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ABSTRACT. -

It is shown that the Regge trajectories  $\alpha(t)$  for the 10<sup>x</sup> and 27 SU(3) representations are negative for  $t \leq 0$ . It is then shown that some SU(6)<sub>W</sub> relations between forward cross sections are in striking contradiction with a Regge pole picture that seems to agree with the experimental data.

It has recently been shown<sup>(1)</sup> that it is possible to derive a class of sum rules for some scattering amplitudes of strong interactions by assuming only the restrictions imposed on the scattering amplitudes by unitarity, analyticity and by a suitable behaviour at high energy suggested by Regge theory. Sum rules for meson-baryon scattering are considered in a series of papers<sup>(2), (3), (4)</sup>, in which it is essentially assumed that the Regge trajectory  $\alpha(t)$  for the exchange in the crossed channel of a meson supposed to belong to the 27 SU(3) representation is negative for  $t \leq 0$ , so that an amplitude which for  $\nu \rightarrow \infty$ , in standard notation, behaves as  $\nu^{\alpha-1}$ , has the right asymptotic behaviour. Such an assumption is considered plausible for the 10, 10<sup>x</sup>, 27 trajectories in the light of our present knowledge of the meson mass spectrum<sup>(4)</sup>. In some of these papers it is shown that an approximate saturation of the superconvergent sum rule gives results which are not in contrast with the experimental data now available about meson-baryon resonances. However a better knowledge of the 10, 10<sup>x</sup>, and 27 seems important for further applications of superconvergence relations.

In the first part of this note we wish to show that there are some reactions experimentally accessible for which it is possible to exchange in the crossed channel only 10<sup>x</sup> and 27 trajectories. The few experimental data now available seem to confirm the validity of the hypothesis  $\alpha(t) < 0$  for  $t \leq 0$ .

2.

Some reactions involving only the  $10^x$  and  $27$  trajectories are listed in Table I, but obviously many others can be found. The general Regge formalism for the hypercharge exchange amplitudes, in the scattering of spin 1/2 and spin 0 particles, has been developed by Überall<sup>(5)</sup>. The momentum transfer distribution for a single pole exchange is given, in notations of Ref. (5), by:

$$(1) \quad \frac{d\sigma}{dt} = (16 \pi s^2)^{-1} \left[ 1 + \cotg^{2\varepsilon} \left( \frac{\pi\alpha}{2} \right) \right] F(t) \left( \frac{s}{s_0} \right)^{2\alpha}$$

where

$$(2) \quad F(t) = b_+^2 - t (M + M')^{-2} \alpha^2 b_-^2$$

is supposed to be a slowly varying function of  $t$  and for our aim can be approximated by a constant.

The value of  $\alpha$  ( $t \approx 0$ ) can be obtained by the ratio of the forward cross section at two different energies under the hypothesis of a single Regge trajectory dominance. Using the available data at 2.24 and 3 GeV<sup>(6, 7)</sup> we obtain for the reaction (a)

$$(3) \quad \alpha(0) = -0.6$$

and for the reaction (d)

TABLE I

A	B	C	D
a	$K^- p \rightarrow \Sigma^- \pi^+$	$10^x, 27$	6, 7, 12
b	$K^- p \rightarrow \Sigma^- \rho^+$	$10^x, 27$	6
c	$K^- p \rightarrow Y^{x-} \pi^+$	$10^x, 27$	7, 13
d	$K^- p \rightarrow \Xi^- K^+$	$27$	6, 7
e	$K^- p \rightarrow \Xi^- K^{*+}$	$27$	6, 7
f	$\pi^- p \rightarrow N^{x-} \pi^+$	$27$	

$$(4) \quad \alpha(0) = -0.6.$$

We have not referred any error because the experimental errors are very large so that the numbers above obtained can give only an order of magnitude and therefore we cannot give much credit to the coincidence of the intercept of the two trajectories. However our main result is that it seems sure that  $\alpha(t) < 0$  for  $t \leq 0$  for both trajectories. To obtain a better check of the Regge behaviour hypothesis for the reactions which we are considering we have tried to fit the experimental  $d\sigma/dt$ . Assuming for the Regge trajectory the form suggested by Pignotti<sup>(8)</sup> which includes some curvature

$$(5) \quad \alpha(t) = -1 + \frac{[1 + \alpha(0)]^2}{[1 + \alpha(0) - \alpha'(0)t]}$$

and assuming also positive signature, as is certainly true in the case of the  $27$  which contributes to the reaction (d), one can fit well enough the experimental data of reaction (a) with  $F(t) = \text{constant} = 5.1 \times 10^2 \text{ mb GeV}^2$  and  $\alpha'(0) = 0.4 \text{ GeV}^{-2}$ .

