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Consequences of Unitary Symmetry for Weak and Electromagnetic Transitions.

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Recent papers [1] have dealt with the introduction of unitary symmetry, *i.e.* invariance under the three-dimensional unitary group, as a convenient approximation in the theory of strong interaction. The strong-interaction Lagrangian is assumed to consist of a part invariant under the unitary group, plus a « correction » breaking the invariance.

In this note we shall examine the properties that follow, for weak and electromagnetic amplitudes, from this hypothesis, together with the violent assumption that the symmetry-breaking « corrections » can be neglected. We do not know under what conditions this last hypothesis can be applied. It might be applicable in the high-energy region — or, better, we know for sure that it is not generally applicable at low energies.

The consequences of violent assumptions are usually far-reaching. Thus we find that the K^0 (or $\overline{K}^0$) electromagnetic form factors must be zero, the charged K form factors must be equal to the charged pion form-factors; there are simple stringent relations betwen the form factors of the baryons (already given by COLEMAN and GLASHOW [2]), of the vector mesons, between the electromagnetic amplitudes from vector to pseudoscalar mesons, between the amplitudes of χ^0 decay and π^0 decay, and between weak interaction amplitudes. These relations depend in part on the particular representation adopted for the baryons.

For instance according to the « eightfold way » the Λ form-factors are one-half of the neutron form factors. They must instead be equal in the Sakata represen-

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(1) S. OGAWA: *Progr. Theor. Phys.*, **21**, 209 (1959); Y. YAMAGOUCHI: *Progr. Theor. Phys. Suppl.* **11**, 37 (1959); M. IKADA, S. OGAWA and Y. OHNUKI: *Progr. Theor. Phys.*, **22**, 715 (1959); **23**, 1073 (1960); I. E. WESS: *Nuovo Cimento*, **15**, 52 (1960); Z. MEKI, M. NAKAGAWA, Y. OHNUKI and S. SAKATA: *Progr. Theor. Phys.*, **23**, 1174 (1960); M. IKEDA, Y. MIYACHI and S. OGAWA: *Progr. Theor. Phys.*, **24**, 569 (1960); M. GELL-MANN: to be published in *Phys. Rev.*; A. SALAM and J. C. WARD: *Nuovo Cimento*, **20**, 419 (1961); Y. NEEMAN: preprint.

(2) S. COLEMAN and S. L. GLASHOW: *Phys. Rev. Lett.*, **6**, 1423 (1961).

tation. If this is true also at low energy a measurement of the magnetic moment would distinguish among the two cases.

1. – The unitary group in three dimensions has 8 generators F_m. Their commutation rules are given, in terms of the totally antisymmetric tensor [3] f_{mnl}, by

$$[F_m, F_n] = if_{mnl}F_l \,. \tag{1}$$

In correspondence to each F_m there is a current $j_m(x)$, conserved as long as unitary simmetry holds. The currents transform according to

$$[F_m, j_n(x)] = if_{mnl}j_l(x) \,. \tag{2}$$

In the theory of Gell-Mann and Neeman (eightfold way), where baryons transform according to the 8-dimensional rapresentation, the electromagnetic current, $j(x)$, is given by

$$j(x) = j_3(x) + \frac{1}{\sqrt{3}}\, j_8(x) \,. \tag{3}$$

From (2) and (3) one sees that F_3, F_8, F_6, and F_7 are the generators of the subgroup that leaves $j(x)$ invariant. Conservation of F_3 and F_8 expresses conservation of I_3 and of hypercharge. We thus limit our considerations to F_6 (F_7 gives the same results). From the identity

$$\langle A\,|[F_6, j(x_1) \cdots j(x_n)]\,|B\rangle = 0 \,;$$

where $j(x_1) \cdots j(x_n)$ is any product of n currents, we derive, if A and B are one-particle states

$$\langle 0\,|[\psi_A, F_6]j \cdots j\,|B\rangle = \langle A\,|j \cdots j[F_6, \psi_B^+]\,|0\rangle \,, \tag{4}$$

where ψ^+, ψ are the relevant creation or annihilation operators, and we have briefly denoted by $j \cdots j$ the product of the currents. In Gell-Mann's « eightfold way » both baryons and mesons are assumed to transform according to the 8-dimensional representation. Therefore, for a suitable choice of the ψ's,

$$[\psi_m, F_n] = if_{mnl}\psi_l \,, \tag{5}$$

and thus (4) relates directly different matrix elements between one particle states. If we apply it to the mesons we obtain, for instance, the following conclusions:

