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Partial breaking of $N = 1,\ D = 10$ supersymmetry

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Abstract

We describe the spontaneous partial breaking of $N = 1,\ D = 10$ supersymmetry to $N = (1, 0),\ d = 6$ and its dimensionally-reduced versions in the framework of nonlinear realizations. The basic Goldstone superfield is $N = (1, 0),\ d = 6$ hypermultiplet superfield satisfying a nonlinear generalization of the standard hypermultiplet constraint. We interpret the generalized constraint as the manifestly worldvolume supersymmetric form of equations of motion of the type I super 5-brane in $D = 10$. The related issues we address are a possible existence of brane extension of off-shell hypermultiplet actions, the possibility to utilize vector $N = (1, 0),\ d = 6$ supermultiplet as the Goldstone one, and the description of $1/4$ breaking of $N = 1,\ D = 11$ supersymmetry.

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1. Introduction. The description of partial breaking of global supersymmetries (PBGS) within the coset approach [1] - [3] received much attention [4] - [13]. Its characteristic feature is that the Goldstone fermionic fields associated with the broken supertranslation generators [14] come out as components of Goldstone multiplets of unbroken SUSY.

The study of different patterns of PBGS in refs. [4] - [13] revealed a few peculiarities of such theories. As applied to the most elaborated case of the $1/2$ partial breaking of $N = 2,\ D = 4$ SUSY, these are as follows.

- There are several inequivalent $N = 1$ Goldstone supermultiplets related with the partial breaking $N = 2 \rightarrow N = 1$: chiral [9], vector [10] and tensor ones [11,13]. These options correspond to different theories.

- The $N = 1$ superfield Goldstone actions can be treated as gauge-fixed, manifestly worldvolume supersymmetric forms of the actions of some BPS superbranes, along the line of refs. [5,6]. The $N = 1$ chiral Goldstone superfield action is recognized as that of the Type I super 3-brane in a flat $D = 6$ background [6]. The $N = 1$ vector Goldstone multiplet action describes a super D3-brane and yields the Born-Infeld (BI) action for the gauge field. In all cases the no-go theorem of [15] is evaded by the general argument of [5].

- In accord with the general features of nonlinear realizations, one can make different $N = 1$ matter actions $N = 2$ supersymmetric by coupling them to Goldstone superfields.

The actions presented in [9] - [11] are nonlinear, "brane" generalizations of familiar off-shell $N = 1$ superfield actions. On the other hand, theories with *linearly* realized $N = 2,\ d = 4$ SUSY admit a good off-shell description, e.g. in harmonic $N = 2$ superspace [16]. It is natural to ask whether some of them can be promoted to those with a nonlinearly realized higher SUSY, say $N = 4$ SUSY, by constructing the formalism of partial breaking of this higher SUSY down to $N = 2$ and identifying some $N = 2$ superfields as the Goldstone ones accompanying this breakdown. Related questions are as to what kind of superbranes could be associated with such theories, whether a brane generalization of the harmonic analyticity [16] underlying ordinary $N = 2$ theories exists, how many different Goldstone $N = 2$ superfields are possible, etc.

In this letter we partly answer these questions. We show that the partial breaking of $N = 1,\ D = 10$ SUSY (amounting to properly central-charge extended $N = 4$ SUSY in $d = 4$ or $N = (1,1)$ SUSY in $d = 6$) down to $N = (1,0),\ d = 6$ SUSY picks out $d = 6$ hypermultiplet as the basic Goldstone superfield. Using the coset space techniques, we find a covariant nonlinear generalization of the standard hypermultiplet constraint in

$N = (1,0),\ d = 6$ superspace [17]. We argue that the generalized constraint encodes a gauge-fixed form of the equations of motion of the super 5-brane in $D = 10$ with manifest worldvolume $N = (1,0),\ d = 6$ SUSY. We give an evidence for the existence of brane extensions of the harmonic analyticity and off-shell hypermultiplet actions. Our relations admit the dimensional reduction by the worldvolume bosonic dimension up to the extreme $N = 8,\ d = 1$ case corresponding to a superparticle in $D = 5$. We elaborate on this simple case in more detail. Finally, we briefly discuss some related questions, in particular, a possibility to apply the PBGS approach to $N = 1,\ D = 11$ SUSY.

2. $N = 1,\ D = 10$ **Poincaré superalgebra in the** $d = 6$ **notation.** From the $d = 6$ viewpoint the $N = 1,\ D = 10$ SUSY algebra is a central-charge extended $N = (1,1)$ Poincaré superalgebra:

$$N = 1,\ D = 10 \quad SUSY \qquad \propto \quad \left\{ Q^i_\alpha, P_{\alpha\beta}, S^{\beta a}, Z^{ia} \right\} \ , \tag{1}$$

where

$$\alpha, \beta = 1, ..., 4 \ , \quad i = 1, 2 \ , \quad a = 1, 2$$

are, respectively, the $d = 6$ spinor ($Spin(1,5)$) indices and the doublet indices of two commuting automorphism $SU(2)$ groups realized on the spinor Q and S generators (see [18] - [20] for the $d = 6$ spinor notation). The basic anticommutation relations read

$$\left\{ Q^i_\alpha, Q^j_\beta \right\} = \epsilon^{ij} P_{\alpha\beta} \ , \quad \left\{ Q^i_\alpha, S^{a\beta} \right\} = \delta^\beta_\alpha Z^{ia} \ , \quad \left\{ S^{a\alpha}, S^{b\beta} \right\} = \epsilon^{ab} P^{\alpha\beta} \ . \tag{2}$$

The $d = 6$ translation generator $P_{\alpha\beta} = -P_{\beta\alpha} = \frac{1}{2}\epsilon_{\alpha\beta\rho\lambda} P^{\rho\lambda}$, together with the "semi-central charge" generator Z^{ia}, form the $D = 10$ translation generator.

To the set (1) one should add the generators of the $D = 10$ Lorentz group $SO(1,9)$

$$SO(1,9) \qquad \propto \quad \left\{ M_{\alpha\beta\ \gamma\delta},\ \ T^{ij},\ \ T^{ab},\ \ K^{\alpha\beta}_{ia} \right\} \ . \tag{3}$$

The generators M and T generate mutually commuting $d = 6$ Lorentz group $SO(1,5)$ and the automorphism (or R-symmetry) group $SO(4) \sim SU(2) \times SU(2)$, the generators K belong to the coset $SO(1,9)/SO(1,5) \times SO(4)$.

