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The CP–conserving contribution to the transverse muon polarization in $K^+ \to \mu^+\nu\gamma$

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Abstract

We present a detailed estimate of the transverse muon polarization (P_T) due to electromagnetic final–state interactions in the decay $K^+ \to \mu^+\nu\gamma$. This CP–conserving effect represents the dominant contribution to P_T within the Standard Model. As a result of an explicit calculation, we find that the CP–conserving contribution to P_T is quite small, typically of $\mathcal{O}(10^{-4})$, essentially due to the suppression factor $\alpha/4\pi^2$. This enforces the sensitivity of P_T in probing extensions of the Standard Model with new sources of time–reversal violations. A brief discussion about possible CP–violating contributions to P_T in the framework of supersymmetric models with unbroken R–parity is also presented.

1 Introduction

The transverse muon polarization in $K^+ \to \pi^0 \mu^+ \nu$ and $K^+ \to \mu^+ \nu \gamma$ decays (P_T) or the component of the muon spin perpendicular to the decay planes, is an interesting observable to search for non–standard sources of time–reversal violations [1]. Measurements of P_T at the level of 10^{-3} are presently undergoing in both channels at the KEK E246 experiment [2], similar sensitivities could in principle be achieved also at DAΦNE [3] and a proposal to reach $\sigma(P_T) \sim 10^{-4}$ exists at BNL [2].

The transverse muon polarization can be different from zero only if the form factors of the corresponding decay amplitude have non–vanishing relative phases. These can be generated either by CP–violating couplings (assuming CPT conservation) or by absorptive effects due to electromagnetic final–state interactions (FSI). The CP–violating contribution is absolutely negligible within the Standard Model (SM) and therefore if detected at the level of 10^{-4} or above it could only be of non–standard origin. In specific New Physics models CP–violating contributions to P_T as large as 10^{-3} can be generated both in $K^+ \to \pi^0 \mu^+ \nu$ (see e.g. [4–6]) and in $K^+ \to \mu^+ \nu \gamma$ (see e.g. [7–9]). As pointed out by Wu and Ng [8], the two modes are complementary in testing possible New Physics scenarios.

Electromagnetic FSI effects in $K^+ \to \pi^0 \mu^+ \nu$ appear only at the two–loop level, leading to a CP–conserving contribution to P_T which is $\mathcal{O}(10^{-6})$ and thus safely negligible [12]. The situation is however different in $K^+ \to \mu^+ \nu \gamma$, where FSI effects are present already at the one–loop level and could potentially lead to $\mathcal{O}(10^{-3})$ effects, like in the case of $K_L \to \pi^- \mu^+ \nu$ [13]. In principle CP–conserving and CP–violating contributions to P_T could be disentangled by a measurement of this quantity in both K^+ and K^- CP–conjugated modes [13], however at present this is not experimentally feasible. A detailed theoretical estimate of the FSI contributions to $P_T(K^+ \to \mu^+ \nu \gamma)$, which is the main purpose of the present letter, is therefore needed in order to identify possible New Physics effects.

The paper is organized as follows: in section 2 we introduce observables and kinematics of $K^+ \to \mu^+ \nu \gamma$, we then proceed in section 3 estimating the dominant FSI contribution to P_T, induced by π^0 exchange. A brief discussion about possible supersymmetric contributions to P_T is presented in section 4 and the results are summarized in the conclusions.

2 Observables and kinematics

Assuming purely left–handed neutrinos, the matrix element of the transition $K^+(p) \rightarrow \mu^+(l)\nu(p_\nu)\gamma(q, \epsilon)$ can be generally decomposed as

$$\mathcal{M} = -\frac{ieG_F}{\sqrt{2}} \sin\theta_c \, \epsilon^*_\alpha \, \bar{u}(p_\nu)(1 + \gamma_5) \left[T_1^\alpha + \gamma_\beta T_2^{\alpha\beta} + i\sigma_{\beta\gamma} T_3^{\alpha\beta\gamma} \right] v(l) \,, \tag{1}$$

