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Precision tests and searches for New Physics with K decays

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Abstract

A short overview of FCNC and CP–violating observables in K decays is presented. Particular attention is paid to the possibility of performing precision tests of flavour dynamics and to the search for New Physics.

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1 Introduction

The study of kaon decays has historically provided one of the richest source of information in the construction of the Standard Model (SM). Above all, let's recall the discovery of P and CP violation, as well as the indirect indication of the existence of charm. Moreover, at present some of the most stringent constraints which any extension of the SM has to face on flavour mixing, CP violation and CPT conservation are derived from kaon physics. But what is even more fascinating is the fact that in the near future, 50 years after their discovery, kaon decays could still offer a valuable and *unique* probe to test the SM and to search for New Physics (NP) [1].

In general, we can separate in three wide classes the observables which it is still very important to measure with increasing accuracy:

1. *Pure NP searches.* The observables belonging to this class are those vanishing or extremely small within the SM, like the widths of the lepton–flavour violating modes ($K_L \to \mu e$, $K \to \pi \mu e, \ldots$) or the transverse muon polarization in $K^+ \to \pi^0 \mu^+ \nu_\mu$ (see e.g. Rizzo in [1] and references therein). The first ones are completely forbidden within the SM whereas the latter is expected to be much smaller than the experimental sensitivity. In these cases a non–vanishing experimental evidence would provide a clear signal for physics beyond the SM, however a positive result is not guaranteed.

2. *Precision SM measurements.* Under this name we group the observables which are completely dominated by SM contributions but are calculable with high accuracy in terms of fundamental parameters. An interesting example in this sector is provided by the $\pi\pi$ scattering lengths, measurable from K_{l4} decays, which can be expressed in terms of the expectation value of the quark condensate in the chiral limit [2]. Similarly, K_{l3} decays provide precise information about the Cabibbo angle and quark–mass ratios [2].

3. *Short–distance observables.* In this category we finally collect the CP–violating and FCNC observables which are calculable with high accuracy in terms of short–distance amplitudes, like the widths of $K \to \pi \nu \bar{\nu}$ decays. This group is probably the most interesting one since it is useful both to test the flavour structure of the SM and also to search for NP. In the following we will concentrate only on this sector, trying to emphasize the cleanliness from long–distance effects and the NP sensitivity of various observables.

2 FCNC rare decays within the SM

The rare transitions $K \to \pi\nu\bar{\nu}$, $K \to \ell^+\ell^-$ and $K \to \pi\ell^+\ell^-$ are naturally good candidates to extract information on the FCNC amplitude $s_L \to d_L f_L \bar{f}_L$ $(f = \nu, \ell)$. Within the SM this amplitude is generated only at the quantum level, through Z–penguin and W–box diagrams, and is particularly interesting because of the dominant role played by the top–quark exchange. Separating the contributions to the amplitude according to the intermediate up–type quark running inside the loop, one can write

$$\mathcal{A}(s_L \to d_L f_L \bar{f}_L) = \sum_{q=u,c,t} V_{qs}^* V_{qd}\, \mathcal{A}_q \ , \tag{1}$$

where V_{ij} denote the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix [3]. The hierarchy of the CKM matrix [4] would favor the first two terms in (1) however the hard GIM mechanism of the parton–level calculation implies $\mathcal{A}_q \sim m_q^2/M_W^2$, which leads to a completely different scenario: assuming the standard phase convention ($\Im V_{us} = \Im V_{ud} = 0$) and expanding the CKM elements in powers of the Cabibbo angle ($\lambda = 0.22$) [4], one finds

$$V_{qs}^* V_{qd}\, \mathcal{A}_q \ \sim \ \left\{ \begin{array}{ll} \mathcal{O}(\lambda^5 m_t^2) \ + i\ \mathcal{O}(\lambda^5 m_t^2) & (q = t)\ , \\ \mathcal{O}(\lambda m_c^2) \ + i\ \mathcal{O}(\lambda^5 m_c^2) & (q = c)\ , \\ \mathcal{O}(\lambda \Lambda_{QCD}^2) & (q = u)\ . \end{array} \right. \tag{2}$$

As can be noticed, the top–quark contribution dominates both real and imaginary parts of the amplitude (the Λ_{QCD}^2 factor in the last line follows from a naive estimate of long–distance effects associated to the up–quark exchange). This implies several interesting consequences for $\mathcal{A}(s_L \to d_L f_L \bar{f}_L)$: i) it is dominated by short–distance dynamics and therefore calculable with high precision in perturbation theory; ii) it is very sensitive to V_{td}, which is one of the less constrained CKM matrix elements; iii) it is likely to have a large CP–violating phase; iv) it is very suppressed within the SM and thus very sensitive to possible NP effects.

