	2	3	4	5	6	7	8	...
G_n	16	48	88	172	320	624	1184	

$$G_n = 2^{\frac{n}{2}}\left(n+2+4 \cdot 2^{\frac{n}{2}}\right) \quad (n \text{ even})$$

$$G_n = 4 \cdot 2^n + \frac{3n+7}{2}\, 2^{\frac{n-1}{2}} \quad (n \text{ odd})$$

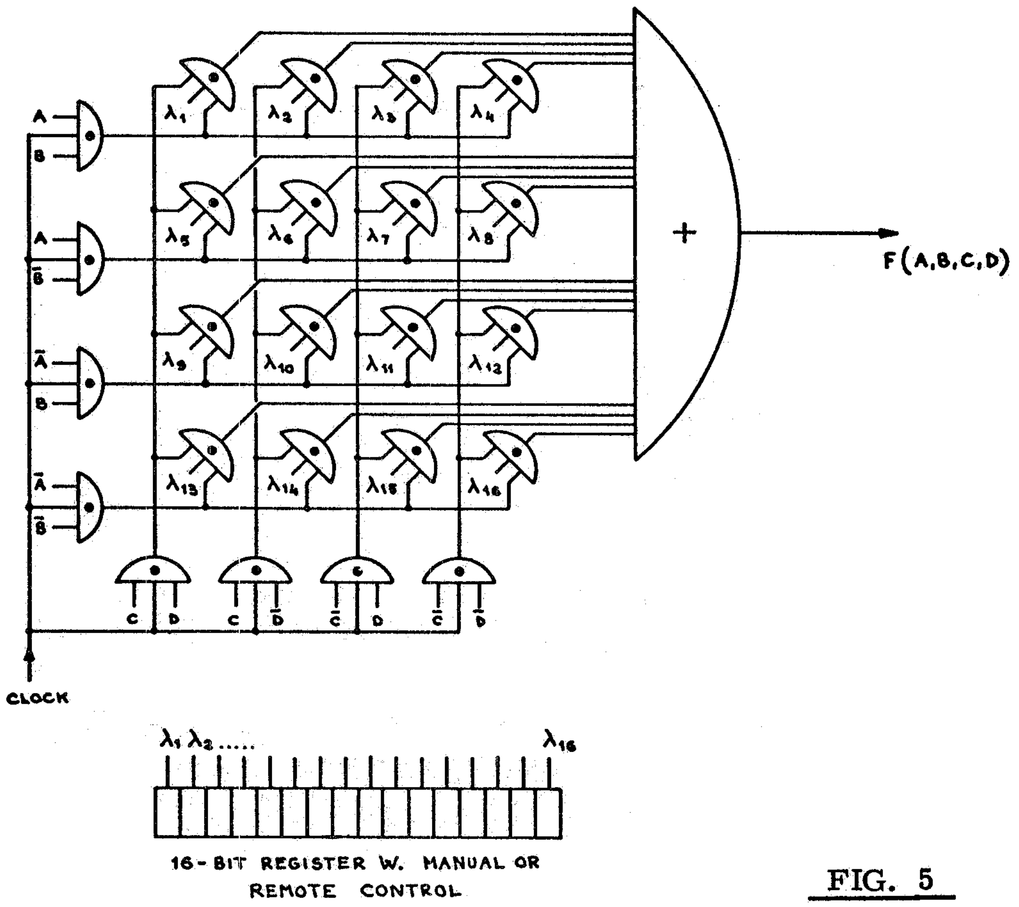


FIG. 5

The reason for different G_n's is the different order S of P and PS schemes. In the examples, the circuit of Fig. 5 is third order (s = 3). The circuit of Fig. 4 is third order too, but could be easily reduced to S = 2 if five-input gates were available.

A third and very interesting configuration is obtained by taking in (10) m = n-1. A typical result is shown in Fig. 6, where a function of five variables is obtained from two 4-variables blocks and a simple mixing stage.