- the K^0 (or $\bar{K}^0$) form factor is always zero (we use also charge conjugation invariance);
- the form factor of K^+ (K^-) is equal to that of π^+ (π^-);
- the amplitude for Compton scattering on $K^\pm$ is equal to the same amplitude of a $\pi^\pm$. For the corresponding amplitudes on neutral mesons one has relations

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of the kind:

$$\sqrt{3}\langle K^0|jj|K^0\rangle = \langle \chi^0|jj|\pi^0\rangle - \sqrt{3}\langle \chi^0|jj|\chi^0\rangle ;$$

$$-\langle K^0|jj|K^0\rangle = \langle \pi^0|jj|\pi^0\rangle - \sqrt{3}\langle \pi^0|jj|\chi^0\rangle .$$

— The amplitude for $\chi^0 \to \gamma+\gamma$ is $1/\sqrt{3}$ times the amplitude for $\pi^0 \to \gamma+\gamma$ etc.
By applying (4) to the baryons one finds

$$(6)\qquad \begin{cases} \langle \Sigma^+|j \cdots j|\Sigma^+\rangle = \langle p|j \cdots j|p\rangle , \\ \langle \Sigma^-|j \cdots j|\Sigma^-\rangle = \langle \Xi^-|j \cdots j|\Xi^-\rangle , \\ \langle \Xi^0|j \cdots j|\Xi^0\rangle = \langle n|j \cdots j|n\rangle - \\ -\dfrac{1}{\sqrt{3}}\langle \Sigma^0|j \cdots j|\Lambda\rangle = \langle n|j \cdots j|n\rangle - \langle \Lambda|j \cdots j|\Lambda\rangle \\ -\sqrt{3}\langle \Lambda|j \cdots j|\Sigma^0\rangle = \langle n|j \cdots j|n\rangle - \langle \Sigma^0|j \cdots j|\Sigma^0\rangle . \end{cases}$$

A relation between the electromagnetic mass splittings,

$$\delta m_{\Xi^-} - \delta m_{\Xi^0} = \delta m_p - \delta m_n + \delta m_{\Sigma^-} - \delta m_{\Sigma^+} ,$$

given by COLEMAN and GLASHOW [2], is contained in (6).

If the current product $j \cdots j$ reduces to a single j one can make further use of the transformation properties of j. A matrix element $\langle A|j_m|B\rangle$ where A, B, j_m all transform according to the eight-dimensional representation can be decomposed as

$$(7)\qquad \langle A|jm|B\rangle = if_{ABm}\,\mathcal{O} + d_{ABm}\,\mathcal{E} ,$$

in terms of the totally antisymmetric tensor f, of the totally symmetric tensor d[3], and of the quantities $\mathcal{O}$ and $\mathcal{E}$.

The identity (7) is similar to the familiar Wigner-Eckart theorem for space rotations. The quantities $\mathcal{O}$ and $\mathcal{E}$ play the role of the so-called reduced matrix elements, and f and d of Clebsch-Gordon coefficients.

The reason why one has two reduced matrix elements in this case is that the reduction of the direct product $8\times8\times8$ contains the representation 1 twice.

Applying (7) to (3) one has

$$(8)\qquad \langle A|j|B\rangle = i\left(f_{AB3} + \frac{1}{\sqrt{3}}f_{AB8}\right)\mathcal{O} + \left(d_{AB3} + \frac{1}{\sqrt{3}}d_{AB8}\right)\mathcal{E} .$$

It is instructive to compare with the corresponding situation when only charge independence is assumed. In that case one has two independent matrix elements,

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(3) A table of the elements of f_{mnl} as well as of d_{mnl} is given in the paper by GELL-MANN [see (1)].

usually called the scalar part and the vector part, which originate directly from the decomposition of j, analogous to (3), into an isovector and an isoscalar part. Here instead, j_3 and j_8 transform both in the same way, according to (2), but each of them originates two reduced matrix elements.

