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P. Di Vecchia and M. Greco : DOUBLE PHOTON EMISSION IN $e^{\pm} e^{-}$ COLLISIONS. -

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(Submitted to Nuovo Cimento for Publication)

INTRODUCTION. -

In the attempt to observe two photon annihilation of electron-positron pairs with "ADA"(1) a number of events were detected in which the two photon coincidences between two lead glass Cerenkov counters could be interpreted as due to the process of double bremsstrahlung

$$e^{+} + e^{-} \rightarrow e^{+} + e^{-} + 2\gamma \tag{1}$$

If the energy of the incident electrons and positrons in sufficiently high, process (1) would dominate over two quanta annihilation by a factor of the order $\alpha^2 (E/m)^2$ because under the conditions of the experiments the four-momentum transfer between the colliding particles of process (1) is smaller than the corresponding quantity for two quanta annihilation. This property of the process (1) allowed one to put it on the list of possible monitoring processes, defined as collision processes between electrons and positrons in which the momentum transfer could be considered sufficiently small, so that its description by quantum electrodynamics with unit form-factors and unmodified propagators could be considered accurate. Besides it seemed that process (1) presented some particular advantages with regard to other monitoring processes(2).

The cross section for process (1) has been calculated previously by M. Bander[4] and by V.N. Bayer and V.M. Galitsky[3], but their results did not agree. We set out to understand the reason for this discrepancy. In view of the lengthiness and complication of the kinematic part of the calculation this investigation was performed along two independent lines by the authors. The results will be compared with those published by other authors, so as to obtain a complete picture of process (1). Our results agree with those obtained by Bayer and Galitsky.

RESULTS. -

The Feynman diagrams contributing to (1) in lowest order are obtained by adding in all possible ways two external photon lines to the two graphs of the elastic scattering and of the annihilation.

In the high energy and small angles approximation only the leading graphs in Fig. 1a have been considered, the contribution of the graphs of Fig. 1b being considered negligible. This is certainly correct for an experiment which looks mainly at the photons in the forward-backward direction, but is not so obvious to hold for the total cross section. The reason that makes the annihilation graphs negligible can be inferred from an inspection of the expression of the photon propagator. Indeed the oder of magnitude of the photon propagator in the annihilation graphs is $E(E-\omega_{1,2})$ in c.m.s., while the propagator of the scattering graphs goes to m^2 (m is the electron mass). The resulting cross section is:

$$\sigma = \frac{(2\pi)^2}{\left[(p_1p_2)^2 - m^4\right]^{1/2}} \int \frac{d^3p_3}{E_3} \int \frac{d^3p_4}{E_4} \int \frac{d^3k_1}{\omega_1} \int \frac{d^3k_2}{\omega_2} \; . \tag{2}$$

$$\cdot \frac{\delta^4(p_1+p_2-p_3-p_4-k_1-k_2)}{(p_1-p_3-k_1)^4} \sum_{spin} \sum_{pol} \frac{|M|^2}{4}$$

Where $p_1(p_2)$ and $p_3(p_4)$ are the four-momenta of the incoming and outgoing electron (positron), k_1, k_2 those of the two photons and the matrix element M is

$$M = \bar{u}(p_3)\, D_\mu\, u(p_1)\, \bar{v}(p_2)\, C^\mu\, v(p_4)\, \frac{i e^4 m^2}{2(2\pi)^5} \tag{3}$$

with

$$D_\mu = \gamma_\mu \frac{\not p_1 - \not k_1 + m}{-2(p_1k_1)} \not e_1 + \not e_1 \frac{\not p_3 + \not k_1 + m}{2(p_3k_1)} \gamma_\mu \; , \tag{4.a}$$

