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Theoretical Discussion of Possible Experiments with Electron-Positron Colliding Beams.

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1. – We discussed recently the possible determination of the pion form factors from the reactions $e^+ + e^- \rightarrow n\pi$ (1). There is at present a definite interest, particularly in Frascati, in the realization of electron-positron colliding beams. In this note we shall briefly present some further theoretical considerations on high energy electron-positron experiments.

2. – High energy e^+-e^- experiments can test the validity of quantum electrodynamics at small distances. There are two other aspects of such experiments that we want to stress:

i) The possibility of exploring form factors of strong interacting particles. These form factors are explored for timelike momentum transfers. Electron scattering experiments — whenever possible — can only explore spacelike momentum tranfers.

ii) The possibility of carrying out consistently a « Panofsky program », *i.e.* the exploration of the spectrum of masses of elementary particles through their interaction with photons. This program can be extended to include the exploration of particular classes of unstable states.

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(1) N. CABIBBO and R. GATTO: *Phys. Rev. Lett.*, **4**, 313 (1960). The same results have also been derived by YUNG SU TSAI: *Phys. Rev.*, **120**, 269 (1960).

3. – Perturbation theory results.

We first list some relevant perturbation theory results. We consider the lowest order graph

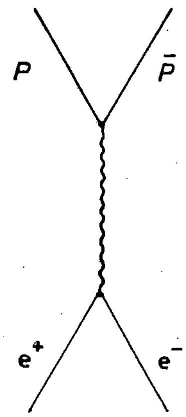

Fig. 1.

where P is a charged spin $\frac{1}{2}$ fermion (f) different from the electron, or a charged spin zero boson (b), or a charged spin 1 boson (B). We call θ the c.m. production angle, α the fine structure constant, $\lambda\!\!\!^{-}$ the wavelength and E the energy of each incident particle in the c.m. system, m_i and β_i the mass and velocity of particle i in the c.m. system. The cross sections are

a) for $e^+ + e^- \to f^+ + f^-$:

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{4}\,\alpha^2 \lambda\!\!\!^{-2} \beta_f \left[\frac{1}{2}(1+\cos^2\theta) + \frac{m_f^2}{2E^2}(1-\cos^2\theta)\right],$$

$$\sigma_{\text{total}} \simeq \frac{1}{m_f^2}(2.1\cdot 10^{-32}\ \text{cm}^2) f(x)\,, \qquad (m_f \text{ in GeV}),$$

with

$$f(x) = \frac{1}{x^2}\left(1-\frac{1}{x^2}\right)^{\frac{1}{2}}\left(1+\frac{1}{2x^2}\right),$$

and

$$x = \frac{E}{m_f}\,;$$

b) for $e^+ + e^- \to b^+ + b^-$

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{16}\,\alpha^2 \lambda\!\!\!^{-2} \beta_b^3 \sin^2\theta\,,$$

$$\sigma_{\text{total}} = \frac{1}{m_b^2}(0.5\cdot 10^{-32}\ \text{cm}^2)\, b(x)\,, \qquad (m_b \text{ in GeV}),$$

with

$$b(x) = \frac{1}{x^2}\left(1 - \frac{1}{x^2}\right)^{\frac{3}{2}},$$

and

$$x = \frac{E}{m_b};$$

c) for $e^+ + e^- \to B^+ + B^-$

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{16}\,\alpha^2 \lambda\!\!\!^{-2} \beta_B^3 \left[2\left(\frac{E}{m_B}\right)^2 (1 + \cos^2\theta) + 3\sin^2\theta\right],$$

$$\sigma_{total} = \frac{1}{m_B^2}\,(2.1\cdot 10^{-32}\ \text{cm}^2) B(x)\,, \qquad (m_B \text{ in GeV}),$$

with

$$B(x) = \frac{3}{4}\left(1 - \frac{1}{x^2}\right)^{\frac{3}{2}}\left(\frac{4}{3} + \frac{1}{x^2}\right),$$

and

$$x = \frac{E}{m_B}.$$

The functions $f(x)$, $b(x)$ and $B(x)$ are reported in Fig. 2. These perturbation theory results are valid only as long as P does not have strong interactions and neglecting radiative corrections.