3. Coset space routine. We are going to construct a nonlinear realization of $N = 1,\ D = 10$ SUSY (together with the $D = 10$ Lorentz group), such that $N = (1,0),\ d = 6$ SUSY remains unbroken. Thus we choose the vacuum stability subgroup to be

$$H \quad \propto \quad \left\{ Q^i_\alpha, P_{\alpha\beta}, T^{ij}, T^{ab}, M_{\alpha\beta\ \gamma\delta} \right\} \ . \tag{4}$$

We put the generators $Q^i_\alpha, P_{\alpha\beta}$ into the coset and associate with them as the coset parameters the coordinates of $N = (1,0),\ d = 6$ superspace

$$Q^i_\alpha \Rightarrow \theta^\alpha_i \ , \quad P_{\alpha\beta} \Rightarrow x^{\alpha\beta} \ . \tag{5}$$

The remaining coset generators, $S^{\alpha a}, Z^{ia}, K^{ia}_{\alpha\beta}$, correspond to genuine spontaneously broken symmetries. The corresponding coset parameters are Goldstone superfields

$$S^{\alpha a} \Rightarrow \Psi_{\alpha a}(x,\theta)\ , \quad Z^{ia} \Rightarrow q_{ia}(x,\theta)\ , \quad K^{ia}_{\alpha\beta} \Rightarrow \Lambda^{\alpha\beta}_{ia}(x,\theta)\ . \tag{6}$$

An element g of the coset space $G/\tilde{H}$, where G is the full supergroup of $N=1,\ D=10$ SUSY (including $SO(1,9)$) and $\tilde{H} = SO(1,5)\times SO(4)$, reads

$$g = e^{x^{\alpha\beta}P_{\alpha\beta}} e^{\theta^\alpha_i Q^i_\alpha} e^{q_{ia}Z^{ia}} e^{\Psi_{a\alpha}S^{a\alpha}} e^{\Lambda^{\alpha\beta}_{ia}K^{ia}_{\alpha\beta}}\ . \tag{7}$$

Acting on (7) from the left by different elements of G with constant parameters, one determines the transformation properties of the coset parameters.

Unbroken supersymmetry ($g_0 = \exp\,(a^{\alpha\beta}P_{\alpha\beta} + \eta^\alpha_i Q^i_\alpha)$):

$$\delta x^{\alpha\beta} = a^{\alpha\beta} + \frac{1}{4}\left(\eta^{i\alpha}\theta^\beta_i - \eta^{i\beta}\theta^\alpha_i\right), \quad \delta\theta^\alpha_i = \eta^\alpha_i\ . \tag{8}$$

Broken supersymmetry ($g_0 = \exp\,(\eta_{a\alpha}S^{a\alpha})$):

$$\delta x^{\alpha\beta} = \frac{1}{4}\epsilon^{\alpha\beta\gamma\delta}\eta^a_\gamma\Psi_{a\delta}, \quad \delta q_{ia} = -\eta_{a\alpha}\theta^\alpha_i, \quad \delta\Psi_{a\alpha} = \eta_{a\alpha}\ . \tag{9}$$

Broken Z-translations ($g_0 = \exp(c_{ia}Z^{ia})$):

$$\delta q^{ia} = c^{ia}\ . \tag{10}$$

The form of broken K transformations is irrelevant for our consideration. The subgroup $\tilde{H}$ is realized as rotations of the $SO(1,5)$ spinor and $SU(2)$ doublet indices.

We see that $N=1,\ D=10$ supergroup as a whole admits a realization on the coordinates of $N=(1,0),\ d=6$ superspace and Goldstone superfields living on this superspace.

The next step is the construction of the left-covariant Cartan 1-forms:

$$g^{-1}dg = \Omega_Q + \Omega_P + \Omega_Z + \Omega_S + \Omega_K + \Omega_{\tilde{H}}\ , \tag{11}$$

where the subscripts denote the relevant generators. We shall actually need only the form Ω_Z

$$\Omega_Z \equiv \Omega^{ia}_Z\, Z_{ia} = \left[(\text{ch}\,\sqrt{\varphi})^{ia}_{jb}\, d\hat{q}^{jb} + \left(\frac{\text{sh}\,\sqrt{\varphi}}{\sqrt{\varphi}}\right)^{ia}_{jb} 2\Lambda^{jb\mu\nu} d\hat{x}_{\mu\nu}\right] Z_{ia}\ , \tag{12}$$

$$\begin{aligned} d\hat{x}^{\alpha\beta} &= dx^{\alpha\beta} - \frac{1}{4}\theta^{i\alpha}d\theta^\beta_i + \frac{1}{4}\theta^{i\beta}d\theta^\alpha_i - \frac{1}{4}\epsilon^{\alpha\beta\mu\nu}\Psi^a_\mu d\Psi_{a\nu}\ , \\ d\hat{q}_{ia} &= dq_{ia} + \Psi_{a\alpha}d\theta^\alpha_i\ , \quad \varphi^{ia}_{jb} \equiv 2\Lambda^{ia\mu\nu}\Lambda_{jb\mu\nu}\ . \end{aligned} \tag{13}$$

4. Inverse Higgs constraints and dynamical equation. By construction, the Cartan form (12) is covariant under all transformations of G realized as left shifts of g. The Goldstone superfields $\Lambda^{\alpha\beta}_{kb}$ and $\Psi_{\alpha a}$ appear inside it *linearly* and so can be covariantly eliminated by the inverse Higgs procedure [21]. This is achieved by imposing the manifestly covariant constraint

$$\Omega_Z = 0\ . \tag{14}$$

It amounts to the following set of equations

$$\widetilde{\Lambda}^{ia}_{\rho\sigma} \equiv -2\left(\frac{\text{th}\,\sqrt{\varphi}}{\sqrt{\varphi}}\right)^{ia}_{jb}\Lambda^{jb}_{\rho\sigma} = (E^{-1})^{\mu\nu}_{\rho\sigma}\,\partial_{\mu\nu}q^{ia} \equiv \nabla_{\rho\sigma}q^{ia}\ , \qquad \Psi_{a\beta} = \frac{1}{2}\,\nabla^k_\beta\, q_{ka}\ , \tag{15}$$