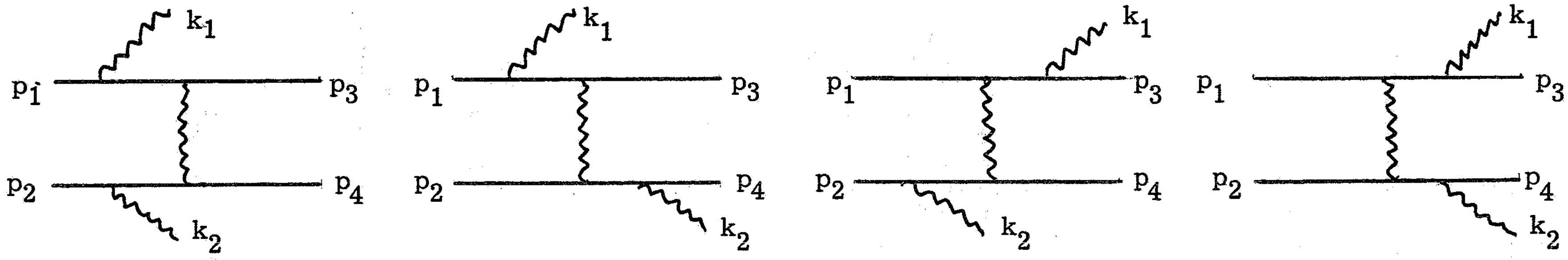

FIG. 1a) - Leading graphs

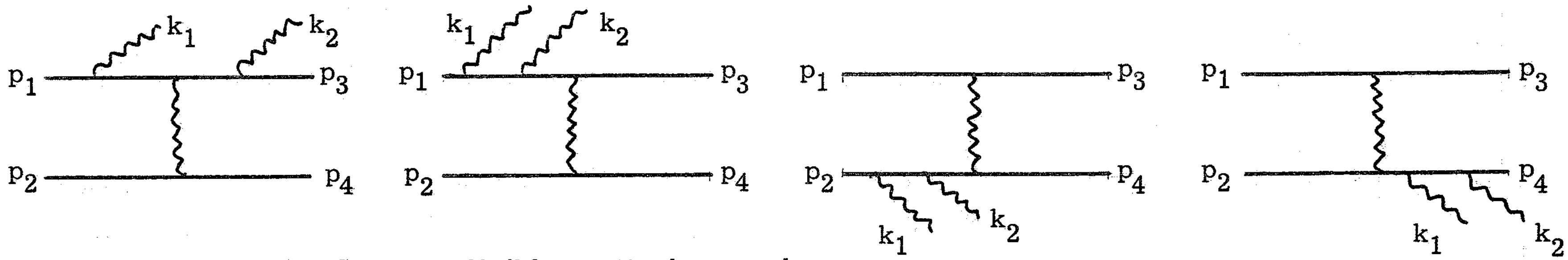

FIG. 1b) - Some negligible scattering graphs

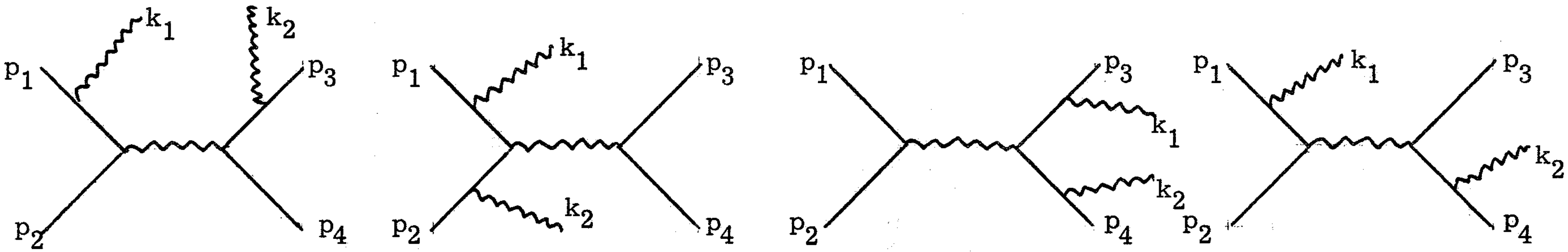

FIG. 1c) - Some negligible annihilation graphs

FIG. 1

(4.b) $$C_\mu = \gamma_\mu \frac{-\not{p}_4 - \not{k}_2 + m}{2(p_4 k_2)} \not{e}_2 + \not{e}_2 \frac{-\not{p}_2 + \not{k}_2 + m}{-2(p_2 k_2)} \gamma_\mu$$

After elimination of the δ -function the cross section (2) becomes:

(5) $$\sigma = \frac{(2\pi)^2}{\left[(p_1p_2)^2 - m^4\right]^{1/2}} \int \frac{d^3k_1}{\omega_1} \int \frac{d^3k_2}{\omega_2} \int d\Omega \left[\frac{d|\vec{p}_3|}{dE_f} \cdot \frac{\vec{p}_3^{\,2}}{E_3E_4(p_1 - p_3 - k_1)^4} \sum_{pol} \sum_{spin} \frac{|M|^2}{4}\right]_{\substack{E_{in} = E_f \\ \vec{P}_{in} = \vec{P}_f}}$$

where $\vec{P}_{in}$ (E_{in}) and $\vec{P}_f$(E_f) are the total momentum (energy) of the system in the initial and final states. The calculation of the integral (5) has been performed in the center of mass frame where $E_1 = E_2 = E$ and $\vec{p}_1 = -\vec{p}_2$. In order to obtain information about the high energy behaviour of the process, the special case in which one photon follows exactly the direction of the incident positron has been treated in detail. The cross section for this forward-backward emission is[2]:

(6) $$d\sigma_{F.B.} = \frac{\alpha^2 r_0^2}{\pi^3} \frac{dx_1}{x_1} \frac{dx_2}{x_2} \left\{ \frac{4}{3}(1 - x_1)(1 - x_2) + (1 - x_1)x_2^2 + (1 - x_2)x_1^2 + x_1^2 x_2^2 \right\} \gamma^4 \, d\Omega_1 \, d\Omega_2 \; .$$

Where $x_{1,2} = \omega_{1,2}/E$ and $d\Omega_{1,2}$ are the elements of solid angle of the two photons. The result (6) is valid only for $d\Omega_{1,2} \ll \pi/\gamma^2$ and this is too strong a limitation on a detection device.

The total section, which gives the energy distribution of the radiation, has been calculated from (5) independently by the two authors, who found respectively:

(7.a) $$d\sigma = \frac{8\alpha^2 r_0^2}{\pi} \frac{dx_1}{x_1} \frac{dx_2}{x_2} \left\{ \frac{9}{4}(1 - x_1)(1 - x_2) + \frac{3}{2}\left[(1 - x_2)x_1^2 + (1 - x_1)x_2^2\right] + x_1^2 x_2^2 \right\} \quad \text{(M.G.)}$$

(7.b)

$$d\sigma = \frac{8\alpha^2 r_o^2}{\pi} \frac{dx_1}{x_1} \frac{dx_2}{x_2} \Big\{ (1-x_1)(1-x_2) \Big[\frac{5}{4} + \frac{7}{8}\zeta(3) \Big] + \Big[x_1^2(1-x_2) + x_2^2(1-x_1) \Big] \Big[\frac{1}{2} + \frac{7}{8}\zeta(3) + \frac{7}{8}\zeta(3) x_1^2 x_2^2 \Big] \Big\} \text{ (P. D. V.)}$$

where $\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3}$.

