S. Tazzari: CONSIDERATIONS ON A LUMINOSITY
MONITOR AT ADONE. -

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NE.

I) -   The reactions that can be used for luminosity monitoring at our
machine are reviewed, trying to establish the attainable accuracy under
various machine conditions.

The luminosity monitor will be looked at more as a machine con
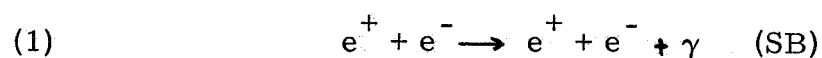trol facility than as a monitor for experiments.

II) - BREMSSTRAHLUNG REACTIONS AND ANNIHILATION AT SMALL
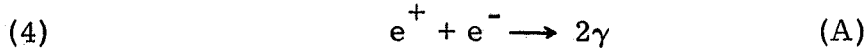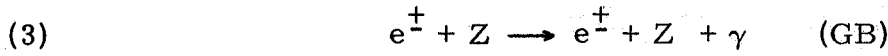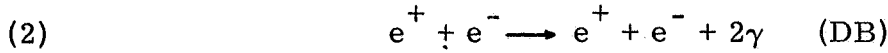ANGLES.

a) Cross sections and counting rates -

To look at these reactions two thin windows are provided in the
doughnut at opposite ends of each straight section. Their angular semi-
aperture with respect to the center of the straight section is of $\sim$ 8 mr
around zero degrees.

The available semiaperture is reduced to $\sim$ 4 mr when the cros
sing angle is 3 mr.

The reactions considered are:

(1) $e^+ + e^- \longrightarrow e^+ + e^- + \gamma$ (SB)

2.

$$(2) \qquad e^+ + e^- \longrightarrow e^+ + e^- + 2\gamma \qquad (DB)$$

$$(3) \qquad e^{\pm} + Z \longrightarrow e^{\pm} + Z + \gamma \qquad (GB)$$

$$(4) \qquad e^+ + e^- \longrightarrow 2\gamma \qquad (A)$$

The cross section for reaction (1) will be taken from reference (1), that for reaction (2) from Bayer and Galitsky[2, 9], whose results have recently been checked by P. Di Vecchia and M. Greco[3].

The counting rates of the two reactions, $\dot{n}_{SB}$ and $\dot{n}_{DB}$, have been plotted versus the ratio $\mathcal{E}$ of the threshold energy, E_γ, to the beam energy, E_M, for different values of E_M, in figures 1 and 2.

The counting rates are proportional to the luminosity L. The energy dependence of L for Adone, once the space charge limit has been attained,

$$(5) \qquad L = \frac{10^{33} \gamma}{3 \times 10^{33}} \quad cm^{-2} \, h^{-1}$$

has been folded into figures 1 and 2 and, for the sake of simplicity, the same $\mathcal{E}$ has been assumed for the two γ rays of reaction (2). The counting rate of reaction (3) is given by

$$(6) \qquad \dot{n}_{GB}(\mathcal{E}) = \alpha \, \sigma_{GB}(\mathcal{E}) Z^2_{AV} p \, N_e \, \frac{\nu_{RF}}{K} \, l$$

where p is the pressure of the residual gas, N_e the number of electrons in the beam, ν_{RF}/K the machine natural frequency, l the length of straight path seen, and α a proportionality constant. The design values of Adone are: $l = 6.5$ m, $p = 10^{-9}$ torr, $N_e = 2 \times 10^{11}$, $\nu_{RF}/K = 2.86 \times 10^6$ S^{-1} the residual gas is assumed to be biatomic with $Z = 7.6$, and the value of α is 3.55×10^{16} cm^{-3}.

$\dot{n}_{GB}(\mathcal{E})$ has been plotted versus $\mathcal{E}$ in fig. 1.

The finite aperture of the viewing ports has been disregarded in calculating $\dot{n}_{SB}$ and $\dot{n}_{GB}$, while it has been folded into $\dot{n}_{DB}$. This can be justified by looking at fig. 3 where the probabilities p(θ) of one (two) γ ray to be emitted between 0^0 and θ (between 0 and θ, and π and $\pi-\theta$) have been plotted. It is assumed that $p_{SB}(\theta) = p_{GB}(\theta)$.

The counting rate of reaction (4) is also strongly affected by the aperture 2 θ^x, of the windows, and for $\theta^x \ll 1$, it is given by

