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ε'/ε IN THE $1/N_c$ EXPANSION

T. Hambye¹ and P. H. Soldan²

¹*INFN, Laboratori Nazionali di Frascati, P.O. Box 13, I-00044 Frascati, Italy*

²*Institut für Physik, Universität Dortmund, D-44221 Dortmund, Germany*

Abstract

We present a new analysis of the ratio ε'/ε which measures the direct CP violation in $K \rightarrow \pi\pi$ decays. We use the $1/N_c$ expansion within the framework of the effective chiral lagrangian for pseudoscalar mesons. From general counting arguments we show that the matrix element of the operator Q_6 is not protected from possible large $1/N_c$ corrections beyond its large- N_c or VSA value. Calculating the $1/N_c$ corrections, we explicitly find that they are large and positive. Our result indicates that a $\Delta I = 1/2$ enhancement is operative for Q_6 similar to the one of Q_1 and Q_2 which dominate the CP conserving amplitude. This enhances ε'/ε and can bring the standard model prediction close to the measured value for central values of the parameters.

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1 Introduction.

Recently, direct CP violation in $K \rightarrow \pi\pi$ decays was observed by the KTeV [1] and NA48 [2] collaborations. The new measurements are in agreement with the results of the NA31 experiment [3]. The present world average for the parameter ε'/ε is [2]

$$\text{Re } \varepsilon'/\varepsilon = (21.2 \pm 4.6) \cdot 10^{-4}, \quad (1)$$

which differs from zero by 4.6 standard deviations. In the standard model CP violation originates in the CKM phase, and direct CP violation is governed by loop diagrams of the penguin type, in which the three quark generations are present. The value of ε'/ε is determined by a non-trivial interplay of the strong, electromagnetic, and weak interactions and depends on almost all SM parameters. This makes its calculation quite complex. The main source of uncertainty in the calculation of the CP ratio is the QCD non-perturbative contribution related to the hadronic nature of the $K \rightarrow \pi\pi$ decays. Using the $\Delta S = 1$ effective hamiltonian,

$$\mathcal{H}_{eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \lambda_u \sum_{i=1}^8 c_i(\mu) Q_i(\mu) \quad (\mu < m_c), \quad (2)$$

the non-perturbative contribution, contained in the hadronic matrix elements of the four-quark operators Q_i , can be separated, at the renormalization scale $\mu \simeq 1 \text{ GeV}$, from the perturbative Wilson coefficients $c_i(\mu) = z_i(\mu) + \tau y_i(\mu)$ (with $\tau = -\lambda_t/\lambda_u$ and $\lambda_q = V_{qs}^* V_{qd}$). Introducing the matrix elements $\langle Q_i \rangle_I \equiv \langle (\pi\pi)_I | Q_i | K \rangle$, ε'/ε can be written as

$$\frac{\varepsilon'}{\varepsilon} = \frac{G_F \omega \text{Im}\lambda_t}{2 |\varepsilon| \text{Re}A_0} \left[\left| \sum_i y_i \langle Q_i \rangle_0 \right| \left(1 - \Omega_{\eta+\eta'} \right) - \frac{1}{\omega} \left| \sum_i y_i \langle Q_i \rangle_2 \right| \right]. \quad (3)$$

$\omega = \text{Re}A_0/\text{Re}A_2 = 22.2$ is the ratio of the CP conserving $K \rightarrow \pi\pi$ isospin amplitudes; $\Omega_{\eta+\eta'}$ takes into account the effect of the isospin breaking in the quark masses. In an exact realization of non-perturbative QCD, the scale dependences of the y_i and $\langle Q_i \rangle_I$ in Eq. (3) must cancel. ε'/ε is dominated by $\langle Q_6 \rangle_0$ and $\langle Q_8 \rangle_2$ which cannot be fixed from the CP conserving data, different from most of the matrix elements of the current-current operators [4,5]. Beside the theoretical uncertainties coming from the calculation of the $\langle Q_i \rangle_I$ and of $\Omega_{\eta+\eta'}$, the analysis of the CP ratio suffers from the uncertainties on the values of various input parameters, in particular of the CKM phase in $\text{Im}\lambda_t$, of $\Lambda_{\text{QCD}} \equiv \Lambda_{\text{MS}}^{(4)}$, and of the strange quark mass. The uncertainty on m_s is especially important due to the density-density structure of Q_6 and Q_8 , which implies that their matrix elements are proportional to the square of the quark condensate and are hence proportional to $1/m_s^2$.