The result (7.a) is given with an error about the order of 5%, and both agree in the limit of low frequencies with the Bloch-Nordisieck cross section $d\sigma_{B.N.}$ (2):

(8) $$d\sigma_{B.N.} = \frac{8\alpha^2 r_o^2}{\pi} \frac{dx_1}{x_1} \frac{dx_2}{x_2} \Big[\frac{5}{4} + \frac{7}{8}\zeta(3) \Big]^{(x)} .$$

The cross sections (6) and (7) have been evaluated in this paper for the case $E_3 \gg m$ where E_3 could be for example the energy of the final electron; the results presented here therefore are wrong only where $E - \omega_{1,2}$ is about the order of m. In each other part of the spectrum the previous results are valid $o(1/\gamma)$ where $\gamma = E/m$. The result (7) agrees with the values of the cross section given by Bayer and Galitsky, which is identical to (7.b).

It has been noted by Sidorov that the cross section (7) can be put in the approximate form:

(9) $$d\sigma \cong \frac{8\alpha^2 r_o^2}{\pi} \frac{dx_1}{x_1} \frac{dx_2}{x_2} F(x_1) F(x_2) ,$$

where F(x) is:

(10) $$F(x) = \frac{3}{2}(1-x) + x^2 .$$

This result is very-important from the experimental point of view: it says that the energy spectrum of one photon is completely independent from the energy spectrum of the other photon. This separability of the cross section has been experimentally confirmed with VEP-1 in Novosibirsk(5).

(x) - The numerical coefficient 2, which appears in a previous paper(2), is not correct and must be substituted with 5/4. The same results have been found by Bayer and Galitsky(6).

The expressions of the forward-backward cross section (6) and of the total cross section (7) allow us to calculate the mean angle $\bar{\Theta}$ of the two photon emission by the process (1):

$$\frac{\gamma^4 \bar{\Theta}^4}{8} =$$

(11) $$= \frac{(1-x_1)(1-x_2)\left[\frac{5}{4}+\frac{7}{8}\zeta(3)\right] + \left[x_1^2(1-x_2)+x_2^2(1-x_1)\right]\left[\frac{1}{2}+\frac{7}{8}\zeta(3)\right] + \frac{7}{8}\zeta(3)x_1^2x_2^2}{\frac{4}{3}(1-x_1)(1-x_2) + x_1^2(1-x_2) + x_2^2(1-x_1) + x_1^2x_2^2}$$

$(\gamma^4\bar{\Theta}^4)/8$ is a slowly varying function of x_1 and x_2, whose values lie between 1.0 and 1.8.

In order to obtain information about the angular behaviour of the emitted photons, we evaluated the cross section $d\sigma(t, x_1, x_2)$ where $t = \gamma\Theta = \gamma R/1$. R is the radius of both the Cerenkov counters and 1 is the distance between a Cerenkov counter and the crossing point of the electron-positron beams. This resulting cross section is:

(12) $$d\sigma(t, x_1 x_2) = \frac{8\alpha^2 r_o^2}{\pi}\frac{dx_1}{x_1}\frac{dx_2}{x_2}\Big\{G_1(t)(1-x_1)(1-x_2) + G_2(t)\left[x_1^2(1-x_2)+x_2^2(1-x_1)\right] + G_3(t)\, x_1^2 x_2^2\Big\}$$

The function G_1, G_2, G_3 are indicated in Table I. This result agrees with an analogous calculation performed by V.N. Bayer et al. (7).

The cross section for emitted photons of energy $\omega_{1,2}/E \geq \varepsilon$ is obtained integrating (12) over x_1 and x_2 and results:

(13) $$\sigma(\varepsilon, t) = \frac{8\alpha^2 r_o^2}{\pi}\Big\{G_1(t)\left[\lg\frac{1}{\varepsilon} - 1 + \varepsilon\right]^2 + G_2(t)(1-\varepsilon^2)\left[\lg\frac{1}{\varepsilon} - 1 + \varepsilon\right] + G_3(t)\left(\frac{1-\varepsilon^2}{2}\right)^2\Big\}$$

A plot of $\sigma(\varepsilon, t)$ is drawn in Fig. 2 for $t \to \infty$. An analogous calculation performed by M. Bander(4) has given the following result:

(14) $$\sigma(\varepsilon, t) = \frac{8\alpha^2 r_o^2}{\pi}\Big\{M_1(t)\left[\lg\frac{1}{\varepsilon} - 1 + \varepsilon\right]^2 + M_2(t)\left[\lg\frac{1}{\varepsilon} - 1 + \varepsilon\right] + \frac{1}{4}M_3(t)\Big\}.$$

TABLE I

t	G_1	G_2	G_3	M_1	M_2	M_3
1	0.081	0.065	0.05	0.16	0.065	0.050
2	0.41	0.31	0.24	0.74	0.31	0.24
3	0.74	0.55	0.41	1.27	0.55	0.41
4	1.02	0.74	0.54	1.70	0.74	0.54
5	1.23	0.88	0.64	2.00	0.88	0.64
6	1.39	0.99	0.71	2.25	0.99	0.71
7	1.52	1.07	0.76	2.41	1.07	0.76
8	1.62	1.14	0.80			
9	1.70	1.19	0.84			
10	1.77	1.23	0.86			
11	1.82	1.27	0.89			
12	1.87	1.30	0.90			
13	1.91	1.32	0.92			
14	1.94	1.34	0.93			

The functions $M_1(t)$, $M_2(t)$, $M_3(t)$ are indicated in Table I. We believe that (14) is valid only in the limit $\varepsilon^2 \ll 1$, because the cross section (14) does not go to zero when ε goes to 1. In this approximation the formulas (13) and (14) present the same analytical form with regard to the dependence on t and ε. Besides the Table I shows an exact coincidence between the functions $G_2(t)$, $G_3(t)$ and $M_2(t)$, $M_3(t)$. The functions $F_1(t)$ and $M_1(t)$, which give the main contribution to the cross section at the low frequencies, present instead an almost constant discrepancy, whose order is of about 50%.

