THE SINGLE REFLECTION REGIME OF X-RAYS TRAVELLING INTO A MONOCAPILLARY

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Abstract

In this manuscript an analysis of the transmission of x-rays through a single monocapillary under the single reflection regime is presented and discussed. Because ray tracing does not allow to explain the experimental data, a first qualitative interpretation of the observed behavior is given in the framework of the wave theory.

Keywords: X-ray, monocapillary, ray optics, wave optics, capillary optics

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1 Introduction

Optical devices like mirrors are usually considered to deflect or shape X-ray beams. Mirrors are based on the total external reflection mechanism. The total external reflection mechanism is well established and explains also the complex behavior such as that of X-rays propagating inside hollow tubes. However, more recently the use of systems of capillaries packed in a definite order to collect and shape X-ray beams was suggested and realised. A detailed analysis of these devices is presented in Ref. [1].

The recent results obtained, analysing the transmission of the X-ray radiation through a capillary system, showed that the radiation distribution behind these devices has a complex character [2]. In particular, by the investigation of the X-ray transmission through a single monocapillary have been noticed that some effects, which are usually neglected in ray optics, may be observed [3,4]. In this work a theoretical analysis of the process of the transmission of X-rays through a monocapillary system under the single reflection regime is given.

2 Ray approximation

Let us consider the case when focusing of X-ray radiation emitted by a point source S is achieved using a monocapillary with a channel diameter of $2r_0$. If the source is positioned on the longitudinal axis of the channel, in the ray approximation the radiation field behind the capillary may be splitted in three regions, illustrated in Fig. 1. The parameters that characterise these regions, are defined by the focusing scheme. The resulting field is determined by both the direct rays travelling through the capillary without any reflection events and those reflected. The reflected rays span the angle $\Delta\theta = \theta_{max} - \theta_{min}$, where θ_{max} and θ_{min} are shown in the layout of Fig. 1. The optimal condition is fulfilled when $\theta_{max} = \theta_c$, where $\theta_c[mrad] \simeq 2.5\,\lambda[\mathring{A}]$ is the critical angle of the total external reflection, at the wavelength λ, for a typical glass material. In this case, the condition of total external reflection $\theta \leq \theta_c$ works for all rays in the $\Delta\theta$ range. In this framework the intensity of the X-ray radiation at each position behind the monocapillary may be defined as

$$I(\theta) = I_1 + I_2, \tag{1}$$

I_1 is the intensity of the direct rays, I_2 is the intensity of the reflected rays, and $I_2(\theta) \propto R_0(\theta)$, where $R_0(\theta)$ is the coefficient of the single X-ray reflection. When $\theta \leq \theta_c$, we have

$$R_0(\theta) \simeq \left| \frac{2\theta}{\sqrt{\varepsilon - 1}} - 1 \right|, \qquad (2)$$

where ε is the dielectric function of the material. Now, if we consider as a first approximation that the field induced by the direct rays at a fixed detector position, is a constant field, the eq.(1) may be written as
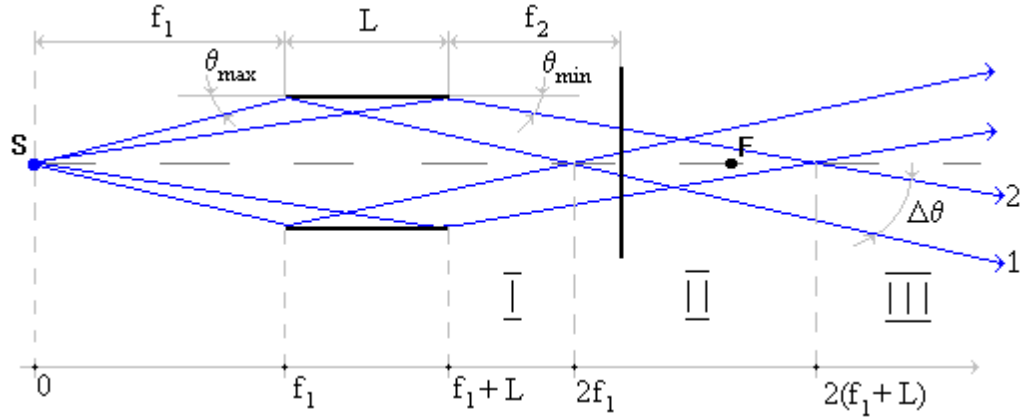
$$I(\theta) \propto const + R(\theta) \qquad (3)$$

