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Standard Model vs New Physics in Rare Kaon Decays*

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Abstract

We present a brief overview of rare K decays, emphasizing the different role of Standard Model and possible New Physics contributions in various channels.

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Being sensitive to flavour dynamics from few MeV up to several TeV, rare kaon decays provide a powerful tool to test the Standard Model (SM) and to search for New Physics (NP). In the following we shall outline some of the most interesting aspects of these decays, starting with the most rare ones, strongly sensitive to NP effects, moving toward processes which are more and more dominated by low-energy dynamics.

1 Lepton-flavour violating modes

Decays like $K_L \to \mu e$ and $K \to \pi\mu e$ are completely forbidden within the SM, where lepton-flavour is conserved, but are also absolutely negligible if we simply extend the model by including only Dirac-type neutrino masses. A positive evidence of any of these processes would therefore unambiguously signal NP, calling for non-minimal extensions of the SM. Moreover, as long as the final state contains at most one pion in addition to the lepton pair, the experimental information on the decay rate can be easily translated into a precise information on the short-distance amplitude $s \to d\mu e$. In this respect we stress that $K_L \to \mu e$ and $K \to \pi\mu e$ provide a complementary information: the first mode is sensitive to pseudoscalar and axial-vector $s \to d$ couplings, whereas the second one is sensitive to scalar, vector an tensor structures.

In exotic scenarios, like R-parity violating SUSY or models with leptoquarks, the $s \to d\mu e$ amplitude can be generated already at tree level. In this case naive power counting suggests that limits on $B(K_L \to \mu e)$ or $B(K \to \pi\mu e)$ at the level of 10^{-11} probe NP scales of the order of 100 TeV [1]. On the other hand, in more "conservative" scenarios where the $s \to d\mu e$ transition can occur only at the one-loop level, it is more appropriate saying that the scale probed is around the (still remarkable !) value of 1 TeV. An interesting example of the second type of scenarios is provided by left-right models with heavy Majorana neutrinos [2].

2 $K \to \pi\nu\bar{\nu}$

These decays are particularly fascinating since on one side, within the SM, their small but non negligible rates are calculable with high accuracy in terms of the less known Cabibbo-Kobayashi-Maskawa (CKM) angles [3]. On the other side, the flavour-changing neutral-current (FCNC) nature implies a strong sensitivity to possible NP contributions, even at very high energy scales.

Within the SM the $s \to d\nu\bar{\nu}$ amplitude is generated only at the quantum level, through Z–penguin and W–box diagrams. Separating the contributions to the amplitude according to the intermediate up-type quark running inside the loop, one can write

$$\mathcal{A}(s \to d\nu\bar{\nu}) = \sum_{q=u,c,t} V_{qs}^* V_{qd} \mathcal{A}_q \sim \begin{cases} \mathcal{O}(\lambda^5 m_t^2) + i\mathcal{O}(\lambda^5 m_t^2) & (q = t) \\ \mathcal{O}(\lambda m_c^2) \ + i\mathcal{O}(\lambda^5 m_c^2) & (q = c) \\ \mathcal{O}(\lambda \Lambda_{QCD}^2) & (q = u) \end{cases} \qquad (1)$$

where V_{ij} denote the elements of the CKM matrix. The hierarchy of these elements [4] would favor up- and charm-quark contributions, however the hard GIM mechanism of the parton-level calculation implies $\mathcal{A}_q \sim m_q^2/M_W^2$, leading to a completely different scenario. As shown on the r.h.s. of (1), where we have employed the standard phase convention ($\Im V_{us} = \Im V_{ud} = 0$) and expanded the CKM matrix in powers of the Cabibbo angle ($\lambda = 0.22$) [4], the top-quark contribution dominates both real and imaginary parts.[1] This structure implies several interesting consequences for $\mathcal{A}(s \to d\nu\bar{\nu})$: it is dominated by short-distance dynamics and therefore calculable with high precision in perturbation theory; it is very sensitive to V_{td}, which is one of the less constrained CKM matrix elements; it is likely to have a large CP-violating phase; it is very suppressed within the SM and thus very sensitive to possible NP effects.