Therefore our results agree completely with those performed by V.M. Bayer and by M. Bander apart from the discrepancy with M. Bander's

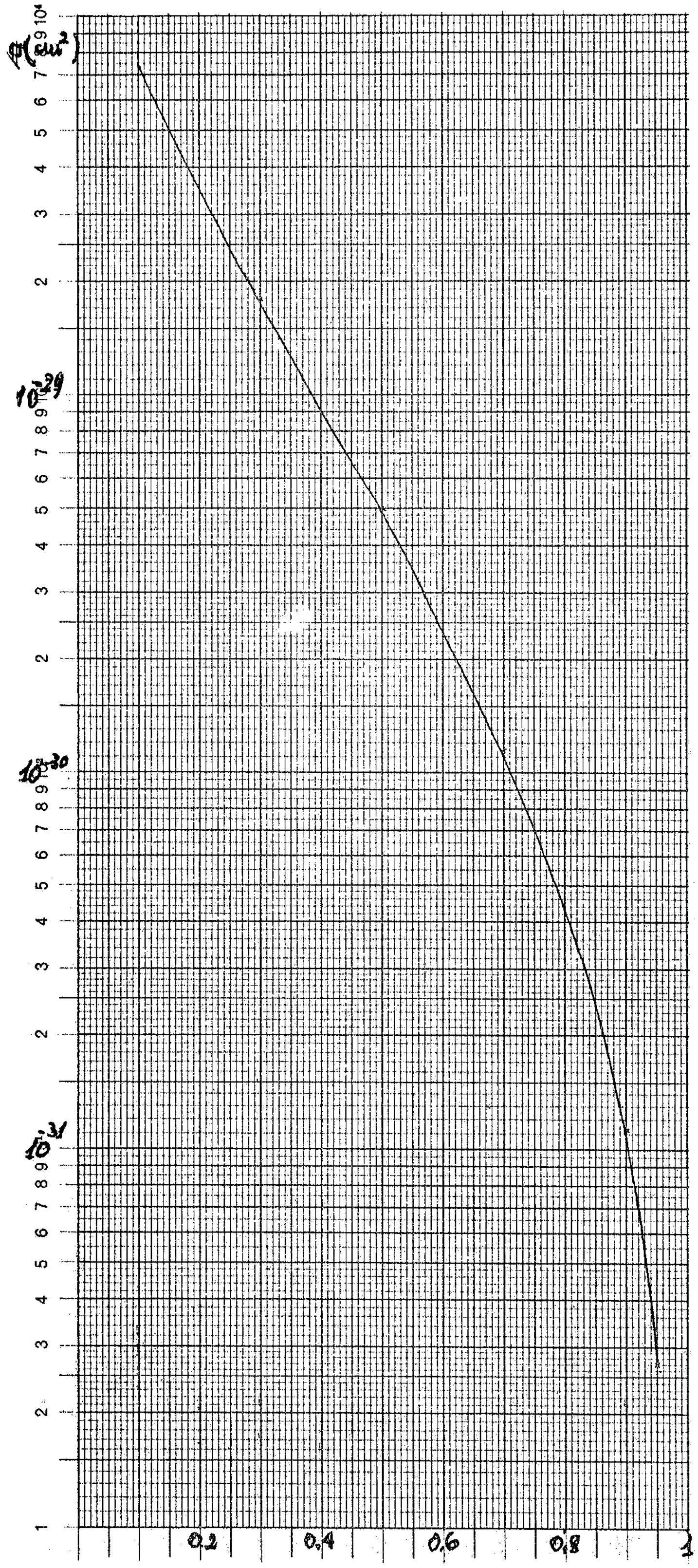

FIG. 2

calculations, concerning the functions $F_1(t)$ and $M_1(t)$.

A remarkable feature of the cross section (7) is the absence of a logarithmical term of the form $\log 2\gamma$, which is so caracteristic of the single bremsstrahlung total cross section(8). This logarithmical term does not appear in the cross section of process (1) because the modulus square of the current matrix element is, because of the trasversality of the emitted photons, proportional to the fourth power of the momentum transfer and the probability of a momentum transfer z is proportional to dz/z^3, so that the differential cross section of the process (1) approaches zero when there is no momentum transfer. The single bremsstrahlung differential cross section behaves as dz/z for low momentum transfer and this creates the logarithmical term. The lack of a logarithmical term puts double bremsstrahlung at a disadvantage opposite the background of the single bremsstrahlung processes created either in collisions with the molecules of the residual gas or in genuine electronpositron collisions(9).

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APPENDIX. -

In this appendix we exibite some details of the calculation of $d\sigma$ and $d\sigma_{F.B.}$, of which before we have gieven only the results. The total cross section (5) can be written in the following form:

$$\sigma = \frac{2\alpha^2 r_o^2 m^6}{E^2(2\pi)^4} \int \omega_1 d\omega_1 d\Omega_1 \int \omega_2 d\omega_2 d\Omega_2 \int d\Omega \,\cdot$$

(A. 1)
$$\cdot \left[X \frac{|\vec{p}_3|^2}{E_3 E_4 (p_1 - p_3 - k_1)^4} \frac{d|\vec{p}_3|}{dE_f} \right]_{\substack{E_{in} = E_f \\ \vec{P}_{in} = \vec{P}_f}}$$

where:

(A. 2)
$$X = \frac{1}{4} \sum_{pol}' Tr\left\{ D_\mu \Lambda_-(p_1) \bar{D}_\nu \Lambda_-(p_3) \right\} Tr\left\{ C^\mu \Lambda_+(p_4) \bar{C}^\nu \Lambda_+(p_2) \right\}$$

Since we consider only the terms which in the limit of high energies do the maximum contribution to the cross section (2), we obtain the following value for the expression (A. 2) by making use of well-known trace theorems[10].