$$\nabla^{(i}_\beta\, q^{k)a} = 0\ . \tag{16}$$

Here

$$\nabla^k_\beta \equiv \mathcal{D}^k_\beta - \frac{1}{4}\,\epsilon^{\rho\lambda\alpha\gamma}(\Psi^b_\alpha \mathcal{D}^k_\beta \Psi_{b\gamma})\,\nabla_{\rho\lambda}\ , \tag{17}$$

$$E^{\rho\sigma}_{\mu\nu} \equiv \frac{1}{2}\left(\delta^\rho_\mu\delta^\sigma_\nu - \delta^\sigma_\mu\delta^\rho_\nu - \frac{1}{2}\epsilon^{\rho\sigma\alpha\beta}\Psi^b_\alpha\partial_{\mu\nu}\Psi_{b\beta}\right)\ , \tag{18}$$

$$\mathcal{D}^j_\beta = \frac{\partial}{\partial\theta^\beta_j} - \frac{1}{2}\,\theta^{j\alpha}\partial_{\alpha\beta}\ , \qquad \{\mathcal{D}^i_\alpha, \mathcal{D}^k_\beta\} = \epsilon^{ik}\partial_{\alpha\beta}\ . \tag{19}$$

It is easy to find the full nonlinear algebra of the covariant derivatives $\nabla^i_\alpha, \nabla_{\rho\beta}$. We explicitly give the anticommutator of spinor derivatives

$$\{\nabla^i_\alpha, \nabla^k_\beta\} \equiv -\nabla^{ik}_{\alpha\beta} = \left[\frac{1}{2}(\delta^\omega_\alpha\delta^\sigma_\beta - \delta^\sigma_\alpha\delta^\omega_\beta)\epsilon^{ki} + \epsilon^{\omega\sigma\gamma\tau}(\nabla^i_\alpha\Psi^d_\gamma)(\nabla^k_\beta\Psi_{d\tau})\right]\nabla_{\omega\sigma}\ . \tag{20}$$

We observe that, besides expressing Goldstone superfields through the only basic one q^{ia}, eq. (14) imposes the nonlinear constraint (16) on this superfield. We recognize it as a nonlinear generalization of the well-known hypermultiplet constraint [17]

$$\mathcal{D}^{(i}_\beta\, q^{k)a} = 0\ . \tag{21}$$

The latter reduces the field content of $q^{ia}(x,\theta)$ to four bosonic and eight fermionic components

$$q^{ia}(x,\theta) \quad\Rightarrow\quad \phi^{ia}(x) + \theta^{\alpha i}\psi^a_\alpha(x) + x\text{-derivatives}\ , \tag{22}$$

and simultaneously puts these fields on shell

$$\Box\phi^{ia}(x) = 0\ , \quad \partial^{\alpha\beta}\psi^a_\beta = 0 \quad \left(\Box \equiv \partial^{\alpha\beta}\partial_{\alpha\beta} = \frac{1}{2}\,\epsilon^{\alpha\beta\mu\nu}\partial_{\alpha\beta}\partial_{\mu\nu}\right)\ . \tag{23}$$

Eq. (16) is expected to yield a nonlinear generalization of the $d = 6$ hypermultiplet irreducibility conditions and equations of motion. It follows from (20) that all superfields obtained by the successive action of ∇^i_α on $\Psi_{a\beta}$ are reduced to ordinary x-derivatives of q^{ia} and $\Psi_{a\beta}$, i.e. these two superfield projections indeed exhaust the irreducible fields content of $q^{ia}(x,\theta)$.

Inspecting how the spontaneously broken nonlinear (super)symmetries (9) - (10) are realized on the components of q^{ia} (at the linearized level), we find that $\phi^{ia}(x)$ and $\psi^a_\alpha(x)$ are just Goldstone fields associated with the broken Z-translations and S- supertranslations, while the Goldstone fields accompanying the spontaneous breakdown of the $SO(1,9)/SO(1,5)\times SO(4)$ transformations, $\partial_{\alpha\beta}\phi^{ia}(x)$, are recognized as the coefficients of the second-order θ monomials in the θ-expansion of $q^{ia}(x,\theta)$.

Thus the only essential Goldstone superfield supporting the partial spontaneous breaking of $N=1,\ D=10$ SUSY down to $N=(1,0),\ d=6$ within the nonlinear realization scheme is the hypermultiplet superfield $q^{ia}(x,\theta)$. It is subjected to the nonlinear dynamical constraint (16) and accommodates all the Goldstone fields associated with the spontaneously broken symmetry generators including those of the $D=10$ Lorentz coset $SO(1,9)/SO(1,5)\times SO(4)$.

Note that the Lorentz Goldstone superfield $\Lambda^{ia}_{\alpha\beta}$ algebraically enters also into the Cartan form $\Omega_S \equiv \Omega_{S\ \alpha b}S^{\alpha b}$, $\Omega_{S\ \alpha b} = d\Psi_{\alpha b} - 2\Lambda_{ib\gamma\alpha}d\theta^{i\gamma} + \ldots$. This could mean that there exists an alternative way to eliminate $\Lambda^{ia}_{\alpha\beta}$, that time in terms of spinor derivative of $\Psi_{\alpha b}$ by equating to zero the appropriate part of the covariant $d\theta$-projection of Ω_S. However, a careful analysis making use of the Maurer-Cartan equations shows that this part identically vanishes upon using the constraint (14) (or eqs. (15), (16)).

It is worth mentioning that the kinematical and dynamical parts of eq. (14) are separately covariant with respect to all hidden symmetries. In other words, eq. (16) is not implied by the formalism of nonlinear realizations, and should be regarded as a dynamical postulate. In the superembedding approach to superbranes [22,23] a similar postulate is known as “the geometro-dynamical principle” or “the basic constraint” (see [23] and references therein). An interplay between the superembedding and PBGS approaches is discussed, e.g., in [12].