The short-distance contributions to $\mathcal{A}(s \to d\nu\bar{\nu})$, within the SM, can be efficiently described by means of a single effective dimension-6 operator: $O_{LL}^{\nu} = (\bar{s}_L \gamma^\mu d_L)(\bar{\nu}_L \gamma_\mu \nu_L)$. The Wilson coefficient of this operator has been calculated by Buchalla and Buras including next-to-leading-order QCD corrections [5] (see also [6,7]), leading to a very precise description of the partonic amplitude. Moreover, the simple structure of O_{LL}^{ν} has two major advantages:

- the relation between partonic and hadronic amplitudes is quite accurate, since the hadronic matrix elements of the $\bar{s}\gamma^\mu d$ current between a kaon and a pion are related by isospin symmetry to those entering K_{l3} decays, which are experimentally well known;

- the lepton pair is produced in a state of definite CP and angular momentum, implying that the leading SM contribution to $K_L \to \pi^0 \nu\bar{\nu}$ is CP violating.

2.1 SM uncertainties

The dominant theoretical error in estimating $B(K^+ \to \pi^+ \nu\bar{\nu})$ is due to the uncertainty of the QCD corrections to the charm contribution (see [7] for an updated discussion), which can be translated into a 5% error in the determination of $|V_{td}|$ from $B(K^+ \to \pi^+ \nu\bar{\nu})$. This uncertainty can be considered as generated by 'intermediate-distance' dynamics;

[1] The Λ_{QCD}^2 factor in the last line of (1) follows from a naive estimate of long-distance effects.

genuine long-distance effects associated to the up quark have been shown to be substantially smaller [8].

The case of $K_L \to \pi^0\nu\bar{\nu}$ is even more clean from the theoretical point of view [9]. Indeed, because of the CP structure, only the imaginary parts in (1) -where the charm contribution is absolutely negligible- contribute to $\mathcal{A}(K_2 \to \pi^0\nu\bar{\nu})$. Thus the dominant direct-CP-violating component of $\mathcal{A}(K_L \to \pi^0\nu\bar{\nu})$ is completely saturated by the top contribution, where the QCD uncertainties are very small (around 1%). Intermediate and long-distance effects in this process are confined only to the indirect-CP-violating contribution [10] and to the CP-conserving one [11] which are both extremely small. Taking into account also the isospin-breaking corrections to the hadronic matrix element [12], one can therefore write a very accurate expression (with a theoretical error around 1%) for $B(K_L \to \pi^0\nu\bar{\nu})$ in terms of short-distance parameters [7,10]:

$$B(K_L \to \pi^0\nu\bar{\nu})_{\rm SM} \;=\; 4.25 \times 10^{-10} \left[\frac{\overline{m}_t(m_t)}{170 \text{ GeV}}\right]^{2.3} \left[\frac{\Im\lambda_t}{\lambda^5}\right]^2 . \tag{2}$$

The high accuracy of the theoretical predictions of $B(K^+ \to \pi^+\nu\bar{\nu})$ and $B(K_L \to \pi^0\nu\bar{\nu})$ in terms of the modulus and the imaginary part of $\lambda_t = V_{ts}^* V_{td}$ could clearly offer the possibility of very interesting tests of the CKM mechanism. Indeed, a measurement of both channels would provide two independent information on the unitarity triangle, which can be probed also by B-physics observables. In particular, as emphasized in [10], the ratio of the two branching ratios could be translated into a clean and complementary determination of $\sin(2\beta)$.

Taking into account all the indirect constraints on V_{ts} and V_{td} obtained within the SM, the present range of the SM predictions for the two branching ratios reads [7]:

$$B(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} \;=\; (0.82 \pm 0.32) \times 10^{-10} , \tag{3}$$

$$B(K_L \to \pi^0\nu\bar{\nu})_{\rm SM} \;=\; (3.1 \pm 1.3) \times 10^{-11} . \tag{4}$$

Moreover, As pointed out recently in [7], a stringent and theoretically clean upper bound on $B(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM}$ can be obtained using only the experimental information on $\Delta M_{B_d}/\Delta M_{B_s}$ to constraint $|V_{td}/V_{ts}|$. In particular, using $(\Delta M_{B_d}/\Delta M_{B_s})^{1/2} < 0.2$ it is found

$$B(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} < 1.67 \times 10^{-10} , \tag{5}$$

which represents a very interesting challenge for the BNL-E787 experiment [13].