(A. 3)
$$X = \sum_{i=1}^{9} A_i$$

where :

$$A_1 = \frac{1}{2m^4} \sum_{pol} \left(\frac{p_1 e_1}{p_1 k_1} - \frac{p_3 e_1}{p_3 k_1} \right)^2 \left(\frac{p_4 e_2}{p_4 k_2} - \frac{p_2 e_2}{p_2 k_2} \right)^2 \cdot$$

$$\cdot \left[(p_1 p_4)(p_2 p_3) + (p_1 p_2)(p_3 p_4) \right]$$

$$A_2 = \frac{1}{2m^4} \sum_{pol} \left(\frac{p_1 e_1}{p_1 k_1} - \frac{p_3 e_1}{p_3 k_1} \right)^2 \left(\frac{p_4 e_2}{p_4 k_2} - \frac{p_2 e_2}{p_2 k_2} \right) \cdot$$

$$\cdot \left[\frac{p_2 e_2}{p_2 k_2} \left((p_1 p_4)(p_3 k_2) + (p_3 p_4)(p_1 k_2) \right) + \right.$$

$$\left. + \frac{p_4 e_2}{p_4 k_2} \left((p_1 k_2)(p_2 p_3) + (p_1 p_2)(p_3 k_2) \right) \right]$$

$$A_3 = \frac{1}{2\,m^4} \sum_{pol} \left(\frac{p_1 e_1}{p_1 k_1} - \frac{p_3 e_1}{p_3 k_1} \right) \left(\frac{p_4 e_2}{p_4 k_2} - \frac{p_2 e_2}{p_2 k_2} \right) \cdot$$

$$\cdot \left\{ - \frac{p_3 e_1}{p_3 k_1} \left[\frac{p_2 e_2}{p_2 k_2} ((p_1 p_4)(k_1 k_2) + (p_1 k_2)(p_4 k_1)) + \right. \right.$$

$$\left. + \frac{p_4 e_2}{p_4 k_2} ((p_1 k_2)(p_2 k_1) + (p_1 p_2)(k_1 k_2)) \right] -$$

$$- \frac{p_1 e_1}{p_1 k_1} \left[\frac{p_2 e_2}{p_2 k_2} ((p_4 k_1)(p_3 k_2) + (k_1 k_2)(p_3 p_4)) + \right.$$

$$\left. \left. + \frac{p_4 e_2}{p_4 k_2} ((k_1 k_2)(p_2 p_3) + (p_2 k_1)(p_3 k_2)) \right] \right\}$$

(A.4)
$$A_4 = \frac{1}{2\,m^4} \sum_{pol} \left(\frac{p_1 e_1}{p_1 k_1} - \frac{p_3 e_1}{p_3 k_1} \right)^2 \left\{ - 2m^2 \frac{(p_1 k_2)(p_3 k_2)}{(p_4 k_2)(p_2 k_2)} - \right.$$

$$\left. - \frac{(p_1 k_2)(p_2 p_3) + (p_1 p_2)(p_3 k_2)}{p_4 k_2} - \frac{(p_1 p_4)(p_3 k_2) + (p_1 k_2)(p_3 p_4)}{p_2 k_2} \right\}$$

$$A_5 = \frac{1}{2\,m^4} \sum_{pol} \left(\frac{p_1 e_1}{p_1 k_1} - \frac{p_3 e_1}{p_3 k_1} \right) \left\{ \frac{p_1 e_1}{p_1 k_1} \left[\frac{(k_1 k_2)(p_2 p_3) + (p_2 k_1)(p_3 k_2)}{p_4 k_2} + \right. \right.$$

$$\left. + \frac{(p_4 k_1)(p_3 k_2) + (k_1 k_2)(p_3 p_4)}{p_2 k_2} \right] +$$

$$+ \frac{p_3 e_1}{p_3 k_1} \left[\frac{(p_1 k_2)(p_2 k_1) + (p_1 p_2)(k_1 k_2)}{p_4 k_2} + \frac{(p_1 p_4)(k_1 k_2) + (p_1 k2)(p_4 k_1)}{p_2 k_2} \right] +$$

$$\left. + 2\,m^2 \left[\frac{(p_1 e_1)}{p_1 k_1} \frac{(k_1 k_2)(p_3 k_2)}{(p_4 k_2)(p_2 k_2)} + \frac{p_3 e_1}{p_3 k_1} \frac{(p_1 k_2)(k_1 k_2)}{(p_2 k_2)(p_4 k_2)} \right] \right\}$$

$$A_6 = \frac{1}{2\,m^4}\left\{ \frac{(k_1k_2)(p_2p_3) + (p_2k_1)(p_3k_4)}{(p_1k_1)(p_4k_2)} + \frac{(p_4k_1)(p_3k_2) + (k_1k_2)(p_3p_4)}{(p_1k_1)(p_2k_2)} + \right.$$

$$+ \frac{(p_1k_2)(p_2k_1) + (p_1p_2)(k_1k_2)}{(p_3k_1)(p_4k_2)} + \frac{(p_1p_4)(k_1k_2) + (p_1k_2)(p_4k_1)}{(p_3k_1)(p_2k_2)} +$$

$$+ 2m^2\left[\frac{(k_1k_2)(p_3k_2)}{(p_1k_1)(p_2k_2)(p_4k_2)} + \frac{(p_1k_2)(k_1k_2)}{(p_3k_1)(p_4k_2)(p_2k_2)} + \right.$$

$$\left. + \frac{(k_1k_2)(p_2k_1)}{(p_1k_1)(p_3k_1)(p_4k_2)} + \frac{(p_4k_1)(k_1k_2)}{(p_1k_1)(p_3k_1)(p_2k_2)} \right] +$$

$$\left. + \frac{2m^4\,(k_1k_2)^2}{(p_1k_1)(p_2k_2)(p_3k_1)(p_4k_2)} \right\}$$

A_7, A_8, A_9 are obtained from A_2, A_4, A_5 by the following substitutions:

(A. 5) $\qquad k_1 \leftrightarrow k_2 \qquad p_1 \leftrightarrow -p_4 \qquad p_3 \leftrightarrow -p_2 \qquad e_1 \leftrightarrow e_2$.