To see which kind of dynamics is hidden in (16), we considered it in the bosonic limit up to the first non-trivial order in fields, the third order. We found that it amounts to the following equation for $\phi^{ia}(x) \equiv q^{ia}(x,\theta)|_{\theta=0}$

$$\Box\phi^{ia} + \frac{1}{2}\ (\partial_{\mu\nu}\phi\cdot\partial_{\rho\lambda}\phi)\,\partial^{\rho\lambda}\partial^{\mu\nu}\phi^{ia} = 0\ , \tag{24}$$

where we omitted three-linear terms containing $\Box$ as they contribute to the next, 5th order, and used the notation $A\cdot B \equiv A^{ia}B_{ia}$. It is easy to see that eq. (24) corresponds to the ”static gauge” form of the bosonic 5-brane Nambu-Goto (NG) action with the induced metric

$$g_{\rho\lambda\ \mu\nu} = \frac{1}{2}\ \left(\epsilon_{\rho\lambda\mu\nu} - \partial_{\rho\lambda}\phi\cdot\partial_{\mu\nu}\phi\right) \equiv \frac{1}{2}\ \left(\epsilon_{\rho\lambda\mu\nu} - d_{\rho\lambda\,\mu\nu}\right)\ , \tag{25}$$

that is

$$\begin{aligned} S_{NG} &= \text{const}\int d^6x\ \left(\sqrt{-\text{det}\,g} - 1\right) \\ &\sim \int d^6x\left\{\text{Tr}\ d - \frac{1}{8}\left(\text{Tr}\ d\right)^2 + \frac{1}{4}\ \text{Tr}\ d^{\,2} + O(d^{\,3})\right\}. \end{aligned} \tag{26}$$

Though it remains to prove that the higher-order corrections are combined into this nice geometric form, the above consideration suggests that this is very likely (in sect. 6 we show this on the simplified $d = 1$ example). Then eq. (16) can be viewed as a manifestly $N = (1, 0),\ d = 6$ worldvolume superymmetric PBGS form of the equations of the scalar super 5-brane in $D = 10$ [24](we use the nomenclature of ref. [25]). So the nonlinear realization description of the partial breaking $N = 1,\ D = 10 \Rightarrow N = (1, 0),\ d = 6$ admits the natural brane interpretation, much in line of the previous studies [5] - [13].

5. Brane extension of harmonic Grassmann analyticity? For further discussion it will be convenient to project all the involved quantities on the $SU(2)$ harmonics $u^{\pm i}, u^{+i}u^-_i = 1$ [16]

$$\theta^\alpha_i\ \Rightarrow\ \theta^{\pm\alpha} = \theta^{\alpha i}u^\pm_i\ ,\ \ \nabla^i_\alpha\ \Rightarrow\ \nabla^\pm_\alpha = \nabla^i_\alpha u^\pm_i\ ,\ \ q^{ia}\ \Rightarrow\ q^{\pm a} = q^{ia}u^\pm_i\ . \tag{27}$$

Then the basic eq. (16) can be rewritten as

$$\nabla^+_\alpha q^{+a} = 0\ . \tag{28}$$

In the standard hypermultiplet case an analogous condition means that q^{+a} lives on an analytic subspace of the full harmonic superspace (x, θ, u), and this was the starting point of construction of off-shell actions for the hypermultiplet in [16].

A difficulty with a similar treatment of (28) stems from the fact that the anticommutator of two ∇^+_α is not vanishing, in contrast to its flat prototype

$$\{\nabla^+_\alpha, \nabla^+_\beta\} = -\epsilon^{\rho\lambda\gamma\tau}(\nabla^+_\alpha\Psi^d_\gamma)\ (\nabla^+_\beta\Psi_{d\ \tau})\nabla_{\rho\lambda} \equiv -F^{++\rho\lambda}_{\alpha\beta}\ \partial_{\rho\lambda}\ . \tag{29}$$

As a result one has an extra integrability condition

$$F^{++\ \rho\lambda}_{\alpha\beta}\partial_{\rho\lambda}q^+_a = 0\ , \tag{30}$$

which could be too strong (e.g., implying q^{ia} to be a constant). We have checked that, up to the seventh order in q^{ia}, this condition is satisfied *identically* as a consequence of the structure of $F^{++\ \rho\lambda}_{\mu\nu}$. It is plausible that this holds to any order and in what follows we can take for granted that (30) produces no new restrictions on q^{+a}.

Then eq. (28) implies, as usual, the existence of an analytic basis in the harmonic superspace where ∇^+_α is reduced to the partial derivative with respect to $\theta^{-\nu}$ (when applied to q^{+a}) and where q^{+a} lives as an unconstrained analytic superfield. The coordinate transformation to this basis should be highly nonlinear in the involved fields.

Instead of trying to find such a change of coordinates, it is easier to seek for a brane generalization of the standard off-shell q^+ action, i.e. for the action yielding in the bosonic sector the whole NG action (26). The possibility that such an action exists for the considered case was noticed in [23]. It is curious that there indeed exists a quartic extension of the standard free q^+ action which correctly reproduces the first terms in (26). It reads

$$\tilde{S}_q \sim \int d\zeta^{(-4)} q^+_a \mathcal{D}^{++} q^{+a} + \alpha \int dZ \, (q^+_a \mathcal{D}^{--} q^{+a})^2 \, . \qquad (31)$$

Here $dZ[du]$ and $d\zeta^{(-4)}$ are the appropriate integration measures over $d = 6$ harmonic superspace and its analytic subspace,

$$dZ = d\zeta^{(-4)} (\mathcal{D}^+)^4 \, , \; d\zeta^{(-4)} = d^6x[du](\mathcal{D}^-)^4 \, , \;\; (\mathcal{D}^\pm)^4 = \frac{1}{4!}\epsilon^{\alpha\beta\gamma\lambda}\mathcal{D}^\pm_\alpha \mathcal{D}^\pm_\beta \mathcal{D}^\pm_\gamma \mathcal{D}^\pm_\lambda \; , \qquad (32)$$

$\mathcal{D}^{\pm\pm} = \partial^{\pm\pm} - 1/2\, \theta^{\pm\alpha}\theta^{\pm\beta}\partial_{\alpha\beta} + \theta^{\pm\alpha}\partial/\partial\theta^{\mp\alpha}$ are harmonic derivatives, α is a dimensionless parameter (we use the same notation for the central-basis and analytic q^{+a}, hoping that this will not lead to confusion). The first term in (31) is the standard free q^+ action. We have found that after eliminating auxiliary fields (beyond expectation, they do not propagate) and making appropriate nonlinear redefinition of the physical bosonic field $\varphi^{ia}(x)$ ($q^{+a}| = \varphi^{ia}u^+_i + \ldots$),

$$\varphi^{ia} = \phi^{ia} - \frac{\alpha}{24}\left\{ \left[(\Box\phi \cdot \phi) - (\partial\phi \cdot \phi) \right] \phi^{ia} - \frac{1}{4}\,(\phi)^2 \Box\phi^{ia} + \frac{1}{2}(\phi \cdot \partial_{\alpha\beta}\phi)\, \partial^{\alpha\beta}\phi^{ia} \right\} + O(\phi^5) \, ,$$

the bosonic part of the component action in (31) in fourth order in fields coincides with (26) under the choice $\alpha = -2/3$.