In all methods the calculation of the hadronic matrix element of Q_6 , which is a peculiar operator, appears to be the most difficult one.

In this talk we present the results we obtained for ε'/ε together with G.O. Köhler and E.A. Paschos [6]. We briefly explain the method used to compute the hadronic matrix elements, and discuss in detail the peculiarities in the calculation of the matrix element of Q_6 .

2 General Framework

To calculate the hadronic matrix elements we start from the effective chiral lagrangian for pseudoscalar mesons which involves an expansion in momenta where terms up to $\mathcal{O}(p^4)$ are included [7]. Keeping only terms of $\mathcal{O}(p^4)$ which contribute, at the order we calculate, to the $K \rightarrow \pi\pi$ amplitudes, for the lagrangian we obtain:

$$\begin{aligned} \mathcal{L}_{eff} = & \frac{f^2}{4} \left(\langle D_\mu U^\dagger D^\mu U \rangle + \frac{\alpha}{4N_c} \langle \ln U^\dagger - \ln U \rangle^2 + \langle \chi U^\dagger + U \chi^\dagger \rangle \right) \\ & + L_5 \langle D_\mu U^\dagger D^\mu U (\chi^\dagger U + U^\dagger \chi) \rangle + L_8 \langle \chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger \rangle, \end{aligned} \quad (4)$$

with $\langle A \rangle$ denoting the trace of A , $\alpha = m_\eta^2 + m_{\eta'}^2 - 2m_K^2$, $\chi = r\mathcal{M}$, and $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$. f and r are parameters related to the pion decay constant F_π and to the quark condensate, with $r = -2\langle \bar{q}q \rangle / f^2$. The complex matrix U is a non-linear representation of the pseudoscalar meson nonet. The conventions and definitions we use are the same as those in [6,8,9].

The method we use is the $1/N_c$ expansion introduced in [10]. In this approach, we expand the matrix elements in powers of the external momenta and of $1/N_c$. From the lagrangian the mesonic representations of the quark currents and densities can be obtained by usual bosonization techniques. It is then straightforward to calculate the tree level (leading- N_c) matrix elements. For the $1/N_c$ corrections to the matrix elements $\langle Q_i \rangle_I$ we calculated chiral loops as described in [8,9]. Especially important to this analysis are the non-factorizable $1/N_c$ corrections, which are UV divergent and must be matched to the short-distance part. They are regularized by a finite cutoff which is identified with the short-distance renormalization scale [8–11]. The definition of the momenta in the loop diagrams, which are not momentum translation invariant, is discussed in detail in [8].

For the Wilson coefficients we use both the leading logarithmic (LO) and the next-to-leading logarithmic (NLO) values [4]. In the pseudoscalar approximation, the matching has to be done below 1 GeV. Values of the y_i at scales $0.6 \text{ GeV} \leq \mu \leq 0.9 \text{ GeV}$ were communicated to us by M. Jamin. The NLO values are scheme dependent and are calculated within naive dimensional regularization (NDR) and in the 't Hooft-Veltman

scheme (HV). The absence of any reference to the renormalization scheme in the low-energy calculation, at this stage, prevents a complete matching at the next-to-leading order [12]. Nevertheless, a comparison of the numerical results obtained from the LO and NLO coefficients is useful as regards estimating the uncertainties and testing the validity of perturbation theory.

3 Analysis of ε'/ε

At next-to-leading order, in the twofold expansion in powers of external momenta and of $1/N_c$, we investigate the tree level contributions from the $\mathcal{O}(p^2)$ and the $\mathcal{O}(p^4)$ lagrangian as well as the one-loop contribution from the $\mathcal{O}(p^2)$ lagrangian. For the matrix elements of the current-current operators, the corresponding terms are $\mathcal{O}(p^2)$, $\mathcal{O}(p^4)$, and $\mathcal{O}(p^2/N_c)$, respectively; for density-density operators they are $\mathcal{O}(p^0)$, $\mathcal{O}(p^2)$, and $\mathcal{O}(p^0/N_c)$.