Figure 1: *The focusing properties of a single monocapillary for the case of $f_1 \geq L$. S is the X-ray source, L is the capillary length, f_1 and f_2 are, respectively, the distances between the source and the monocapillary, and between the monocapillary and the film.*

Hence, an intense radiation ring in the background of a general uniform field produced by the direct rays, should be observed behind the monocapillary (Fig. 2). This condition is characterised by ring sizes defined by the relations

$$\begin{cases} r_1 &= \left| r_0 - (L + f_2)\theta_{max} \right|, \\ r_2 &= \left| r_0 - f_2\theta_{min} \right| \end{cases} \qquad (4)$$

or, taking into account that

$$\theta_{max} = \frac{r_0}{f_1}, \quad \theta_{min} = \frac{r_0}{f_1 + L}, \qquad (5)$$
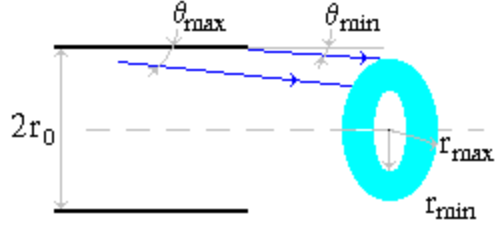
by the following ones

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Figure 2: *Sketch of the X-ray distribution behind a monocapillary of diameter* $2r_0$. *The radii* r_{min} *and* r_{max} *correspond to* r_1 *and* r_2 *for the 1st zone, and to* r_2 *and* r_1 *for the 3rd zone.*

$$
\begin{cases}
r_1/r_0 & = \quad |1 - (L + f_2)/f_1| \\
r_2/r_0 & = \quad |1 - f_2/(L + f_1)|
\end{cases}
\tag{6}
$$

Evidently, we have two limit cases

$$
\begin{cases}
r_1 & = \quad 0 \qquad at \qquad f_1 = L + f_2, \\
r_2 & = \quad 0 \qquad at \qquad f_2 = L + f_1
\end{cases}
\tag{7}
$$

that define the boundaries for the 2nd zone. A more accurate analysis shows that in the 1st zone of Fig. 1, from the capillary end to the point $f_1 = L + f_2$, one has to observe a light ring distribution with a central minimum. In the 2nd region, from $f_1 = L + f_2$ to $f_2 = f_1 + L$, the ring contracts so as to observe a maximum at the centre due to the composition of the reflected rays. However, the highest intensity is at the geometrical focus F, where the condition $f_1 = f_2$ is satisfied. In the 3rd zone, from the point $f_2 = f_1 + L$ to ∞, the reflected rays move away, and as a result we should obtain a ring with a minimum at the centre. The axial distribution of X-rays behind the capillary is qualitatively presented for the three regions in Fig. 3.

