



LABORATORI NAZIONALI DI FRASCATI
SIS-Pubblicazioni

LNF-99/018(p)
13 Luglio 1999

TRANSITION RADIATION IN THE PRE-WAVE ZONE

V.A.Verzilov

INFN, Laboratori Nazionali di Frascati, P.O. Box 13, I-00044 Frascati, Italy

Abstract

The wave zone condition for TR produced on the boundary between a metal and vacuum is examined. It is shown that in both forward and backward directions the wave zone sets in at the same distance, that is of the order of the formation length for forward TR. The features of backward TR in the pre-wave zone are considered.

PACS:41.60.-m; 41.75.Ht

Submitted to Physics Letters A

It is a widely accepted point of view that transition radiation (TR) in the forward direction is formed over the so-called *formation length*, that is the distance needed for the particle Coulomb field and radiation field to separate due to the difference in their velocities of propagation [1]. In the relativistic regime the formation length exceeds the radiation wavelength in orders of magnitude. For TR produced on the boundary between a metal and vacuum in the optical range, which is only the object under consideration in this paper, the formation length is roughly $\lambda\gamma^2$, where λ is a wavelength and γ is the relativistic factor. In the case of backward TR the particle and radiation fields overlap only within a wavelength from the target surface.

Sometimes this argument is exploited to conclude that *wave zone* (or far-field) conditions are different for forward and backward TR [2]. Meanwhile, it is actually not true in regards to the radiation itself. Both backward and forward TR evolve, before reaching the wave zone, over the same distance determined by the transverse dimension of the electromagnetic field of a moving particle. In fact, while the particle itself can certainly be considered a point, its Coulomb field Fourier-component, involved in the radiation at the wavelength λ , occupies a finite space, outer border of which scales in the transverse (to the particle trajectory) plane roughly as $\lambda\gamma$.

Since, eventually, the source of TR is the atomic electron currents induced on the metallic surface by the incident particle Coulomb field, its size is that of the field. The last statement is, however, valid for the infinite boundary. For a target of the finite size this does not hold if the particle field exceeds the transverse dimension of the target. Then, strong variations in the radiation properties are expected [3] at wavelengths for which the parameter $\lambda\gamma$ is larger than the target radius.

Nevertheless, as long as the target size has no effect on the radiation, one can consider a portion of the target surface with an extension of the order of $\lambda\gamma$ around the particle trajectory as the TR source. Though rather artificial for the first view, the concept of the source for TR was found very useful for making clear the physical arguments determining the wave zone condition.

The term "wave zone" is preferred in this paper to the widespread "far-field" notation mainly to avoid use of the "near-field" one, which is typically addressed to the domain of a close vicinity (within a wavelength) to the source, where quasistatical, i.e. of non-radiation nature, phenomena may occur. Instead, the term "pre-wave zone" seemed more appropriate for our purpose, that was to give an outline on the problem of TR in the pre-wave zone with an emphasis on its physical side.

We shall start from simple phase relations and consider an extended coherent source of a radiation. Let O and S be two points on the source surface separated by a distance ρ (Fig. 1). Generally, waves emitted by these points at the same phase arrive at an arbitrary

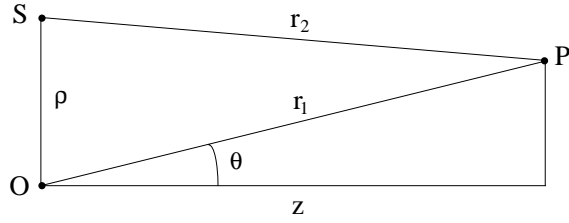


Figure 1: Waves emitted by two different points O and S of the source arrive at any arbitrary point P with a phase difference $\Delta\varphi = k(r_2 - r_1)$.

observation point P with a phase difference $\Delta\varphi = k(r_2 - r_1)$, where $k = 2\pi/\lambda$ is the wave vector. Assuming the observation point to be reasonably far from the source, so that $r_1, r_2 \gg \rho$, it becomes

$$\Delta\varphi \approx k \left(\frac{\rho^2}{2r_1} - \frac{\boldsymbol{\rho} \cdot \mathbf{r}_1}{r_1} \right). \quad (1)$$

The second term in Eq. (1) does not depend on r_1 and, hence, defines the phase relations in the wave zone. Then, the first one gives a first order (with respect to the parameter ρ/r_1) correction to the wave zone phase map and its contribution increases as the point P moves towards the source. For any source points between O and S the radiation is in the wave zone at P if this term is small enough

$$k \frac{\rho^2}{2r_1} \ll \pi. \quad (2)$$

Waves emitted by points outside the region of a radius ρ , given by Eq. (2), exhibit more or less evident interference, distorting the wave zone behaviour. Thus, for the radiation of the whole source to be in the wave zone, Eq. (2) must be fulfilled for all the points of the source. The obvious conclusion is the larger the dimension of the source the farther the wave zone from it. The distance from the source to the wave zone boundary grows quadratically with the source size.