$$(7) \qquad \dot{n}_A = \frac{L^x}{3 \times 10^3} \, \frac{\pi \, r_0^2}{2\gamma} \, \ln(\theta^{x2} \, \gamma^2 + 1).$$
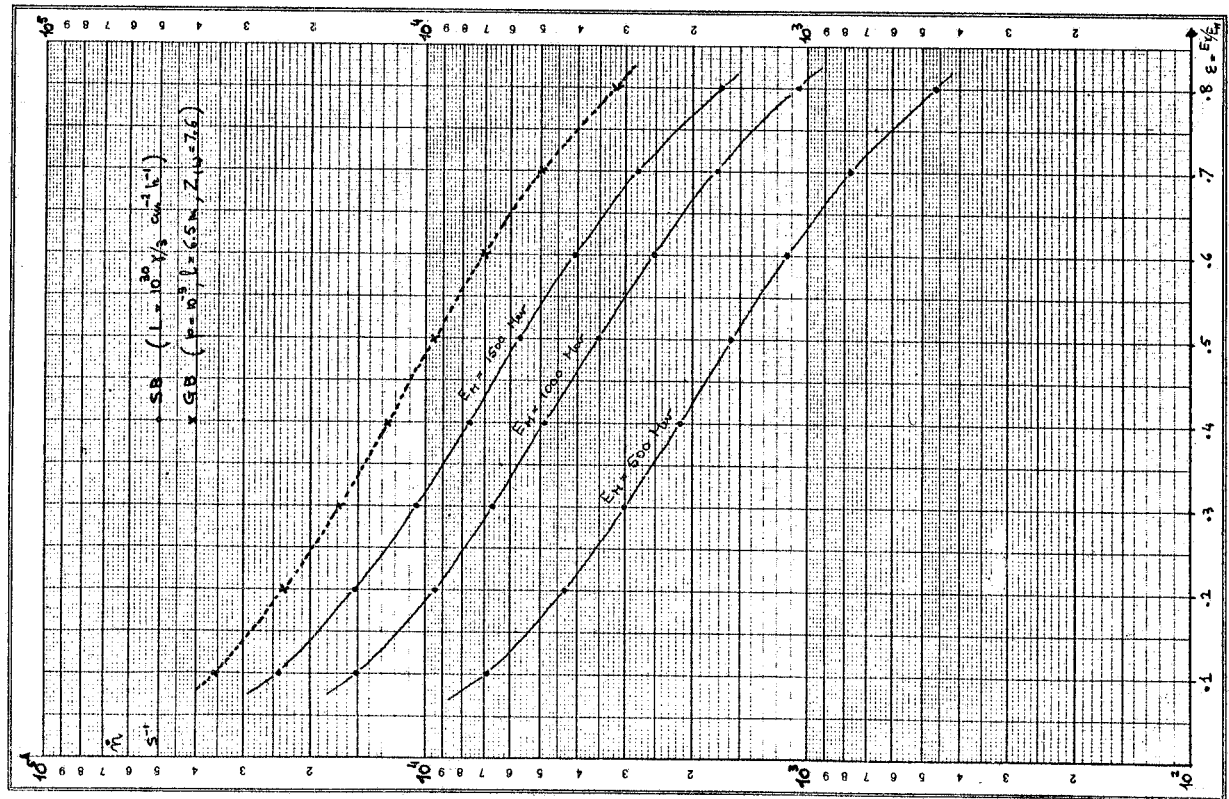