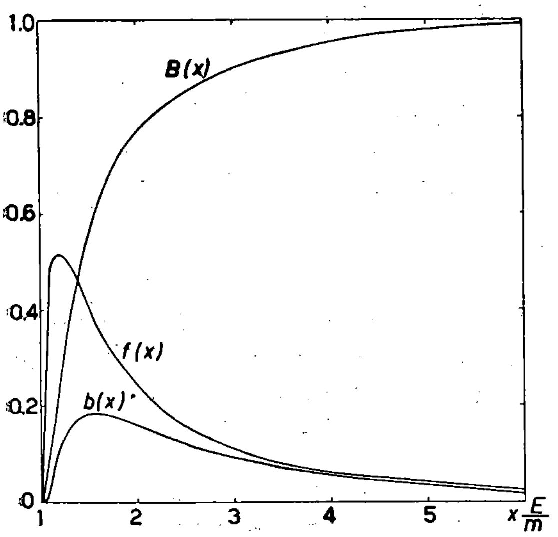

Fig. 2.

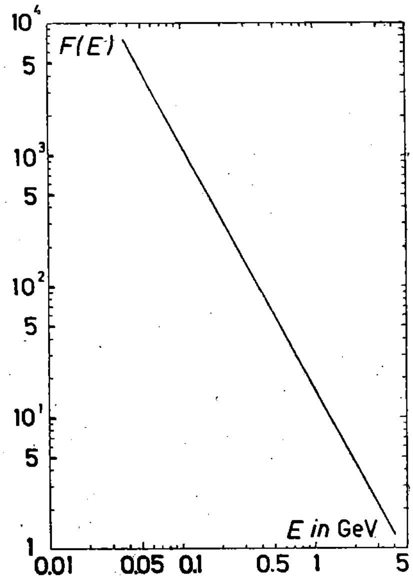

Fig. 3.

The total cross-section for $e^+ + e^- \to \gamma + \gamma$ is given by

$$\sigma_{total} \simeq (3.2\cdot 10^{-32}\ \text{cm}^2) F(E)\,,$$

where $F(E)$, with E in GeV, is reported in Fig. 3.

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4. – Production of strong-interacting particles.

We consider the lowest order graph in the electromagnetic interaction but including all strong interaction effects.

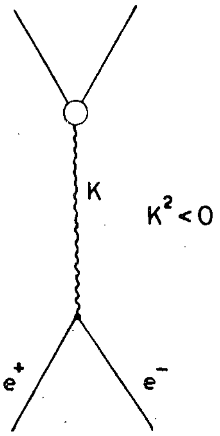

Fig. 4.

The form of the $PP\gamma$ vertex is only limited by Lorentz- and gauge-invariance.

a) for $e^+ + e^- \to f + \bar{f}$ where f is a charged or neutral fermion of spin $\frac{1}{2}$

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{8}\,\alpha^2 \lambda^2 \beta_f F(\cos\theta)\,,$$

where

$$F(\cos\theta) = |F_1^{(f)}(-4E^2) + \mu_f F_2^{(f)}(-4E^2)|^2(1+\cos^2\theta) + \\ + \sin^2\theta \left| \frac{m_f}{E} F_1^{(f)}(-4E^2)|^2 + \frac{E}{m_f}\mu_f^2 |F_2^{(f)}(-4E^2) \right|^2 .$$

Here μ_f is the static anomalous magnetic moment of f, and $F_1^{(f)}(k^2)$, $F_2^{(f)}(k^2)$ are the analytical continuation of the electric and magnetic form factors of f for negative values of k^2. The situation is illustrated in the following graph for the special case of the isotopic vector part of the nucleon electromagnetic vertex.