where we have factored–out explicitly the dependence on the Fermi constant (G_F) and the Cabibbo angle (θ_c). Within the SM, the tensors T_i are given by

$$T_1^\alpha = m_\mu f_K \left(\frac{p^\alpha}{q\cdot p} - \frac{l^\alpha}{q\cdot l} \right) \,, \tag{2}$$

$$T_2^{\alpha\beta} = \frac{F_A}{m_K} \left(g^{\alpha\beta} q\cdot p - p^\alpha q^\beta \right) - i\frac{F_V}{m_K} \epsilon^{\alpha\beta\rho\delta} q_\rho p_\delta \,, \tag{3}$$

$$T_3^{\alpha\beta\gamma} = m_\mu f_K \frac{g^{\alpha\gamma} q^\beta}{2q\cdot l} \,, \tag{4}$$

where $f_K \simeq 160$ MeV is the usual kaon decay constant and $F_{V,A}$ are form factors associated with the time–ordered product of electromagnetic and weak currents [10]. The one–loop estimate of $F_{V,A}$ in chiral perturbation theory leads to the constant values $F_V = -0.095$ and $F_A = -0.042$, which are consistent with experimental data [10]. As long as electromagnetic FSI and $\mathcal{O}(G_F^2)$ electroweak corrections are neglected, f_K and $F_{V,A}$ are real. On the other hand, new sources of CP violation beyond the Cabibbo–Kobayashi–Maskawa (CKM) matrix [11] could generate non–negligible phases to these form factors [8].

Introducing the adimensional variables

$$x = \frac{2p\cdot q}{m_K^2} \,, \qquad y = \frac{2p\cdot l}{m_K^2} \qquad \text{and} \qquad r_\mu = \frac{m_\mu^2}{m_K^2} \,, \tag{5}$$

the Dalitz plot distribution of the decay can be written as

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}x\mathrm{d}y} = \frac{m_K}{256\pi^3} \sum_{pol(\nu,\gamma,\mu)} |\mathcal{M}|^2 = \frac{G_F^2 m_K^5 \alpha \sin^2\theta_c}{32\pi^2} \rho(x, y) \,, \tag{6}$$

with the boundary conditions

$$0 \le \quad x \quad \le 1 - r_\mu \,,$$
$$\frac{r_\mu + (1 - x)^2}{1 - x} \le \quad y \quad \le 1 + r_\mu \,. \tag{7}$$

The full expression of the adimensional function $\rho(x, y)$, calculated with the T_i in (2 - 4), can be found in [8]. The inner bremsstrahlung (IB) contribution, obtained in the limit $F_V = F_A = 0$, is given by

$$\rho_{F_V=F_A=0}(x, y) \doteq \rho_{IB}(x, y) = 2r_\mu \frac{f_K^2}{m_K^2} \left(\frac{1 - y + r_\mu}{x^2(x + y - 1 - r_\mu)} \right)$$
$$\times \left(x^2 + 2(1 - x)(1 - r_\mu) - \frac{2xr_\mu(1 - r_\mu)}{x + y - 1 - r_\mu} \right) . \tag{8}$$

The triple product defining the transverse muon polarization in $K^+ \to \mu^+ \nu \gamma$ is

$$\mathcal{P}_T \doteq \frac{\vec{s} \cdot (\vec{q} \times \vec{l})}{|\vec{q} \times \vec{l}|} , \tag{9}$$

where $\vec{q}$, $\vec{l}$ and $\vec{s}$ denote the spatial components in the kaon rest frame of q, l and the spin vector of the muon s, respectively. As usual, the latter obeys the relations $s \cdot l = 0$ and $s^2 = -1$. In terms of four–dimensional vectors we can write

$$\mathcal{P}_T = \frac{2}{m_K^3} \frac{\epsilon_{\alpha\beta\gamma\delta} p^\alpha s^\beta q^\gamma l^\delta}{F(x, y)} \qquad (\epsilon_{0123} = +1) , \tag{10}$$

where

$$F(x, y) = \sqrt{(1 - y + r_\mu)\left[(1 - x)(x + y - 1) - r_\mu\right]} . \tag{11}$$