In the case of TR the size of the source is of the order of $\lambda\gamma$. The characteristic angle of emission is normally small $\theta \sim 1/\gamma$ and so $r_1 \approx z$. As a result, the wave zone condition for TR is written as follows

$$z \gg \lambda\gamma^2. \quad (3)$$

One of the properties of the wave zone is that the source is seen as a quasipoint one from there. This is necessary to represent the radiation field by a simple spherical wave. Thus, in the wave zone one must require a smallness of the parameter $\lambda\gamma$ with respect to the transverse dimension of the TR spot, that is $z\theta \sim z/\gamma$, and we arrive again at Eq. (3).

However, it should be noted that at any z the wave zone condition breaks down for the small range of angles

$$\theta \leq \frac{\lambda\gamma}{z}. \quad (4)$$

Within this range the size of the TR source plays a role.

Given arguments are equally applicable to the forward and backward cases. This means that if one "switched off" the Coulomb field of a particle once it had gone out of the target material, no difference would be observed in the TR field evolution in the both forward and backward directions. To avoid complications associated with the contribution from the Coulomb field, from this moment on, we limit ourselves only to the case of backward TR.

So, touching upon the mathematical aspect of the problem, let us consider backward TR emerging when a normally incident particle with a charge q and a velocity $v \rightarrow c$ hits a perfectly conducting infinite boundary $z = 0$. In this case only transverse components of the particle field are essential, which are [1]:

$$E_{x,y}^q(z, \varkappa, \omega) = -\frac{4\pi i q}{v} \frac{\varkappa_{x,y}}{\varkappa^2 + \alpha^2} e^{-iz\omega/v}, \quad (5)$$

where $\alpha = \omega/v\gamma$ and \varkappa is the transverse component of the wave vector. The radiation field is obtained by satisfying the boundary condition for tangential components

$$E_{x,y}^q + E_{x,y}^r = 0. \quad (6)$$

In the spatial representation:

$$E_{x,y}^r(z, \rho, \omega) = -\frac{2q}{v} n_{x,y} \int_0^\infty \frac{\varkappa^2 d\varkappa}{\varkappa^2 + \alpha^2} J_1(\varkappa\rho) e^{iz\sqrt{k^2 - \varkappa^2}}, \quad (7)$$

where \mathbf{n} is the unit vector lying in the x,y - plane and directed from the z -axes to the observation point and J_1 is the Bessel function of the first kind. The spatial-spectral distribution of TR, that is the radiation power per unit of the frequency and per unit of the transversal area valid at any distance (except, perhaps, very short ones) can be written in the form

$$\frac{d^2W}{d\omega d\mathbf{u}} = \frac{q^2}{\pi^2 c} |\Phi(u, w, \gamma)|^2, \quad (8)$$

where dimensionless variables $u = k\rho$ and $w = kz$ are used and

$$\Phi(u, w, \gamma) = \int_0^\infty \frac{t^2 dt}{t^2 + \gamma^{-2}} J_1(ut) e^{iw\sqrt{1-t^2}}. \quad (9)$$

It is very convenient and even useful to consider the integral in Eq. (9) in the context of the stationary phase method. According to this method, at large distances $w \gg 1$, it can be approximated by the contribution from the close vicinity of a single point where the derivative of the phase function (with respect to t) in the integral vanishes. The phase of the Bessel function must be included in the phase function at that. The size of the domain around this so-called "stationary" point t_s , giving the dominant contribution to the integral, is of the order of $1/\sqrt{w}$. Evaluation of the integral essentially depends on whether this quantity is much smaller compared to $1/\gamma$, which is a measure of the variation rate of the fractional term. If this holds true the fraction may be approximated by its value at t_s and the rest integral is readily evaluated resulting in the classical wave zone expression. It is easy to see that the condition $1/\sqrt{w} \ll 1/\gamma$, is fully equivalent to Eq. (3).