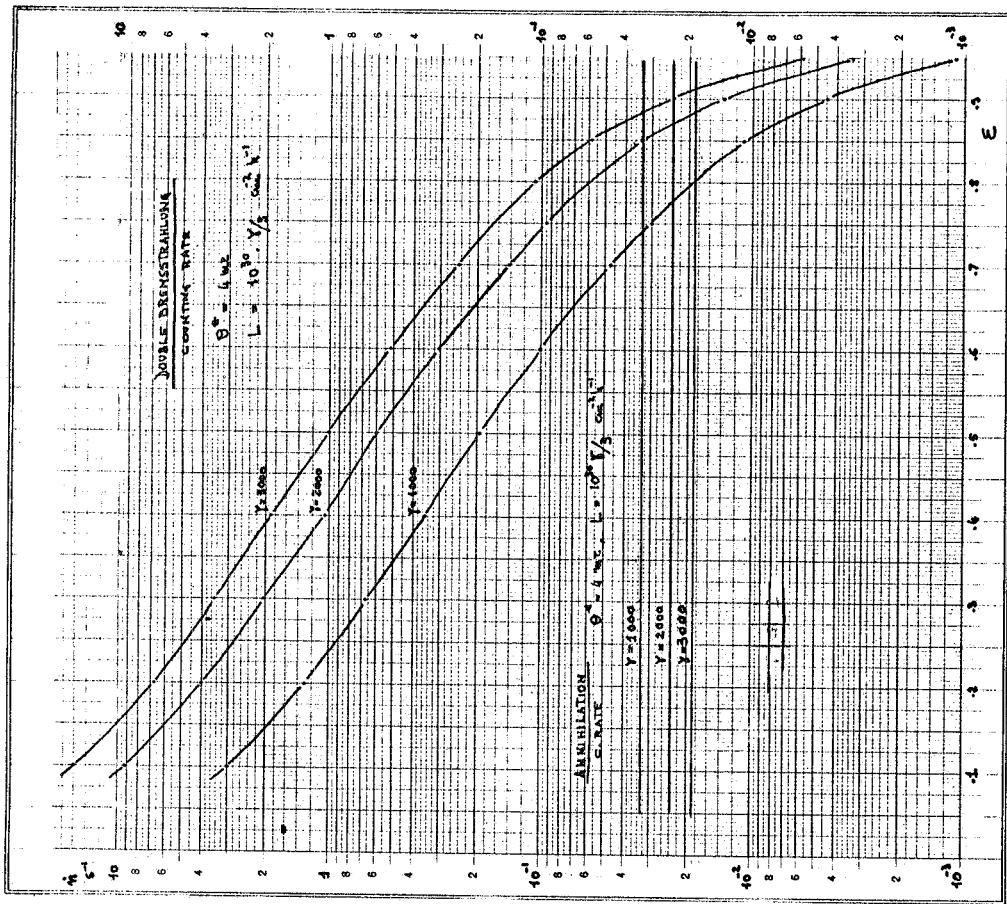
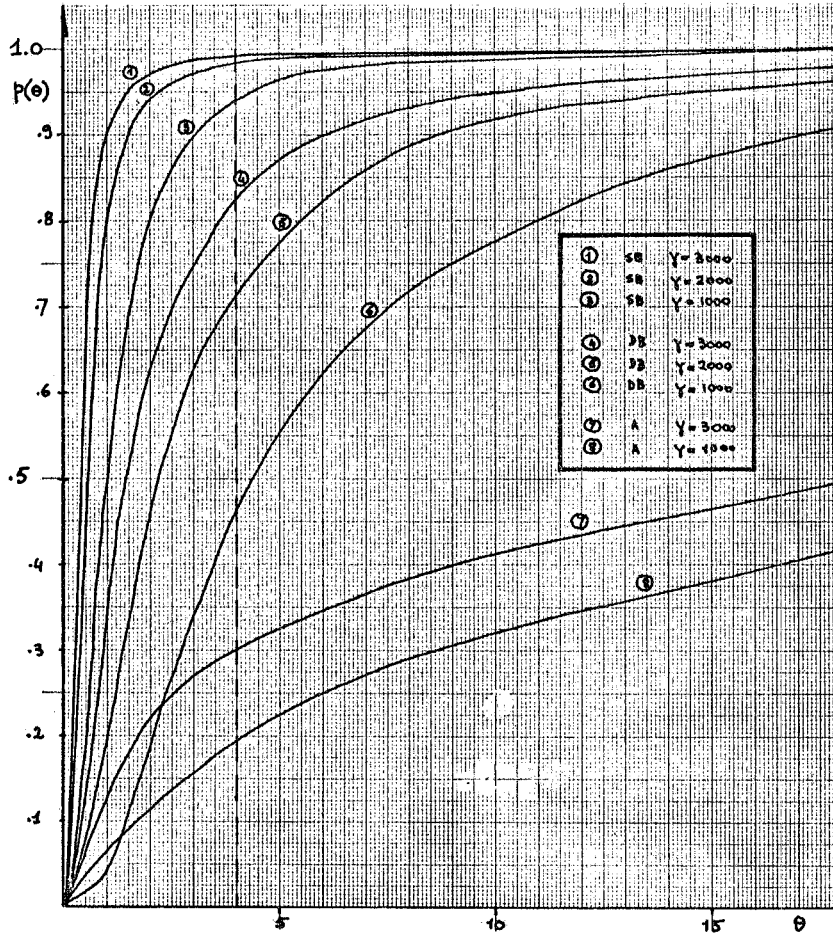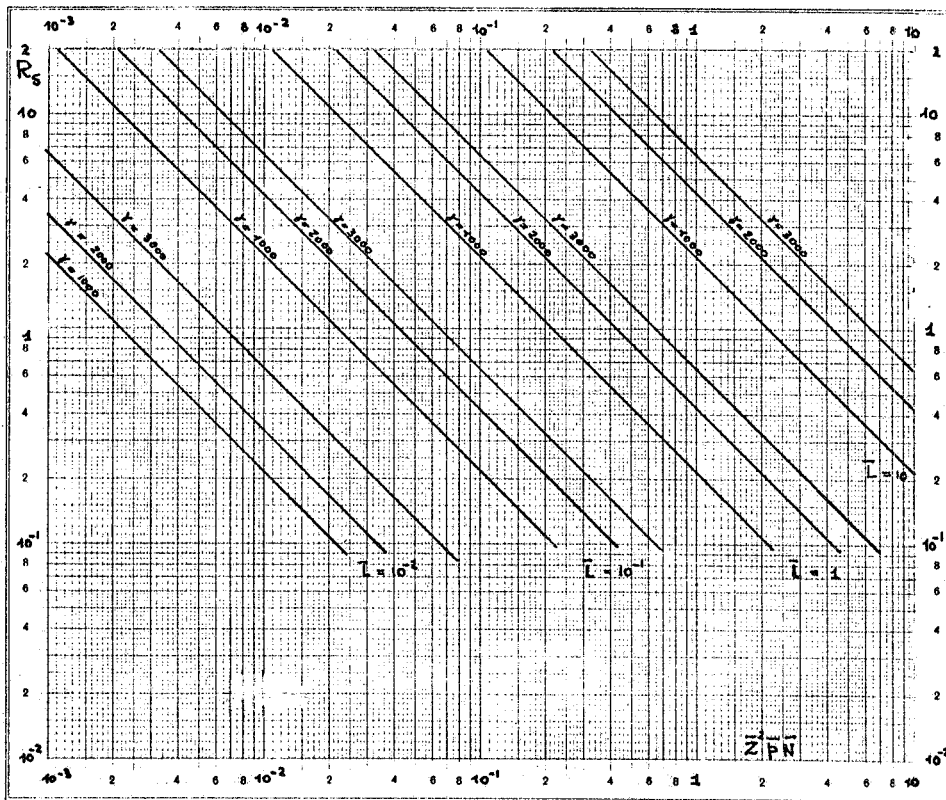FIG. 2