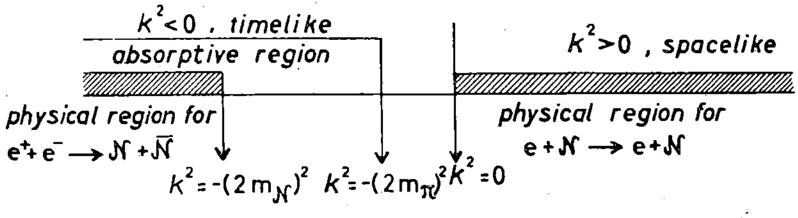

Fig. 5.

In the graph we have reported the physical regions and the absorptive region on the k^2 real axis.

The form factors F_1 and F_2 have an imaginary part along the absorptive cut. Therefore one can have a polarization of the fermion in $e^+ + e^- \to f + \bar{f}$, normal to the production plane (there can be no polarization of the fermion, neglecting radiative corrections, in the scattering $e + f \to e + f$, as follows from time reversal arguments). The polarization $P(\theta)$ is along $\boldsymbol{p}_f \wedge \boldsymbol{p}_{e^+}$ and is given by

$$F(\theta)P(\theta) = -\sin(2\theta)\beta_f^2 \frac{E}{m_f} \operatorname{Im}\{F_1^{*(f)}(-4E^2)\mu_f F_2^{(f)}(-4E^2)\}\,.$$

The polarization of $\bar{f}$ is $-P(\theta)$ as follows directly from TCP.

b) In $e^+ + e^- \to b + \bar{b}$ where b is a charged or neutral boson of spin zero

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{16}\,\alpha^2 \lambda\!\!\!^{-2} \beta_f^3 |F(-4E^2)|^2 \sin^2\theta\,,$$

where $F(k^2)$ is the form factor of b. In general the $\gamma P\bar{P}$ vertex is described in terms of an isotopic vector amplitude and an isotopic scalar amplitude (for pion there is only the vector amplitude, for Λ only the scalar amplitude). The absorptive cut starts at $k^2 = -(2m_\pi)^2$ for the isotopic scalar amplitude, and at $k^2 = -(3m_\pi)^2$ for the isotopic vector amplitude, except for the $\gamma\Sigma\Sigma$ vertex, for which the $\Sigma \to \Lambda + \pi$ transformation produces a lowering of the threshold (²).

c) For $e^+ + e^- \to B + \bar{B}$ where B is a neutral or charged spin one meson

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{16}\,\alpha^2 \lambda\!\!\!^{-2} \beta^3 \left\{ 2\left(\frac{E}{m_B}\right)^2 |F_1(-4E^2) + \mu F_2(-4E^2) + \varepsilon F_3(-4E^2)|^2 (1+\cos\theta) + \right.$$

$$\left. + \sin^2\theta \left[2\left| F_1(-4E^2) + 2\left(\frac{E}{m_B}\right)^2 \varepsilon F_3(-4E^2)\right|^2 + \left| F_1(-4E^2) + 2\left(\frac{E}{m_B}\right)^2 \mu F_2(-4E^2)\right|^2 \right] \right\},$$

where F_1, F_2 and F_3 are form factors, μ is the anomalous magnetic moment and ε is the quadrupole moment of B.

5. – Effects of the proposed $T = 1$, $J = 1$ pion-pion resonance.

Except for $e^+ + e^- \to \pi^+ + \pi^-$ and $e^+ + e^- \to \pi^+ + \pi^- + \pi^0$ the threshold for production is at higher k^2 than the threshold of the absorptive region. This circumstance will make theoretical predictions very difficult. The reactions $e^+ + e^- \to \pi^+ + \pi^-$ and $e^+ + e^- \to \pi^+ + \pi^- + \pi^0$ have already been discussed (¹). The pion-pion resonance, in the form proposed by FRAZER and FULCO (³), would produce a cross-section $\sim 8.3 \cdot 10^{-31}\ \text{cm}^2$, 17 times bigger than the perturbation theory value at the resonance.