2.2 Beyond the SM: general considerations

As far as we are interested only in $K \to \pi\nu\bar{\nu}$ decays, we can roughly distinguish the extensions of the SM into two big groups: those involving new sources of quark-flavour mixing (like generic SUSY extensions of the SM, models with new generations of quarks, etc. . .) and those where the quark mixing is still ruled by the CKM matrix (like the 2-Higgs-doublet model of type II, constrained SUSY models, etc. . .). In the second case NP contributions are typically smaller than SM ones at the amplitude level (see e.g. [14,15] for some recent discussions). On the other hand, in the first case it is possible to overcome the $\mathcal{O}(\lambda^5)$ suppression of the dominant SM amplitude. If this is the case, it is then easy to generate sizable enhancements of $K \to \pi\nu\bar{\nu}$ rates (see e.g. [16] and [17]).

Concerning $K_L \to \pi^0\nu\bar{\nu}$, it is worthwhile to emphasize that if lepton-flavor is not conserved [18,19] or right-handed neutrinos are involved [20], then new CP-conserving contributions could in principle arise.

Interestingly, despite the variety of NP models, it is possible to derive a model-independent relation among the widths of the three neutrino modes [18]. Indeed, the isospin structure of any $s \to d$ operator bilinear in the quark fields implies

$$\Gamma(K^+ \to \pi^+\nu\bar{\nu}) = \Gamma(K_L \to \pi^0\nu\bar{\nu}) + \Gamma(K_S \to \pi^0\nu\bar{\nu}) \,, \tag{6}$$

up to small isospin-breaking corrections, which then leads to

$$B(K_L \to \pi^0\nu\bar{\nu}) < \frac{\tau_{K_L}}{\tau_{K^+}} B(K^+ \to \pi^+\nu\bar{\nu}) \simeq 4.2 B(K^+ \to \pi^+\nu\bar{\nu}) \,. \tag{7}$$

Any experimental limit on $B(K_L \to \pi^0\nu\bar{\nu})$ below this bound can be translated into a non-trivial dynamical information on the structure of the $s \to d\nu\bar{\nu}$ amplitude.

2.3 SUSY contributions and the $Z\bar{s}d$ vertex

We will now discuss in more detail the possible modifications of $K \to \pi\nu\bar{\nu}$ decays in the framework of a generic low-energy supersymmetric extension of the SM, which represents a very attractive possibility from the theoretical point of view [21]. Similarly to the SM, also in this case FCNC amplitudes are generated only at the quantum level, provided we assume unbroken R parity and minimal particle content. However, in addition to the standard penguin and box diagrams, also their corresponding superpartners, generated by gaugino-squarks loops, play an important role. In particular, the chargino-up-squarks diagrams provide the potentially dominant non-SM effect to the $s \to d\nu\bar{\nu}$ amplitude [22]. Moreover, in the limit where the average mass of SUSY particles is substantially larger than M_W, the penguin diagrams tend to dominate over the box ones and the dominant SUSY effect can be encoded through an effective $Z\bar{s}d$ coupling [16,23].

The flavour structure of a generic SUSY model is quite complicated and a convenient model-independent parameterization of the various flavour-mixing terms is provided by the so-called mass-insertion approximation [25]. This consists of choosing a simple basis for the gauge interactions and, in that basis, to perform a perturbative expansion of the squark mass matrices around their diagonal. Employing a squark basis where all quark-squark-gaugino vertices involving down-type quarks are flavor diagonal, it is found that the potentially dominant SUSY contribution to the $Z\bar{s}d$ vertex arises from the double mixing $(\tilde{u}_L^d - \tilde{t}_R) \times (\tilde{t}_R - \tilde{u}_L^s)$ [16]. Indirect bounds on these mixing terms dictated by vacuum-stability, neutral-meson mixing and $b \to s\gamma$ leave open the possibility of large effects [16]. More stringent constraints can be obtained employing stronger theoretical assumptions on the flavour structure of the SUSY model [23]. However, the possibility of sizable modifications of $K \to \pi\nu\bar{\nu}$ widths (including enhancements of more than one order of magnitude in the case of $K_L \to \pi^0\nu\bar{\nu}$) cannot be excluded a priori.