FIG. 1

4.

FIG. 3

FIG. 4

$\dot{n}_A$ is plotted in fig. 2 and $p_A(\theta)$ in fig. 3.

b) S i g n a l   t o   n o i s e   r a t i o -

Let us define:

$$\bar{L} = \frac{L(cm^{-2}h^{-1})}{10^{33}}, \qquad \bar{p} = \frac{P(torr)}{10^{-9}},$$

$$(8) \qquad \bar{N}_e = \frac{N_e}{2 \times 10^{11}}, \qquad \bar{\gamma} = \frac{\gamma}{3 \times 10^3}$$

$$\bar{Z} = \frac{Z}{7.6} \quad \text{(for a biatomic gas)},$$

and, since the cross sections of reactions (1), (2), (3) are practically independent of energy, let us introduce the counting rates $n^{(0)}$ corresponding to $\bar{L}$, $\bar{Z}$, $\bar{p}$, $\bar{N}_e$, $\bar{\gamma} = 1$, and $\mathcal{E} = 0.1$. One has

$$(9) \qquad \begin{aligned} \dot{n}^{(0)}_{SB} &\simeq 2.4 \times 10^4 \, s^{-1} \\ \dot{n}^{(0)}_{DB} &\simeq 1.6 \times 10^1 \, s^{-1} \\ \dot{n}^{(0)}_{GB} &\simeq 3.6 \times 10^4 \, s^{-1} \end{aligned}$$

The additional hypothesis will be made that N_e be the same for the two beams. In practice, if the space charge limit is attained, luminosity is proportional to the intensity of the lesser beam[5] so that the assumption is not too restrictive.

The counting rates will then be given by:

$$(9') \qquad \begin{aligned} \dot{n}_{SB} &\simeq \bar{L} \, \dot{n}^{(0)}_{SB} \\ \dot{n}_{DB} &\simeq \bar{L}(1 - \frac{a}{\gamma}) \, \dot{n}^{(0)}_{DB} \qquad (a = 0.19) \\ \dot{n}_{GB} &\simeq \bar{Z}^2 \, \bar{p} \, \bar{N} \, \dot{n}^{(0)}_{GB} \end{aligned}$$

The factor $(1 - \frac{a}{\gamma})$ in n_{DB} takes into account (to first order) the finite aperture of the viewing ports, assuming $\theta^x = 4$ mr.

Using (9') the signal to background ratios for reactions (1) and (2) are:

$$R_S = \frac{\dot{n}_{SB}}{\dot{n}_{GB}} = \frac{\bar{L}}{\bar{Z}^2\,\bar{p}\bar{N}}\;\frac{\dot{n}_{SB}^{(0)}}{\dot{n}_{GB}^{(0)}} \approx 0.65\,\frac{\bar{L}}{\bar{Z}^2\,\bar{p}\bar{N}}$$

(10)

$$R_D = \frac{\dot{n}_{DB}}{(\dot{n}_{SB}+\dot{n}_{GB})^2\,\tau_{RF}} = \frac{\bar{L}(1-a/\bar{\gamma})\,\dot{n}_{DB}^{(0)}}{\left[\bar{L}\dot{n}_{SB}^{(0)} + \bar{Z}^2\,\bar{p}\bar{N}\dot{n}_{GB}^{(0)}\right]^2\,\tau_{RF}}$$

if

$$\left[\bar{L}\dot{n}_{SB}^{(0)} + \bar{Z}^2\,\bar{p}\bar{N}\dot{n}_{GB}^{(0)}\right] \ll \nu_{RF}$$

R_S and R_D have been plotted as functions of $\bar{Z}^2\,\bar{p}\bar{N}$ and for several values of L and γ in figg. 4 and 5. The energy dependence of $\bar{L}$ as given by (5) and (8) has been included but it must be borne in mind that this is correct only if the space charge limit is attained[5].

It is to be noted that R_S and R_D are almost independent of ε, so that the fact that (9) pertain to $\varepsilon = 0.1$ is not too restrictive.

c) Measurements of L and errors involved -

We will consider measuring two γ's in coincidence from reaction (2) or single γ's (on one side only) from reaction (1).

In the first instance the number of coincidences with one arm de layed by one full turn provides a continuous measure of background. In the second case background has to be measured, by removing the crossing, and then subtracted out. The measurement of reaction (4) will be discussed later on.

The counting rates time dependence due to lifetime will be neglec ted.

Calling S_D the number of DB events, once the background has been subtracted out, one has

(11)
$$S_D = L\,\sigma_{DB}\,(\varepsilon,\,\theta^x)\,T$$

where T is the time.

