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Supersymmetric contributions to direct CP violation in $K \to \pi\pi\gamma$ decays*

G. Colangelo[1], G. Isidori[2], J. Portolés[3]

[1] *Inst. für Theor. Physik der Universität Zürich, CH-8057 Zürich, Switzerland*
[2] *INFN, Laboratori Nazionali di Frascati, P.O. Box 13, I–00044 Frascati, Italy*
[3] *Departament de Física Teòrica (I.F.I.C.)*
C.S.I.C.–Universitat de València, Apdo. 2085, E–46071 València, Spain

Abstract

We analyze the supersymmetric contributions to direct–CP–violating observables in $K \to \pi\pi\gamma$ decays induced by gluino–mediated magnetic–penguin operators. We find that $\epsilon'_{+-\gamma}$ and the differential width asymmetry of $K^\pm \to \pi^\pm\pi^0\gamma$ decays could be substantially enhanced with respect to their Standard Model values, especially in the scenario where ϵ'/ϵ is dominated by supersymmetric contributions. These observables could therefore provide a useful tool to search for New Physics effects in $|\Delta S| = 1$ transitions, complementary to ϵ'/ϵ and rare decays.

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1 Introduction

The phenomenon of CP violation is one of the least tested aspects of the Standard Model (SM) and represents one of the sectors where a large sensitivity to possible New Physics (NP) effects can be expected. An important step forward in understanding the nature of this phenomenon has recently been achieved by the KTeV and NA48 collaborations, obtaining the following measurements of direct CP violation in $K^0(\bar{K}^0) \to 2\pi$ decays:

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \begin{cases} (28.0 \pm 4.1) \times 10^{-4} & [1]\ , \\ (18.5 \pm 7.3) \times 10^{-4} & [2]\ . \end{cases} \tag{1}$$

These results, together with the earlier finding by NA31 [3], clearly establish the existence of direct CP violation, as generally predicted by the SM. However, an intriguing aspect of this new measurement is that the values in (1) tend to be larger than most SM estimates [4,5]. Unfortunately the theoretical predictions of ϵ'/ϵ are affected by large uncertainties, mainly of non–perturbative origin, and it is possible that the experimental values above are still compatible with the SM expectations (see, in particular, Ref. [5]). Nonetheless, it is clear that after these new experimental results the chances of sizable NP contributions in ϵ'/ϵ have increased substantially.

Among other possible NP scenarios, low energy supersymmetry [6] represents one of the most interesting and consistent extensions of the Standard Model. In generic supersymmetric models, the large number of new particles carrying flavor quantum numbers would naturally lead to large effects in CP–violating and flavor–changing neutral–current (FCNC) amplitudes [7,8]. Actually, in this context the problem is not how to generate large CP–violating effects, but rather how to avoid dangerous corrections to small quantities like ϵ_K or Δm_K, which seem to be consistent with their SM expectations. However, as discussed recently in [9–11], in specific supersymmetric scenarios it is possible to generate non–standard $\mathcal{O}(10^{-3})$ contributions to ϵ'/ϵ without getting troubles with the experimental constraints of other CP and FCNC processes.

From a phenomenological point of view, the supersymmetric sources of a sizable enhancement of ϵ'/ϵ which can avoid fine–tuning problems in $|\Delta S| = 2$ amplitudes, are basically two [11]: a large $\bar{s}dG$ vertex induced by the chromomagnetic operator [10] and an enhanced $\bar{s}dZ$ vertex [12]. Since the problem of non–perturbative uncertainties in the estimate of ϵ'/ϵ is typically worse in the case of supersymmetric contributions, it is very useful to identify other observables which could clearly signal the manifestation of either of these two mechanisms. As discussed in [11,13], in the case of the enhanced $\bar{s}dZ$ vertex there is a strong correlation between ϵ'/ϵ and the theoretically–clean $K \to \pi\nu\bar{\nu}$ widths. The scenario where ϵ'/ϵ receives sizable supersymmetric corrections via the $\bar{s}dZ$ vertex could therefore be clearly excluded or confirmed by future precise experiments on rare decays.