At shorter distances $w \leq \gamma^2$, i.e. in the pre-wave zone, the situation is different because the poles $t = \pm i/\gamma$ of the fraction turn out to interfere with the contribution from the stationary point. Therefore, a proper account of these singularities should be taken in the complex plane. Furthermore, the poles are close to the integration domain boundary. In terms of the stationary phase method this case is classified as three close critical points in the two-fold integral problem [4], consideration of which anyway goes beyond the scope of this paper.

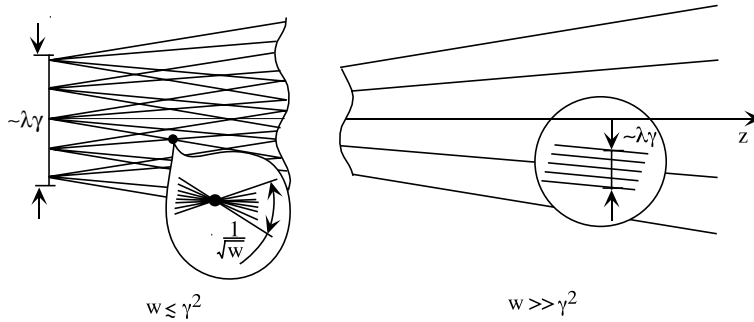


Figure 2: The waves propagating in different directions in the pre-wave zone strongly overlap and interfere. In the wave zone they are well separated similar to those from the point source.

Though the stationary phase method does not provide an easy way to evaluate the problem it allows a very clear geometrical interpretation (Fig. 2). At first we note that the integration variable is actually an angular variable $t = \sin \theta \approx \theta$ referring to an angle which rays (or wave vectors) make with the z - axes. The fact, that the value of the integral of Eq. (9) is essentially determined by the contribution of the stationary point and its vicinity, can be interpreted in a way that the field at an observation point $P(u,w)$ is created by a bundle of rays with an angular spread $\Delta\theta \sim 1/\sqrt{w}$ passing through this

point. In the pre-wave zone $\Delta\theta$ is large and waves propagating in different directions are strongly overlap and interfere. On the contrary, in the wave zone $\Delta\theta \ll 1/\gamma$ and the rays are well separated similar to those from a point source, though in a given direction there is not a single ray but a bundle of parallel ones with a transverse dimension $\sim \lambda\gamma$, small compared to the TR spot $\sim z/\gamma$.

Figure 2 also aids to clarify the angular features of TR in the pre-wave zone. First of all we draw attention to the point that two definitions may be formally applied to the radiation angular distribution. These are the energy emitted in the given direction and the energy emitted in the given cone (solid angle). Being equivalent in the wave zone, they differ in the pre-wave one. This fact is crucial for understanding though it can be a source of confusion. The classical Frank formula

$$\frac{d^2W}{d\omega d\Omega} = \frac{q^2}{\pi^2 c} \frac{\beta \sin^2 \theta}{(1 - \beta^2 \cos^2 \theta)}, \quad (10)$$

$\beta = v/c$, gives the energy emitted in a fixed direction. It can be obtained from Eq. (8) by integrating over the variable u across any infinite transverse plane. Thus, the TR energy flux in a given direction proves to be invariant at any distance from the source. But as a consequence of such the integration, the information about the spatial distribution of the radiation turns out to be lost. In other words, from only Eq. (10) it is impossible to find out how TR evolves in the space within the pre-wave zone.

The energy emitted in a given cone is obtained from Eq. (8) upon integrating over t and introducing corresponding angular variables instead of spatial ones. The angular distribution derived in this way keeps all the spatial information but, as shown below, it varies with the distance from the source approaching Eq. (10) in the wave zone.

A direct analytical evaluation of Eq. (9) seems to be a complex problem. Alternatively to the evaluation of the integral, the following approximate procedure can be suggested. First of all we note that, in spite of the infinite upper limit of integration, the dominant contribution to the integral is given by the range of t from 0 to a few times of $1/\gamma \ll 1$ and the function Φ can be written as

$$\Phi \approx e^{i\omega} \int_0^\infty \frac{t^2 dt}{t^2 + \gamma^{-2}} J_1(ut) e^{i\omega t^2/2}. \quad (11)$$

Then we expand the fraction as the Taylor series around a point $t = k/\gamma$, were k is a positive real number.

$$\frac{t^2}{t^2 + \gamma^{-2}} \approx \frac{k^2}{1 + k^2} + \sum_{n=1}^{\infty} \sum_{p=0}^n A_n^p t^{n-p}, \quad (12)$$

$$A_n^p = \sum_{m=0}^{n-1} \frac{(-1)^{n+m+p-1} \binom{n}{p} \binom{n+1}{n-2m} k^{n+p-2m} \gamma^{n-p}}{(1+k^2)^{n+1}}. \quad (13)$$

In Eq. (13) the symbol $\binom{n}{m}$ stands for the binomial coefficients.