Analytical formulas for all matrix elements at these orders are given in [8,9]. Among them four are particularly interesting and important:

$$\begin{aligned} \langle Q_1 \rangle_0 &= -\frac{1}{\sqrt{3}} F_\pi (m_K^2 - m_\pi^2) \left[1 + \frac{4\hat{L}_5^r}{F_\pi^2} m_\pi^2 + \frac{1}{(4\pi)^2 F_\pi^2} \right. \\ &\quad \left. \times \left(6\Lambda_c^2 - \left(\frac{1}{2} m_K^2 + 6m_\pi^2 \right) \log \Lambda_c^2 + \dots \right) \right], \end{aligned} \quad (5)$$

$$\begin{aligned} \langle Q_2 \rangle_0 &= \frac{2}{\sqrt{3}} F_\pi (m_K^2 - m_\pi^2) \left[1 + \frac{4\hat{L}_5^r}{F_\pi^2} m_\pi^2 + \frac{1}{(4\pi)^2 F_\pi^2} \right. \\ &\quad \left. \times \left(\frac{15}{4} \Lambda_c^2 + \left(\frac{11}{8} m_K^2 - \frac{15}{4} m_\pi^2 \right) \log \Lambda_c^2 + \dots \right) \right], \end{aligned} \quad (6)$$

$$\langle Q_6 \rangle_0 = -\frac{4\sqrt{3}}{F_\pi} R^2 (m_K^2 - m_\pi^2) \left[\hat{L}_5^r - \frac{3}{16(4\pi)^2} \log \Lambda_c^2 + \dots \right], \quad (7)$$

$$\begin{aligned} \langle Q_8 \rangle_2 &= \frac{\sqrt{3}}{2\sqrt{2}} F_\pi R^2 \left[1 + \frac{8m_K^2}{F_\pi^2} \left(\hat{L}_5^r - 2\hat{L}_8^r \right) - \frac{4m_\pi^2}{F_\pi^2} \left(3\hat{L}_5^r - 8\hat{L}_8^r \right) \right. \\ &\quad \left. - \frac{1}{(4\pi)^2 F_\pi^2} \left(m_K^2 - m_\pi^2 + \frac{2}{3} \alpha \right) \log \Lambda_c^2 + \dots \right], \end{aligned} \quad (8)$$

with $R \equiv 2m_K^2/(m_s + m_d)$. The ellipses denote finite terms which are not written explicitly here. The constants \hat{L}_5^r and \hat{L}_8^r are renormalized couplings whose values are $\hat{L}_5^r = 2.07 \cdot 10^{-3}$ and $\hat{L}_8^r = 1.09 \cdot 10^{-3}$. Eqs. (5) - (8) have several interesting properties [8,9]. First, the VSA values for $\langle Q_1 \rangle_0$ and $\langle Q_2 \rangle_0$ are far too small to account for the large $\Delta I = 1/2$ enhancement observed in the CP conserving amplitudes. Using the large- N_c limit [$B_1^{(1/2)} = 3.05$, $B_2^{(1/2)} = 1.22$; $B_i^{(\Delta I)} \equiv \text{Re} \langle Q_i \rangle_I / \langle Q_i \rangle_I^{\text{VSA}}$] improves the agreement

between theory and experiment, but it still provides a gross underestimate. However, the non-factorizable $1/N_c$ corrections in Eqs. (5) and (6) contain quadratically divergent terms which are not suppressed with respect to the tree level contribution, since they bring about a factor of $\Delta \equiv \Lambda_c^2/(4\pi F_\pi)^2$ and have large prefactors. Varying Λ_c between 600 and 900 MeV, $B_1^{(1/2)}$ and $B_2^{(1/2)}$ take the values 8.2 – 14.2 and 2.9 – 4.6, respectively. Quadratic terms in $\langle Q_1 \rangle_0$ and $\langle Q_2 \rangle_0$ produce a large enhancement which brings the $\Delta I = 1/2$ amplitude in agreement with the data [9]. Corrections beyond the chiral limit ($m_q = 0$) in Eqs. (5) - (6) are suppressed by a factor of $\delta = m_{K,\pi}^2/(4\pi F_\pi)^2 \simeq 20\%$ and were found to be small.