The width of a ring is determined both by the beam going through the capillary without suffer any reflection (the direct beam)

$$
\Delta f_{sp} = 2 \left(r_0 + f_2 \theta_{min} \right) \quad at \quad f_2 \leq f^*
\tag{8}
$$

and by the reflected beam

$$
\Delta f_{sp} = 2 \left(f_2 - f_1 + L \right) \theta_{max} \quad at \quad f_2 > f^*,
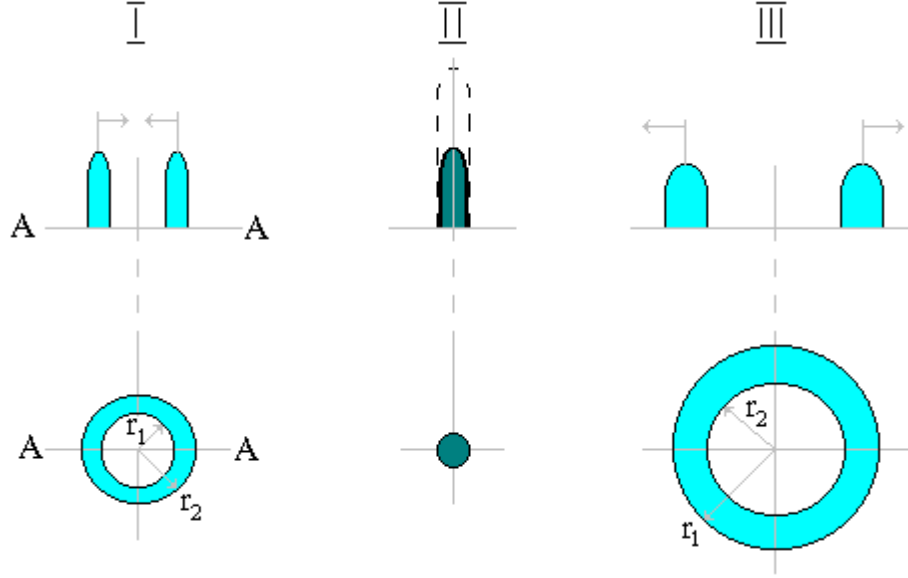\tag{9}
$$

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Figure 3: *The axial distributions of X-rays behind the monocapillary. The ring expands moving away the exit of the monocapillary. For the 2nd zone the distribution at the focal plane is essentially a spot with maximum intensity along the monocapillary axis.*

where $f^* = (r_0 + (f_1 - L)\,\theta_{max})/(\Delta\theta)$, f^* is located in a place of the 3rd region out of Fig. 1 . Because the angular width of the beam behind the monocapillary, is defined by the expression

$$\Delta\theta = \frac{r_0\,L}{f_1\,(f_1 + L)},\tag{10}$$

in the limit $f_1 \simeq L$, that corresponds to the standard experimental conditions, $f^* \simeq 2L$. Actually, if the distance between the radiation source and the monocapillary entrance is of the order of the capillary length, a spot due to the reflected part of the beam, occurs at the third zone. As a consequence, the width of the ring increases.