After the substitution of Eq. (12) into Eq. (11), the remaining integral can be evaluated in terms of the hypergeometric function ${}_1F_1$ resulting in the following expression for Φ :

$$\Phi \approx e^{iw} \left\{ \frac{k^2}{1+k^2} \frac{1 - e^{iu^2/2w}}{u} + u \sum_{n=1}^{\infty} \sum_{p=0}^n B_n^p \frac{{}_1F_1 \left(\frac{n-p}{2} + 1; 2; i \frac{u^2}{2w} \right)}{(iw)^{(n-p)/2+1}} \right\}, \quad (14)$$

$$B_n^p = A_n^p 2^{(n-p)/2-1} \Gamma \left(\frac{n-p}{2} + 1 \right). \quad (15)$$

In a practical realization of the above algorithm a due attention should be paid to the proper choice of the k value and the number of terms to retain in the sum over n .

The following results of numerical calculations do illustrate the properties of backward TR in the pre-wave zone. Fig. 3 shows the energy flux in the unit solid angle at different distances from the source versus an angular variable $x = u/w\gamma$. In the relativistic regime $x \approx \theta\gamma$. Since the distance is given in units of γ^2 , the results are independent of the particle energy. The strong change in the form of the curves is clearly observed

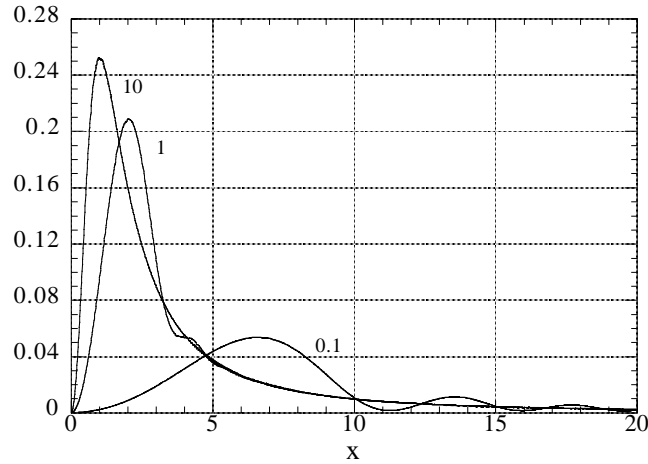


Figure 3: TR energy flux in an unit solid angle $\frac{\pi^2 c}{q^2} \frac{d^2 W}{d\omega dx}$. Numbers by the curves are distances w from the source in units of γ^2 .

with the variation of either the distance from the source or the wavelength. Distributions

become wider at shorter distances and longer wavelengths revealing a clear interference structure.

The well-known wave zone property of the TR angular distribution is that it peaks at an angle $x = \gamma$. This is no longer the case in the pre-wave zone if one considers the energy emitted in a cone. In Fig. 4 the peak position x_{peak} with respect to the center $x = 0$ is given as a function of the distance from the source.

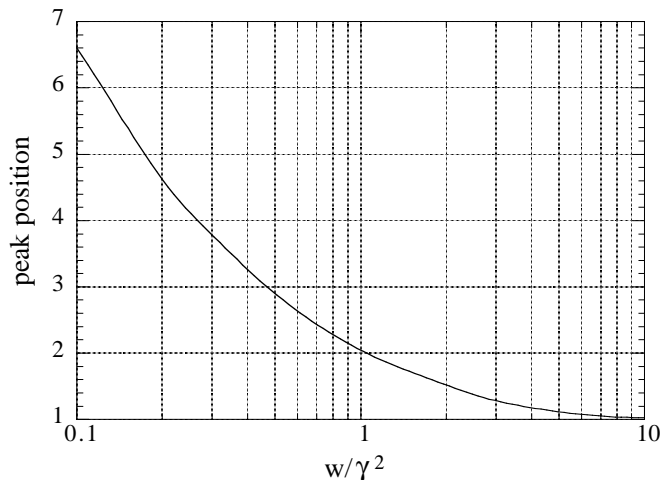


Figure 4: Peak position versus the parameter w/γ^2 .