More difficult to identify is the case where ϵ'/ϵ receives sizable contributions by the chromomagnetic operator. Indeed this non–standard effect would be present mainly in non–leptonic processes. However, since there is a strict correlation between the chromomagnetic operator ($\sim \bar{s}\sigma^{\mu\nu}t^a dG^a_{\mu\nu}$) and the magnetic penguin contributing to the $s \to d\gamma$ transition ($\sim \bar{s}\sigma^{\mu\nu}dF_{\mu\nu}$), interesting consequences of this scenario could in principle be observed in processes with real photons or e^+e^- pairs in the final state. As shown in [11], an example of such processes is provided by the $K_L \to \pi^0 e^+e^-$ decay. In this letter we analyze the consequences of this scenario in $K \to \pi\pi\gamma$ decays, focusing on the possible enhancements of direct–CP–violating observables. As we will show, these can provide complementary information to rare decays.

The paper is organized as follows: in Section 2 we recall the structure of supersymmetric contributions to magnetic operators and their impact on ϵ'/ϵ. In Section 3 we estimate the matrix element of the tensor current, necessary to evaluate CP–violating effects in $K \to \pi\pi\gamma$ decays. The general decomposition of $K \to \pi\pi\gamma$ amplitudes and the estimate of the supersymmetric contributions to $\epsilon'_{+-\gamma}$ is given in Section 4, while in Section 5 we discuss the charge asymmetry in $K^\pm \to \pi^\pm\pi^0\gamma$ decays. Finally in Section 6 we summarize our results.

2 Gluino contributions to magnetic operators and ϵ'/ϵ

A useful framework to evaluate supersymmetric contributions to CP–violating and FCNC processes is provided by the mass–insertion approximation [8]. This consists in choosing a simple flavor–basis for the gauge interactions and, in that basis, to perform a perturbative expansion of the squark mass matrices around their diagonal. Gluino–mediated amplitudes usually provide the dominant effect, therefore the basis typically adopted is the one where the gluino–quark–squark vertices are flavor–diagonal.

A detailed discussion of the leading terms generated by gluino exchange in the framework of the mass–insertion approximation can be found in [14,15]. Given the strong constraints from $|\Delta S| = 2$ processes, it is found that only the dimension–5 magnetic operators induced by $\tilde{d}_{L(R)} - \tilde{s}_{R(L)}$ mixing could lead to sizable CP–violating effects in $|\Delta S| = 1$ amplitudes avoiding fine–tuning problems. These operators can be written as [14]

$$\mathcal{H}^{(5)}_{eff} = \frac{(\delta^D_{RL})_{21}}{m_{\tilde{g}}} \left[\tilde{C}_7(x_{gq}) \bar{s}_R \sigma^{\mu\nu} d_L \hat{F}_{\mu\nu} + \tilde{C}_8(x_{gq}) \bar{s}_R \sigma^{\mu\nu} \hat{G}_{\mu\nu} d_L \right]$$

$$+\frac{(\delta^D_{LR})_{21}}{m_{\tilde g}}\left[\widetilde C_7(x_{gq})\bar s_L\sigma^{\mu\nu}d_R\hat F_{\mu\nu}+\widetilde C_8(x_{gq})\bar s_L\sigma^{\mu\nu}\hat G_{\mu\nu}d_R\right]+\text{h.c.}\ , \tag{2}$$

where $\hat G_{\mu\nu}=gt^aG^a_{\mu\nu}$, $\hat F_{\mu\nu}=eF_{\mu\nu}$,

$$(\delta^D_{AB})_{ij}=(\delta^D_{BA})^*_{ji}=(M^2_D)_{\tilde q^i_A\tilde q^j_B}/m^2_{\tilde d}\ , \tag{3}$$