Interestingly a non-standard $Z\bar{s}d$ vertex can be generated also in non-SUSY extensions of the SM (see e.g. [26]). It is therefore useful trying to constraint this scenario in a model-independent way. At present the best direct limits on the $Z\bar{s}d$ vertex are dictated by $K_L \to \mu^+\mu^-$ [18,16,24], bounding the real part of the coupling, and $\Re(\epsilon'/\epsilon)$ [24], constraining the imaginary one. Unfortunately in both cases the bounds are not very accurate, being affected by sizable hadronic uncertainties. Concerning ϵ'/ϵ, it is worthwhile to mention that the non-standard $Z\bar{s}d$ vertex could provide an explanation for the apparent discrepancy between $(\epsilon'/\epsilon)_{\rm exp}$ and $(\epsilon'/\epsilon)_{\rm SM}$ [23,27], even if it is certainly too early to make definite statement in this respect [28]. In the future the situation could become much more clear with precise determinations of both real and imaginary part of the $Z_{\bar{s}d}$ coupling by means of $\Gamma(K^+ \to \pi^+\nu\bar{\nu})$ and $\Gamma(K_L \to \pi^0\nu\bar{\nu})$. Note that if we only use the present constraints from $K_L \to \mu^+\mu^-$ and $\Re(\epsilon'/\epsilon)$, we cannot exclude enhancements up to one order of magnitude for $\Gamma(K_L \to \pi^0\nu\bar{\nu})$ and up to a factor ~ 3 for $\Gamma(K^+ \to \pi^+\nu\bar{\nu})$ [23,24].

3 $K \to \pi\ell^+\ell^-$ and $K \to \ell^+\ell^-$

Similarly to $K \to \pi\nu\bar{\nu}$ decays, the short-distance contributions to $K \to \pi\ell^+\ell^-$ and $K \to \ell^+\ell^-$ are calculable with high accuracy and are potentially sensitive to NP effects. However, in these processes the size of long-distance contributions is usually much larger due to the presence of electromagnetic interactions. Only in few cases (mainly in CP-violating observables) long-distance contributions are suppressed and it is possible to extract the interesting short-distance information.

3.1 $K \to \pi \ell^+ \ell^-$

The single-photon exchange amplitude, dominated by long-distance dynamics, provides the largest contribution to the CP-allowed transitions $K^+ \to \pi^+ \ell^+ \ell^-$ and $K_S \to \pi^0 \ell^+ \ell^-$. The former has been observed, both in the electron and in the muon mode, whereas only an upper bound of about 10^{-6} exists on $B(K_S \to \pi^0 e^+ e^-)$ [30]. This amplitude can be described in a model-independent way in terms of two form factors, $W_+(z)$ and $W_S(z)$, defined by [29]

$$i \int d^4x e^{iqx} \langle \pi(p) | T \left\{ J^\mu_{\rm elm}(x) \mathcal{L}_{\Delta S=1}(0) \right\} | K_i(k) \rangle = \frac{W_i(z)}{(4\pi)^2} \left[z(k+p)^\mu - (1-r_\pi^2) q^\mu \right] , \qquad (8)$$

where $q = k-p$, $z = q^2/M_K^2$ and $r_\pi = M_\pi/M_K$. The two form factors are non singular at $z = 0$ and, due to gauge invariance, vanish to lowest order in Chiral Perturbation Theory (CHPT) [31]. Beyond lowest order one can identify two separate contributions to the $W_i(z)$: a non-local term, $W_i^{\pi\pi}(z)$, due to the $K \to 3\pi \to \pi\gamma^*$ scattering, and a local term, $W_i^{\rm pol}(z)$, that encodes the contributions of unknown low-energy constants (to be determined by data) [29]. At $\mathcal{O}(p^4)$ the local term is simply a constant, whereas at $\mathcal{O}(p^6)$ also a term linear in z arises. We note, however, that already at $\mathcal{O}(p^4)$ chiral symmetry alone does not help to relate W_S and W_+, or K_S and K^+ decays [31].

Recent results on $K^+ \to \pi^+ e^+ e^-$ and $K^+ \to \pi^+ \mu^+ \mu^-$ by BNL-E865 [32] indicates very clearly that, due to a large linear slope, the $\mathcal{O}(p^4)$ expression of $W_+(z)$ is not sufficient to describe experimental data. This should not be consider as a failure of CHPT, rather as an indication that large $\mathcal{O}(p^6)$ contributions are present in this channel.[2] Indeed the $\mathcal{O}(p^6)$ expression of $W_+(z)$ seems to fit well data. Interestingly, this is not only due to a new free parameter appearing at $\mathcal{O}(p^6)$, but it is also due to the presence of the non-local term. The evidence of the latter provides a real significant test of the CHPT approach.