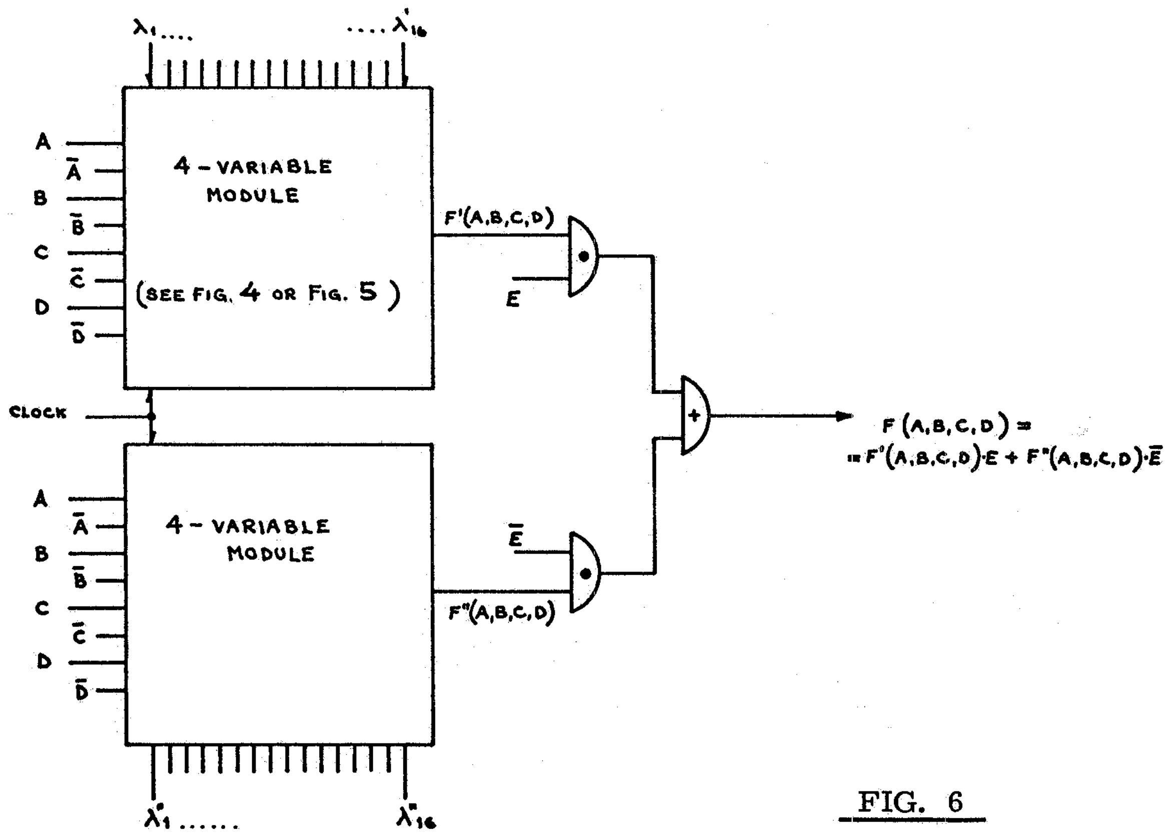


FIG. 6

We have thus reached a remarkable result: one can build, for instance, correlation modules for n = 4 variables only, and then obtain functions of n = 5, 6, ... variables by small auxiliary units.

However, when compared to P and PS configurations for n = 5, 6, .. the logic of Fig. 6 shows more propagation delay.

4. THE COMPLETE SYSTEM. -

Besides decision circuits, a logic system must include memory elements and input-output devices.

A complete apparatus for multicounter experiments, which includes the decision modules outlined in sec. 3, is shown schematically in Fig. 7.

When a master is obtained, the value ("0" or "1") of each variable, either involved in the master or not, can be stored in binary memory elements (flip-flops).

Any needed function of stored variables can then be performed, without undergoing the time limitations of the master.