The short–distance contribution to $\mathcal{A}(s_L \to d_L f_L \bar{f}_L)$ can be efficiently described by means of a single effective dimension–6 operator: $O_{LL}^f = (\bar{s}_L \gamma^\mu d_L)(\bar{f}_L \gamma_\mu f_L)$. The Wilson coefficients of O_{LL}^f have been calculated by Buchalla and Buras including next–to–leading–order QCD corrections [5] (see also [6,7]), leading to a very precise description of the partonic amplitude. Moreover, the simple structure of O_{LL}^f has two major advantages: i) the relation between partonic and hadronic amplitudes in the above mentioned rare decays is quite accurate, since the hadronic matrix elements of the $(\bar{s}_L \gamma^\mu d_L)$ current between a kaon and a pion (or the vacuum) are related by isospin symmetry to those entering K_{l3} (or K_{l2}) decays, which are experimentally well known; ii) the lepton pair is produced in a state of definite CP and angular momentum ($J^{CP} = 1^-$) implying, for instance, that the leading contribution of $\mathcal{A}(s_L \to d_L f_L \bar{f}_L)$ to $K_L \to \pi^0 f \bar{f}$ is CP violating.

The short–distance contributions of the $s_L \to d_L f_L \bar{f}_L$ amplitude to $K \to \pi\nu\bar{\nu}$, $K \to \ell^+\ell^-$ and $K \to \pi\ell^+\ell^-$ is therefore very well under control. The remaining question to address in order to quantify their potential in testing flavour dynamics is the estimate of other possible contributions. For instance in the case of $K \to \pi\ell^+\ell^-$ an important role is certainly played by the $s_L \to d_L \ell_V \bar{\ell}_V$ amplitude, due to electromagnetic interactions. Then in all decays there is the question of possible long–distance contaminations. In the following we shall discuss in more detail the potential sources of uncertainties for the various channels.

2.1 $K \to \pi\nu\bar{\nu}$

These modes are particularly clean since neutrinos couple to quarks only via W and Z exchange, thus the only non–vanishing contribution to the decay is provided by the $s_L \to d_L \nu_L \bar{\nu}_L$ amplitude discussed above.

In the charged channel ($K^+ \to \pi^+\nu\bar{\nu}$) the dominant theoretical error is related to the uncertainty of the QCD corrections to $\mathcal{A}_c$ (see [7] for an updated discussion), which can be translated into a 5% error in the determination of $|V_{td}|$ from $B(K^+ \to \pi^+\nu\bar{\nu})$. This QCD uncertainty can be considered as generated by 'intermediate–distance' dynamics; genuine long–distance effects associated to $\mathcal{A}_u$ have been shown to be substantially smaller [8].

The case of $K_L \to \pi^0\nu\bar{\nu}$ is even more clean from the theoretical point of view [9]. Indeed, because of the CP structure, the leading contribution to the decay amplitude generated by dimension–6 operators is proportional to the imaginary parts in (1). This implies that in the dominant (direct–CP–violating) part of the amplitude the charm contribution is completely negligible with respect to the top one, where the uncertainty of the QCD corrections is around 1%. Intermediate and long–distance contributions to this decay are essentially confined only to the indirect–CP–violating contribution ($K_L \to K_S \to \pi^0\nu\bar{\nu}$ [10]) and to the CP–conserving one (generated at short distances by higher–dimensional operators [11]) which are both extremely small. Taking into account also the isospin–breaking corrections to the hadronic matrix elements [12], one can therefore write a very accurate expression (with a theoretical error around 1%) for $B(K_L \to \pi^0\nu\bar{\nu})$ in terms of short–distance parameters [7,10]:

$$B(K_L \to \pi^0\nu\bar{\nu})_{SM} \;=\; 4.25 \times 10^{-10} \left[\frac{\overline{m}_t(m_t)}{170 \text{ GeV}}\right]^{2.3} \left[\frac{\Im\lambda_t}{\lambda^5}\right]^2 . \qquad (3)$$

The high accuracy of the theoretical predictions of $B(K^+ \to \pi^+\nu\bar{\nu})$ and $B(K_L \to$

$\pi^0\nu\bar{\nu}$) in terms of the modulus and the imaginary part of $\lambda_t = V_{ts}^* V_{td}$ could clearly offer the possibility of very interesting tests of the CKM mechanism. Indeed, a measurement of both channels would provide two independent information on the unitarity triangle (or equivalently on the ρ–η plane [4]), which can be probed also by B–physics observables. In particular, as emphasized in [7], the ratio of the two branching ratios could be translated into a determination of $\sin(2\beta)$, the CP–violating observable measurable in a clean way also from $B^0(\bar{B}^0) \to J/\Psi K_S$. A comparison of the two measurements would then provide a very powerful tool to search for NP.