The behavior is analogue if we consider an extended source. Obviously, for an extended source we have to change, in our description, the angles θ_{min} and θ_{max} which essentially affects the length of the second zone. Although, it should be noticed that θ_{max} increases while θ_{min} decreases, it is more important to underline that, in this framework a minimum in the intensity distribution should be observed along the monocapillary axis at the third zone. In other words, in the case of an extended source the distribution behind a monocapillary still has a ring distribution.
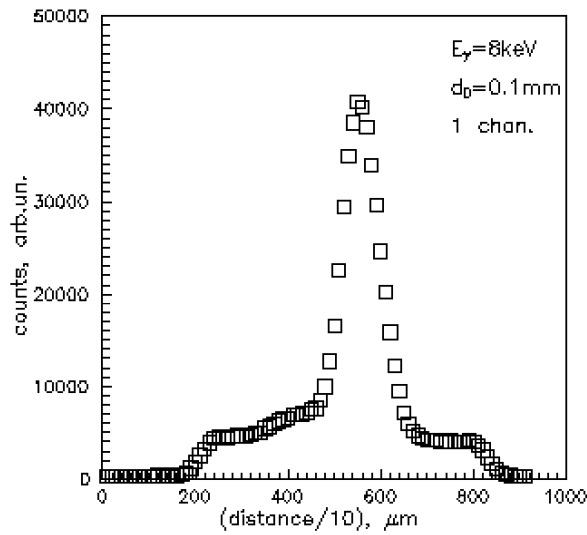
Figure 4: *The X-ray distribution of the 3rd zone in the transverse plane behind a single monocapillary (reproduced from Ref.[3]).*
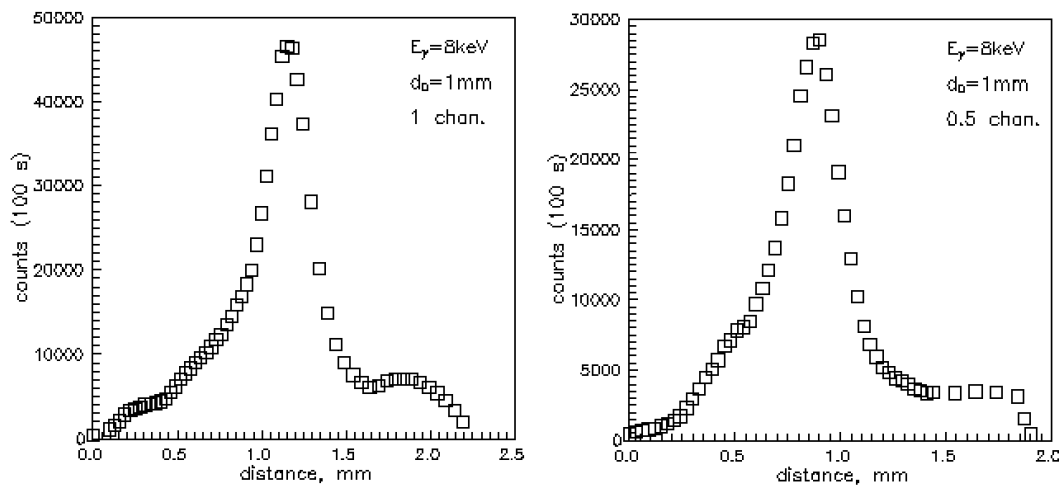

Figure 5: *The X-ray distributions of the 3rd zone in the transverse plane behind a single monocapillary. The left distribution was obtained with an opened capillary entrance while the right distribution with a masked (half) capillary entrance (reproduced from Ref.[3]).*

3   Comparison between experiments and simulations

Experiments on the X-rays transmission by a single monocapillary under the single re-flection condition are described in detail in Ref. [3] where the intensity distribution of the radiation behind the monocapillary for the three regions above described, has been inves-

tigated.

From this investigation it is possible to claim that while the results of the first two zones are in good agreement with the theoretical estimations, in the 3rd zone, there is a large disagreement. In fact, as shown in Fig. 4, a maximum in the transverse plane is clearly observed at centre of the X-ray distribution.

A background, due both to the direct and reflected beams is also clearly observed. It is interesting to note also that, because the maximum appears at large distance from the 3rd zone along the longitudinal monocapillary axis, the transverse dimension of the maximum is limited. To clarify the origin of the central maximum in the 3rd zone, experiments were carried out both using a monocapillary with one opened entrance end (Fig. 5, left) and with an half masked entrance end (Fig. 5, right). The comparison shows that if we mask half of the entrance, the corresponding portion of the beam disappears. In addition, although a strong peak is still observed, the central peak intensity decreases demonstrating that the maximum is determined by the waves reflected by points close each other on the inner capillary surface.

In order to better understand the experimental data, we calculated the distributions of the X-rays transmitted by a monocapillary both in the ray and in the wave approximations. The computer simulations of the distributions in the 3rd zone are shown in Fig. 6. In the ray optics approximation (left figure) one obtains an image that may be satisfactory explained by the superposition of the direct beam, i.e. X-rays travelling through the capillary without reflection, and the portion of the reflected beam under the single reflection regime. As expected, the distribution has a minimum on the longitudinal axis of the capillary. However, calculations made in the wave approximation (right panel) shows that a different and less intense maximum should appear at the centre of the radiation distribution. This comparison supports a model that considers the presence of interference processes, coming from different area of the reflecting walls of capillary, that determines the resulting intensity.