For the operators Q_6 and Q_8 , values rather close to the VSA [$B_6^{(1/2)} = B_8^{(3/2)} = 1$] are used in the literature. As a result the experimental range for ε'/ε can be accommodated in the standard model only if there is a conspiracy of the input parameters m_s , $\Omega_{\eta+\eta'}$, $\text{Im}\lambda_t$, and Λ_{QCD} (see e.g. [5]). The fact that the VSA fails completely in explaining the $\Delta I = 1/2$ rule therefore raises the question whether it can be used for Q_6 and Q_8 . In fact, at the present stage of the calculation, the case of $\langle Q_6 \rangle_0$ and $\langle Q_8 \rangle_2$ is different from that of $\langle Q_{1,2} \rangle_0$. The leading- N_c values are very close to the corresponding VSA values. Moreover, the non-factorizable loop corrections in Eqs. (7) and (8), which are of $\mathcal{O}(p^0/N_c)$, are found to be only logarithmically divergent [8]. Consequently, in the case of $\langle Q_8 \rangle_2$ they are suppressed by a factor of δ compared to the leading $\mathcal{O}(p^0)$ term and are expected to be of the order of 20% to 50% depending on the prefactors. We note that Eq. (8) is a full leading plus next-to-leading order analysis of the Q_8 matrix element. The case of $B_6^{(1/2)}$ is more complicated since the $\mathcal{O}(p^0)$ term vanishes for Q_6 . Nevertheless, the non-factorizable loop corrections to this term remain and have to be matched to the short-distance part of the amplitudes [8]. These $\mathcal{O}(p^0/N_c)$ non-factorizable corrections must be considered at the same level, in the twofold expansion, as the $\mathcal{O}(p^2)$ tree contribution. Consequently, a value of $B_6^{(1/2)}$ around one [which corresponds to the $\mathcal{O}(p^2)$ term alone] is not a priori expected. However, numerically it turns out that the $\mathcal{O}(p^0/N_c)$ contribution is only moderate. This property can be understood from the $(U^\dagger)_{dq}(U)_{qs}$ structure of the Q_6 operator which vanishes to $\mathcal{O}(p^0)$ implying that the factorizable and non-factorizable $\mathcal{O}(p^0/N_c)$ contributions cancel to a large extent [8]. This explains why the deviations we observe with respect to the VSA values are smaller than for Q_1 and Q_2 and why in particular for Q_6 to $\mathcal{O}(p^0/N_c)$ we do not observe a $\Delta I = 1/2$ enhancement. Varying Λ_c between 600 and 900 MeV, $B_6^{(1/2)}$ and $B_8^{(3/2)}$ take the values 1.10 – 0.72 and 0.64 – 0.42, respectively. $B_6^{(1/2)}$ and $B_8^{(3/2)}$ are therefore more efficiently protected from possible large $1/N_c$ corrections of the $\mathcal{O}(p^2)$ lagrangian than $B_{1,2}^{(1/2)}$. The effect of the $\mathcal{O}(p^0/N_c)$ term is however important for $B_6^{(1/2)}$ as for $B_8^{(3/2)}$ because it gives rise to a noticeable dependence on the cutoff scale [8]. We note that $B_8^{(3/2)}$ shows a scale dependence which is very

similar to the one of $B_6^{(1/2)}$ leading to a stable ratio $B_6^{(1/2)}/B_8^{(3/2)} \simeq 1.72$ for Λ_c between 600 and 900 MeV. The $\mathcal{O}(p^0/N_c)$ corrections consequently make the cancellation of Q_6 and Q_8 in ε'/ε less effective, but the values of the matrix elements are reduced. Hence we obtain values for ε'/ε [6] close to the ones found with the VSA values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$; in particular, for central values of the input parameters [$m_s(1 \text{ GeV})=150 \text{ MeV}$, $\Omega_{\eta+\eta'} = 0.25$, $\text{Im}\lambda_t = 1.33 \cdot 10^{-4}$, and $\Lambda_{\text{QCD}} = 325 \text{ MeV}$, see [6] and references therein] the values for the CP ratio are significantly smaller than the data.