The modulus squared of $\mathcal{M}(K^+ \to \mu^+ \nu \gamma)$, summed over photon and neutrino polarizations, can be decomposed as

$$\sum_{pol(\nu,\gamma)} |\mathcal{M}|^2 = e^2 G_F^2 m_K^4 \sin^2 \theta_c \left[\rho(x, y) + \mathcal{P}_T \sigma(x, y) + \mathcal{O}(s \cdot p, s \cdot q)\right] , \tag{12}$$

therefore the ratio $\sigma(x, y)/\rho(x, y)$ is usually referred as the distribution of the transverse muon polarization [7,8]. We note, however, that this notation is misleading since $\sigma(x, y)/\rho(x, y)$ is not directly an observable. What can be measured in realistic experiments is, for instance, the expectation value of $\mathcal{P}_T$ averaged over a Dalitz plot region S, which is given by

$$\langle P_T \rangle_S \doteq \frac{\int_S d\Phi \sum_{pol(\nu,\gamma)} |\mathcal{M}|^2 \, \mathcal{P}_T}{\int_S d\Phi \sum_{pol(\nu,\gamma)} |\mathcal{M}|^2} = \frac{\int_S dxdy \, \sigma(x, y)G(x, y)}{\int_S dxdy \, \rho(x, y)} , \tag{13}$$

where[1]

$$G(y) = \frac{1}{2\beta^2}\left[1 + \frac{\beta^2 - 1}{2\beta} \log\left(\frac{1 + \beta}{1 - \beta}\right)\right] \qquad \text{and} \qquad \beta = \sqrt{1 - 4r_\mu/y^2} . \tag{14}$$

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[1] $d\Phi$ indicates the differential phase–space element and the second identity in (13) follows from integration over angular variables.

In the limit where the area S reduces to a point in the Dalitz plot, the integrals on the r.h.s. of (13) drop out and we obtain

$$P_T(x,y) = \frac{\sigma(x,y)G(x,y)}{\rho(x,y)} \; . \tag{15}$$

As it can be easily verified, $\sigma(x,y)$ is non vanishing only if the T_i have non–trivial phases. Assuming that New Physics contributions generate complex CP–violating phases for the form factors f_K and $F_{V,A}$ of (2 - 4), we obtain

$$\sigma_{NP}(x,y) = -2\sqrt{r_\mu}F(x,y)\left[\Im\left(\frac{f_K F_V^*}{m_K}\right)f_V(x,y) + \Im\left(\frac{f_K F_A^*}{m_K}\right)f_A(x,y)\right] \; , \tag{16}$$

where

$$\begin{aligned}
f_V(x,y) &= \frac{2-x-y}{x+y-1-r_\mu} \; , \\
f_A(x,y) &= \frac{(2-x)(x+y)-2(1+r_\mu)}{x(x+y-1-r_\mu)} \; ,
\end{aligned} \tag{17}$$

in agreement with the results of [7,8].

3 The CP–conserving contribution to P_T

As anticipated in the introduction, the CP–conserving contribution to P_T is generated by electromagnetic FSI which induce non–trivial phases in the T_i. The leading rescattering diagram contributing in $K^+ \to \mu^+\nu\gamma$ is the one shown in Fig. 1. Other absorptive contributions appearing only at the two–loop level can be safely neglected [12].

Since we are interested only in the absorptive contribution of the diagram in Fig. 1, the result is finite and can be calculated in terms of on–shell amplitudes for $\pi^0 \to \gamma\gamma$ and $K^+ \to \pi^0\mu^+\nu$. In our conventions these are given by

$$\mathcal{A}\left(\pi^0(p_\pi) \to \gamma(\epsilon_1,q_1)\gamma(\epsilon_2,q_2)\right) = +\frac{ie^2}{2\sqrt{2}\pi^2 f_\pi}\epsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu q_1^\nu \epsilon_2^\rho q_2^\sigma \; , \tag{18}$$

$$\mathcal{A}\left(K^+(p) \to \pi^0(p_\pi)\mu^+(l)\nu(p_\nu)\right) = -\frac{iG_F}{2}\sin\theta_c \; [f_+(p+p_\pi)^\mu + f_-(p-p_\pi)^\mu]$$
$$\times \bar{u}(p_\nu)(1+\gamma_5)\gamma_\mu v(l) \; . \tag{19}$$