This observation suggests the existence of the q^+ action with the whole static-gauge NG action in the bosonic sector. Clearly, the superfield equations of motion following from it, together with the analyticity condition, should amount to the basic nonlinear constraint (16) (or (28)). This action should be $N = (1,0), \; d = 6$ ($N = 2, \; d = 4$) counterpart of the Goldstone chiral superfield action of ref. [9,11]. Possible existence of such a brane analog of the free off-shell q^+ action raises the question what could be brane analogs of q^+ actions with interaction. The latter yield hyper-Kähler sigma models in their bosonic sector. Presumably, their brane extensions could correspond to super 5-branes on non-trivial curved backgrounds.

All such actions, being generalizations of off-shell q^+ actions, should necessarily involve infinite sets of auxiliary fields. They could provide an interesting alternative to the standard Green-Schwarz-type lagrangian description of superbranes [6,24,26]. It would be important to find the symmetry principles behind their structure. In the next section we present further evidence in favour of the existence of such actions.

6. $N=2\,,\ D=5$ **superparticle.** All the relations presented so far admit simple dimensional reduction to the $d=5$ and further $d=4\,, ...1$ worldvolumes by neglecting dependence on the corresponding worldvolume coordinates (in the Green-Schwarz approach this amounts to the "double dimensional reduction"). One gets in this way manifestly worldvolume supersymmetric superfield equations of super 4-brane in $D=9$, super 3-brane in $D=8$, supermembrane in $D=7$ and so on, up to $N=2$ superparticle in $D=5$. They all have 8 manifest and 8 nonlinearly realized supersymmetries. Here we illustrate our consideration on the example of $N=2\,,\ D=5$ superparticle.

In this case the basic anticommutation relations (2) become

$$\left\{Q^i_\alpha, Q^j_\beta\right\} = \epsilon^{ij}\Omega_{\alpha\beta}P\ , \quad \left\{Q^i_\alpha, S^{a\beta}\right\} = \delta^\beta_\alpha Z^{ia}\ , \quad \left\{S^{a\alpha}, S^{b\beta}\right\} = -\epsilon^{ab}\Omega^{\alpha\beta}P\ . \tag{33}$$

The full automorphism group of (33) is the product $Spin(1,4)\times Spin(5)$; the first factor is the target $D=5$ Lorentz group which acts on the indices i, a

$$SO(1,4)\sim Spin(1,4) \qquad \propto \quad \left\{T^{ij}, T^{ab}, K^{ia}\right\}\ , \tag{34}$$

and $Spin(5)$ acts on the spinor indices. In (33), $\Omega_{\alpha\beta} = -\Omega_{\beta\alpha}$ is the invariant $Spin(5)$ symplectic metric allowing to raise and lower the spinor indices ($\Omega^{\alpha\beta} \;=\; -\frac{1}{2}\epsilon^{\alpha\beta\rho\gamma}\Omega_{\rho\gamma}$ $\Omega_{\alpha\beta}\Omega^{\beta\gamma} = \delta^\gamma_\alpha$), P is the worldline translations operator.

Basically, the reduction to the case at hand is accomplished via the substitution $\partial_{\alpha\beta} = \Omega_{\alpha\beta}\partial_t$, where $t = \Omega_{\alpha\beta}x^{\alpha\beta} = -\Omega^{\alpha\beta}x_{\alpha\beta}$ is the worldline coordinate. The relations (15), (16) (in the notation using $SU(2)$ harmonics) preserve their form,

$$\nabla^+_\alpha q^{+a} = 0\ , \qquad \Psi^a_\beta = \nabla^+_\beta q^{-a} = -\nabla^-_\beta q^{+a}\ , \qquad \tilde\Lambda^{\pm a} = E^{-1}\partial_t q^{\pm a}\ , \tag{35}$$

with

$$\begin{aligned} \nabla^\pm_\alpha &= \mathcal{D}^\pm_\alpha - \frac{1}{2}\Psi^{b\beta}\mathcal{D}^\pm_\alpha\Psi_{b\beta}E^{-1}\partial_t\ , \quad E = 1 - \frac{1}{2}\Psi^{a\alpha}\partial_t\Psi_{a\alpha}\ , \\ \{\nabla^+_\alpha, \nabla^-_\beta\} &= \nabla^{+-}_{\alpha\beta} = F_{\alpha\beta}\partial_t\ , \quad F_{\alpha\beta} = -E^{-1}\left[\Omega_{\alpha\beta} + \nabla^+_\alpha\Psi^{b\rho}\nabla^-_\beta\Psi_{b\rho}\right]\ . \end{aligned} \tag{36}$$

Acting on Ψ^a_β in (35) by covariant derivatives, one finds

$$\nabla^+_\alpha\Psi^a_\beta = -F_{\alpha\beta}\partial_t q^{+a}\ , \qquad \nabla^-_\alpha\Psi^a_\beta = F_{\beta\alpha}\partial_t q^{-a}\ , \tag{37}$$

whence it follows, in particular, that

$$\{\nabla^+_\alpha, \nabla^+_\beta\} = 0\,. \tag{38}$$

Looking at the matrix $F_{\alpha\beta}$, one observes that eqs. (37) are the system of nonlinear equations for the unknowns $\nabla^\pm_\alpha \Psi^a_\beta$. In the given simplified case it can be explicitly solved, and, further, the explicit expression for Ψ^a_β in terms of $q^{\pm b}$ can be found. For our purposes it is enough to give the solution in the bosonic limit, with all fermions discarded

$$\nabla^\pm_\alpha \Psi^a_\beta \mid = \mathcal{D}^\pm_\alpha \Psi^a_\beta = 2\Omega_{\alpha\beta}\,\frac{1}{1+\sqrt{1-v^2}}\,\partial_t q^{\pm a}\,, \quad (v^{ia} \equiv \sqrt{2}\,\partial_t q^{ia})\,. \tag{39}$$