Before of performing the integrations in (A. 1) over $d\Omega$, $d\Omega_1$, $d\Omega_2$ the following substitutions must be done:

(A. 6)

$$\vec{p}_4 = -\vec{p}_3 - \vec{k}_1 - \vec{k}_2 \qquad\qquad E_4 = 2E - \omega_1 - \omega_2 - E_3$$

$$E_3 = \frac{2E^2 - 2E\omega_1 - 2E\omega_2 + \omega_1\omega_2 - \vec{k}_1\vec{k}_2}{2E - \omega_1 - \omega_2 - \vec{\beta}_3\vec{k}_1 + \vec{\beta}_3\vec{k}_2}$$

where according to the previous approximations β and β_3 result:

(A. 7) $$\beta = 1 - \frac{m^2}{2E^2} \qquad\qquad \beta_3 = 1 - \frac{m^2}{2(E - \omega_1)^2}$$

Since the energy of the incident particles is very high, the maximum contribution to the integral (A. 1) is obtained when one photon follows the incident electron, the other the incident positron and the scattering angles of the electron and positron are very small.

Therefore it is possible to develop the integrand in a series around the points $\theta = \theta_1 = 0$, $\theta_2 = \pi$ so that in the integral (A. 1) are considered only the contributions around the pick of the differential cross section. Owing to this development the angles θ, θ_1, θ_2 can be considered as vectors (Fig. 3b) of the plane orthogonal to the colliding beams and the cross section (A.1) results:

$$\sigma = \frac{\alpha^2 r_0^2}{\pi^4} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \int \frac{d\vec{z}}{z^4} \int d\vec{x} \left\{ 2(1-x_1) I(\vec{x}, \vec{z}) + \right. \tag{A.8}$$

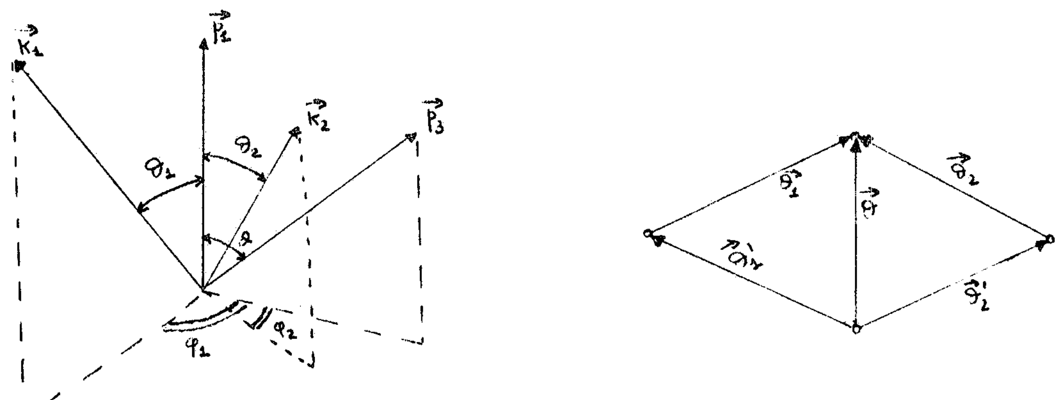
$$\left. + x_1^2 F(\vec{x}, \vec{z}) \right\} \int d\vec{y} \left\{ 2(1-x_2) I(\vec{y}, \vec{z}) + x_2^2 F(\vec{y}, \vec{z}) \right\}$$

where :

$$\vec{x} = \gamma \vec{\theta}_1 \qquad \vec{y} = \gamma \vec{\theta}_2 \qquad \vec{z} = \gamma \left[(1-x_1)\vec{\theta} + x_1 \vec{\theta}_1 \right]$$

$$I(\vec{x}, \vec{z}) = \frac{x^2}{(1+x^2)^2} + \frac{(\vec{x}-\vec{z})^2}{\left[1+(\vec{x}-\vec{z})^2\right]^2} - \frac{2\vec{x}(\vec{x}-\vec{z})}{(1-x^2)\left[1+(\vec{x}-\vec{z})^2\right]} \tag{A.9}$$

$$F(\vec{x}, \vec{z}) = \frac{z^2}{(1+x^2)\left[1+(\vec{x}-\vec{z})^2\right]}$$



a) b)

FIG. 3

If we eliminate the integrations over dx_1, dx_2, $d\vec{x}$ and $d\vec{y}$, the forward-backward cross section $d\sigma_{F.B.}$ can be obtained provided that we put $\vec{x} = \vec{y} = 0$ and integrate over $\vec{x}$. By making use of polar coordinates it is easy to perform the integration over the two photon azimuths $\varphi_{1,2}$ and then on x and y from 0 to $\gamma\Theta$. The resulting cross section is:

$$d\sigma = \frac{2\alpha^2 r_o^2}{\pi} \frac{dx_1}{x_1} \frac{dx_2}{x_2} \int_0^\infty \frac{dz}{z^3} \left\{ 4(1-x_1) J(z,u) + \right. \tag{A.10}$$

$$\left. + x_1^2 G(z,u) \right\} \left\{ 4(1-x_2) J(z,u) + x_2^2 G(z,u) \right\}$$

where:

$$J(z,u) = \frac{2+z^2}{2z\sqrt{z^2+4}} \lg \frac{(z+\sqrt{z^2+4})^4}{4\left\{z\left[z+\sqrt{z^2+4}\right]\left[A(z,u)-1+u(4+z^2)\right. +4A(z,u)\right\}} -$$

$$- \frac{3}{4} + \frac{1}{2} u + \frac{(2+z^2)u-1}{4A(z,u)}$$

$$G(z,u) = \frac{z}{\sqrt{z^2+4}} \lg \frac{(z+\sqrt{z^2+4})^4}{4\left\{z\left[z+\sqrt{z^2+4}\right]\left[A(z,u)-1+u(4+z^2)\right] +4A(z,u)\right\}} \tag{A.11}$$

$$A(z,u) = \left[1 - 2z^2 u + u^2 z^2 (z^2+4)\right]^{1/2}$$

$$u = \frac{1}{1+t^2} = \frac{1}{1+\gamma^2 \Theta^2}$$

Because of the convergence of the integral (A.10) the upper limit of integration has been set equal to infinity and the calculation has been accouplished analytically only if $u=0$. Otherwive the integration over z has been performed numerically.

Instead of develop the integrand of (A.1) in a series around the forward-backward direction, we perform the angular integration in the other following way. Let us introduce a slight different definition of the various angles, as in Fig. 4.

By making use of the (A.6) we find the following expression for the photon propagator:

$$(p_1 - p_3 - k_1)^4 =$$

$$= \frac{4(\sigma - \delta\cos\theta + \rho\,\mathrm{sen}\,\theta)^2}{(2E-\omega_1-\omega_2)+\beta_3\cos\theta(\omega_1\cos\theta_1-\omega_2\cos\theta_2)+\beta_3\,\mathrm{sen}\,\theta[\omega_1\,\mathrm{sen}\,\theta_1\cos\phi+\omega_2\,\mathrm{sen}\,\theta_2\cos(\phi-\chi)]\}^2} \tag{A.12}$$

where:

$$\sigma = 2(E-\omega_1)^2(E-\omega_2) + \left[(p_1-k_1)-m^2\right]\left[2E-\omega_1-\omega_2\right]$$

$$\delta = \Big\{2(E-\omega_1)(E-\omega_2)(p_1-\omega_1\cos\theta_1) - \left[(p_1k_1)-m^2\right]\left[\omega_1\cos\theta_1-\omega_2\cos\theta_2\right]\Big\}\beta_3$$

$$\rho = \Big\{2(E-\omega_1)(E-\omega_2)\,\omega_1\,\mathrm{sen}\,\theta_1\cos\phi + \left[(p_1k_1)-m^2\right]\left[\omega_1\,\mathrm{sen}\,\theta_1\cos\phi+\omega_2\,\mathrm{sen}\,\theta_2\cos(\phi-\chi)\right]\Big\}\beta_3$$

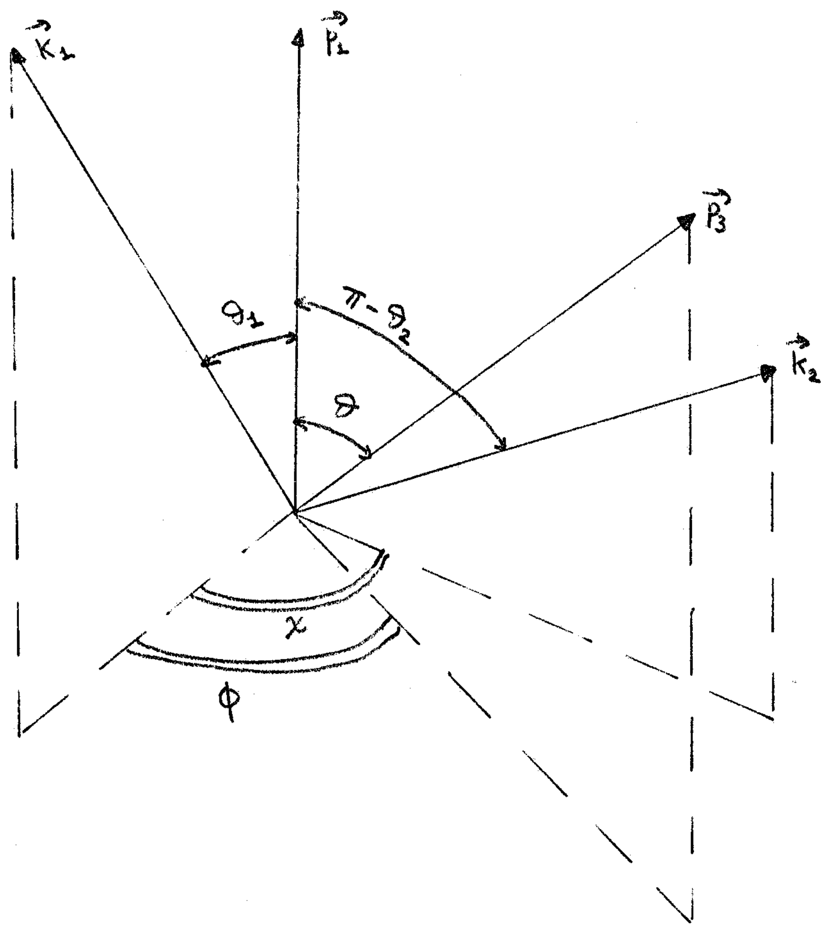


FIG. 4

By substituting (A.12) into (A.1), in order to perform the integration over the anomaly θ of the outgoing electron, we must calculate some integrals of this kind:

$$\text{(A. 13)} \quad \int_0^{\pi} \frac{\operatorname{sen}\theta \, d\theta \, P(\cos\theta, \operatorname{sen}\theta)}{(\sigma - \delta\cos\theta + \rho\operatorname{sen}\theta)^2 (\alpha - u\cos\theta - v\operatorname{sen}\theta)^h (\alpha' - u'\cos\theta - v'\operatorname{sen}\theta)^k}$$

