Laboratori Nazionali di Frascati

LNF-61/28 (1961)

G. Putzolu: RADIATIVE CORRECTIONS TO PION PRODUCTION IN $e^+ - e^-$ COLLISIONS.

Estratto dal: Nuovo Cimento, 20, 542 (1961)

IL NUOVO CIMENTO VOL. XX, N. 3 1° Maggio 1961

# Radiative Corrections to Pion Production in $e^+$-$e^-$ Collisions.

G. PUTZOLU
*Laboratori Nazionali del CNEN - Frascati*

(ricevuto il 16 Febbraio 1961)

**Summary.** — Radiative corrections to the processes $e^+$-$e^- \to n$ pions are evaluated in relation to the planned colliding beam experiments. A theorem is proved which shows the possibility of separating experimentally the contribution of the $\gamma$-$n$ pion vertex from the contribution of the $2\gamma$-$n$ pion vertex.

---

In a recent work CABIBBO and GATTO (1) have discussed the possibility of direct measurement of the form factors of the photon-pion vertex through processes of the type:

$$e^+ + e^- \to n \text{ pions}. \tag{1}$$

They have also obtained in the first electromagnetic approximation the expression of the corresponding cross-section in the center-of-mass system. We will here examine in which way their results can be modified if the radiative corrections in the second electromagnetic approximations are taken into account. The general situation is then characterized by Feynman's diagrams of Fig. 1, where $F$ and $G$ represent the « complete » vertices $\gamma$-$n\pi$ and $2\gamma$-$n\pi$; that is they correspond to the sum of all the diagrams with external lines of this type.

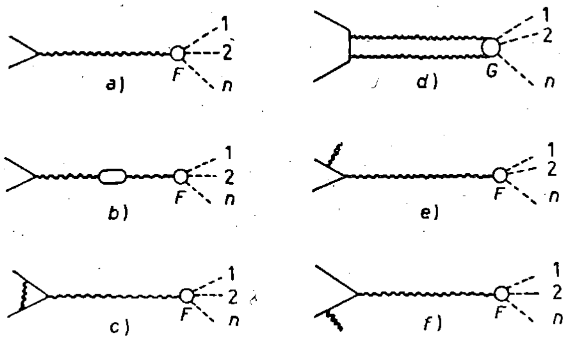

Fig. 1.

Experiments on processes of type (1) should precisely measure $F$ and $G$.

---

(1) N. CABIBBO and R. GATTO: *Phys. Rev. Lett.*, 4, 313 (1960).

1854

We have neglected the diagrams corresponding to emission of photons from the second vertex. Taking into account the difference of mass between pions and electrons, their contribution is probably negligible in comparison with the contribution from the diagrams (1*e*) and (1*f*).

The corrections corresponding to diagrams 1*b*), 1*c*), 1*e*), 1*f*) do not involve new photon-pion vertices and therefore they will be later calculated with the usual techniques.

The correction corresponding to the diagram with exchange of two virtual photons (Fig. 1*d*) contains the new unknown vertex, 2 $\gamma$-pions.

On the basis of general considerations we will show that it is easy to isolate experimentally the contributions to the cross-sections from the graphs with exchange of two photons; *i.e.* it is possible to distinguish the contributions of the two form factors. Our proof is based on three points:

*a*) In processes of type (1) the initial state, being eigenstate of the charge with eigenvalue 0, is a superposition of two eigenstates of the charge-parity operator $C$ relative to eigenvalues $+1$ and $-1$:

$$|i\rangle = |i^+\rangle + |i^-\rangle \quad \text{with} \quad C|i^\pm\rangle = \pm|i^\pm\rangle . \tag{2}$$

*b*) Calling $S'$ and $S''$ the terms of the operator $S$ corresponding to diagrams of Fig. 1*a* and 1*d*) respectively we have

$$S'|i^+\rangle = 0 , \qquad S''|i^-\rangle = 0 . \tag{3}$$

In fact for instance

$$S'|i^+\rangle = (ie)^2\int dx_1\, dx_2\, i\Delta_f(x_1, x_2) J_\pi(x_1)|0\rangle \langle 0|J_e(x_2)|i^+\rangle = 0 ,$$

since

$$\langle 0|J_e(x_2)|i^+\rangle = \langle 0C^{-1}|C J_e(x_2)C^{-1}|Ci^+\rangle = -\langle 0|J_e(x_2)|i^+\rangle .$$

In the same way it is shown that $S''|i^-\rangle = 0$.

*c*) We will now show that if the set of final states $F$ distinguished from the measurement is an invariant subspace for the operator $C$, the contribution to the cross-section of the interference term between the matrix elements corresponding to the diagrams of Fg. 1*a*) and 1*d*) is zero. In fact in this case we can assume as basis in $F$ a set of vectors $|f_s\rangle$ wich are eigenstates of $C$.

$$C|f_s\rangle = \pm|f_s\rangle . \tag{5}$$

The contribution to the transition probability in $e^6$ from the initial state $|i\rangle$

to the set of final states $F$, due to the diagrams with exchange of two photons is

$$(6)\quad \begin{cases} 2\,\mathrm{Re}\{\sum_s \langle i|S'|f_s\rangle\langle f_s|S''|i\rangle\} = \\ 2\,\mathrm{Re}\{\sum_s \langle i^-|S'|f_s\rangle\langle f_s|S''|i^+\rangle\} = \\ 2\,\mathrm{Re}\{\sum_s \langle i^- C^{-1}|CS'C^{-1}|Cf_s\rangle\langle f_s C^{-1}|CS''C^{-1}|Ci^+\rangle\} = \\ -2\,\mathrm{Re}\{\sum_s \langle iS'|f_s\rangle\langle f_s|S''|i\rangle\} = 0\,. \end{cases}$$

Let us now examine to wich experimental situation does the condition that $F$ be an invariant subspace of $C$ correspond. It is evidently sufficient that the apparatus revealing the final particles do not distinguish the $\pi^+$ from the $\pi^-$. This condition can certainly be verified in the experiments with intersecting beams in project at Stanford and at Frascati.