Taking into account all the indirect constraints on V_{ts}^* and V_{td} obtained within the SM, the present range of the SM predictions for the two branching ratios is given by [7]:

$$
\begin{aligned}
B(K^+ \to \pi^+\nu\bar{\nu})_{SM} &= (0.82 \pm 0.32) \times 10^{-10} \,, && (4)\\
B(K_L \to \pi^0\nu\bar{\nu})_{SM} &= (3.1 \pm 1.3) \times 10^{-11} \,, && (5)
\end{aligned}
$$

to be compared with the recent experimental results:

$$
\begin{aligned}
B(K^+ \to \pi^+\nu\bar{\nu}) &= 4.2^{+9.7}_{-3.5} \times 10^{-10}\ [13] \,, && (6)\\
B(K_L \to \pi^0\nu\bar{\nu}) &< 1.6 \times 10^{-6}\ [14] \,. && (7)
\end{aligned}
$$

2.2 $K \to \ell^+\ell^-$ and $K \to \pi\ell^+\ell^-$

In the decays involving charged leptons the problem of long–distance effects becomes much more important because of the presence of electromagnetic interactions. In general we can distinguish three classes of electromagnetic long–distance amplitudes:

1. *One–photon exchange.* This mechanism provides the by far dominant contribution to the CP–allowed transitions $K^+ \to \pi^+\ell^+\ell^-$ and $K_S \to \pi^0\ell^+\ell^-$ [15] (see [16] for an updated discussion). The former has been observed, both in the electron and in the muon mode, whereas only an upper bound of about 10^{-6} exists on $B(K_S \to \pi^0 e^+e^-)$ [17]. Unfortunately chiral symmetry alone does not help to relate $B(K^+ \to \pi^+\ell^+\ell^-)$ and $B(K_S \to \pi^0 e^+e^-)$, and without model–dependent assumptions one can only set a theoretical upper bound of about 10^{-8} on the latter [16].

   In the case of $K_L \to \pi^0\ell^+\ell^-$ the long–distance part of the one–photon exchange amplitude is forbidden by CP invariance but it contributes to the decay via K_L–K_S mixing, leading to

$$
B(K_L \to \pi^0 e^+e^-)_{CPV-ind} = 3 \times 10^{-3}\, B(K_S \to \pi^0 e^+e^-) \,. \qquad (8)
$$

On the other hand, the direct–CP–violating part of the decay amplitude is very similar to the one of $K_L \to \pi^0\nu\bar{\nu}$ but for the fact that it receives an additional short–distance contribution by the photon penguin. This theoretically clean part of the amplitude leads to [18]

$$B(K_L \to \pi^0 e^+e^-)^{SM}_{CPV-dir} \; = \; 0.69 \times 10^{-10} \; \left[\frac{\overline{m}_t(m_t)}{170 \text{ GeV}}\right]^2 \; \left[\frac{\Im\lambda_t}{\lambda^5}\right]^2 \; . \qquad (9)$$

The two CP–violating components of the $K_L \to \pi^0 e^+e^-$ amplitude will in general interfere. Given the present uncertainty on $B(K_S \to \pi^0 e^+e^-)$, at the moment we can only set the rough upper limit

$$B(K_L \to \pi^0 e^+e^-)^{SM}_{CPV-tot} \; \lesssim \; \text{few} \times 10^{-11} \qquad (10)$$

on the sum of all the CP–violating contributions to this mode (the present experimental upper bound is about two orders of magnitude larger [17]). We stress, however, that the phases of the two CP–violating amplitudes are well know. Thus if $B(K_S \to \pi^0 e^+e^-)$ will be measured, it will be possible to determine the interference between direct and indirect CP–violating components of $B(K_L \to \pi^0 e^+e^-)_{CPV}$ up to a sign ambiguity.

2. *Two–photon exchange in S wave.* This amplitude plays an important role in $K_L \to \ell^+\ell^-$ transitions. In the $K_L \to e^+e^-$ case it is by far the dominant contribution and it can be estimated with a relatively good accuracy in terms of $\Gamma(K_L \to \gamma\gamma)$. This leads to the prediction $B(K_L \to e^+e^-) \sim 9 \times 10^{-12}$ [19] which recently seems to have been confirmed by the four $K_L \to e^+e^-$ events observed at BNL–E871 [20].