L can be calculated from (11) the error being

(12)
$$\left(\frac{\Delta L}{L}\right)^2 = \left(\frac{\Delta S_D}{S_D}\right)^2 + \left\{\alpha_{\varepsilon D}^2\left(\frac{\Delta\varepsilon}{\varepsilon}\right)^2 + \alpha_{\theta D}^2\left(\frac{\Delta\theta}{\theta}\right)^2\right\} = \Delta_{SD}^2 + \Delta_D^2$$
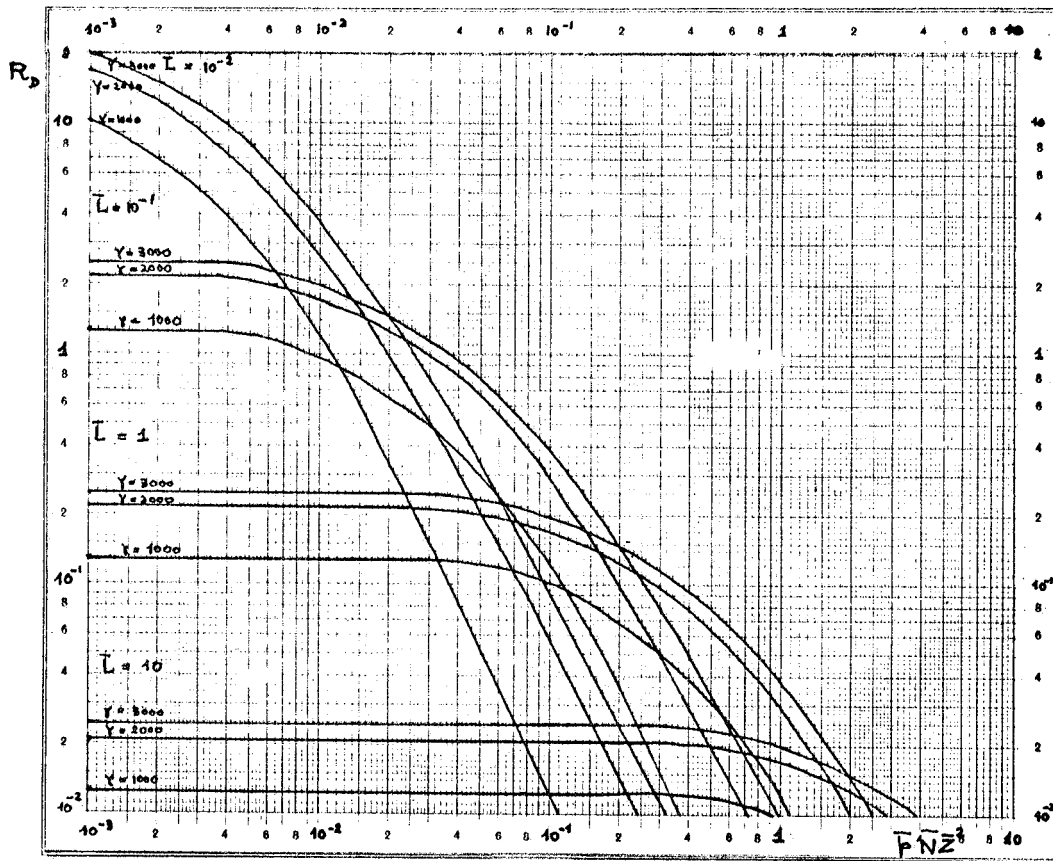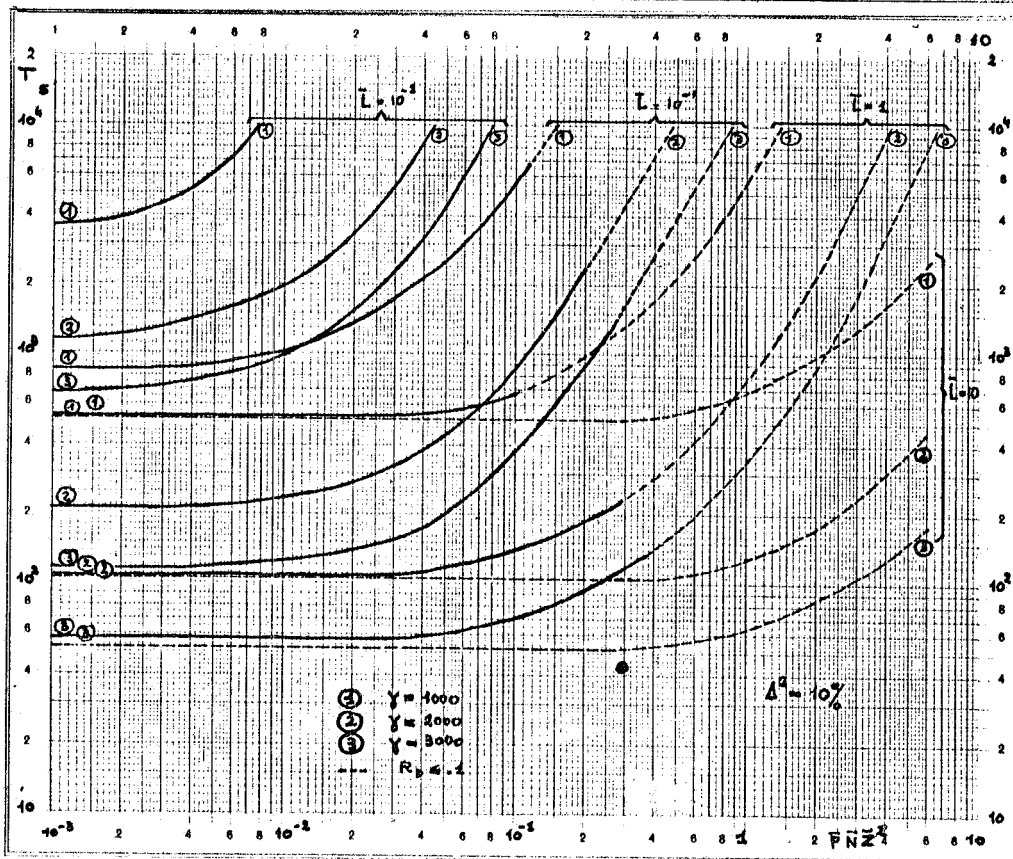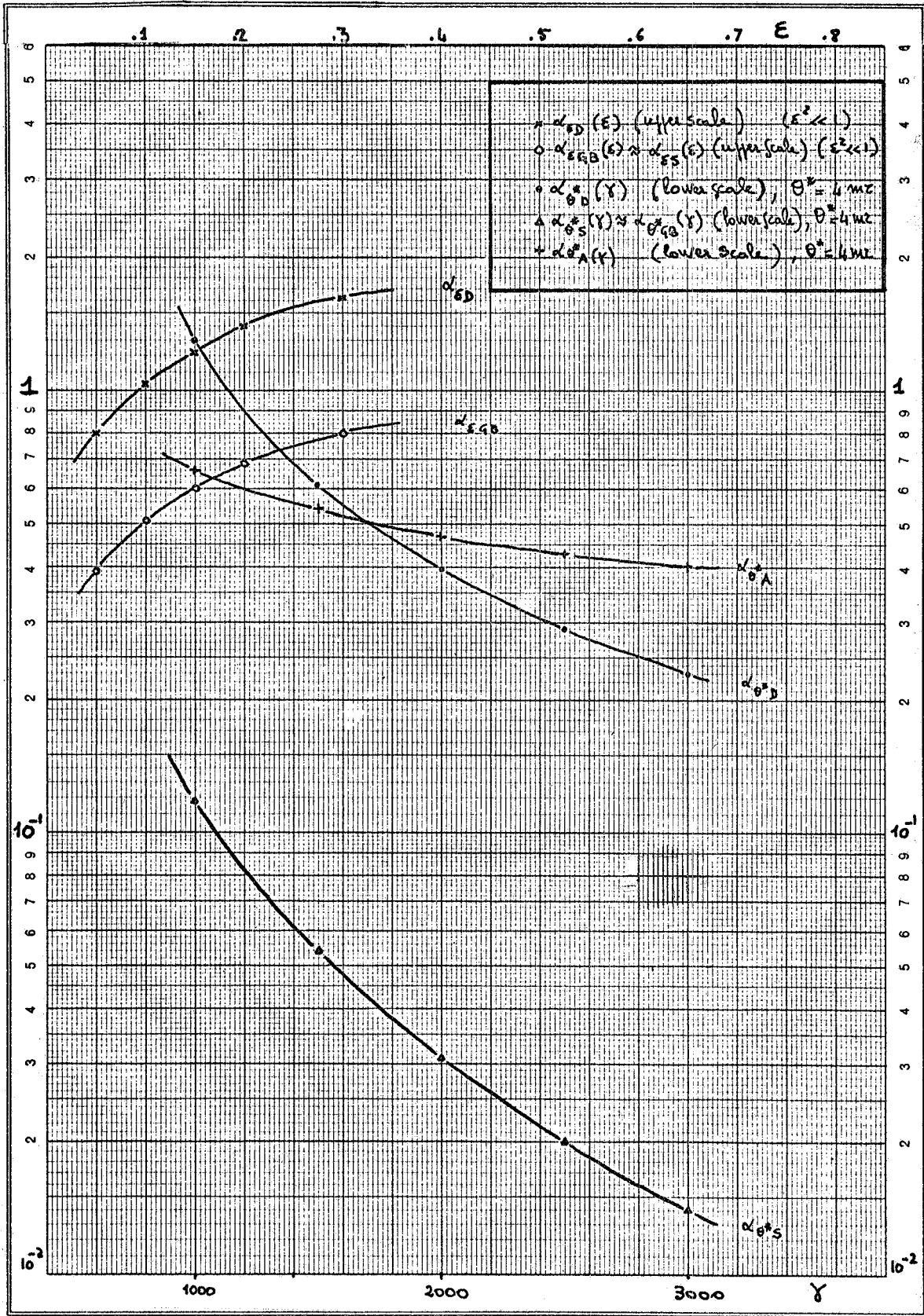
7.