$m_{\tilde d}$ is the average down–squark mass, $m_{\tilde g}$ is the gluino mass and $x_{gq}=m^2_{\tilde g}/m^2_{\tilde d}$. Neglecting QCD corrections, the Wilson coefficients $\widetilde C_{7,8}(x_{gq})$ are given by [11,14]

$$\widetilde C_7(x)=-\frac{\alpha_s}{24\pi}F_0(x)\ ,\qquad \widetilde C_7(1)=-\frac{1}{108}\frac{\alpha_s}{\pi}\ , \tag{4}$$

$$\widetilde C_8(x)=\frac{\alpha_s}{8\pi}G_0(x)\ ,\qquad \widetilde C_8(1)=-\frac{5}{144}\frac{\alpha_s}{\pi}\ , \tag{5}$$

with

$$G_0(x)=\frac{x(22-20x-2x^2+16x\ln(x)-x^2\ln(x)+9\ln(x))}{3(1-x)^4}\ , \tag{6}$$

$$F_0(x)=\frac{4x(1+4x-5x^2+4x\ln(x)+2x^2\ln(x))}{3(1-x)^4}\ . \tag{7}$$

Due to the smallness of the electric charge, the contribution generated by $\mathcal{H}^{(5)}_{eff}$ to $\mathrm{Re}(\epsilon'/\epsilon)$ is dominated by the terms proportional to $\widetilde C_8$. This can be written as [11]

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)^{\rm SUSY}_G=P_G\,\mathrm{Im}\Lambda^-_g\ , \tag{8}$$

where

$$\Lambda^-_g=\left[(\delta^D_{LR})_{21}-(\delta^D_{LR})^*_{12}\right]G_0(x_{gq}) \tag{9}$$

and[1]

$$\begin{aligned}P_G&=\frac{11}{64}\frac{\omega}{|\epsilon|\mathrm{Re}(A_0)}\frac{m^2_\pi m^2_K}{F_\pi(m_s+m_d)}\frac{\alpha_s(m_{\tilde g})}{\pi}\frac{1}{m_{\tilde g}}\eta B_G\\ &\simeq 2.4\times10^2B_G\left(\frac{137\ \mathrm{MeV}}{m_s+m_d}\right)\left(\frac{500\ \mathrm{GeV}}{m_{\tilde g}}\right)\left(\frac{\alpha_s(m_{\tilde g})}{\alpha_s(500\ \mathrm{GeV})}\right)^{\frac{23}{21}}\ .\end{aligned} \tag{10}$$

The expression (8) has been obtained neglecting the mixing induced by QCD corrections between $\widetilde C_8$ and the Wilson coefficients of the SM $|\Delta S|=1$ effective Hamiltonian. This

[1] Following [16], here we adopt a normalization of $K\to(2\pi)_I$ amplitudes such that $\mathrm{Re}(A_0)^{\rm exp}=2.72\times10^{-7}$ GeV and we employ the notation $F_\pi=92.4$ MeV. Note that both these conventions differ from those adopted in [11]. Moreover $\omega^{-1}=(\mathrm{Re}(A_0)/\mathrm{Re}(A_2))_{exp}=22.2\pm0.1$ is the $\Delta I=1/2$ rule enhancement factor.

is a good approximation if $\widetilde{C}_8$ is sufficiently large: in this case the renormalization–group evolution of $\widetilde{C}_8$ is almost diagonal and is taken into account by the factor [11]

$$\eta = \left(\frac{\alpha_s(m_{\tilde{g}})}{\alpha_s(m_t)}\right)^{\frac{2}{21}} \left(\frac{\alpha_s(m_t)}{\alpha_s(m_b)}\right)^{\frac{2}{23}} \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{\frac{2}{25}} \simeq 0.89 \left(\frac{\alpha_s(m_{\tilde{g}})}{\alpha_s(500 \text{ GeV})}\right)^{\frac{2}{21}} . \quad (11)$$

In (10) we have not explicitly shown the scale dependence of quark masses and B_G, which are evaluated at $\mu = m_c$. The parameter B_G, expected to be $\mathcal{O}(1)$ for a renormalization scale $\mu \sim 1$ GeV, is defined by

$$\langle (\pi\pi)_{I=0} | \bar{s}_R \sigma^{\mu\nu} \hat{G}_{\mu\nu} d_L | K^0 \rangle (\mu) = \frac{11}{4\sqrt{2}} \frac{m_\pi^2}{F_\pi} \frac{m_K^2}{m_s(\mu) + m_d(\mu)} B_G(\mu) . \quad (12)$$