   More interesting from the short–distance point of view is the case of $K_L \to \mu^+\mu^-$. Here the two–photon long–distance amplitude is still large but the short–distance one, generated by the real part of $\mathcal{A}(s_L \to d_L\mu_L\bar{\mu}_L)$ and thus sensitive to $\Re V_{td}$ [5], is comparable in size. Unfortunately the dispersive part of the two–photon contribution is much more difficult to be estimated in this case, due to the stronger sensitivity to the $K_L \to \gamma^*\gamma^*$ form factor. Despite the precise experimental determination of $B(K_L \to \mu^+\mu^-)$, the present constraints on $\Re V_{td}$ from this observable are not very interesting [21]. Nonetheless, the measurement of $B(K_L \to \mu^+\mu^-)$ is still useful to put stringent bounds on possible NP contributions. Moreover, we stress that the uncertainty of the $K_L \to \gamma^*\gamma^* \to \mu^+\mu^-$ amplitude could be partially decreased in the future by precise experimental information on the form factors of $K_L \to \gamma\ell^+\ell^-$ and $K_L \to e^+e^-\mu^+\mu^-$ decays, especially if these would be consistent with the parameterization of the $K_L \to \gamma^*\gamma^*$ form factor proposed in [21].

3. *Two–photon exchange in D wave.* This final amplitude (the smallest of the three) is interesting since it produces a non–helicity–suppressed CP–conserving contribution to $K_L \to \pi^0 e^+ e^-$ [22]. This contribution does not interfere with the CP–violating one in the total rate and leads to $B(K_L \to \pi^0 e^+ e^-)_{CPC} \sim$ few $\times$ 10^{-12}. At the moment it is not easy to perform accurate predictions of $B(K_L \to \pi^0 e^+ e^-)_{CPC}$, however, precise experimental information on the di–photon spectrum of $K_L \to \pi^0 \gamma\gamma$ at low $m_{\gamma\gamma}$ could help to clarify the situation [22]. Moreover, the Dalitz plot distribution of CPV and CPC contributions to $K_L \to \pi^0 e^+ e^-$ are substantially different: in the first case the e^+e^- pair is in a P wave, whereas in the latter it is in a D wave. Thus in principle it is possible to extract experimentally the interesting $B(K_L \to \pi^0 e^+ e^-)_{CPV}$ from an observation of various $K_L \to \pi^0 e^+ e^-$ events.

3 $K \to \pi\nu\bar{\nu}$ and $K_L \to \pi^0 e^+ e^-$ beyond the SM

As we have seen in the previous section, the branching ratios of $K_L \to \pi^0\nu\bar{\nu}$, $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0 e^+ e^-$[1] could give us valuable and precise information about flavour mixing. Within the SM this is ruled by the CKM mechanism, which implies the strong $\mathcal{O}(\lambda^5)$ suppression of $\mathcal{A}(s_L \to d_L f_L \bar{f}_L)$ and leads to the small predictions in (4–5) and (10). It is therefore natural to expect that these observables are very sensitive to possible extensions of the SM in the flavour sector.

As long as we are interested only in NP effects to rare FCNC processes, we can roughly distinguish the extensions of the SM into two big groups: those involving new sources of flavour mixing (like generic SUSY extensions of the SM, models with new generations of quarks, etc...) and those where the flavour mixing is still ruled by the CKM matrix (like the 2–Higgs–doublet model of type II, constrained SUSY models, etc...). In the second case the effect to rare decays is typically small, at most of the same order of magnitude as the SM contribution (see e.g. [23,24] for some recent discussions). On the other hand, in the first case it is easy to generate sizable effects, leading to large enhancements with respect to the SM rates (see e.g. [25] and [26]).

Interestingly, despite the variety of NP models, it is possible to derive a model–independent relation among the widths of the three neutrino modes [27]. Indeed, the isospin structure of any $s \to d$ operator bilinear in the quark fields implies

$$\Gamma(K^+ \to \pi^+\nu\bar{\nu}) = \Gamma(K_L \to \pi^0\nu\bar{\nu}) + \Gamma(K_S \to \pi^0\nu\bar{\nu}) \,, \tag{11}$$

[1] The measurement of $B(K_L \to \pi^0 e^+ e^-)$ should be supplemented by a Dalitz plot analysis and a determination or a stringent experimental bound on $B(K_S \to \pi^0 e^+ e^-)$.

up to small isospin–breaking corrections, which then leads to

$$B(K_L \to \pi^0 \nu\bar{\nu}) \ < \ \frac{\tau_{K_L}}{\tau_{K^+}} B(K^+ \to \pi^+ \nu\bar{\nu}) \ . \tag{12}$$

Any experimental limit on $B(K_L \to \pi^0 \nu\bar{\nu})$ below this bound can be translated into a non–trivial dynamical information on the structure of the $s \to d\nu\bar{\nu}$ amplitude. Using the experimental result in (6), the present model–independent bound on $B(K_L \to \pi^0 \nu\bar{\nu})$ is about 6×10^{-9} (more than two orders of magnitude larger than the SM value!).