With (8) one fonds directly

$$(9)\quad \left\{\begin{array}{lll} (6) & \langle \Sigma^0|j|\Sigma^0\rangle & = \frac{1}{3}\mathscr{E}\,, \\ (3) & \langle \Lambda^0|j|\Lambda^0\rangle & = -\frac{1}{3}\mathscr{E}\,, \\ (4) & \langle \Sigma^0|j|\Lambda^0\rangle & = \frac{1}{\sqrt{3}}\mathscr{E}\,, \\ (7) & \langle \Sigma^-|j|\Sigma^-\rangle & = \frac{1}{3}\mathscr{E} - \mathscr{O}\,, \\ (9) & \langle \Xi^-|j|\Xi^-\rangle & = \frac{1}{3}\mathscr{E} - \mathscr{O}\,, \\ (8) & \langle \Xi^0|j|\Xi^0\rangle & = -\frac{2}{3}\mathscr{E}\,, \\ (1) & \langle p|j|p\rangle & = \frac{1}{3}\mathscr{E} + \mathscr{O}\,, \\ (2) & \langle n|j|n\rangle & = -\frac{2}{3}\mathscr{E}\,, \\ (5) & \langle \Sigma^+|j|\Sigma^+\rangle & = \frac{1}{3}\mathscr{E} + \mathscr{O}\,. \end{array}\right.$$

From (9) one obtains the relations between the anomalous magnetic moments given by COLEMAN and GLASHOW [2,4]. We recall that one has (denoting explicitly the tensor indices)

$$\mathscr{O}^\mu = \overline{u}(p_f)\,\mathscr{O}_1(K^2)\gamma^\mu + \mathscr{O}_2(K^2)\sigma^{\mu\nu}K^\nu]u(p_i)\,,$$

with $K = p_f - p_i$, and similarly for $\mathscr{E}^\mu$.

Relations similar to (9) hold for the form factors of the postulated vector mesons or for the amplitudes of radiative transitions between vector mesons and pseudoscalar mesons. Thus one finds, for instance,

$$\langle \pi_0'|j|\chi_0\rangle = \langle \chi_0'|j|\pi^0\rangle = -\frac{1}{\sqrt{3}}\langle \chi_0'|j|\chi_0\rangle = \frac{1}{3}\langle \chi_0'|j|\pi_0\rangle = -\frac{2}{\sqrt{3}}\langle K_0'|j|K^0\rangle\,,$$

$$\langle \pi'^+|j|\pi^+\rangle = \langle K'^+|j|K^+\rangle\,,$$

where, for instance, π_0' is the vector meson with the same isospin properties of π^0. Furthermore K'^+ (K'^-) has the same form factors as π'^+ (π'^-), and the form factors of neutral vector mesons are all zero.

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(4) Consistency with the relations of Coleman and Glashow requires an additional minus-sign in ther definition of the Σ-Λ transition moment.

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2. – If one assumes that the weak currents are simply linear combinations of the currents j_m ([5]), one can apply (7) to derive relations between amplitudes for leptonic decays, always in the same spirit of neglecting that part of the strong Lagrangian that violates unitary simmetry. Thus, the $\Delta S=+1$, $\Delta Q=+1$ weak current could be of the form $g(j_4+ij_5)$ where g is a constant. One then has the decomposition

$$g\langle A|j_4+ij_5|B\rangle = (if_{AB4}-f_{AB5})\,\mathcal{O}'+(d_{AB4}+id_{AB5})\,\mathcal{E}', \tag{10}$$

where $\mathcal{O}'$ and $\mathcal{E}'$ are $g\mathcal{O}$ and $g\mathcal{E}$.