The constraint in (35) implies the following equation (once again, with all fermions omitted)

$$\begin{aligned}\Big[\Big(\nabla^{+-}_{\beta\nu}\nabla^{+-}_{\rho\gamma} + \nabla^{+-}_{\nu\gamma}\nabla^{+-}_{\rho\beta} - \nabla^{+-}_{\beta\gamma}\nabla^{+-}_{\rho\nu}\Big) - \{\nabla^+_\rho, [\nabla^-_\gamma, \nabla^{+-}_{\beta\nu}]\}\Big]\, q^+_a \\ +\{\nabla^+_\rho, [\nabla^+_\beta, \nabla^{+-}_{\nu\gamma}]\} q^-_a = 0\,. \end{aligned}\tag{40}$$

A straightforward calculation shows that the terms with spinor derivatives in this relation identically vanish, while the term within the parenthesis yields, modulo an overall scalar factor, the dynamical equation for $q^{ia}(t)$ (we write it in terms $v^{ia} = \sqrt{2}\,\partial_t q^{ia}$)

$$\partial_t v^{ia} E^{kb}_{ia} \equiv \partial_t v^{ia}\left(I^{kb}_{ia} + \frac{\partial_t v_{ia}\partial_t v^{kb}}{1-v^2+\sqrt{1-v^2}}\right) = 0\,. \tag{41}$$

After multiplying from the right by the matrix E and using

$$(E^2)^{ia}_{kb} = I^{ia}_{kb} + \frac{\partial_t v^{ia}\partial_t v_{kb}}{1-v^2}\,, \tag{42}$$

one rewrites (41), up to a scalar factor, in the form

$$\frac{d}{dt}\left(\frac{v^{ia}}{\sqrt{1-v^2}}\right) = 0\,, \tag{43}$$

that is recognized as the equation of motion corresponding to the static-gauge form of the NG action for the massive particle in $M^{1,4}$

$$S \sim \int dt\sqrt{1-v^2}\,. \tag{44}$$

Although, due to the specificity of the $d = 1$ case, the above bosonic equation actually amounts to the free one $\partial_t v^{ik} = 0$, [1] we expect that in the non-trivial $d > 1$

[1] Nonetheless, the action corresponding to this equation should be just (44), because it is the unique bosonic action that respects the nonlinearly realized $SO(1,4)/SO(4)$ hidden symmetry of the constraint for q^{ia} in (35).

cases the constraint (35) yields the equation of motion for q^{ia} in the form similar to (41), and it takes the standard NG form only after rotating the free target space index by an appropriate field-dependent non-degenerate matrix. Actually, when we performed the lowest-order computation outlined in sect. 4, we met just this peculiarity.

Finally, we address the issue of existence of the off-shell harmonic analytic action for this simplest system. Since in the present case the integrability condition (38) is valid generically (not only when applied on q^{+a}), the analytic basis definitely exists. Like in the $d = 6$ case, we shall try to construct the action directly in the analytic harmonic $d = 1$ superspace $\zeta = (t, \theta^{+\alpha}, u^{\pm i})$. We start from the $d = 1$ reduction of the action (31) (with $d^6x \to dt$ in eq. (32))

$$S_q \sim S_0 + S_1 = \int d\zeta^{(-4)} q^+_a \mathcal{D}^{++} q^{+a} + \alpha \int dZ\, A^2 \tag{45}$$

$$A = q^+_a \mathcal{D}^{--} q^{+a} \equiv q^+ \cdot \mathcal{D}^{--} q^+ \,. \tag{46}$$

We vary this action with respect to the lowest-order part of the broken SUSY transformation

$$\delta_{(0)} q^+_a = c^+_a = \eta^\alpha_a \theta^+_\alpha \,. \tag{47}$$

The free part in (45) is obviously invariant while the quartic part is not

$$\delta_{(0)} S_1 = \alpha \int dZ \left\{ L^{-3} \cdot \mathcal{D}^{++} q^+ + 2A\, (q^+ \cdot c^-) \right\} \,. \tag{48}$$

Here

$$L^{-3a} = c^{-a}\; q^+ \cdot (\mathcal{D}^{--})^2 q^+ + q^{+a}\; c^- \cdot (\mathcal{D}^{--})^2 q^+ + 2\mathcal{D}^{--} q^{+a}\; c^- \cdot \mathcal{D}^{--} q^+ \,,\; c^-_a = \eta^\alpha_a \theta^-_\alpha \,. \tag{49}$$

The second term in (48) vanishes as a consequence of analyticity of q^+, while the first term can be cancelled by the appropriate analyticity-preserving variation of q^+ in the free part of the action

$$\delta_{(1)} q^+_a = -\alpha (\mathcal{D}^+)^4 L^{-3}_a \,. \tag{50}$$

Thus the first nonlinear term in the variation of q^+ under the hidden SUSY is also uniquely defined. Already at this step we observe an important phenomenon. Commuting $\delta_{(1)}$ with $\delta_{(0)}$, one immediately finds that, to the first order in q^+, the correct closure $\sim \partial_t q^+_a$ for the broken SUSY is achieved only modulo equations of motion. In other words, it is *impossible* to keep off shell both hidden and manifest SUSY's, the best we can gain is the off-shell world-line $N = 8$ SUSY [2].

[2]This situation is quite similar to the formulation of $N = 4\,,\; D = 4$ super Yang-Mills theory via unconstrained harmonic $N = 2\,,\; D = 4$ superfields [27]: only the manifest $N = 2$ SUSY is off-shell in such a formulation.

The next steps in our recursion procedure are to compute the variation $\delta_{(1)}S_1$ and to look for the sixth-order correction S_2, such that $\delta_{(0)}S_2$ cancels $\delta_{(1)}S_1$. We proceed from the most general sixth-order Lagrangian density of the dimension -4, local in harmonics and having zero harmonic $U(1)$ charge. A part of its variation has the form $\sim \mathcal{D}^{++}q^+_a$ and hence can be cancelled by the appropriate shift of q^+ in the free action (it is of the fourth order in q^+). The remaining part is required to cancel $\delta_{(1)}S_1$. This requirement, together with that of on-shell closure of the hidden SUSY to the third order in q^+, uniquely (up to a freedom in the choice of independent structures in the Lagrangian) fix S_2 to be

$$
\begin{aligned}
S_2 &= \frac{2\alpha^2}{5}\int dZ\,\Big\{2\,AB^{++}(\mathcal{D}^{--}q^+\cdot\mathcal{D}^{--}\partial_t\,q^+) - \frac{3}{2}\,A(\mathcal{D}^{--}B^{++})^2 \\
&\quad - 7\,\mathcal{D}^{--}A\,\mathcal{D}^{--}B^{++}\,B^{++} - \frac{7}{2}\,A\,\partial_t\,A\,\mathcal{D}^{--}B^{++}\Big\}\ , \qquad (51)
\end{aligned}
$$