Unfortunately there is no analog model–independent bound for $K_L \to \pi^0 e^+ e^-$. However, to compare the NP sensitivity of $K_L \to \pi^0 \nu\bar{\nu}$ and $K_L \to \pi^0 e^+ e^-$, we note that in the specific scenario where the dominant contribution to both processes is generated by an effective $Z\bar{s}d$ vertex, one expects $B(K_L \to \pi^0 e^+ e^-) \simeq B(K_L \to \pi^0 \nu\bar{\nu})/6$ [25].

3.1 Supersymmetric contributions

We will now discuss in more detail the rare FCNC transitions in the framework of a low–energy supersymmetric extension of the SM –with unbroken R parity, minimal particle content and generic flavour couplings– which represents a very attractive possibility from the theoretical point of view. Similarly to the SM, also in this case FCNC amplitudes are generated only at the quantum level. However, in addition to the standard penguin and box diagrams, also their corresponding superpartners, generated by gaugino–squarks loops, play an important role. In particular, the chargino–up–squarks diagrams provide the potentially dominant non–SM effect to the $s \to d\nu\bar{\nu}(\ell^+\ell^-)$ amplitude [28]. Moreover, in the limit where the average mass of SUSY particles (M_S) is substantially larger than M_W, the penguin diagrams tend to dominate over the box ones and the dominant SUSY effect can be encoded through an effective $Z\bar{s}_L d_L$ coupling [25].

The flavour structure of a generic SUSY model is quite complicated and a convenient way to parametrize the various flavour—mixing terms is provided by the so–called mass–insertion approximation [29]. This consists of choosing a simple basis for the gauge interactions and, in that basis, to perform a perturbative expansion of the squark mass matrices around their diagonal. The same approach could be employed also within the SM, rotating for instance the u^i_L fields and choosing the basis where the $W - d_L - u^d_L$ coupling is diagonal. In this case it would be easy to verify that the dominant contribution to the $Z\bar{s}_L d_L$ vertex is generated at the second order in the mass expansion by a double $q^i_L - q^j_R$ mixing, namely $(u^d_L - t_R) \times (t_R - u^s_L)$. The two off–diagonal mass terms would indeed be proportional to $m_t V_{td}$ and $m_t V^*_{ts}$. As shown in [25], this "second–order structure" remains valid also for the SUSY (chargino–up–squarks) contributions. In this case the situation is slightly more complicated due to the interplay between the standard CKM matrix (ruling the higgsino–$q_L^i - \tilde{q}_R^j$ vertex) and a new matrix responsible for the $\tilde{q}_L^i - \tilde{q}_R^j$ mixing [28]. It is indeed possible to consider terms with a single off–diagonal CKM element and a single $\tilde{q}_L^i - \tilde{q}_R^j$ mixing. However, in perfect analogy with the SM case, the potentially dominant SUSY contribution arises from the double mixing $(\tilde{u}_L^d - \tilde{t}_R) \times (\tilde{t}_R - \tilde{u}_L^s)$ [25]. This leads to an effective $Z \bar{s}_L d_L$ vertex proportional to

$$\tilde{\lambda}_t = \frac{(\widetilde{M}_U^2)_{s_L t_R} (\widetilde{M}_U^2)_{t_R d_L}}{M_S^4} \,, \tag{13}$$

which can be considered as the analog of the SM factor $\lambda_t (m_t^2/M_W^2)$.

The phenomenological constraints on $\tilde{\lambda}_t$ can be divided into two groups:

1. indirect M_S–dependent bounds on $(\widetilde{M}_U^2)_{s_L t_R}$ and $(\widetilde{M}_U^2)_{t_R d_L}$, dictated mainly by vacuum–stability, neutral–meson mixing ($K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$ and $B^0 - \bar{B}^0$) and $b \to s\gamma$;

2. direct limits on the $Z \bar{s}_L d_L$ coupling dictated by $K_L \to \mu^+\mu^-$ and $\Re(\epsilon'/\epsilon)$, constraining $\Re\tilde{\lambda}_t$ and $\Im\tilde{\lambda}_t$, respectively.

In a wide range of M_S (0.5 TeV $\lesssim M_S \lesssim 1$ TeV) the first type of bounds are rather weak and leave open the possibility for large effects in rare decays. In particular, $\Gamma(K^+ \to \pi^+ \nu\bar{\nu})$ could be enhanced up to one order of magnitude with respect to the SM prediction, whereas for $\Gamma(K_L \to \pi^0 \nu\bar{\nu})$ and $\Gamma(K_L \to \pi^0 e^+ e^-)$ the enhancement could even be higher [25]. Concerning the direct constraints, the bound on $\Re\tilde{\lambda}_t$ from $K_L \to \mu^+\mu^-$ is certainly quite stringent [30], however one could still generate the above large enhancements with an almost imaginary $\tilde{\lambda}_t$ (actually this is a necessary condition to enhance the CP–violating modes).