The explicit analytical expression of $\mathcal{M}_{FSI}$ thus obtained is rather complicated and will not be given here, but it simplifies considerably in the limit $m_{\pi^0}^2/m_K^2 \to 0$. Considering the interference of $\mathcal{M}_{FSI}(m_{\pi^0} = 0)$ with the leading matrix element in (1) leads to

$$\sigma_{FSI}(x,y) = \frac{\alpha}{4\pi^2} \sqrt{r_\mu}F(x,y)\frac{f_K}{f_\pi}\left[f_+\sigma_1 + f_-\sigma_2 + \frac{m_K(F_A - F_V)}{f_K}f_-\sigma_3\right] \; , \tag{20}$$

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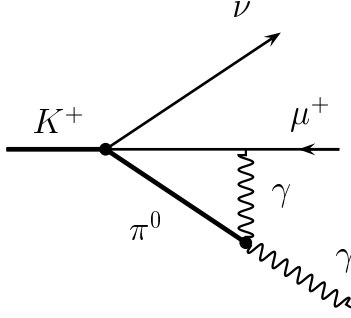Figure 1: The leading rescattering diagram contributing to the transverse muon polarization in $K^+ \to \mu^+ \nu \gamma$.

where

$$\sigma_1 = \frac{1}{4x(1-x-y)^2}(4 + r_\mu - 4x - 4y)(1 + r_\mu - 2x + x^2 - y + xy) , \quad (21)$$

$$\sigma_2 = \frac{r_\mu}{4x(1-x-y)^2}(1 + r_\mu - 2r_\mu(x+y) - x^2 - 3y + xy + 2y^2) , \quad (22)$$

$$\sigma_3 = \frac{1}{4(1-x-y)}(1 + r_\mu - y)(x + y - 1 - r_\mu) . \quad (23)$$

The value of P_T obtained inserting $\sigma_{FSI}(x,y)$ into (15) is plotted in Fig. 2 as a function of x and y. A comparison of the full result obtained with $m_\pi^0 \neq 0$ and the approximate one for $m_\pi^0 = 0$ is shown in Fig. 3. Both figures have been obtained using $f_+ = 1$, $f_- = 0$ and $f_K/f_\pi = 1.21$, in addition to the numerical values of $F_{A,V}$ specified in section 2. It is worthwhile to mention that the analytical result in (20) is very general and could be easily extended to the case of other radiative semileptonic processes, like for instance the $B^+ \to \tau^+ \nu \gamma$ decay.