where

$$
B^{++} = q^+\cdot\partial_t\,q^+\ . \qquad (52)
$$

To simplify this expression, let us treat B^{++} as the analytic potential of some composite $N=8\,,\ d=1$ (dimensionally reduced $N=(1,0)\,,\ d=6$) vector multiplet [16] and introduce a non-analytic potential B^{--} by the standard relation [20]

$$
\mathcal{D}^{--}B^{++} - \mathcal{D}^{++}B^{--} = 0\ . \qquad (53)
$$

We substitute it into (51), make use of the identity

$$
2B^{++} = \mathcal{D}^0 B^{++} = [\mathcal{D}^{++},\mathcal{D}^{--}]B^{++} = (\mathcal{D}^{++})^2 B^{--} - \mathcal{D}^{--}\mathcal{D}^{++}B^{++}\ ,
$$

and integrate by parts with respect to $\mathcal{D}^{++}$. In the course of this computation we omit all terms of the form $\sim(\mathcal{D}^{++}q^+\cdot F^{-3})$ as they can be absorbed into the redefinition of q^+ (the relevant shift is of the fifth order in q^+ and so does not affect S_1). The final answer for S_2 is as follows

$$
S_2 = 2\alpha^2\int dZ\ A\,B^{--}\,B^{++}\ . \qquad (54)
$$

The existence of this sixth-order term is a non-trivial fact and can be regarded as a strong indication that the full harmonic action for this $D=5$ superparticle (and its higher-dimensional counterparts) exists. The form of S_2 (54) is rather suggestive: it looks like the harmonic superspace action of the composite $N=8\,,\ d=1$ vector multiplet B^{++} in some background specified by the superfield A, both B^{++} and A being functions of the Goldstone hypermultiplet superfield q^+_a. This analogy could provide a hint of how to construct the full action. Also, it seems to imply a link with $N=8$ super D0-brane the worldline supermultiplet of which is just $N=8\,,\ d=1$ vector multiplet.

7. Concluding remarks. Besides the already mentioned problems for the future study, we list here a few other ones.

It is interesting to inquire whether some other $N = (1,0),\ d = 6$ supermultiplets can be given the Goldstone interpretation and to which patterns of PBGS they could be relevant. The simplest one is the vector multiplet [18] comprising the fields $A_{[\mu\nu]}(x), \lambda^{\mu}_i(x), Y^{(ik)}(x)$. As the fermionic field λ (a candidate for Goldstino) is of the same $d = 6$ chirality as the Grassmann coordinate $\theta^{i\alpha}$, this multiplet can serve as the Goldstone one for the PBGS pattern $N = (2,0),\ d = 6 \rightarrow N = (1,0),\ d = 6$. By analogy with ref.[10], one can expect that the related theory is a manifestly $N = (1,0),\ d = 6$ supersymmetric BI theory with the hidden nonlinearly realized rest of $N = (2,0),\ d = 6$ SUSY (or $N = 4$ BI theory with hidden extra $N = 2$ SUSY in $D = 4$). It is expected to yield a manifestly worldvolume supersymmetric PBGS description of super D5-brane in $D = 6$ [3].

The $N = (1,0)\,,\ d = 6$ hypermultiplet parametrize transverse directions also in a special kind of super 5-brane in $D = 10$, the heterotic 5-brane obtained as a solitonic solution in the heterotic string theory [29]. It was argued in [30] that for quantum consistency of this solitonic 5-brane some extra worldsurface supermultiplets should be added, in particular, an $SU(2)$ gauge vector $N = (1,0)\ \ d = 6$ multiplet. It would be interesting to understand the necessity of such additional $d = 6$ multiplets within the PBGS approach. Note that the simple scalar super 5-brane to which our attention was limited here corresponds to another solution to the equations of the heterotic string theory, the "neutral solution" [31].

It is intriguing to examine from the PBGS point of view $N = 1,\ D = 11$ (or the type IIA $N = 2,\ D = 10$) SUSY. The $N = (1,0),\ d = 6$ superfield framework is suitable for studying the $1/4$ breaking of this SUSY. Let us see what happens in the linearized approximation.

From the $d = 6$ point of view, promoting $N = 1,\ D = 10$ SUSY to $D = 11$ amounts to adding one more bosonic translation generator P_{11}, two supertranslation generators of opposite chiralities $Q^{i\alpha}, S^a_\beta$ and two extra Lorentz generators U^{ia} and $W_{\alpha\beta} = -W_{\beta\alpha}$. The latter extend $SO(1,9)$ to $SO(1,10)$ and belong to the cosets $SO(5)/SO(4)$ and $SO(1,6)/SO(1,5)$. We still wish to have $N = (1,0),\ d = 6$ SUSY as the only unbroken one, so we should add to the already incorporated Goldstone superfields several new ones associated with the extra generators

$$P_{11} \Rightarrow \Phi(x,\theta)\,,\ Q^{i\alpha} \Rightarrow \eta_{i\alpha}(x,\theta)\,,\ S^a_\alpha \Rightarrow \xi^\alpha_a(x,\theta)\,,$$
$$U^{ia} \Rightarrow u_{ia}(x,\theta)\,,\ W_{\alpha\beta} \Rightarrow v^{\alpha\beta}(x,\theta)\,. \tag{55}$$

[3]These proposals were originally made in [28].

At the linearized level, the standard coset techniques yield the following expressions for the covariant $d\theta$-projections of the Cartan 1-forms related to the newly introduced (super)translations generators

$$P_{11} \Rightarrow \mathcal{D}^i_\alpha \Phi + \eta^i_\alpha \,, \quad Q^{i\alpha} \Rightarrow \mathcal{D}^i_\alpha \eta_{j\beta} + 2\delta^i_j v_{\alpha\beta} \,, \quad S^a_\alpha \Rightarrow \mathcal{D}^i_\alpha \xi^\beta_a - \delta^\alpha_\beta u^i_a \,. \tag{56}$$