Buras and Silvestrini recently claimed that the possibility of a large $\Im\tilde{\lambda}_t$ is substantially reduced by the constraints from $\Re(\epsilon'/\epsilon)$ [30]. According to these authors, the enhancement of the rare widths can be at most of one order of magnitude in $\Gamma(K_L \to \pi^0 e^+ e^-)$ and not more than a factor ~ 3 in $\Gamma(K^+ \to \pi^+ \nu\bar{\nu})$. We agree with them that in principle the measurement of $\Re(\epsilon'/\epsilon)$ provides a bound on $\Im\tilde{\lambda}_t$, however we are more skeptical about the precise value of this bound at present. As we shall discuss more extensively in the next section, the problem with $\Re(\epsilon'/\epsilon)$ is that on one side the SM prediction is affected by large theoretical uncertainties, on the other side this observable is sensitive also to other SUSY effects, which could partially cancel the contribution of $\Im\tilde{\lambda}_t$. In addition, even the experimental results concerning $\Re(\epsilon'/\epsilon)$ are not very clear at present [17]. Probably the situation will improve in the future, but at the moment the extraction of bounds on the $Z \bar{s}_L d_L$ vertex from $\Re(\epsilon'/\epsilon)$ requires some additional assumptions. On the contrary, we stress that the direct constraints which could be obtained from the rare decays, even if less stringent, would be much more clear from the theoretical point of view.

4 ϵ'/ϵ within and beyond the SM

The ϵ'/ϵ parameter can be defined as

$$\frac{\epsilon'}{\epsilon} = \frac{e^{i(\pi/2+\delta_2-\delta_0)}}{\epsilon}\frac{\omega}{\sqrt{2}}\left[\frac{\Im A_2}{\Re A_2}-\frac{\Im A_0}{\Re A_0}\right] , \tag{14}$$

where $A_{0,2}$ denote the $K^0 \to (2\pi)_{0,2}$ amplitudes, $\delta_{0,2}$ the corresponding strong phases, $\omega = \Re A_2/\Re A_0 \simeq 1/22$ and ϵ is the standard $\Delta S = 2$ CP–violating term. A measurement of ϵ'/ϵ can provide very interesting information about the global symmetries of the SM. Indeed, as it is well known, an evidence for $\epsilon'/\epsilon \neq 0$ would be a clear signal of direct CP violation [31]. Moreover, given that arg$(\epsilon) = \pi/4 \simeq \pi/2 + \delta_2 - \delta_0$, the phase of ϵ'/ϵ is almost vanishing, implying $|\Im(\epsilon'/\epsilon)| \ll |\Re(\epsilon'/\epsilon)|$. This relation can be modified only by adding CPT non–invariant terms in $K \to 2\pi$ amplitudes and thus can be used to test CPT invariance [31].

More problematic is the question of what kind of short–distance information can be extracted from ϵ'/ϵ and thus to what extent this observable can be used to perform precision tests of the SM in the flavour sector. Similarly to the rare $FCNC$ transitions, also the weak phases of A_0 and A_2 are generated only at the quantum level and are very sensitive to the structure of the CKM matrix. The short–distance information about these amplitudes are usually encoded in the Wilson coefficients of appropriate four–quark operators, which can be calculated with a good accuracy down to scales $\mu \gtrsim m_c$ [32,33]. However, contrary to the rare decays, in the case of $K \to 2\pi$ transitions it is very difficult to evaluate the hadronic matrix elements of the effective operators.

At the quark level $\Im A_0$ is dominated by the gluon penguin whereas $\Im A_2$ by the electroweak ones. In both cases the dominant contribution is provided by four–quark operators of the type $(\bar{s}^\alpha_L\gamma^\mu d^\beta_L)\sum_q y_q(\bar{q}^\beta_R\gamma_\mu q^\alpha_R)$, namely O_6 for $\Im A_0$ ($y_q = 1$) and O_8 for $\Im A_2$ ($y_q = e_q$), which have enhanced matrix elements in the chiral limit. A useful approximate expression for $\Re(\epsilon'/\epsilon)$ can be obtained by showing explicitly the dependence on the matrix elements of these two operators [30,34]:

$$\Re\left(\frac{\epsilon'}{\epsilon}\right)_{SM} = \left[-1.4 + 8.2\left(R_s B_6^{(1/2)}\right) - 4.0\left(R_s B_8^{(3/2)}\right)\right] \times \Im\lambda_t \, . \tag{15}$$

Here $R_s = [158\ \mathrm{MeV}/(m_s(m_c)+m_d(m_c))]^2$ shows the leading dependence on the quark masses of the two matrix elements, whereas their actual value is hidden in the B–factors

$B_6^{(1/2)}$ and $B_8^{(3/2)}$, expected to be positive and $\mathcal{O}(1)$. The uncertainty in the numerical coefficients of (15) is expected to be around or below 20% [34] (see also Buras in [1]).