3 Matrix elements of the tensor current

Contrary to the case of ϵ'/ϵ, the $\widetilde{C}_7$ terms of $\mathcal{H}_{eff}^{(5)}$ could play an important role in CP–violating observables of $K \to \pi\pi\gamma$ decays. In order to evaluate their impact, we need to estimate the matrix elements of the $\bar{s}_{R(L)} \sigma^{\mu\nu} d_{L(R)}$ current between kaon and pion states. Given the Lorentz structure and the transformation properties under CP and $SU(3)_L \times SU(3)_R$, the lowest–order chiral realization of the tensor current can be written as

$$\bar{s}_R \sigma_{\mu\nu} d_L \longrightarrow -i \frac{a_T F_\pi^2}{2} \left[\partial_\mu U^\dagger \partial_\nu U U^\dagger - \partial_\nu U^\dagger \partial_\mu U U^\dagger \right]_{23} , \quad (13)$$

$$\bar{s}_L \sigma_{\mu\nu} d_R \longrightarrow -i \frac{a_T F_\pi^2}{2} \left[\partial_\mu U \partial_\nu U^\dagger U - \partial_\nu U \partial_\mu U^\dagger U \right]_{23} , \quad (14)$$

where we have neglected terms proportional to the Levi-Civita tensor, $\epsilon_{\mu\nu\rho\sigma}$, not interesting to the present analysis. Here U is the usual chiral field (we follow the notation of [16]) and a_T is an unknown coupling.

To obtain a first estimate of a_T we proceed by differentiating and using the e.o.m. on both sides of (13-14). In this way on the l.h.s. we obtain some terms whose chiral realization is well known, namely the $\bar{s}_{L(R)} \gamma^\mu d_{L(R)}$ currents. Identifying the corresponding terms on the r.h.s. we then obtain

$$a_T = \frac{m_s + m_d}{m_K^2} . \quad (15)$$

Unfortunately, it is not possible to repeat this identification for all the quark bilinears which appear on the l.h.s. This shows that Eq. (15) is not to be trusted literally. The same conclusion can also be reached by noting that the scale dependence of the tensor current is not the same as that of the scalar bilinear. Eq. (15) would therefore give the wrong scale dependence of the matrix elements of the tensor current, and, strictly speaking, cannot be correct. On the other hand, we find Eq. (15) instructive, in the sense that it shows that the coefficient a_T (which has dimensions of the inverse of a mass) must be proportional to the inverse of the scale of chiral symmetry breaking, with a numerical coefficient of $\mathcal{O}(1)$.

An additional indication on the value of a_T can be obtained by evaluating the $\langle K|\bar{s}\sigma^{\mu\nu}d|\pi\rangle$ matrix element in the limit where the strange quark mass is very heavy ($m_s \gg \Lambda_{QCD}$). The value of a_T thus determined can be written as [17]

$$|a_T| \simeq \frac{1}{2m_K}\left[f_+(q^2) + \mathcal{O}(f_-)\right] , \tag{16}$$

where $f_\pm(q^2)$ are the form factors of the vector current. Obviously, this result can be trusted even less than Eq. (15). On the other hand it shows that if we vary the strange quark mass, and approach its physical value from above, we get a value of a_T which is numerically close to that obtained with chiral arguments. We believe that this serves as an independent check of the order of magnitude, and gives us confidence that the real value of a_T cannot be too different from the estimates presented here. A further independent estimate of $|a_T|$ very close to the one in (16) can be obtained also in the framework of vector meson dominance, as in [18]. Given these results, for simplicity we shall assume in the following

$$a_T = \frac{B_T}{2m_K} , \tag{17}$$

where B_T is a dimensionless parameter expected to be of $\mathcal{O}(1)$. Note, however, that Eq. (17) does not show the correct chiral behaviour, which should rather be read from (15). Both the correct dependence on the quark masses, and on the QCD renormalization scale are assumed to be hidden inside B_T.