In the case of reaction (d) we obtain  $F(t) = 54 \text{ mb GeV}^2$ ; however the typical Regge behaviour of  $d\sigma/dt$  is not so evident and this suggests that the relevant Regge trajectory is very flat:  $\alpha'(0) \sim 0$ .

Using the trajectories found before and with a suitable choice of the residues it is possible to account for the behaviour of  $d\sigma/dt$  for the other reactions listed.

We again emphasize that our results are essentially qualitative. Owing to the importance of the problem it should be interesting to obtain other experimental data at higher energies and with better statistics.

We turn now to another problem strictly connected to our previous discussion. It is known<sup>(9)</sup> that the relativistic group  $SU(6)_W$  gives many relations between the squares of matrix elements in the forward (or backward) direction for two body or quasi-two body reactions. In particular there are 36 processes whose amplitudes are, according to  $SU(6)_W$ , simple multiples of one basic amplitude  $D$  (see Table II of Ref. (9)). While some of these relations in the forward direction seem well satisfied at low energies<sup>(10)</sup> (compare also Fig. 1 of Ref. (11)), for some of the few others that have been tested experimentally the overall disagreement is of a factor 100 or more<sup>(11)</sup>, and it seems that the disagreement grows as the energy increases.

To make a more definite analysis let us consider two examples, which seem particularly expressive.  $SU(6)_W$  gives in the forward direction the following relations

$$(6) \quad d\sigma \left[ \pi^- p \longrightarrow \pi^- N^{*+} \right] = \frac{1}{12} d\sigma \left[ \pi^- p \longrightarrow \pi^+ N^{*-} \right]$$

$$(7) \quad d\sigma \left[ K^- p \longrightarrow \pi^- Y^{*+} \right] = \frac{1}{4} d\sigma \left[ K^- p \longrightarrow \pi^+ Y^{*-} \right]$$

4.

We note that these predictions are consistent with a Regge pole theory, only if the intercepts of the trajectories dominating the high energy behaviour of the two reactions are equal. According to our previous consideration this is not the case for both relations (6), (7).

Consider first relation (6): in the crossed channel of the left-hand reaction we can exchange the  $\rho$  trajectory so that the cross section is expected to go as  $\sim s^{-1}$  at high energy; in the right-hand reaction we can exchange only a 27 trajectory which is negative for  $t \leq 0$  as we have shown, so that the cross section vanishes very rapidly at increasing energies and relation (6) cannot be satisfied. Relation (7) can be discussed in a similar way<sup>(x)</sup>: the leading trajectory for the left-hand reaction is the  $K^*$  one, for which it is well known that  $\alpha_{K^*}(0) > 0$ , while for the right-hand reaction the leading trajectory belongs to the 10\*: once again we find that the right-hand reaction is much more depressed than the left-hand one, in striking contradiction with the  $SU(6)_W$  predictions. Many similar contradictions can be found by comparing the  $SU(6)_W$  predictions given in Table II of Ref. (9) with the expected Regge behaviour.

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(x) - For an experimental check of relation (7) compare Table II of Ref. (11), where a very strong disagreement is found.

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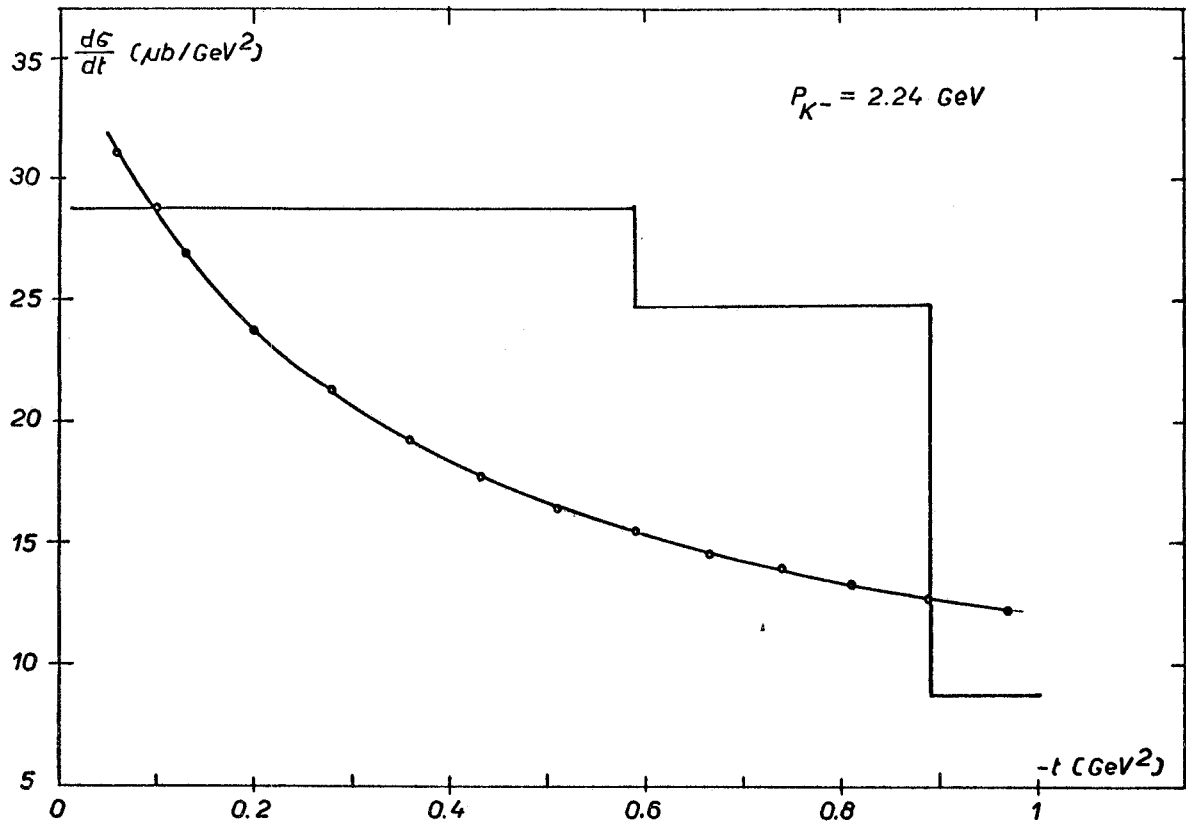