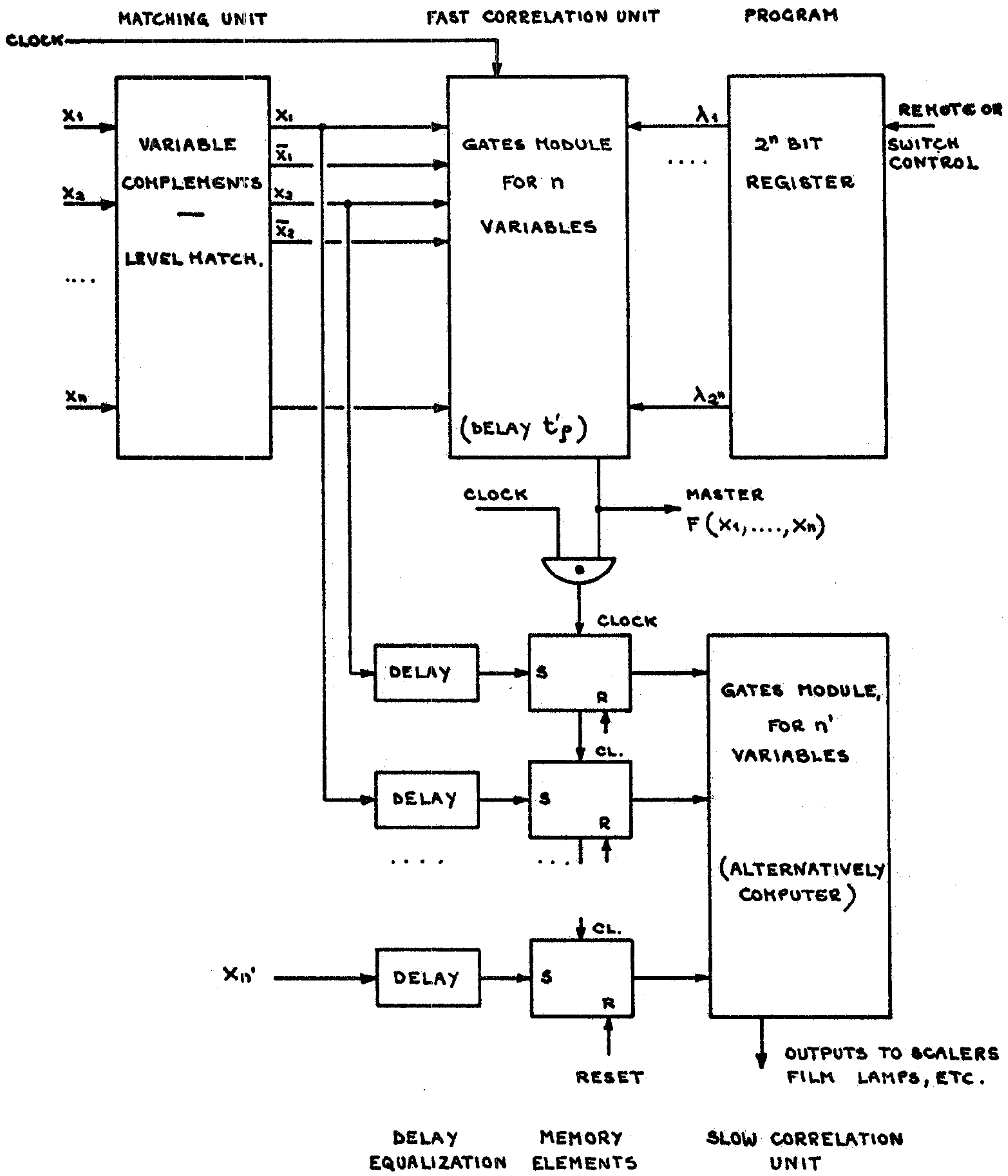


FIG. 7

It is important to note, however, that the logic modules that perform slow correlations are identical to the fast ones, and that it does not pay to build them with slower components.

It is obviously interesting to examine some charachteristics of this system. Let us consider, as an example, a fast correlation module for $n = 4$ variables (which is not for from a practical choice).

Whether the scheme is P-type or PS-type (see Fig. 4, 5), and since no other circuit components than micrologics are required, the cost is related to the following unitary costs

- about 400 ÷ 1000 £ /gate-input
- about 2.300 £ /bistable register element.

As for mounting difficulties, a rough evaluation shows that the circuit of Fig. 5, for instance, requires eleven DUAL IN LINE elements(x). They can be conveniently mounted on a 5 x 5 in printed card, and located in a twin (owing to panel space for $\lambda_1, \ldots, \lambda_{16}$ switches) NIM module.

The intrinsic gate resolving time is between 5 and 10 nsec FWHM, varying with the type of element and manufacturer.

5. CONCLUSION. -

Integrated circuit elements have already attained speed ranges (clock repetion frequency = 50 ÷ 100 Mc/sec) like those required for many high energy experiments electronics. Furthermore, they are still in their young age. Dramatic developments are to be awaited, as ultra-fast computers cameon. On the other side, there are little symptoms for great improvements of conventional electronic modules.

These were the starting points of this paper, whose aim is to show that instead of trying to convert to integrated circuity the usual modules, one can change the logical organization of experimental block schemes.

The result is a new "correlation unit" which can give, in a time not much longer than required for a coincidence/anti-coincidence operation, the most general logic function of n variables; this unit may be computer programmed.

This performance, unattainable from conventional modules, seems on the contrary simple enough to be obtained from integrated elements, and offers an exciting improvement of logic flexibility at little cost and complexity increase.

(x) - These are 14-leads elements, each .77 x 3 x .2 in. Much smaller packages are available, but they require special soldering equipment.