4 $K \to \pi\pi\gamma$ amplitudes and $\epsilon'_{+-\gamma}$

The most general form, dictated by gauge and Lorentz invariance, for the transition amplitude $K(p_K) \to \pi_1(p_1)\pi_2(p_2)\gamma(\epsilon, q)$ is given by

$$A(K \to \pi\pi\gamma) = \epsilon^*_\mu \left[E(z_i)(qp_1p_2^\mu - qp_2p_1^\mu) + M(z_i)\epsilon^{\mu\nu\rho\sigma}p_{1\nu}p_{2\rho}q_\sigma\right]/m_K^3, \tag{18}$$

where E and M, known as electric and magnetic amplitudes, are dimensionless functions of

$$z_i = \frac{p_i q}{m_K^2} \quad (i = 1, 2) \qquad \text{and} \qquad z_3 = z_1 + z_2 = \frac{p_K q}{m_K^2} \tag{19}$$

(only two of the z_i's are independent). Following [16] we can decompose the electric amplitude as $E = E_{IB} + E_{DE}$, where

$$E_{IB}(z_i) = \frac{eA(K \to \pi_1\pi_2)}{M_K z_3}\left(\frac{Q_2}{z_2} - \frac{Q_1}{z_1}\right) \tag{20}$$

is the well–known bremsstrahlung contribution (eQ_i denotes the electric charge of the pion π_i). Furthermore, we can expand the direct–emission amplitudes E_{DE} and M as

$$
\begin{aligned}
E_{DE}(z_i) &= E_1 + O\left[(z_1 - z_2)\right] \ , &(21)\\
M(z_i) &= M_1 + O\left[(z_1 - z_2)\right] \ , &(22)
\end{aligned}
$$

where the higher order terms in $(z_1 - z_2)$ can be safely neglected due to the phase–space suppression.

The first CP violating observable we shall consider is

$$
\eta_{+-\gamma} = \frac{A(K_L \to \pi^+\pi^-\gamma)_{E_{IB}+E_1}}{A(K_S \to \pi^+\pi^-\gamma)_{E_{IB}+E_1}} \ . \tag{23}
$$

Due to the vanishing of direct emission amplitudes, at small photon energies $\eta_{+-\gamma}$ tends to the usual $K \to 2\pi$ parameter $\eta_{+-} = A(K_L \to \pi^+\pi^-)/A(K_S \to \pi^+\pi^-)$. On the other hand, the difference $(\eta_{+-\gamma} - \eta_{+-})$, that vanishes for $E_\gamma \to 0$, is an independent index of direct CP violation. Following [16] we can write

$$
\epsilon'_{+-\gamma} \ = \ \eta_{+-\gamma} - \eta_{+-} \ = \ i\frac{e^{i(\delta_n - \delta_0)} m_K z_+ z_-}{e\sqrt{2}\mathrm{Re}A_0} \left(\mathrm{Im}A_0 \frac{\mathrm{Re}E_n}{\mathrm{Re}A_0} - \mathrm{Im}E_n \right) \ , \tag{24}
$$

where on the r.h.s. we have neglected small contributions suppressed by $\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$ $= 0.045$ and the following decomposition has been employed

$$
E_1(K^0) = \frac{1}{\sqrt{2}} \, e^{i\delta_n} E_n \ , \qquad\qquad (p_1, p_2) \equiv (p_+, p_-) \ . \tag{25}
$$

Assuming that the dominant SUSY contribution to the CP–violating phase of E_n is generated by the magnetic photon operator we find

$$
\mathrm{Im}\,(E_n)^{\mathrm{SUSY}} = -\frac{e m_K^2}{12 F_\pi} \frac{\alpha_s(m_{\tilde g})}{\pi} \frac{\eta^2 B_T}{m_{\tilde g}} \left[\frac{F_0(x_{gq})}{G_0(x_{gq})} + 8(1 - \eta^{-1}) \right] \mathrm{Im}\Lambda_g^- \ . \tag{26}
$$