In the $K_L \to \pi^0 \ell^+ \ell^-$ decay the long-distance part of the single-photon exchange amplitude is forbidden by CP invariance but it contributes to the processes via K_L-K_S mixing, leading to

$$B(K_L \to \pi^0 e^+ e^-)_{\rm CPV-ind} \;=\; 3 \times 10^{-3}\; B(K_S \to \pi^0 e^+ e^-) \; . \qquad (9)$$

On the other hand, the direct-CP-violating part of the decay amplitude is very similar to the one of $K_L \to \pi^0 \nu \bar{\nu}$ but for the fact that it receives an additional short-distance

[2] This should not surprise since in this mode sizable next-to-leading order contributions could arise due to vector-meson exchange.

contribution due to the photon penguin. Within the SM, this theoretically clean part of the amplitude leads to [33]

$$B(K_L \to \pi^0 e^+ e^-)^{\rm SM}_{\rm CPV-dir} \;=\; 0.67 \times 10^{-10} \; \left[\frac{\overline{m}_t(m_t)}{170~{\rm GeV}} \right]^2 \; \left[\frac{\Im \lambda_t}{\lambda^5} \right]^2 , \tag{10}$$

and, similarly to the case of $B(K_L \to \pi^0 \nu\bar\nu)$, it could be substantially enhanced by SUSY contributions [16,23]. The two CP-violating components of the $K_L \to \pi^0 e^+ e^-$ amplitude will in general interfere. Given the present uncertainty on $B(K_S \to \pi^0 e^+ e^-)$, at the moment we can only set the rough upper limit

$$B(K_L \to \pi^0 e^+ e^-)^{\rm SM}_{\rm CPV-tot} \;\lesssim\; {\rm few} \times 10^{-11} \tag{11}$$

on the sum of all the CP-violating contributions to this mode [29]. We stress, however, that the phases of the two CP-violating amplitudes are well know. Thus if $B(K_S \to \pi^0 e^+ e^-)$ will be measured, it will be possible to determine the interference between direct and indirect CP-violating components of $B(K_L \to \pi^0 e^+ e^-)_{\rm CPV}$ up to a sign ambiguity. Finally, it is worth to note that an evidence of $B(K_L \to \pi^0 e^+ e^-)_{\rm CPV}$ above the 10^{-10} level, possible within specific supersymmetric scenarios [23], would be a clear signal of physics beyond the SM.

An additional contribution to $K_L \to \pi^0 \ell^+ \ell^-$ decays is generated by the CP-conserving processes $K_L \to \pi^0 \gamma\gamma \to \pi^0 \ell^+ \ell^-$ [34]. This however does not interfere with the CP-violating amplitude and, as we shall discuss in the next section, it is quite small ($\lesssim\; 4 \times 10^{-12}$) in the case of $K_L \to \pi^0 e^+ e^-$.

3.2 $K_L \to l^+ l^-$

The two-photon intermediate state plays an important role in $K_L \to \ell^+ \ell^-$ transitions. This is by far the dominant contribution in $K_L \to e^+ e^-$, where the dispersive integral of the $K_L \to \gamma\gamma \to l^+ l^-$ loop is dominated by the term proportional to $\log(m_K^2/m_e^2)$. The presence of this large logarithm implies also that $\Gamma(K_L \to e^+ e^-)$ can be estimated with a relatively good accuracy in terms of $\Gamma(K_L \to \gamma\gamma)$, yielding to the prediction $B(K_L \to e^+ e^-) \sim 9 \times 10^{-12}$ [35] which recently seems to have been confirmed by the four events observed at BNL-E871 [36].

More interesting from the short-distance point of view is the case of $K_L \to \mu^+ \mu^-$. Here the two-photon long-distance amplitude is not enhanced by large logs and it is almost comparable in size with the short-distance one, sensitive to $\Re V_{td}$ [5]. Unfortunately the dispersive part of the two-photon contribution is much more difficult to be estimated in this case, due to the stronger sensitivity to the $K_L \to \gamma^* \gamma^*$ form factor. Despite the precise experimental determination of $B(K_L \to \mu^+\mu^-)$, the present constraints on $\Re V_{td}$ from this observable are not very stringent [37]. Nonetheless, the measurement of $B(K_L \to \mu^+\mu^-)$ is still useful to put significant bounds on possible NP contributions. Moreover, we stress that the uncertainty of the $K_L \to \gamma^*\gamma^* \to \mu^+\mu^-$ amplitude could be partially decreased in the future by precise experimental information on the form factors of $K_L \to \gamma\ell^+\ell^-$ and $K_L \to e^+e^-\mu^+\mu^-$ decays, especially if these would be consistent with the general parameterization of the $K_L \to \gamma^*\gamma^*$ vertex proposed in [37].