(²) R. KARPLUS, C. SUMMERFIELD and E. WICHMAN: *Phys. Rev.*, **111**, 1187 (1958).
(³) W. A. FRANZER and I. R. FULCO: *Phys. Rev.*, **117**, 1609 (1960).

A favorable process, from the point of view of theoretical analysis, is $e^+ + e^- \to \pi^0 + \gamma$, as shown in the following graph where the different regions along the k^2 axis, for the vertex $\pi^0 \to \gamma + \gamma$, when one γ has mass $-k^2$ are indicated.

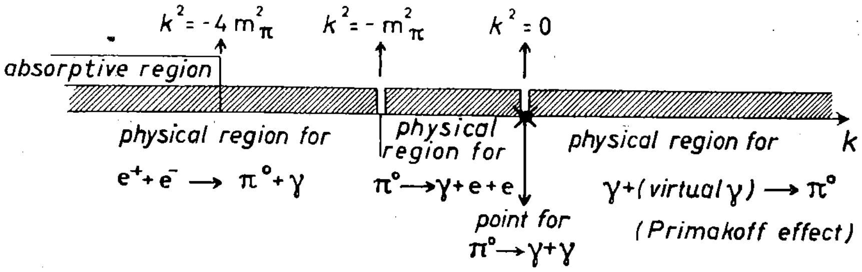

Fig. 6.

The cross section is

$$\frac{d\sigma}{d(\cos\theta)} = \pi\alpha \frac{\tau^{-1}}{m_\pi^3} \beta_\pi^3 (1 + \cos^2\theta) \left| \frac{G(-4E^2)}{G(0)} \right|^2,$$

where $G(k^2)$ is the form factor for $\pi^0 \to \gamma + \gamma$. τ^{-1} is the inverse of the $\pi^0 \to \gamma + \gamma$ lifetime and it is proportional to $|G(0)|^2$. We have used dispersion relations to calculate $G(-4E^2)$, assuming that only the resonant $T = 1$, $J = 1$, 2π state contributes to the absorptive part and using the $\gamma + \pi \to \pi + \pi$ amplitude given by WONG [4]. The result is shown in Fig. 7. The total cross-section for $e^+ + e^- \to \pi^0 + \gamma$ is given by $2.75 \cdot 10^{-35}\,\text{cm}^2\; u_R(x)$ with the resonance model. In perturbation theory $(G \equiv G(0))$ it is given by $2.75 \cdot 10^{-35}\,\text{cm}^2 \cdot u(x)$. We have assumed $2.2 \cdot 10^{-16}$ s for the π^0 lifetime. For the charged pion form factor we have used the form proposed by FRAZER and FULCO. With the form proposed by BOWCOCK, COTTINGHAM and LAURIÉ the maximum of $u_R(x)$ would be at higher x ($x \sim 4.8$) and 2.1 times bigger.