Then using (8) to express both $\mathrm{Im}\Lambda_g^-$ and $(\mathrm{Im}A_0)_G^{\mathrm{SUSY}}$ in terms of $\mathrm{Re}(\epsilon'/\epsilon)_G^{\mathrm{SUSY}}$, we obtain

$$
\left(\frac{\epsilon'_{+-\gamma}}{\epsilon} \right)^{\mathrm{SUSY}} = \frac{e^{i(\delta_n - \delta_0 + \pi/4)} z_+ z_-}{\omega} \left[R_{FG} - \frac{m_K \mathrm{Re}E_n}{e\mathrm{Re}A_0} \right] \mathrm{Re} \left(\frac{\epsilon'}{\epsilon} \right)_G^{\mathrm{SUSY}} \ , \tag{27}
$$

where

$$
\begin{aligned}
R_{FG} &= \frac{16}{33\sqrt{2}} \frac{m_K(m_s + m_d)}{m_\pi^2} \eta \frac{B_T}{B_G} \left[\frac{F_0(x_{gq})}{G_0(x_{gq})} + 8(1 - \eta^{-1}) \right] &(28)\\
&\simeq -1.9 \frac{B_T}{B_G} \left(\frac{m_s + m_d}{137\ \mathrm{MeV}} \right) \qquad (\text{for } m_{\tilde g} = 500\ \mathrm{GeV}, x_{gq} = 1) \ .
\end{aligned}
$$

Unfortunately at the moment there are no precise experimental informations about ReE_n, however naive chiral counting suggests $m_K \mathrm{Re} E_n/(eReA_0) \ll 1$ [16]. Neglecting this contribution in (27), assuming $|B_T/B_G| \leq 1$, $x_{gq} \leq 1.3$ [19] and $(m_s + m_d) \leq 158$ MeV, we finally obtain

$$\left| \frac{\epsilon'_{+-\gamma}}{\epsilon} \right|^{\mathrm{SUSY}} \leq \; 50\; z_+ z_- \;\mathrm{Re} \left(\frac{\epsilon'}{\epsilon} \right)^{\mathrm{SUSY}}_G \leq \; 0.15\; z_+ z_- \;, \tag{29}$$

where the last inequality has been obtained imposing $\mathrm{Re}(\epsilon'/\epsilon)^{\mathrm{SUSY}}_G \leq 3 \times 10^{-3}$. Note that the sensitivity of this result to the value of $m_{\tilde{g}}$ and $m_{\tilde{d}}$ is very small: they enter only through the F_0/G_0 ratio and the factor η in (28).

Interestingly the upper bound (29) is substantially larger (almost one order of magnitude) with respect to the corresponding one obtained within the Standard Model [16]. A large value of $\epsilon'_{+-\gamma}/\epsilon$ could therefore offer a clean signature of the scenario where ϵ'/ϵ is dominated by supersymmetric magnetic–type contributions. Moreover, we notice that $\epsilon'_{+-\gamma}/\epsilon$ is generated by the interference of two $\Delta I = 1/2$ amplitudes (it is indeed enhanced by ω^{-1} with respect to ϵ'/ϵ) and therefore, contrary to ϵ'/ϵ or $K_L \to \pi^0 e^+ e^-$, it is almost insensitive to possible new–physics effects in the $\bar{s}dZ$ vertex.

Finally, we stress that the correlation between gluino–mediated contributions to ϵ'/ϵ and $\epsilon'_{+-\gamma}/\epsilon$ is clearer than the corresponding one between ϵ'/ϵ and $B(K_L \to \pi^0 e^+ e^-)$ [11]. Indeed, due to the different number of pions in the final state, the supersymmetric coupling ruling the effect in $K_L \to \pi^0 e^+ e^-$ is not exactly the same as in ϵ'/ϵ and $\epsilon'_{+-\gamma}/\epsilon$ [11].