FIG. 5

FIG. 7

8.

FIG. 6

where $\alpha_{\varepsilon D}$, $\alpha_{\theta D}$ are plotted in fig. 6. ($\alpha_{\theta D}$ was obtained, for $\theta \approx \theta^x$, by means of a first order fit).

Recalling the proposed procedure the purely statistical error can be obtained from the following formula:

$$(13) \qquad \Delta^2_{SD} = (\frac{\Delta S_D}{S_D})^2 = \frac{1}{S_D} \frac{2+R_D}{R_D} = \frac{1}{n_{DB}T} \frac{2+R_D}{R_D}$$

provided R_D is large enough ($R_D \gtrsim 0.1$) that the error due to possible efficiency differences between the prompt and the delayed channel can be kept much smaller than ΔS_D.

Errors on time have been neglected.

We have plotted in fig. 7 the time necessary for ΔS_D to be reduced to 10% as a function of $\bar{Z}^2 \bar{p} \bar{N}$, and for several values of $\bar{L}$.

Let us now call S_S the SB counting rate and B_S its background (GB). We then have

$$(14) \qquad S_S = L \sigma_{SB}(\varepsilon) p_{SB}(\theta) T$$

$$B_S = KpnTZ^2 \qquad\qquad K = const.$$

Recalling the proposed procedure, indexing with "o" quantities pertaining to the background measurement, assuming $K = K_o$ (in particular neglecting errors due to trajectory differences between the two measurements), introducing the assumption that ion trapping effects are not important (least p and p_o may be very much different and not simply measurable with the machine vacuum-gauges) and neglecting errors on time and on the expression of σ_{SB}, we obtain

$$(15) \qquad (\frac{\Delta L}{L})^2 = \left[\frac{1}{S_S}(1 + \frac{1}{R_S}) + \frac{1}{R_S^2} \frac{1}{B_{S0}} \right] + \left\{ \alpha^2_{\varepsilon S}(\frac{\Delta \varepsilon}{\varepsilon})^2 + \alpha^2_{\theta S}(\frac{\Delta \theta}{\theta})^2 + \right.$$

$$\left. + \frac{1}{R_S^2} \left[(\frac{\Delta \rho_N}{\rho_N})^2 + (\frac{\Delta \rho_p}{\rho_p})^2 \right] \right\} = \Delta^2_{SS} + \Delta^2_S$$

where

$$\rho_N = \frac{T \bar{N} \bar{Z}^2}{T_o \bar{N}_o \bar{Z}_o^2} ; \qquad \rho_p = \frac{\bar{p}}{\bar{p}_o} ;$$

10.

and $\alpha_{\epsilon S}$, $\alpha_{\theta S}$ are plotted in fig. 6; where again Δ_{SS} is purely statistical and can, this time, be neglected provided L $\gtrsim 10^{-3}$.

As far as reaction (4) is concerned the line spectrum of the emitted γ's allows, in principle, the signal to noise ratio to be made very high, and the error on the threshold to be eliminated[9].

The error on L can still be written as

$$(\frac{\Delta L}{L})^2 = \Delta^2_{SA} + \Delta^2_A$$

where Δ^2_{SA} is given by the analogous of (13) and

$$\Delta^2_A \approx \alpha^2_{\theta*A} (\frac{\Delta\theta}{\theta})^2$$

$\alpha_{\theta*A}$ is plotted in fig. 6.

By writing

$$\dot{n}_A(\mathcal{E}_1) = \alpha\, \dot{n}_{DB}(\mathcal{E}_2) \qquad R_A(\mathcal{E}_1) = \beta R_D,$$

where $R_A(\mathcal{E})$ is the ratio of annihilation to DB plus the chances from SB and GB, the statistical error can be written as

$$\Delta^2_{SA}(\mathcal{E}_1) = \frac{1}{\alpha\, \dot{n}_{DB}(\mathcal{E}_2)T} \frac{2+\beta R_D}{\beta R_D}$$

If one assumes $\mathcal{E}_1 = 0.95$, $\mathcal{E}_2 = 0.1$ one gets (see fig. 2) $\alpha \approx 10^{-2}$, while

$$\beta = \frac{R_A(\mathcal{E}_1)}{R_D} = \frac{\dot{n}_A(\mathcal{E}_1)}{\dot{n}_D(\mathcal{E}_1)(1+\frac{1}{R_D})} \frac{1}{R_D} \lesssim \frac{\dot{n}_A(\mathcal{E}_1)}{\dot{n}_D(\mathcal{E}_1)} \approx 10$$

so that, for the same counting time one always has

$$\Delta^2_{SA}(\mathcal{E}_1) \gg \Delta^2_{SD}(\mathcal{E}_2).$$

d) - Summing up we observe that some parameters, like Z, that only experiment will allow us to determine, are crucial, and also that it is difficult to give now a reliable estimate of all the errors involved. It appears though that it will be hard, using these reactions, to attain accu-

racies of better than $\sim 10\%$.

Single bremsstrahlung is the most attractive reaction from the point of view of counting rate, but the background subtraction procedure is troublesome. At present the DB signal to background ratio (at around 0°) seems too bad to make the reaction a good monitor (at least under normal machine conditions) although the background subtraction is very convenient. The annihilation reaction seems to be at a still greater disadvantage.

III) - $e^+ e^-$ SCATTERING (SC).

It seems feasible to measure this reaction, at small angles, to an accuracy of the order of a few percent and with a good counting rate[7] ($\sim 1 \cdot$ L s^{-1}), with counters of ~ 2 x 20 cm^2 at $\sim 5^o$ and 1 m from the interaction region, the only dangerous background coming from particles lost along the ring. If we assume that the most effective ones are those lost on the straight section walls[6, 8], we calculate that their number, n_e, is for Adone

(16) $$\dot{n}_e \cong 150 \ \bar{p} \ mA^{-1} \ s^{-1}.$$

Now assume that the experimental apparatus is such that the average probability that a single shower fires one of its arms is η, while the average probability that it fires both arms is γ.

The values of η and γ were estimated[10] at a primary energy of 1000 MeV and for secondary electrons of energy $\gtrsim 100$ MeV and found to be

$$\eta \approx 10^{-2} \qquad \gamma \approx 3 \times 10^{-7}$$

to within a factor of ~ 5.

It follows that, a full current (100 mA/beam)

$$\dot{n}_e \ \gamma \approx 10^{-2} \dot{n}_{SC} \qquad \dot{n}_e^2 \ \eta^2 \ \tau \approx 10^{-2} \dot{n}_{SC}$$

No shielding has been considered. It appears that by adding same more stringent requirements on energy or collinearity the backgrounds can be reduced to very tolerable values (note that $\dot{n}_e \ \gamma$ can not be subtracted out by means of delayed coincidences).