FIG. 1a

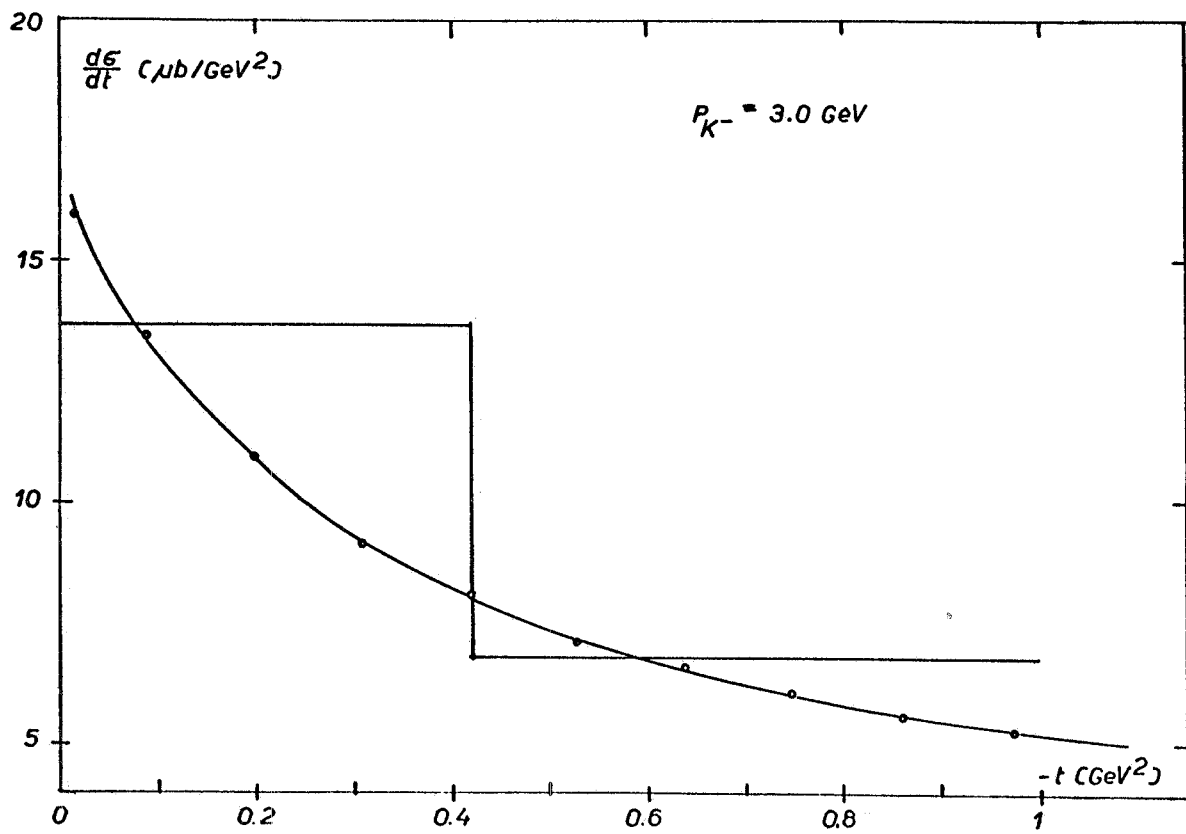


FIG. 1b