5 Charge asymmetry in $K^\pm \to \pi^\pm \pi^0 \gamma$

A very clean observable of direct CP violation is provided by the asymmetry between $K^+ \to \pi^+ \pi^0 \gamma$ and $K^- \to \pi^- \pi^0 \gamma$ decay widths [16,18,20–22]. The decay rates of $K^\pm \to \pi^\pm \pi^0 \gamma$ are conveniently expressed in terms of T_c^*, the kinetic energy of the charged pion in the kaon rest frame, and $W^2 = (qp_K)(qp_\pm)/(m^2_{\pi^+} m^2_K)$. Factorizing the IB differential width, one can write [23]

$$\begin{aligned} \frac{\partial^2 \Gamma}{\partial T_c^* \partial W^2} \;=\; & \frac{\partial^2 \Gamma_{IB}}{\partial T_c^* \partial W^2} \left\{ 1 \;+\; 2\, \frac{m^2_{\pi^+}}{m_K} \mathrm{Re} \left(\frac{E_{DE}}{eA} \right) W^2 \right. \\ & \left. + \frac{m^4_{\pi^+}}{m^2_K} \left(\left| \frac{E_{DE}}{eA} \right|^2 + \left| \frac{M}{eA} \right|^2 \right) W^4 \right\} , \end{aligned} \tag{30}$$

where $A \equiv A(K^\pm \to \pi^\pm \pi^0)$. Since the linear term in W^2 is sensitive to the interference between the IB amplitude and the first electric dipole term E_1, it is convenient to introduce a direct–CP–violating observable Ω, defined as follows

$$\frac{\partial^2\Gamma^+/\partial T_c^*\partial W^2 \ - \ \partial^2\Gamma^-/\partial T_c^*\partial W^2}{\partial^2\Gamma^+/\partial T_c^*\partial W^2 \ + \ \partial^2\Gamma^-/\partial T_c^*\partial W^2} \ = \ \Omega\, W^2 \ \ + \mathcal{O}\left(\frac{m_\pi^4}{m_K^4} W^4\right) . \qquad (31)$$

Setting $(p_1, p_2) \equiv (p_\pm, p_0)$ and factorizing the strong phases analogously to (25) we write [16],

$$E_1(K^\pm) = e^{i\delta_1}\ E_c\ , \qquad\qquad E_{IB}(K^\pm) = -e^{i\delta_2}\ \frac{3e\mathrm{Re}(A_2)}{2m_K z_\pm z_3} . \qquad (32)$$

Assuming, as in the neutral channel, that the magnetic photon operator gives the dominant SUSY contributions to the CP–violating phase of E_c, we find

$$\mathrm{Im}(E_c)^{\text{SUSY}} = \mathrm{Im}(E_n)^{\text{SUSY}}\ , \qquad (33)$$

where $\mathrm{Im}(E_n)^{\text{SUSY}}$ is given in (26). Substituting this result in (30) we finally obtain

$$\begin{aligned} \Omega^{\text{SUSY}} \ &= \ \frac{64}{99}\frac{|\epsilon|}{\omega^2}\frac{m_s + m_d}{m_K}\ \sin(\delta_1 - \delta_2)\, \eta\, \frac{B_T}{B_G} \\ &\quad \times \left[\frac{F_0(x_{gq})}{G_0(x_{gq})}\ +\ 8(1-\eta^{-1})\right]\ \mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)_G^{\text{SUSY}} . \end{aligned} \qquad (34)$$

Since the dominant CP–conserving $K^\pm \to \pi^\pm\pi^0\gamma$ amplitude is a $\Delta I = 3/2$ transition, Ω is enhanced by a factor ω^{-2} with respect to ϵ'. This enhancement, however, is partially compensated by the fact that the strong phase-difference appearing in (34) is quite small $(\delta_1 - \delta_2) \simeq 10^\circ$ [24].[2] Employing the same assumptions adopted in Eq. (29) and using $\sin(\delta_1 - \delta_2) \leq 0.2$ we find

$$|\Omega|^{\text{SUSY}}\ \leq\ 0.077\ \mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)_G^{\text{SUSY}} \leq 2.3\times 10^{-4}\ . \qquad (35)$$

Similarly to the case of $(\epsilon'_{+-\gamma}/\epsilon)^{\text{SUSY}}$, also the result in (35) is substantially larger than what expected within the Standard Model [16].[3]

Since the kinetic variable W^2 can reach values of $\mathcal{O}(1)$ [21], the result (35) implies that in a specific region of the Dalitz plot, the asymmetry between $K^+ \to \pi^+\pi^0\gamma$ and $K^- \to \pi^-\pi^0\gamma$ distributions can be of $\mathcal{O}(10^{-4})$. A much smaller value is obtained performing a wide integration over the phase space. For instance integrating over W and T_c^*

[2] While $\delta_2(m_K) \simeq -7^\circ$ [24], in principle the δ_1 phase shift should be input with a dependence in the integration variables. This is however beyond the accuracy required by the present analysis.