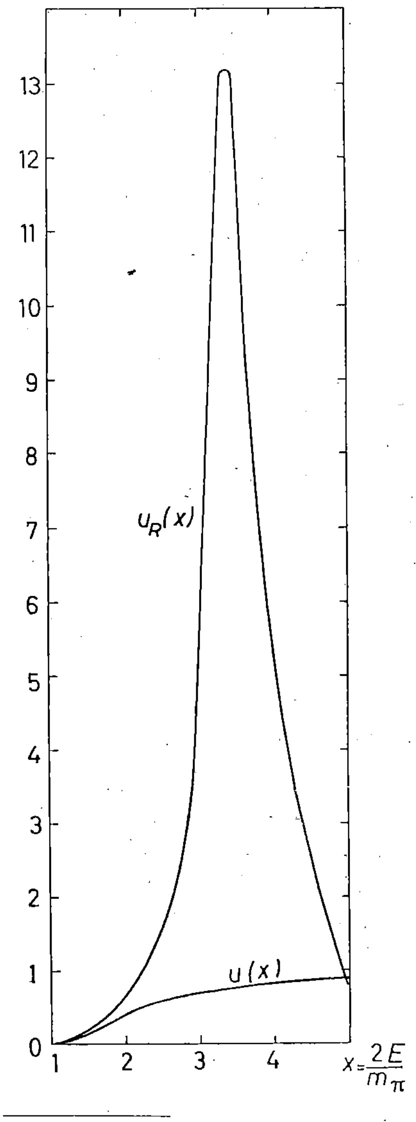

Fig. 7. – The cross section for $e^+ + e^- \to \pi^0 + \gamma$ is $2.75 \cdot 10^{-35}\, u_R(x)\,\text{cm}^2$ with Frazer-Fulco resonance; it is $2.75 \cdot 10^{-35} u(x)$ if the energy dependence of the form factor is neglected. The assumed π^0 lifetime is $2.2 \cdot 10^{-16}$ s.

[4] A. WANG; *Phys. Rev. Lett.*, **5**, 70 (1960).

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6. – General discussion of the possible resonances. Detection of Nambu's neutral vector meson.

In the last section we discussed two possibilities of direct detection of the suggested $T=1$, $J=1$, π-π resonance from e^+-e^- collisions. In this section we shall show that e^+-e^- collisions are very suitable for detecting possible neutral resonant states with $J=1$ and charge conjugation number $C=-1$ (and of course, with zero nucleonic number and zero strangeness). We consider the effect of the resonance in the reaction $e^++e^-\to$(final state) to originate from a graph of the type

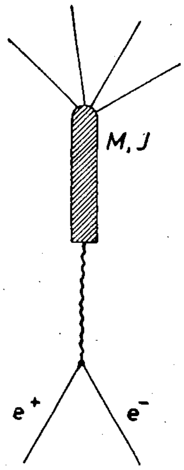

Fig. 8.

where the resonant state of mass M and spin J (0 or 1) decays into the final state f with a branching ratio B_f of its total rate Γ. The cross-section curve has to be folded with the experimental resolution curve that we approximate with a rectangle of width $2\Delta E$. We consider three cases: *a*) narrow resonance, $2\Delta E>\Gamma$; *b*) wide resonance $2\Delta E<\Gamma$; *c*) intermediate case, $2\Delta E\approx\Gamma$. The contribution to $e^++e^-\to$ $\to$(final state), from the resonance, for an experiment at the resonance with spread ΔE is then approximately given: for *a*) by $\sigma_{av}\simeq 2\pi\lambda\!\!\!^{-2}(\pi/4)(2J+1)B_iB_f(\Gamma/2\Delta E)$; for *b*) by $\sigma_{max}=\pi\lambda\!\!\!^{-2}(2J+1)B_iB_f$; for *c*) both formulae can safely be applied since they coincide in this case apart from a factor $\pi/2$. A typical comparison can be made for instance with $e^++e^-\to\mu^++\mu^-$, the cross-section of which is $(\pi/3)\alpha^2\lambda\!\!\!^{-2}$ for $E\gg m_\mu$. Practically, for intensities of the order $(10^{10}\div10^{11})$ electrons or positrons, the discussion can be limited to those decay modes for which $B_f\simeq1$. Therefore the relevant quantities are: $\Gamma_i/\Delta E$ in case *a*); Γ_i/Γ in case *b*) and, of course, any of these two quantities in case *c*). From charge conjugation and gauge invariance one sees that Γ_i (rate into e^++e^-) is proportional to: $\alpha^4m_e^2$ for $J=0$, $C=1$; $\alpha^6m_e^2$ for $J=0$, $C=-1$; α^4 for $J=1$, $C=1$; α^2 for $J=1$, $C=-1$.