As it can be read off from Fig. 2, the FSI contribution to $P_T(K^+ \to \mu^+ \nu \gamma)$ is of $\mathcal{O}(10^{-4})$ all over the phase space. This result, mainly due to the strong suppression factor $\alpha/4\pi^2$ in (20), implies that an experimental evidence of P_T at the level of 10^{-3} would be a clear signal of physics beyond the SM. Moreover, we note that the peak of $(P_T)_{FSI}$ near the boundary $y = 1 + r_\mu$ is simply due to the vanishing of the bremsstrahlung contribution in $\rho(x,y)$ (see Eq. (8)): if both F_A and F_V were vanishing, $(P_T)_{FSI}$ would have an unphysical divergence (occurring in a region with almost no events) for $y \to 1 + r_\mu$.
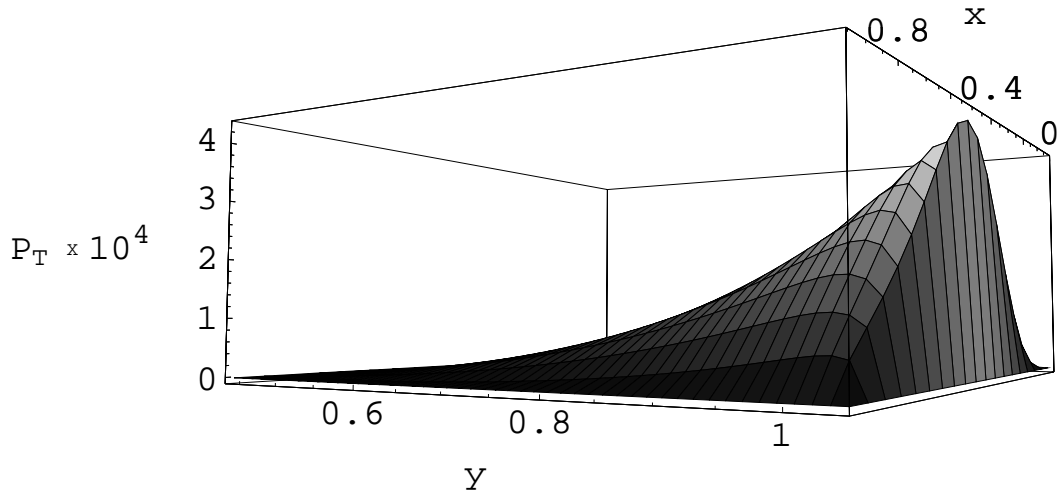
Figure 2: Distribution of the transverse muon polarization, as defined in (15), resulting from FSI and evaluated in the limit $m_{\pi^0} = 0$.
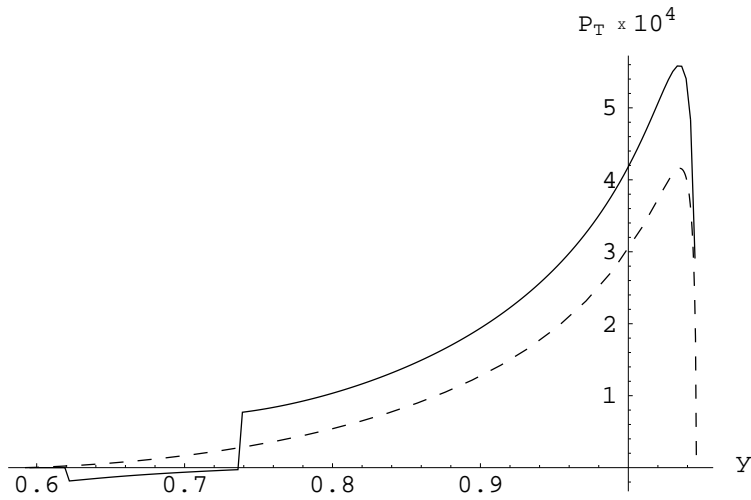

Figure 3: Distribution of the transverse muon polarization resulting from FSI for fixed $x = 0.5$ as a function of y. The solid curve correspond the the complete calculation, the dashed one is obtained in the limit $m_{\pi^0} = 0$.

4 Supersymmetric CP–violating contributions to P_T

Supersymmetric extensions of the SM with unbroken R–parity, minimal particle content and generic flavour couplings represent a very attractive possibility from the theoretical point of view. In this context several new CP–violating phases are introduced (see e.g. [14] and references therein), therefore sizable T–violating contributions to P_T could in principle be expected. The question we shall try to answer in the following is what is the maximum value of these possible T–violating effects.

Supersymmetric contributions to the transverse muon polarization in $K^+ \to \mu^+ \nu \gamma$ have been extensively discussed by Wu and Ng [8]. According to these authors, the possibly largest contribution results from the effective $W \bar{s}_R u_R$ vertex induced by squark–gluino penguins. In the framework of the mass–insertion approximation and employing the so–called super CKM basis [14], the leading contribution to the effective $W \bar{s}_R u_R$ vertex is generated at the second order in the mass expansion by double $\tilde{q}_L^i - \tilde{q}_R^j$ mixing. Indeed, in this framework the result of [8] can be re–written as