Where $P(\cos\theta, \operatorname{sen}\theta)$ is a polynomial in $\cos\theta$, and h, k can take any value among 0, 1, 2. The terms $(\alpha - h\cos\theta - v\operatorname{sen}\theta)$, $(\alpha' - u'\cos\theta - v'\operatorname{sen}\theta)$ result from the expressions of $(p_4 k_2)$ and $(p_3 k_1)$. With the transformation $z = tg(\theta/2)$ we obtain some integrals of the form:

$$\text{(A. 14)} \quad I_{\alpha, hk} = \int_0^{\infty} \frac{z^{\alpha} \, dz}{(z - z_1)^2 (z - z_2)^2 (z - z_3)^h (z - z_4)^h (z - z_5)^k (z - z_6)^k}$$

where α is an integer and

$$z_{1,2} = \frac{-\rho \pm i\sqrt{\sigma^2 - \delta^2 - \rho^2}}{\sigma + \delta} \qquad z_{3,4} = \frac{v \pm i\sqrt{\alpha^2 - u^2 - v^2}}{\alpha + u}$$

$$z_{5,6} = \frac{v' \pm i\sqrt{\alpha'^2 - u'^2 - v'^2}}{\alpha' - u'}$$

The integration can be performed in the complex plane of variable z : the path is shown in Fig. 5.

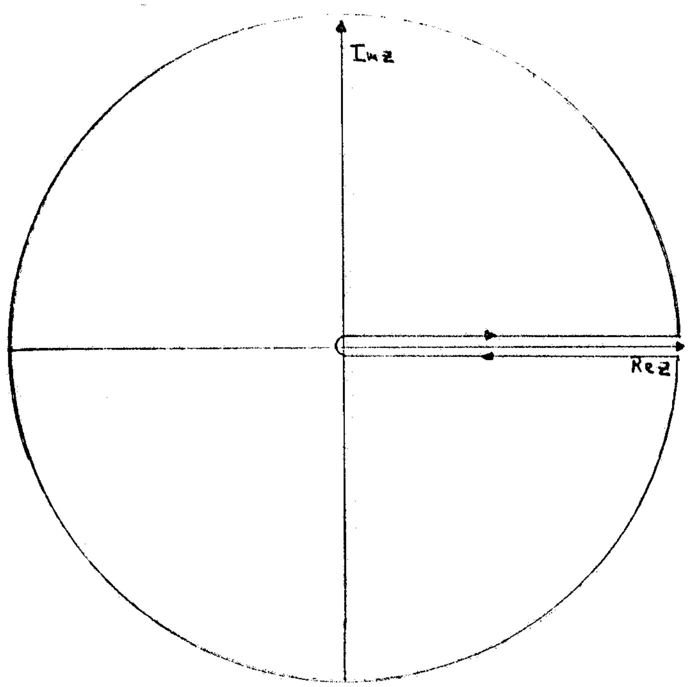


FIG. 5

We obtain:

$$I_{\alpha,hk} = - \frac{1}{\cos \pi\alpha} \frac{d}{d\alpha} \left[\sum_{i=1}^{6} \mathrm{Res}(z_i) \right]. \qquad \text{(A. 15)}$$

For $\theta_1 = \theta_2 = 0$ the function resulting from (A. 15), after integration over θ, is independent from ϕ. This result therefore gives the forward-backward cross section.

For the general case, we have to perform the integration over the azimuthal angle ϕ. This can be accomplished in a rather difficult manner, because the large number of terms which come from the integration over θ, make the calculation very cumbersome. The result is expressed by the following formula:

$$d\sigma = \frac{8\alpha^2 r_o^2}{\pi} \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} \frac{d(\cos\theta_1)\, d(\cos\theta_2)}{(1-\beta\cos\theta_1)^2(1-\beta\cos\theta_2)^2} \frac{1}{4\gamma^4} \cdot$$

$$\text{(A. 16)} \qquad \cdot \left\{ \frac{9}{4}\left(1-\frac{\omega_1}{E}\right)\left(1-\frac{\omega_2}{E}\right) + \left(1-\frac{\omega_1}{E}\right)\left(\frac{\omega_2}{E}\right)^2 \left[2 - \frac{1}{2\gamma^2(1-\beta\cos\theta_2)}\right] + \right.$$

$$+ \left(1-\frac{\omega_2}{E}\right)\left(\frac{\omega_1}{E}\right)^2 \left[2 - \frac{1}{2\gamma^2(1-\beta\cos\theta_1)}\right] + \left(\frac{\omega_1}{E}\right)^2\left(\frac{\omega_2}{E}\right)^2 \cdot$$

$$\left. \cdot \left[\frac{3}{4} + \left(1 - \frac{1}{2\gamma^2(1-\beta\cos\theta_1)}\right)\left(1 - \frac{}{2\gamma^2(1-\beta\cos\theta_2)}\right)\right] \right\} + B \; .$$

The term B is not integrated over ϕ, because of its very complicated analytic expression. However it does not contribute to the total cross section; which is obtained performing the integration on θ_1 and θ_2 in the first part of the right hand side of (A. 16). The dependence of B on θ_1 and θ_2 is of the form $R(\cos\theta_1, \cos\theta_2)\, \lg(\cos\theta_1, \cos\theta_2)$, ($R(\cos\theta_1, \cos\theta_2)$ and $\lg(\cos\theta_1, \cos\theta_2)$ are a rational and a logarithmical function of $\cos\theta_1$ and $\cos\theta_2$) and do contribute to the angular distributions.