One observes that the Goldstone superfields η^i_a, $v_{\alpha\beta}$, u^{ia} (like $\Lambda^{ia}_{\alpha\beta}$ and Ψ^a_β) can be covariantly eliminated by equating to zero appropriate parts of the above projections of Cartan forms. On the other hand, the superfields Φ, ξ^α_a can be shown to never appear linearly (without derivatives on them) in any Cartan form. So in the given case the set of unremovable Goldstone superfields enlarges to $\{q^{ia}, \Phi, \xi^\alpha_a\}$. New superfields are reducible and we should impose on them proper constraints similar to the constraint (16) for q^{ia}. By analogy with the $D = 10$ case we assume that the covariant elimination of the redundant Goldstone superfields and imposing constraints on the essential ones are simultaneously effected by equating to zero full $d\theta$-projections (56) of the translation and supertranslation Cartan forms (or the covariant nonlinear versions of (56) in the full nonlinear case). As the result of such a procedure at the considered linearized level one gets the following expressions for the new redundant Goldstone superfields

$$v_{\alpha\beta} = -\frac{1}{4}\partial_{\alpha\beta}\Phi \,, \qquad \eta^i_\alpha = -\mathcal{D}^i_\alpha \Phi \,, \qquad u^i_a = \frac{1}{4}\mathcal{D}^i_\beta \xi^\beta_a \,, \tag{57}$$

and, simultaneously, the following constraints for the new unremovable ones

$$\text{(a)} \quad \mathcal{D}^{(i}_\beta \mathcal{D}^{k)}_\alpha \Phi = 0 \,; \qquad \text{(b)} \quad \mathcal{D}^i_\beta \xi^\alpha_a - \frac{1}{4}\,\delta^\alpha_\beta\, \mathcal{D}^i_\gamma \xi^\gamma_a = 0 \,. \tag{58}$$

The constraint (58a) is immediately recognized as the one defining the self-dual tensor $N = (1,0),\ d = 6$ supermultiplet in the field-strength formulation [32]. This constraint leaves the Goldstone fermion $\eta^i_\alpha(x)$, a scalar $\phi(x) = \Phi|$ (it parametrizes the broken eleventh direction) and a self-dual field strength $F_{(\alpha\beta)}(x)$ as the only irreducible fields in $\Phi(x,\theta)$ and puts all them on shell. The Goldstone superfields q^{ia}, Φ are naturally unified into a $N = (2,0)$ self-dual multiplet which is known to be the worldvolume multiplet of the M5-brane [23,26,33]. This nicely matches with the fact that these two $N = (1,0)$ multiplets realize the $1/2$ spontaneous breaking of $N = 1,\ D = 11$ SUSY down to $N = (2,0),\ d = 6$ SUSY $\propto \{Q^i_\alpha, S^a_\beta, P_{\alpha\beta}, so(1,5) \oplus so(4)\}$.

It is the remaining Goldstone superfield ξ^α_a which executes further breaking of this $N = (2,0),\ d = 6$ SUSY down to $N = (1,0)$. Surprisingly, the constraint (58b) turns out to be too strong: it reduces $\xi^\alpha_b(x,\theta)$ to a few bosonic and fermionic constants

$$\text{eq.(47b)} \quad \Rightarrow \quad \xi^\alpha_b(x,\theta) = \xi^\alpha_b(x) - \theta^{\alpha k}\, u_{kb} - \tfrac{1}{2}(\theta^{\alpha i}\theta^\beta_i)\, \phi_{\beta b} \,, \tag{59}$$

$$\partial_{\alpha\beta}\, u_{bi} = \partial_{\alpha\beta}\, \phi_{b\gamma} = 0 \,,\ \partial_{\gamma\beta}\, \xi^\alpha_b(x) + \delta^\alpha_\beta \phi_{\gamma b} - \delta^\alpha_\gamma \phi_{\beta b} = 0 \ \Rightarrow\ \xi^\alpha_b(x) = \xi^\alpha_b + 2\, x^{\alpha\gamma}\phi_{\gamma b} \tag{60}$$

Nevertheless, this constraint is the only one which *(i)* is linear in $\mathcal{D}^i_\alpha$ and *(ii)* enjoys all linearly realized symmetries. We still do not know how to interpret this. Possible ways out are, e.g., to impose some alternative constraint of higher order in derivatives, or to retain the linearity in $\mathcal{D}^i_\alpha$ but to allow an explicit breaking of the $D = 11$ Lorentz symmetry and, simultaneously, of manifest $SO(4)$ symmetry, say, down to the diagonal $SU(2)$ subgroup. In this case there arises a possibility to impose on ξ^α_b the constraints identifying it with a superfield strength of $N = (1, 0),\ d = 6$ Maxwell multiplet [18] (they can be chosen on- or off-shell). Of course, there remains a difficult problem of correct generalization to the full nonlinear case [10].

Curiously, the constants in (59), (60) have true conformal dimensions and index structure for being parameters of some specific coset of superconformal extension of the $N = (2, 0),\ d = 6$ super Poincaré group, the supergroup $OSp(6, 2|4)$ [33]. Indeed, ξ^α_b are going to be the parameters of the second Poincaré supertranslations, u_{ia} the parameters of the coset $SO(5)/SO(4)$ and $\phi_{\gamma b}$ the parameters of one of two special supersymmetries.

It is interesting to analyze from a similar standpoint also the type IIB $N = 2,\ D = 10$ SUSY. It can be argued that its $1/2$ breaking should be realized on the $N = (1, 0)$ hypermultiplet and $N = (1, 0)$ Maxwell field strength superfields as the Goldstone ones. Together they form an on-shell $N = (1, 1)\ ,\ d = 6$ Maxwell-Goldstone multiplet. Further breaking to $N = (1, 0),\ d = 6$ SUSY in this case requires an extra essential fermionic Goldstone $N = (1, 0)$ superfield $\nu^b_\alpha(x, \theta)$ constrained in an appropriate way. We failed to find a proper candidate for such superfield and constraints among the known $N = (1, 0)\ ,\ d = 6$ multiplets.

Finally, it is desirable to further clarify the relationships between the PBGS and superembedding approaches. They seem to be complementary to each other. The PBGS approach deals from the beginning with a minimal set of Goldstone superfields accommodating the physical brane degrees of freedom and it offers systematic techniques to deduce the transformation laws of these superfields under hidden nonlinear symmetries. On the other hand, superembedding approach allows one to classify physical worldvolume supermultiplets related to various superbranes and, under some assumptions (e.g., “geometro-dynamical principle”), to learn whether these multiplets are on- or off-shell. In particular, the linearized analysis of the $N = 1,\ D = 10$ super 5-brane in ref. [23] (in the framework of conventional superspace) picks out just the $d = 6$ hypermultiplet as a physical multiplet and predicts it to be on-shell.

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