It seems reasonable to assume $\Delta E\sim1$ MeV. Then in case *a*) only resonances $J=1$, $C=-1$ would produce relatively large effects. The same conclusion applies also to cases *b*) and *c*) since $\Gamma\gtrsim\Delta E$ implies a fast decay by strong interaction or at most by single γ emission ($\Gamma\propto\alpha$). A $J=1$, $C=-1$ meson with $T=1$ can decay rapidly into pions. The resonant effects discussed in the last chapter belong to this type. A $J=1$, $C=-1$ meson with $T=0$, called ρ^0, has been proposed in particular by NAMBU ([5]), and calculations have been made by HUFF ([6]). We don't assume any direct coupling of ρ^0 with electrons. Following HUFF we examine in detail three special cases:

1) $m_\rho = m_\pi$: only $\rho^0 \to e^+ + e^-$ is possible, with $\Gamma \simeq 10^{16}\,s^{-1}$, giving for $e^+ + e^- \to \rho^0 \to e^+ + e^-$ a $\sigma_{av} \simeq 7.8 \cdot 10^{-30}\,cm^2$. The cross-section for $e^+ + e^- \to \gamma + \gamma$ at this energy is $\simeq 7.5 \cdot 10^{-29}\,cm^2$.

2) $m_\rho = 2m_\pi$: $\rho^0 \to \pi^0 + \gamma$, $\to e^+ + e^-$, $\to \mu^+ + \mu^-$ are possible with $B_{\pi^0\gamma} \simeq 1$, $B_{e^+e^-} \simeq \simeq B_{\mu^+\mu^-} \simeq 10^{-2}$ and $\Gamma \simeq 1.5 \cdot 10^{-19}\,s$. Then for $e^+ + e^- \to \rho^0 \to \pi^0 + \gamma$, $\sigma_{av} \simeq 1.3 \cdot 10^{-29}\,cm^2$, and for $e^+ + e^- \to \rho^0 \to e^+ + e^-$, or $\to \mu^+ + \mu^-$, $\sigma_{av} \simeq 10^{-31}\,cm^2$; to be compared with the cross-section for $e^+ + e^- \to \mu^+ + \mu^-$, at that energy $\simeq 0.86 \cdot 10^{-30}$. The perturbation theory estimate for $e^+ + e^- \to \pi^0 + \gamma$ gives $2.3 \cdot 10^{-35}\,cm^2$.

3) $m_\rho = 3m_\pi$: we assume rates $\sim 10^{21}\,s^{-1}$ for $\rho^0 \to \pi^0 + \gamma$, $\to 2\pi + \gamma$ and $B \simeq 10^{-3}$ for $\rho^0 \to e^+ + e^{-3}$, $\to \mu^+ + \mu^-$. The main effect is in $e^+ + e^- \to \pi^0 + \gamma$ or $2\pi + \gamma$ giving $\sigma_{av} \simeq 10^{-29}\,cm^2$, to be compared with the perturbation theory estimate $\simeq 3.8 \cdot 10^{-35}\,cm^2$ for $e^+ + e^- \to \pi^0 + \gamma$ at this energy.

7. – Problems concerning weak interactions. Detection of the intermediate vector meson.

We pointed out the relevance of $e^+ + e^-$ experiments for a Panofsky program. A charged fermion f weakly interacting and with mass $> m_K$ would hardly have been detected if there is a selection rule forbidding the mixed couplings $(fe\gamma)$ or $(f\mu\gamma)$. A very interesting possibility is the detection of semiweak-interacting boson B with mass $> m_K$, as suggested, for instance, for mediating weak interactions. The relevant cross-sections are given in Section **3**. If B has an anomalous magnetic moment μ_B the differential cross section becomes