4   Wave model

Usually in a general scattering process, the target surface may be considered as the possible source of a secondary wave. A secondary wave diffuses into the space with a well definite angular distribution that strongly depends on the parameters of both the illuminating beam and the target area [5,6]. In the limit $\lambda \to 0$ the secondary wave propagates at the definite angle θ with a negligible divergence ($\Delta\theta \to 0$), so that the angles of incidence and reflection of this wave are determined by Snell's law. In this approximation (geometrical approximation), we do not expect to observe any maximum in the distribution on the axis of the monocapillary. But actually, as shown both by theoretical and experimental
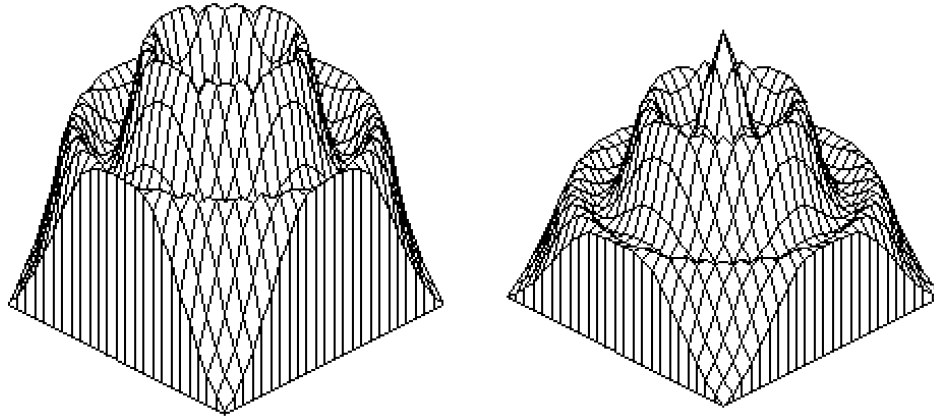
Figure 6: *The simulation of an X-ray distribution behind a single monocapillary calcu-*
lated for the 3rd region: in the ray approximation (left plot) and in the wave approximation
(right plot).

investigations, if we change the incidence angle, also the shape of the scattered radiation
distribution qualitatively changes. In particular, the fraction of the X-ray reflected to an-
gles less than the incidence angle grows up if we increase the incidence angle of the beam.
If we assume a small correlation radia of surface roughness, we obtain that a portion of the
reflected beam spreads to angles significantly lower than the incidence angle [5]. Hence,
these rays may originate the superposition pattern observed in the 3rd zone.

To evaluate the interference contribution of the rays travelling inside a monocapil-
lary, with diameter $d_0 \equiv 2r_0$, we may consider a monocapillary formed by a number of
equal annular strips having widths of $2l'$. Let us also consider the interference between
two rays reflected from a narrow annular strip. In this case the inner surface of the strip is
the source of the secondary waves (see Fig. 7). The effect may be changed by considering
two imaginary sources S_1 and S_2 arranged at the distance D by the detection plane (film).
Obviously, because the strip represents a ring of width $2l'$, the imaginary sources will also
form a ring (Fig. 7 shows one cross-section of the system). The path difference between
two rays emitted by S_1 and S_2 at the position having the coordinate h in the transverse
plane is equal to