However, since the leading $\mathcal{O}(p^0)$ contribution vanishes for Q_6 , corrections from higher order terms beyond the $\mathcal{O}(p^2)$ and $\mathcal{O}(p^0/N_c)$ are expected to be large [6]. The terms of $\mathcal{O}(p^2)$ and $\mathcal{O}(p^0/N_c)$ correspond to the lowest (non-vanishing) order, and the calculation of the next order terms is very desirable. In the twofold expansion, the higher order corrections to the matrix element of Q_6 are of orders: $\mathcal{O}(p^4)$, $\mathcal{O}(p^0/N_c^2)$, and $\mathcal{O}(p^2/N_c)$. A full calculation of these terms is beyond the scope of our study. In particular, higher order terms in the p^2 expansion, which are chirally suppressed, cannot be calculated because the low-energy couplings in the $\mathcal{O}(p^6)$ lagrangian are very uncertain or even unknown. In [6] we investigated the $\mathcal{O}(p^2/N_c)$ contribution, i.e., the $1/N_c$ correction at the next order in the chiral expansion, because it brings about, for the first time, quadratic corrections on the cutoff. We remind the readers that for the CP conserving amplitude it is mainly the (quadratic) $\mathcal{O}(p^2/N_c)$ corrections which bring to $\langle Q_{1,2} \rangle_0$ a large enhancement relative to the (leading- N_c) $\mathcal{O}(p^2)$ values. As the leading- N_c value for Q_6 is also $\mathcal{O}(p^2)$ we cannot a priori exclude that the value of $\langle Q_6 \rangle_0$ is largely affected by $\mathcal{O}(p^2/N_c)$ corrections too, removing from Q_6 the property observed to $\mathcal{O}(p^0/N_c)$, to be protected from large $1/N_c$ corrections. As explained above, quadratic $\mathcal{O}(p^2/N_c)$ corrections are proportional to the factor $\Delta \equiv \Lambda_c^2/(4\pi F_\pi)^2$ relative to the $\mathcal{O}(p^2)$ tree level contribution. Different is the case of Q_8 since its leading- N_c value is $\mathcal{O}(p^0)$ at lowest order in the chiral expansion. Quadratic terms for Q_8 are consequently chirally suppressed with respect to the leading- N_c value.

Calculating the quadratic term of $\mathcal{O}(p^2/N_c)$ for matrix element of Q_6 and adding it to the $\mathcal{O}(p^2)$ and $\mathcal{O}(p^0/N_c)$ result in Eq. (7) we obtain:

$$\langle Q_6 \rangle_0 = -\frac{4\sqrt{3}}{F_\pi} R^2 (m_K^2 - m_\pi^2) \left[\hat{L}_5^r \left(1 + \frac{3}{2} \frac{\Lambda_c^2}{(4\pi)^2 F_\pi^2} \right) - \frac{3}{16} \frac{\log \Lambda_c^2}{(4\pi)^2} + \dots \right]. \quad (9)$$

The result for the $\mathcal{O}(p^2/N_c)$ term we already presented in [13]. Numerically, we observe a large positive correction from the quadratic term in Eq. (9). The slope of this correction is qualitatively consistent and welcome since it compensates for the logarithmic decrease at $\mathcal{O}(p^0/N_c)$. Varying Λ_c between 600 and 900 MeV, the $B_6^{(1/2)}$ factor takes the values 1.50 – 1.62. The approximate stability of $B_6^{(1/2)}$ is in accordance with the perturbative

evolution, since the non-diagonal dependence of y_6 on the renormalization scale, i.e., the one beyond the leading- N_c scale dependence of R^2 in Eq. (9), was found to be small. The quadratic term of $\mathcal{O}(p^2/N_c)$ is of the same magnitude as the $\mathcal{O}(p^2)$ tree term. Q_6 is a $\Delta I = 1/2$ operator, and the enhancement of $\langle Q_6 \rangle_0$ indicates that at the level of the $1/N_c$ corrections the dynamics of the $\Delta I = 1/2$ rule applies to Q_6 as to $Q_{1,2}$. One might however note that the enhancement observed for Q_6 is smaller than for Q_1 and Q_2 .

Using the quoted values for $B_6^{(1/2)}$ together with the full leading plus next-to-leading order B factors for the remaining operators [6] we calculated ε'/ε for central values of m_s , $\Omega_{\eta+\eta'}$, $\text{Im}\lambda_t$, and Λ_{QCD} . The results for the three sets of Wilson coefficients LO, NDR, and HV and for Λ_c between 600 and 900 MeV are given in Tab. 1. The numbers are obtained with two different methods for analyzing the sensitivity on the imaginary part coming from the final states interactions. In the first case, we use the real part of our calculation and the phenomenological phases $\delta_0 = (34.2 \pm 2.2)^\circ$ and $\delta_2 = (-6.9 \pm 0.2)^\circ$ [14], and replace $|\sum_i y_i \langle Q_i \rangle_I|$ in Eq. (3) by $\sum_i y_i \text{Re}\langle Q_i \rangle_I / \cos \delta_I$ [15]. In the second case, we use only the real part assuming zero phases. The latter case is very close to the results we would get if we used the small imaginary part obtained at the one-loop level [6]. Collecting together the LO, NDR, and HV results for the two cases and for central values of the parameters, we find the following (conservative) range:

$$7.0 \cdot 10^{-4} \leq \varepsilon'/\varepsilon (\text{central}) \leq 24.7 \cdot 10^{-4},$$

which is in the ball park of the experimental result in Eq. (1). Performing a complete scanning of the parameters [$125 \text{ MeV} \leq m_s(1 \text{ GeV}) \leq 175 \text{ MeV}$, $0.15 \leq \Omega_{\eta+\eta'} \leq 0.35$, $1.04 \cdot 10^{-4} \leq \text{Im}\lambda_t \leq 1.63 \cdot 10^{-4}$, and $245 \text{ MeV} \leq \Lambda_{\text{QCD}} \leq 405 \text{ MeV}$] we obtain $2.2 \cdot 10^{-4} \leq \varepsilon'/\varepsilon (\text{scanned}) \leq 63.2 \cdot 10^{-4}$ (see Tab. 2). The numerical values in the tables can be compared with the results of the Munich, Trieste, and Rome groups [5,15,16]. The values for $B_6^{(1/2)}$ can also be compared with [17]. The large ranges reported in Tab. 2 can be traced back to the large ranges of the input parameters. The parameters, to a large extent, act multiplicatively, and the larger range for ε'/ε is due to the fact that the central value(s) for the ratio are enhanced roughly by a factor of two compared to the results obtained with B factors for Q_6 and Q_8 close to the VSA. More accurate information on the parameters, from theory and experiment, will restrict the values for the CP ratio.

4 Summary

We have shown that the operator Q_6 , similar to Q_1 and Q_2 , is not protected from large $1/N_c$ corrections coming from quadratic terms of $\mathcal{O}(p^2/N_c)$. From general counting arguments we have good indications that among the various next-to-leading order terms in

	Case 1	Case 2
LO	$19.5 \leq \varepsilon'/\varepsilon \leq 24.7$	$14.8 \leq \varepsilon'/\varepsilon \leq 19.4$
NDR	$16.1 \leq \varepsilon'/\varepsilon \leq 23.4$	$12.5 \leq \varepsilon'/\varepsilon \leq 18.3$
HV	$9.3 \leq \varepsilon'/\varepsilon \leq 19.3$	$7.0 \leq \varepsilon'/\varepsilon \leq 14.9$
LO + NDR + HV	$9.3 \leq \varepsilon'/\varepsilon \leq 24.7$	$7.0 \leq \varepsilon'/\varepsilon \leq 19.4$

Table 1: Central ranges for ε'/ε (in units of 10^{-4}) as explained in the text.

	Case 1	Case 2
LO	$8.0 \leq \varepsilon'/\varepsilon \leq 62.1$	$6.1 \leq \varepsilon'/\varepsilon \leq 48.5$
NDR	$6.8 \leq \varepsilon'/\varepsilon \leq 63.9$	$5.2 \leq \varepsilon'/\varepsilon \leq 49.8$
HV	$2.8 \leq \varepsilon'/\varepsilon \leq 49.8$	$2.2 \leq \varepsilon'/\varepsilon \leq 38.5$
LO + NDR + HV	$2.8 \leq \varepsilon'/\varepsilon \leq 63.9$	$2.2 \leq \varepsilon'/\varepsilon \leq 49.8$

Table 2: Same as in Tab. 1, but for the complete scanning of the parameters.

the p^2 and $1/N_c$ expansions they are the dominant ones. Calculating those terms we find that they enhance $B_6^{(1/2)}$ and bring ε'/ε much closer to the data for central values of the parameters. We obtain a quadratic evolution for Q_6 which indicates that a $\Delta I = 1/2$ enhancement is operative for Q_6 similar to the one of Q_1 and Q_2 . $B_8^{(3/2)}$ is expected to be affected much less by terms of $\mathcal{O}(p^2/N_c)$ due to an extra p^2 suppression factor relative to the leading $\mathcal{O}(p^0)$ tree term.

One should recall that our analysis of the $\mathcal{O}(p^2/N_c)$ terms for Q_6 is performed in the chiral limit. It would be desirable to calculate the corrections beyond the chiral limit, from logarithms and finite terms. It would also be interesting to investigate the effect of higher resonances. Each of the additional effects separately is not expected to counteract largely the enhancement found for $B_6^{(1/2)}$. Nevertheless, in the extreme (and unlikely) case where all these effects would come with the same sign a significant modification of the result cannot be excluded formally. In order to reduce the scheme dependence in the result for ε'/ε , appropriate subtractions would be necessary (see [17,18]).

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