Various estimates of R_s and of the B–factors can be found in the literature, leading to results for $\Re(\epsilon'/\epsilon)_{SM}$ which range essentially between 0 and 3×10^{-3} [34–36]. Certainly some non–perturbative techniques are more reliable than others, however in all cases it is very difficult to provide quantitative estimates of the errors, especially in the case of the B–factors. Lattice results, for example, are based on the lowest–order chiral relation between $\langle K|O_i|2\pi\rangle$ and $\langle K|O_i|\pi\rangle$, and could be affected by sizable corrections due to next–to–leading terms in the chiral expansion. Interesting progress in calculating hadronic matrix elements have recently been made in the framework of the $1/N_c$ expansion [37,38], nonetheless even there we are still far from precise results, especially in the case of O_6 and O_8.

Given the above considerations, it is clear that at present $\Re(\epsilon'/\epsilon)$ cannot be used to perform precision tests of the SM. In the context of NP scenarios, one can generally expect two main effects in $\Re(\epsilon'/\epsilon)$: i) a modification of the phase of the gluon–penguin amplitude and thus of $\Im A_0$, ii) a modification of the phase of the electroweak–penguin amplitude and thus of $\Im A_2$. As we have shown in the previous section, the second effect could be bounded independently also from the rare processes $K_L \to \pi^0\nu\bar{\nu}$ and $K_L \to \pi^0 e^+e^+$. In the future $\Re(\epsilon'/\epsilon)$ could therefore provide an interesting complementary window for NP searches in $\Delta S = 1$ amplitudes. However, this would require better experimental bounds on both rare modes and $\Re(\epsilon'/\epsilon)$ and, possibly, also better theoretical control on the B–factors.

5 Conclusions

The $K \to \pi\nu\bar{\nu}$ decays provide a unique opportunity to perform high precision tests of CP violation and flavour mixing, both within and beyond the SM. In some NP scenarios, even in the case of generic supersymmetric extensions of the SM, sizable enhancements to $B(K \to \pi\nu\bar{\nu})$ are possible and, if detected, these could provide the first evidence for physics beyond the SM. However, even if NP will be discovered before via direct searches, we stress that precise measurements of these rare modes will provide unique information about the flavour structure of any extension of the SM.

Among the $K \to X_d \ell^+\ell^-$ decays, the most interesting one from the short–distance point of view is probably $K_L \to \pi^0 e^+e^-$. In order to extract precise information from this mode, the measurement of its decay rate should be accompanied by a Dalitz plot analysis and a determination or a stringent experimental bound on $B(K_S \to \pi^0 e^+e^-)$.

Accurate measurements of $\Re(\epsilon'/\epsilon)$ and $\Im(\epsilon'/\epsilon)$ will provide interesting information about the global symmetries of the SM (especially if $\Re(\epsilon'/\epsilon)$ were found to be clearly different form zero). However, given the large theoretical uncertainty, at present $\Re(\epsilon'/\epsilon)$ is not very useful to perform precision tests of the model.

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References

[1] For interesting recent reviews see e.g.: A.J. Buras, TUM-HEP-316-98 [hep-ph/9806471]; T. Rizzo, SLAC-PUB-7936 [hep-ph/9809526]; Y. Nir, WIS-98-29-DPP [hep-ph/9810520]; G. Burdman, MADPH-98-1093 [hep-ph/9811457].

[2] See e.g. J. Bijnens *et al.* [hep-ph/9411311] and M. Knecht *et al.* [hep-ph/9411259] in *The Second Daphne Physics Handbook*, eds. L. Maiani, G. Pancheri and N. Paver (SIS–Frascati, 1995).

[3] N. Cabibbo, *Phys. Rev. Lett.* **10** 531 (1963); M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49** 652 (1973).

[4] L. Wolfenstein, *Phys. Rev. Lett.* **51** 1945 (1983).

[5] G. Buchalla and A.J. Buras, *Nucl. Phys.* **B398** 285 (1993); *Nucl. Phys.* **B400** 225 (1993); *Nucl. Phys.* **B412** 106 (1994).