For the simplest processes of type (1), that is $e^+ + e^- \to \pi^+ + \pi^-$, it follows that, apart from particular experimental situations, the term of the cross-section in $e^6$ due to the diagram $d)$ is an odd function of $\cos\theta$ (where $\theta$ is the angle between the momenta of the electron and of the $\pi^-$). Therefore it does not contribute to the differential cross-section for $\theta = 90°$ and to the total cross-section. As instead the contribution of the other diagrams with exchange of one photon are even functions of $\cos\theta$, it is possible to separate the two types of contributions in an experience with variable $\theta$; this means that it is possible to measure separately the form factors for the vertices $2\pi$-$\gamma$ and $2\pi$-$2\gamma$. The above conclusion is valid also for similar processes like $e^+ + e^- \to \mu^+ + \mu^-$, $e^+ + e^- \to K^+ + K^-$, etc. Thus in an experiment $e^+ + e^- \to \mu^+ + \mu^-$ it is possible to distinguish the contribution to the cross-section in $e^6$ due to the diagrams wich are unimportant for a check of renormalization theory, from the contributions of vertex and self-energy corrections.

Keeping in mind the above result, we have calculated the expression of the cross-section to be used if one wants to measure the form factor of the « complete » vertex $\gamma$-$n\pi$ through an experience of type (1).

The formula is valid for the conditions stated above, *i.e.* no contribution from the diagram with vertex $2\gamma$-$n\pi$; we have taken into account the fact that the electrons are certainly relativistic, and we have assumed that the maximum energy $\varepsilon$ of the bremsstrahlung photons (see Fig. 1$e$ and 1$f$) is small with reference to the energy of the emitting particles.

In this hypothesis the expression of the correction is independent of the specific form of vertex $\gamma$-$n\pi$.

Using the usual techniques to calculate radiative corrections we find

$$(7)\qquad \mathrm{d}\sigma_n^{(1)} = \mathrm{d}\sigma_n^{(0)}(1 + \delta_{SE} + \delta_V + \delta_B)\,,$$

where $d\sigma_n^{(0)}$ and $d\sigma_n^{(1)}$ are the cross-sections in the first and in the second electromagnetic approximation of process (1), while $\delta_{SE}$, $\delta_V$ and $\delta_B$ are the percentage corrections due to diagrams of Fig. 1*b*), 1*c*), 1*e*) and 1*f*). Expressions for $d\sigma_n^{(0)}$ are given in (1); for the corrections we have obtained

$$\text{(8)} \quad \begin{cases} \delta_{SE} = \dfrac{2\alpha}{\pi}\dfrac{2}{3}\left\{\ln\dfrac{2E}{m} - \dfrac{5}{6}\right\}, \\ \delta_V = -\dfrac{2\alpha}{\pi}\left\{\left(1 - 2\ln\dfrac{2E}{m}\right)\ln\dfrac{\lambda}{m} + \left(\ln\dfrac{2E}{m}\right)^2 - \dfrac{3}{2}\ln\dfrac{2E}{m} + 1 + \dfrac{\pi^2}{6}\right\}, \end{cases}$$

$$\text{(9)} \quad \delta_B = -\frac{2\alpha}{\pi}\left\{\left(1 - 2\ln\frac{2E}{m}\right)\ln\frac{2\varepsilon}{\lambda} + \left(\ln\frac{2E}{m}\right)^2 - \ln\frac{2E}{m} + \frac{\pi^2}{6}\right\},$$

where $\alpha$ is the fine structure constant, $E$ is the energy and $m$ the mass of the electron (or positron), $\varepsilon$ is the maximum energy of the bremsstrahlung photons, $\lambda$ is the fictitious mass of the photon, that disappears in the expression for global correction.

It is to be noted that (7) and (8) are also valid for $e^+ + e^- \to \mu^+ + \mu^-$, if $d\sigma^{(0)}$ and $d\sigma^{(1)}$ are the corresponding cross-sections in $e^4$ and $e^6$, neglecting radiative corrections due to the $\mu$-mesons. To take them into account it is sufficient to replace in (7) $(1 + \delta_{SE} + \delta_V + \delta_B)$ with $(1 + \delta_{SE} + \delta_V + \delta_B + \delta_{SE}^{(\mu)} + \delta_V^{(\mu)} + \delta_B^{(\mu)})$, where the $\delta^{(\mu)}$'s are obtained from the $\delta$'s by substituting the electron's mass with the $\mu$-meson's one.

In this formula the effects due to creation by the intermediate photon of virtual particles heavier than electrons have been neglected; a most important contribution could come from the two-pion intermediate states [2,3].

In agreement with [3] they are negligible.

* * *

We are indebted to Professor B. Touschek for helpful assistance and encouragement.

(2) L. M. Brown and F. Calogero: *Phys. Rev. Lett.*, **4**, 315 (1960).
(3) Yung-Su-Tsai: *Phys. Rev.*, **120**, 269 (1960).

## RIASSUNTO

Sono state calcolate le correzioni radiative ai processi $e^+ + e^- \to n$ pioni in relazione agli esperimenti a fasci incrociati in progetto. Viene dimostrato un teorema che indica la possibilità di separare sperimentalmente il contributo del vertice $\gamma$-$n$ pioni dal contributo del vertice $2\gamma$-$n$ pioni.