$$\Im \left(\frac{f_K F_V^*}{m_K} \right)_{W \bar{s}_R u_R} = -\frac{\alpha_s(M_{\tilde{q}})}{18 \pi \sin \theta_c} \left[x^2 I_0(x) \right] \Im \left[\left(\delta_{LR}^D \right)_{23} \left(\delta_{LR}^U \right)_{13}^* \right] , \qquad (24)$$

where $M_{\tilde{q}}$ ($M_{\tilde{g}}$) indicates the average squark (gluino) mass, $x = M_{\tilde{q}}^2 / M_{\tilde{g}}^2$, $I_0(x)$ is defined in [8][2] and, denoting by $M_{[U,D]}^2$ the squark mass matrices,

$$\left(\delta_{LR}^{[U,D]} \right)_{ab} = \left(M_{[U,D]}^2 \right)_{a_R b_L} / M_{\tilde{q}}^2 . \qquad (25)$$

Interestingly, the double–mixing mechanism leading to (24) is very similar to the one occurring in the chargino–squark contribution to the effective $Z \bar{s}_L d_L$ vertex, relevant for rare decays and recently discussed in [15]. In both cases the supersymmetric contribution is potentially large since: i) there is no explicit $1/M_{\tilde{q}}$ suppression in the amplitudes, ii) there are no suppressed CKM matrix elements and iii) the third generation of squarks is involved. However, the charged–current nature of the $W \bar{s}_R u_R$ amplitude implies that one of the two $\tilde{q}_L^i - \tilde{q}_R^j$ mixing terms must be in the down sector. This makes a substantial difference among $W \bar{s}_R u_R$ and $Z \bar{s}_L d_L$ amplitudes, indeed the coupling $(\delta_{LR}^D)_{23}$ appearing in (24) is much more constrained than the corresponding one relevant for the $Z \bar{s}_L d_L$ vertex, namely $(\delta_{LR}^U)_{23}$. Vacuum–stability bounds [17] and phenomenological constraints from the $b \to s \gamma$ decay [16] naturally leads to

$$\left| \left(\delta_{LR}^D \right)_{23} \left(\delta_{LR}^U \right)_{13}^* \right| \lesssim O(10^{-2}) . \qquad (26)$$

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[2] $I_0(1) = 1$ and $[x^2 I_0(x)] \sim 1$ for reasonable values of x.

Therefore, unless extremely fine–tuned mechanisms are invoked, we estimate the r.h.s. of (24) to be at most of $\mathcal{O}(10^{-4})$. Using Eq. (16) we then obtain an upper bound for the contribution of the gluino–mediated $W\bar{s}_R u_R$ vertex to $|P_T(K^+ \to \mu^+\nu\gamma)|$ of about 10^{-4}, that is more than two orders of magnitude smaller than reported in [8].

Similar comments apply to the other mechanism discussed in [8], namely the effective $H\bar{s}_L u_R$ vertex. Also in this case the constraints imposed by the $b \to s\gamma$ transition put severe fine–tuning problems. We then conclude that in the framework of minimal R–parity conserving supersymmetric models it is very unnatural to generate contributions to $P_T(K^+ \to \mu^+\nu\gamma)$ larger than 10^{-4}. Analogous conclusions have already been obtained by Fabbrichesi and Vissani in the case of $K^+ \to \pi^0\mu^+\nu$ [6].

5 Conclusions

We have presented a detailed estimate of the transverse muon polarization due to electromagnetic final–state interactions in the decay $K^+ \to \mu^+\nu\gamma$. Our analysis shows that $(P_T)_{FSI}$ is of $\mathcal{O}(10^{-4})$ all over the phase space and that the largest values are obtained in the less populated region of the Dalitz plot. These results imply that an experimental evidence of P_T at the level of 10^{-3}, or above, would be a clear signal of physics beyond the SM.

We have further argued why it is very difficult to accommodate values of $|P_T(K^+ \to \mu^+\nu\gamma)|$ larger than 10^{-4} in the framework of minimal R–parity conserving supersymmetric models. If a non–vanishing value of $P_T(K^+ \to \mu^+\nu\gamma)$ were observed in the forthcoming experiments, this could be more likely understood in the framework of R–parity breaking models [9] or models with a non–standard Higgs sector [7].

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