[6] M. Misiak and J. Urban, TUM-HEP-336/98 [hep-ph/9901278].

[7] G. Buchalla and A.J. Buras, CERN-TH/98-369 [hep-ph/9901288].

[8] M. Lu and M. Wise, *Phys. Lett.* **B324** 461 (1994).

[9] L. Littenberg, *Phys. Rev.* **D39** 3322 (1989).

[10] G. Buchalla and A.J. Buras, *Phys. Rev.* **D54** 6782 (1996).

[11] G. Buchalla and G. Isidori, *Phys. Lett.* **B440** 170 (1998).

[12] W.J. Marciano and Z. Parsa, *Phys. Rev.* **D53** R1 (1996).

[13] S. Adler *et al.* (E787 Collab.), *Phys. Rev. Lett.* **79** 2204 (1997); S. Kettel, these proceedings.

[14] J. Adams *et al.* (KTeV Collab.), EFI-98-21 [hep-ex/9806007]; K. Hanagaki, these proceedings.

[15] G. Ecker, A. Pich and E. de Rafael, *Nucl. Phys.* **B291** 692 (1987).

[16] G. D'Ambrosio, G. Ecker, G. Isidori and J. Portolés, *J. High Energy Phys.* **08** 004 (1998) [hep-ph/9808289].

[17] C. Caso *et al.* (Review of Particle Properties), *Eur. Phys. J.* **C3** 1 (1998).

[18] A. Buras, M.E. Lautenbacher, M. Misiak and M. Münz, *Nucl. Phys.* **B423** 349 (1994).

[19] G. Valencia, *Nucl. Phys.* **B517** 339 (1998); G. Dumm and A. Pich, *Phys. Rev. Lett.* **80** 4633 (1998).

[20] D. Ambrose *et al.* (E871 Collab.), *Phys. Rev. Lett.* **81** 4309 (1998).

[21] G. D'Ambrosio, G. Isidori and J. Portolés, *Phys. Lett.* **B423** 385 (1998).

[22] L.M. Sehgal, *Phys. Rev.* **D38** 808 (1988); L. Cappiello, G. D'Ambrosio and M. Miragliuolo, *Phys. Lett.* **B298** 423 (1993); P. Heiliger and L.M. Sehgal, *Phys. Rev.* **D47** 4920 (1993); A.G. Cohen, G. Ecker and A. Pich, *Phys. Lett.* **B304** 347 (1993); J.F. Donoghue and F. Gabbiani, *Phys. Rev.* **D51** 2187 (1995); G. D'Ambrosio and J. Portolés, *Nucl. Phys.* **B492** 417 (1997).

[23] G. Burdman, *Phys. Rev.* **D59** 035001 (1999).

[24] G.-C. Cho, *Eur. Phys. J.* **C5** 525 (1998); T. Goto, Y. Okada and Y. Shimizu, *Phys. Rev.* **D58** 094006 (1998).

[25] G. Colangelo and G. Isidori, *J. High Energy Phys.* **09** 009 (1998) [hep-ph/9808487].

[26] T. Hattori, T. Hausike and S. Wakaizumi, TOKUSHIMA-98-01 [hep-ph/9804412].

[27] Y. Grossman and Y. Nir, *Phys. Lett.* **B398** 163 (1997).

[28] Y. Nir and M. Worah, *Phys. Lett.* **B243** 326 (1998); A.J. Buras, A. Romanino and L. Silvestrini, *Nucl. Phys.* **520** 3 (1998).

[29] L.J. Hall, V.A. Kostelecky and S. Rabi, *Nucl. Phys.* **267** 415 (1986).

[30] A.J. Buras and L. Silvestrini, TUM-HEP-334/98 [hep-ph/9811471].

[31] See e.g. L. Maiani in *The Second Daphne Physics Handbook*, eds. L. Maiani, G. Pancheri and N. Paver (SIS–Frascati, 1995).

[32] A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz, *Nucl. Phys.* **B400** 37 (1993); A.J. Buras, M. Jamin and M.E. Lautenbacher, *Nucl. Phys.* **B400** 75 (1993).

[33] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, *Nucl. Phys.* **B415** 403 (1994).

[34] A.J. Buras, M. Jamin and M.E. Lautenbacher, *Phys. Lett.* **B389** 749 (1996).

[35] M. Ciuchini *et al.*, *Z. Phys.* **C68** 239 (1995)

[36] S. Bertolini, M. Fabbrichesi and J.O. Eeg, SISSA-19-98-EP [hep-ph/9802405].

[37] T. Hambye *et al.*, *Phys. Rev.* **D58** 014017 (1998).

[38] M. Knecht, S. Peris and E. de Rafael, CPT-98-P-3734 [hep-ph/9812471].