[3] An asymmetry at the level of 10^{-4} between $K^+ \to \pi^+\pi^0\gamma$ and $K^- \to \pi^-\pi^0\gamma$ widths was claimed in [22] already within the Standard Model. This result was however clearly overestimated as discussed in [16,18].

in the interval 55 MeV $\leq T_c^* \leq 90$ MeV [25], leads to

$$\delta\Gamma = \frac{\Gamma(K^+ \to \pi^+\pi^0\gamma)\ -\ \Gamma(K^- \to \pi^-\pi^0\gamma)}{\Gamma(K^+ \to \pi^+\pi^0\gamma)\ +\ \Gamma(K^- \to \pi^-\pi^0\gamma)} \leq 3\times 10^{-3}\ \text{Re}\left(\frac{\epsilon'}{\epsilon}\right)^{\text{SUSY}}_{G}\ . \tag{36}$$

As pointed out in [22], we finally note that CPT invariance allows us to connect, at the first order in α_{em}, the charge asymmetry of the total widths in $K^\pm \to \pi^\pm\pi^0\gamma$ to the one in $K^\pm \to \pi^\pm\pi^0$. The relation is given by

$$\begin{aligned} \frac{\Gamma(K^+ \to \pi^+\pi^0)\ -\ \Gamma(K^- \to \pi^-\pi^0)}{\Gamma(K^+ \to \pi^+\pi^0)\ +\ \Gamma(K^- \to \pi^-\pi^0)} &= -\frac{B(K^+ \to \pi^+\pi^0\gamma)}{B(K^+ \to \pi^+\pi^0)}\,\delta\Gamma \\ &\simeq -1.3\times 10^{-3}\,\delta\Gamma\ . \end{aligned} \tag{37}$$

that, through (36), leads to an asymmetry of $\mathcal{O}(10^{-9})$ for the non–radiative process.

6 Conclusions

The unexpectedly large values of Re(ϵ'/ϵ) recently put forward by the KTeV and the NA48 collaborations need a better theoretical understanding. The difference from most SM estimates could be explained either with unknown (but standard) non–perturbative effects or with New Physics. Since the theoretical improvements in the calculation of the non–perturbative effects may require a long time, it is worth looking for other observables that could confirm or exclude the New–Physics origin of the observed direct CP violation.

In this letter we have pointed out a strict correlation between the SUSY contributions to the chromomagnetic operator, affecting ϵ'/ϵ, and the magnetic $sd\gamma$ operator contributing to $K \to \pi\pi\gamma$ amplitudes. We have searched for direct–CP–violating observables in the latter processes which may get enhanced by a large coefficient in front of the magnetic–penguin operator.

First we have considered $K_{L,S} \to \pi^+\pi^-\gamma$ decays and concluded that the ratio $\epsilon'_{+-\gamma}/\epsilon$ is presumably enhanced over its SM value in the scenario where ϵ'/ϵ is dominated by gluino–mediated supersymmetric amplitudes. In particular for large photon energies $|\epsilon'_{+-\gamma}/\epsilon|$ could reach values of $\mathcal{O}(0.5\%)$. In the $K^\pm \to \pi^\pm\pi^0\gamma$ modes we have studied the charge asymmetry of the decay distributions. We have found that also this clean direct–CP–violating observable could be enhanced by supersymmetric effects, reaching values of $\mathcal{O}(10^{-4})$ in specific phase–space regions.

In both cases the results found imply that a more detailed experimental investigation of CP violation in $K \to \pi\pi\gamma$ decays is well worth the effort. Interestingly, this investigation could already be started with existing experimental facilities like KTeV, NA48 and KLOE. Finally, we stress that the major theoretical uncertainty in the present analysis comes from the ratio of hadronic matrix elements B_T/B_G. We hope that this quantity could be pinned down more precisely in the future with lattice–QCD calculations.

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