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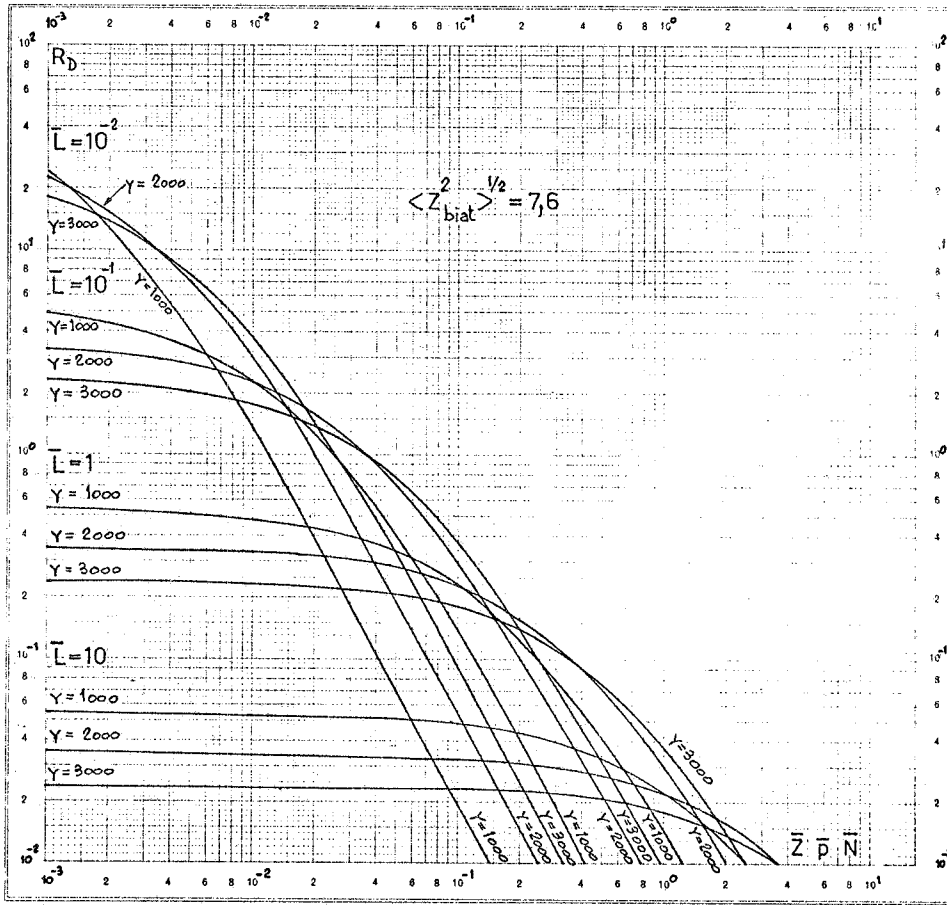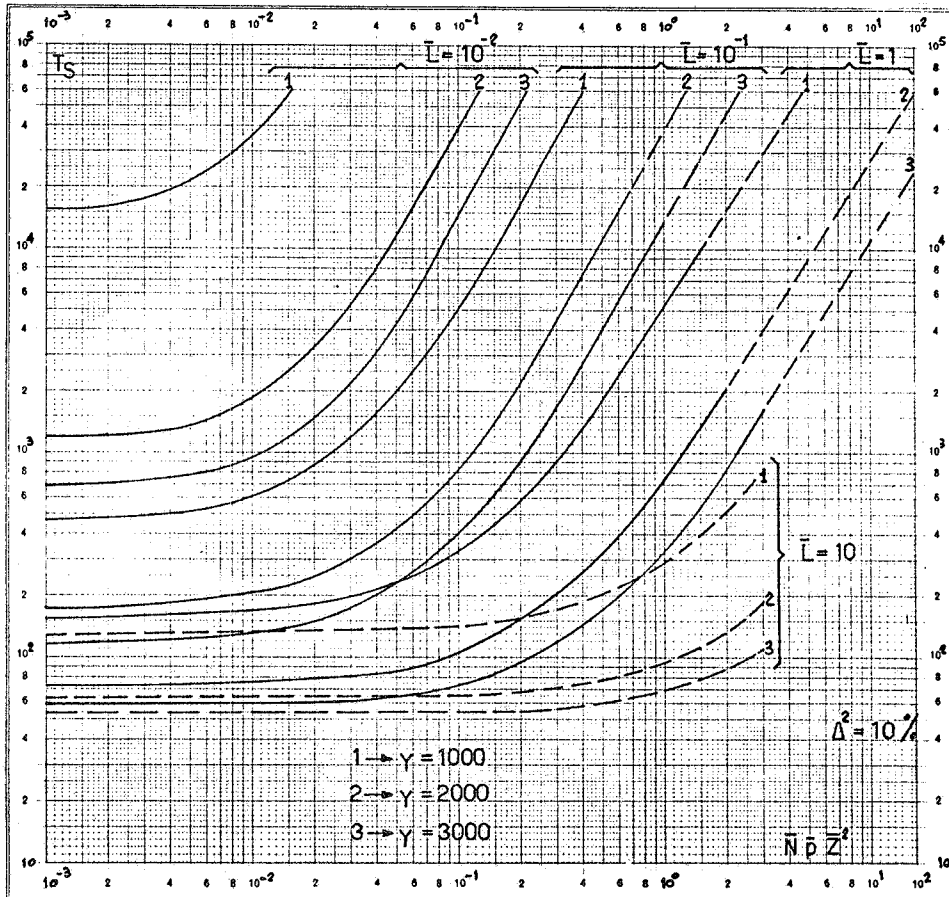S. Tazzari: Considerations on a luminosity monitor at Adone. -

FIG. 5

FIG. 7