$$\Delta = h \frac{d_0}{D} \,, \tag{11}$$

and because of the small glancing angles, $\theta \leq \theta_c$, the transverse wave mainly defines the
scattering processes (the longitudinal wave remains practically unchanged). As a conse-
quence, because the wavelength λ may be approximated with the transverse wavelength
$\lambda_\perp$, the intensity of the radiation from the *i*-th strip at the point h may be written using the
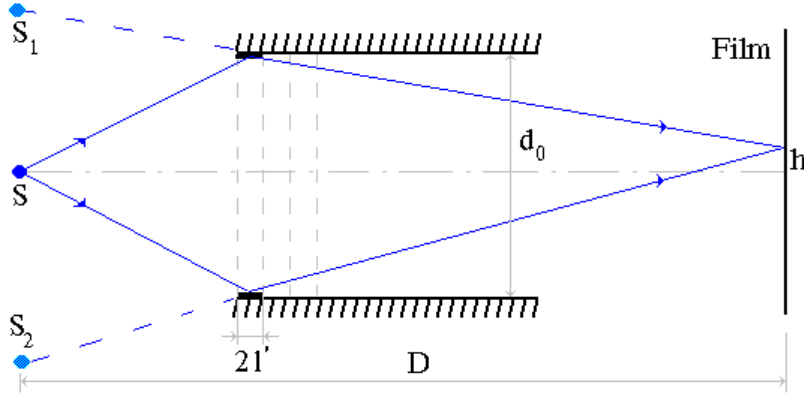
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Figure 7: *Scheme of the interference of X-rays travelling into a monocapillary.*

following relation

$$\frac{dI_i}{dx} \propto 1 + \cos(k_\perp \Delta) = 1 + \cos\left(\frac{2\pi}{\lambda_\perp}\frac{hd_0}{D}\right) , \qquad (12)$$

or

$$I_i(h) \propto 2l_i + \frac{B}{\pi}\sin\left(\frac{2\pi l_i}{B}\right)\cos\left(\frac{2\pi h}{B}\right) \qquad (13)$$

where $l \approx l'\theta$ is the transverse size of the strip,

$$B = \frac{D}{d_0}\lambda_\perp \qquad (14)$$

the distance between the interference rings on the film (an "interference width"). In order to account for the influence of the whole capillary, approximated by a cylinder, it is necessary to add all the contributions of the strips and then integrate over the azimuthal angle (the last step may be omitted due to the axial symmetry):

$$I(h) = \sum I_i(h) , \qquad (15)$$

where $l_i \approx \Delta L\,\theta_i$, $\theta_i \in [\theta_{min}, \theta_{max}]$.

An estimation of the interference width can be obtained using the eq.(14), where $\lambda_\perp$ is replaced by λ/θ,

$$B \simeq \frac{D\lambda}{d_0\theta}$$

Using the following values: $D = 30\,cm$, $d_0 = 0.05\,cm$, $\theta \approx 10^{-3}\,rad$ - we obtain $B \simeq 60\mu m$. Actually this value may be significantly higher for smaller value of d_0.

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Indeed, we have to underline here that the interference width B is inversely proportional to the "channel" size d_0, i.e., the interference width increases while decreasing the channel diameter. In addition, because the experimental investigations with a single monocapillary showed that the interference pattern do not disappear masking an half of its end [3,4], we demonstrated that the waves contributing to the central maximum, are only those reflected by one of the capillary halfs. Moreover, the transverse plane distance between the waves considered has to be smaller than the capillary diameter d_0.

5 Conclusion

We have investigated the behaviour of X-ray travelling into capillaries both with theoretical model and experimental methods. In the single reflection regime, the presence of X-ray distribution maximum at the monocapillary axis at the far third zone demonstrated the complex character of the distribution behind a monocapillary. The investigation of the transmission process through a single monocapillary shows, in particular, that some features of this process are not reproduced in the framework of the ray approximation. Altough the ray tracing method is very useful for any estimation of the efficiency of these devices, for a more detailed analysis of the shape of the focus it is necessary to consider the wave theory.

In addition, to describe the process, it is necessary to take into account the angular distribution of the reflected beam. Because the understanding of the phenomena certainly may have an impact in the development of the capillary optics technology, additional experimental and theoretical efforts of this particular optical devices are necessary.

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