If one assumes universality in the coupling of the weak currents to the leptons the $\Delta S=0$, $\Delta Q=+1$ weak current would be $g(j_1+ij_2)$, with the same g as in (10). But then the rates for hyperon leptonic decays would be much larger than observed. Therefore the use of the universality hypothesis is, at least, inconvenient, in such a scheme; of course, the hypothesis may be true, but masked by strong renormalization effects. We do not therefore insist on the relations between matrix elements of different currents. From (10) one finds

$$\left\{\begin{aligned}
g\langle \Xi^-|j_4+ij_5|\Lambda\rangle &= \frac{1}{\sqrt{2}}\left(\sqrt{3}\,\mathcal{O}'-\frac{1}{\sqrt{3}}\,\mathcal{E}'\right),\\
g\langle \Sigma^-j_4+ij_5|n\rangle &= -\,\mathcal{O}'+\mathcal{E}',\\
g\langle \Sigma^0|j_4+ij_5|p\rangle &= \frac{1}{\sqrt{2}}(-\,\mathcal{O}'+\mathcal{E}'),\\
g\langle \Lambda|j_4+ij_5|p\rangle &= \frac{1}{\sqrt{2}}\left(-\sqrt{3}\,\mathcal{O}'-\frac{1}{\sqrt{3}}\,\mathcal{E}'\right),\\
g\langle \Xi^-|j_4+ij_5|\Sigma^0\rangle &= \frac{1}{\sqrt{2}}(\mathcal{O}'+\mathcal{E}'),\\
g\langle \Xi^0|j_4+ij_5|\Sigma^+\rangle &= \mathcal{O}'+\mathcal{E}'.
\end{aligned}\right. \tag{11}$$

3. – Still in the same spirit of neglecting violations of unitary simmetry we can easily establish that, in the limit of zero momentum transfer, $\mathcal{E}'\to 0$ (*i.e.* the form factor multiplying γ_μ in the expansion of $\mathcal{E}'$ is zero for $K^2=0$). In fact in the limit of zero momentum transfer the relevant matrix element is proportional to $\langle A|F_4+iF_5|B\rangle$, since the generators F_m are also the space integrals of the fourth component of j_m (and, of course, are conserved if j_m is divergenceless). However

$$\langle A|F_4+iF_5|B\rangle = \langle 0|[\psi_A, F_4+iF_5]|B\rangle = if_{AB4}-f_{AB5}, \tag{12}$$

showing that $\mathcal{E}'(0)=0$ and $\mathcal{O}'(0)=g$.

(*) If strange currents violating $\Delta S=\Delta Q$ exist (preliminary report from the Padua-Wisconsin group) this possibility is lost, at least in the framework of the three dimensional unitary group.

4. – There are other versions of the models based on unitary symmetry differing mainly in the representation of the baryons. In the model by Gell-Mann and Neeman the eight known baryons are the basis of an eight-dimensional representation. In the original Sakata model (1) three baryons p, n, Λ, are the basis of a three dimensional representation, while other baryons belong to higher representations (to the 15-dimensional, or to both the 15 — and the 6 — dimensional representation). One can easily extend the considerations we have made in the previous sections to the Sakata model. One has however to be careful in identifying properly the currents. The electromagnetic current in the Sakata model is no longer given by (3) but it is given instead by

$$j(x) = j_3(x) + \frac{1}{\sqrt{3}} j_8(x) + \frac{1}{3} j_0(x) \,. \tag{13}$$

Note the addition of $\frac{1}{3} j_0(x)$, proportional to the baryonic current, which belongs to the one-dimensional representation, and therefore does not transform according to (2). This circumstance again brings two independent « reduced » matrix elements and we find the relation

$$\langle n | j | n \rangle = \langle \Lambda | j | \Lambda \rangle \,, \tag{14}$$

i.e., neutron and Λ have the same form factors and anomalous moment.

In contrast to this results, the eightfold way gives $\langle n | j | n \rangle = 2 \langle \Lambda | j | \Lambda \rangle$. The Λ anomalous magnetic moment will presumably be measured rather soon by precession in a strong magnetic field (6).

(6) This experiment is under development at Brookhaven and at CERN.

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