The spectrum of TR in the wave zone does not depend on any parameters: the wavelength, angle of emission, particle energy. In the pre-wave zone the spectrum varies from point to point in space being a complex function of many factors. Fig. 5 shows spectra of TR calculated for "detectors" of different apertures. All the spectra exhibit a reduction in the intensity for long wavelengths. The effect is weaker for larger detector apertures and becomes clearly marked for higher particle energies. The dependence on the aperture size indicates that low frequencies are not fully lost. A redistribution of the frequency contents, as a result of interference, rather takes place; the central part of the radiation fan is depleted of low frequencies.

In the conclusion we note that the TR properties in the pre-wave zone are very different from those in the wave zone. The wave zone sets in at quite macroscopic distances moving away from the TR source linearly with the wavelength and quadratically with the particle energy. Therefore, the pre-wave zone effects may be of practical importance at long wavelengths, e.g. in experiments on far-infrared coherent TR [5,6], or in measurements at high particle energies [7].

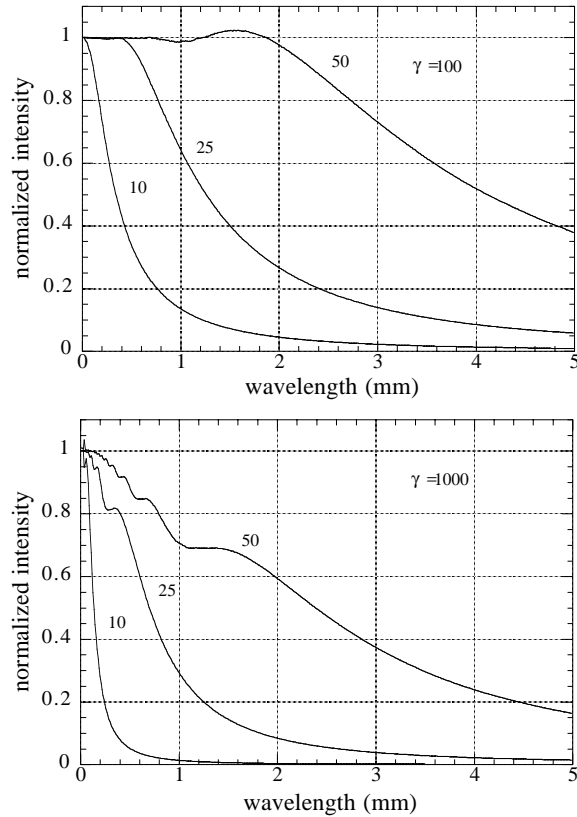


Figure 5: Normalized pre-wave zone TR spectra a distance of 1 m from the source integrated over the "detector" apertures given in millimeters next to the curves.

References

- [1] V.L.Ginzburg and V.N.Tsytoovich, *Transition radiation and transition scattering* (Adam Higler, Bristol, 1990).
- [2] X.Artru, R.Chehab, K.Honkavaara, A. Variola, Nucl.Instrum. and Meth. in Phys. Res. B145 (1998) 160.
- [3] N.F.Shul'ga and S.N.Dobrovol'skii, Pis'ma Zh. Eksp. Teor. Fiz. 65 (1997) 581 [JETP Lett. 65 (1997) 611]. N.F.Shul'ga, S.N.Dobrovol'skii and V.G.Syshchenko, Nucl.Instr. and Meth. in Phys. Res. B 145 (1998) 180.
- [4] V.A.Borovikov, *Uniform stationary phase method*, IEE Electromagnetic Waves Series 40, (The Institution of Electrical Engineers, London, 1994).
- [5] Y.Shibata, K.Ishi, T.Takahashi et al., Phys. Rev. A 44 (1991) R3449. Y.Shibata, K.Ishi, T.Takahashi et al., Phys. Rev. E 49 (1991) 785.

- [6] U.Happek, A.J.Sievers and E.B.Blum, Phys. Rev. Lett. 67 (1991) 2962.
- [7] P.Catras, W.P.Leemans, E.Esarey et al., *Beam profile measurement at 30 GeV using optical transition radiation*, to appear in Proc. of 1999 Particle Accelerator Conference, New York, USA, March-April 1999.