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{16}\alpha^2 \lambda\!\!\!^{-2} \beta^3 \left\{ 2(1+\mu_B)^2 \left(\frac{E}{m_B}\right)^2 (1+\cos^2\theta) + \left(2 + \left(\frac{E}{m_B}\right)^4 (1-\beta^2+2\mu_B)^2\right)\sin^2\theta \right\}.$$

Note that this formula gives a cross-section that goes to a constant for $E \gg m_B$ if $\mu_B = 0$, or increases $\propto E^2$ if $\mu_B \neq 0$. In both cases unitarity is violated at high energy. From angular momentum and charge conjugation considerations the cross-section should not exceed $2\pi\lambda\!\!\!^{-2}$ if unitarity is respected, neglecting radiative corrections. In fact in this case only the ${}^3S_1$ and ${}^3D_1$ waves can interact, giving a maximum interaction of $2\pi\lambda\!\!\!^{-2}$ after summing over polarizations. It follows that, even for $\mu_B = 0$, unitarity is violated at high energy (precisely for $E > 340 m_B$). Another interesting possibility is to look for a semiweakly coupled B^0, which would give rise to resonances in $e^+ + e^- \to e^+ + e^-$, $\to \mu^+ + \mu^-$, $\to \pi^+ + \pi^-$, etc. The formulae of the preceding section, case *a*) can then be applied. On the basis of the semiweak coupling we assume $\Gamma \simeq 5 \cdot 10^{17}\,s^{-1}$ with branching ratios $\sim \frac{1}{5}$ for $B^0 \to e^+ + e^-$, or $B^0 \to \mu^+ + \mu^+$. For $e^+ + e^- \to X^0 \to \mu^+ + \mu^-$ one has then $\bar{\sigma}_{av} \simeq 2\pi\lambda\!\!\!^{-2}(2.6 \cdot 10^{-5})$ which is about 3 times bigger than $e^+ + e^- \to \mu^+ + \mu^-$ with $\sigma \simeq 2\pi\lambda\!\!\!^{-2}(0.88 \cdot 10^{-5})$ for $E \gg m_\mu$.

([5]) Y. NAMBU; *Phys. Rev.*, **106**, 1366, (1957).
([6]) R. W. HUFF: *Phys. Rev.*, **112**, 1021. (1958).

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The contribution from the local weak interactions to, for instance, $e^+ + e^- \to \mu^+ + \mu^-$ is very small at low energy but increases fast with energy. The addition to the lowest order electromagnetic matrix element of a term

$$(2\pi)^{-1} (\sqrt{8}G)[\bar{u}(\mu^-)\gamma_\mu \tfrac{1}{2}(1+\gamma_5)v(\mu^+)][\bar{v}(e^+)\gamma_\mu \tfrac{1}{2}(1+\gamma_5)u(e^-)]\,,$$

modifies the cross-section formula to

$$\frac{\mathrm{d}\sigma}{\mathrm{d}(\cos\theta)} = \frac{\pi}{8}\,\alpha^2 \lambda\!\!\!{}^{-2}\,[(1+\cos^2\theta)(1+\varepsilon+\varepsilon^2) + 2(\varepsilon+\varepsilon^2)\cos\theta]\,,$$

where $\varepsilon \simeq 6.2\cdot 10^{-4}(E/m_{\mathcal{N}})^2$, with $m_{\mathcal{N}}$ = nucleon mass, and we have taken for G the value of the Fermi constant. A $\cos\theta$ term in the differential cross-section also arises from graphs with two photons exchanged. More uniquely a longitudinal polarization of the $\mu^\pm$ given by

$$P^\pm = \pm(\varepsilon+\varepsilon^2)\,\frac{(1+\cos\theta)^2,}{(1+\cos^2\theta)+(\varepsilon+\varepsilon^2)(1+\cos\theta)^2}\,,$$

would indicate the presence of weak interactions. However ε becomes ~ 1 only for colliding beams each of ~ 30 GeV.

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