4 Two-photon processes

$K \to \pi\gamma\gamma$ and $K \to \gamma\gamma$ decays are completely dominated by long-distance dynamics and therefore not particularly useful to search for NP. However, these modes are interesting on one side to perform precision tests of CHPT, on the other side to estimate long-distance corrections to the $\ell^+\ell^-$ channels (see e.g. [38] and references therein).

Among the CHPT tests, an important role is played by $K_S \to \gamma\gamma$. The first non-vanishing contribution to this process arises at $\mathcal{O}(p^4)$ and, being generated only by a finite loop amplitude, is completely determined [39]. Since in this channel vector meson exchange contributions are not allowed, and unitarity corrections are automatically included in the $\mathcal{O}(p^2)$ coupling [38], we expect that the $\mathcal{O}(p^4)$ result provides a good approximation to the full amplitude. This is confirmed by present data [30], but a more precise determination of the branching ratio is need in order to perform a more stringent test.

Similarly to the $K_S \to \gamma\gamma$ case, also the leading non-vanishing contribution to $K_L \to \pi^0\gamma\gamma$ arises only at $\mathcal{O}(p^4)$ and is completely determined [40]. However, in this case large $\mathcal{O}(p^6)$ corrections can be expected due to both unitarity corrections and vector meson exchange contributions. Indeed the $\mathcal{O}(p^4)$ prediction for $B(K_L \to \pi^0\gamma\gamma)$ turns out to be substantially smaller (more than a factor 2) than the experimental findings [38]. After the inclusion of unitarity corrections and vector meson exchange contributions, both spectrum and branching ratio of this decay can be expressed in terms of a single unknown coupling: a_V [41]. The recent KTeV measurement [42] has shown that the determination of a_V from both spectrum and branching ratio of $K_L \to \pi^0\gamma\gamma$ leads to the same value, $a_V = -0.72 \pm 0.08$, providing an important consistency check of this approach.

As anticipated, the $K_L \to \pi^0\gamma\gamma$ amplitude is also interesting since it produces a CP-conserving contribution to $K_L \to \pi^0\ell^+\ell^-$ [41]. For $\ell = e$ the leading $O(p^4)$ contribution is helicity suppressed and only the $O(p^6)$ amplitude with the two photons in $J = 2$ leads to a non-vanishing $B(K_L \to \pi^0e^+e^-)_{\rm CPC}$ [34]. Given the recent experimental result [42], this should not exceed 4×10^{-12} [41]. Moreover, we stress that the Dalitz plot distribution of CP-conserving and CP-violating contributions to $K_L \to \pi^0e^+e^-$ are substantially different: in the first case the e^+e^- pair is in a state of $J=1$, whereas in the latter has $J=2$. Thus in principle it is possible to extract the interesting $B(K_L \to \pi^0 e^+ e^-)_{\rm CPV}$ from a Dalitz plot analysis of the decay. On the other hand, the CP-conserving contribution is enhanced and more difficult to be subtracted in the case of $K_L \to \pi^0 \mu^+ \mu^-$, where the helicity suppression of the leading $O(p^4)$ contribution (photons in $J=0$) is much less effective (see Heiliger and Sehgal in [41]).

5 Conclusions

Rare K decays provide a unique opportunity to perform high precision tests of CP violation and flavour mixing, both within and beyond the SM.

A special role is undoubtedly played by $K \to \pi \nu \bar{\nu}$ decays. In some NP scenarios sizable enhancements to the branching ratios of these modes are possible and, if detected, these would provide the first evidence for physics beyond the SM. Nevertheless, even in absence of such enhancements, precise measurements of $K \to \pi \nu \bar{\nu}$ widths will lead to unique information about the flavour structure of any extension of the SM.

Among decays into a $\ell^+\ell^-$ pair, the most interesting one from the short-distance point of view is probably $K_L \to \pi^0 e^+ e^-$. However, in order to extract precise information from this mode an experimental determination (or a stringent upper bound) on $B(K_S \to \pi^0 e^+ e^-)$ is also necessary.

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