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Do cosmic rays perturb the operation of a large resonant spherical detector of gravitational waves?

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Abstract

The sensitivity of resonant gravitational wave detectors is reviewed. The effect of cosmic rays on a large spherical detector is considered. It is shown that the sensitivity to short bursts, to monochromatic and to stochastic GW is not significantly degraded by cosmic rays. For a two-detector experiment, only one detector needs to be installed in an underground laboratory. This supports the idea to install a resonant detector at sea-level near a GW interferometer.

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1 – INTRODUCTION

The purpose of this note is to evaluate the effect of cosmic rays on a resonant GW detector. This is important for deciding whether or not an underground laboratory, which gives a nearly perfect shielding, is needed when the resonant detectors will reach a sensitivity so good to respond very much to cosmic rays. In particular the new proposals for very massive spherical detectors should take into consideration the problem of a possible degradation of the sensitivity by a cosmic ray effect.

Before dealing with this problem we have considered useful to overview the present situation with the resonant detectors, so to help the design and discussion on the more advanced and very massive resonant GW antennas.

2 – SENSITIVITY OF A RESONANT DETECTOR FOR GRAVITATIONAL WAVES

The sensitivity of a cylindrical resonant detector for gravitational waves (GW) can be expressed [1)] at a level of signal-to-noise ratio (SNR) equal to unity by means of the formula

$$S_h(f_o) = \frac{\pi}{2}\frac{kT_e}{MQv^2}\frac{1}{f_o} \tag{1}$$

with the following meaning for symbols:

f_o	resonance frequency of the detector
$S_h(f_o)$	the power spectrum at the resonance frequency of the gravitational potential h that can be detected with SNR=1 (spectral sensitivity).
T_e	the thermodynamic temperature of the detector plus a term due to the transducer back-action. This is negligible when a SQUID amplifier is used.
M	mass of the bar
Q	quality factor of the detector
v	sound velocity in the bar material

We notice that eq. (1) does not depend on the transducer characteristics, provided the back-action is neglected.

The frequency bandwidth of the detector is given by

$$\Delta f = \frac{f_o}{Q}\frac{4\,T_e}{T_{eff}} \tag{2}$$

where T_{eff} is the noise temperature for burst detection, that is the average value of the noise after applying to the GW data a filter matched to delta-like signals. It can be shown [1)] that the frequency bandwidth depends heavily on the transducer and associated electronics. It turns out that Δf can approach 100 Hz if one is able to operate the detector near the quantum limit, that is $T_{eff} \approx 10^{-7}$ K.

From eqs. (1) and (2) one can derive the antenna sensitivity to various types of GW.

For bursts with duration τ_g the sensitivity (we do not forget, with SNR=1) is given by

$$h \approx \frac{1}{\tau_g} \sqrt{\frac{S_h}{2 \pi \Delta f}} \tag{3}$$

Another useful formula can be derived by eqs. (1) and (2)

$$h \approx \frac{L}{\tau_g v^2} \sqrt{\frac{k T_{eff}}{M}} \tag{4}$$

For monochromatic GW, integrating over a continuous time t_m, the minimum wave amplitude that can be detected with SNR=1 is

$$h = \sqrt{\frac{2 S_h}{t_m}} \tag{5}$$

Finally eq.(1) gives immediately the sensitivity to a GW stochastic background in terms of an upper limit only, since it is practically impossible to subtract from the measured power spectrum the contribution due to noise. In order to measure the stochastic background one needs to cross-correlate the output of two antennas[1,2], obtaining the measurement of the cross-spectrum

$$S_h(f) = \frac{\sqrt{S_{1h}(f)\, S_{2h}(f)}}{\sqrt{t_m\, \delta f}} \tag{6}$$

where t_m is the total time of crosscorrelation and δf is the frequency bandwidth in common between the two detectors. The standard deviation of this measurement is equal [2] to the same S_h given by eq. (6).

From the measured S_h we can calculate the value of Ω, the ratio between the GW energy density and the energy density needed for a close Universe, using the following formula

$$\Omega = \frac{4 \pi^2}{3} \frac{f^3}{H^2} S_h(f) \tag{7}$$

where H is the Hubble constant.

We notice that it is important to have a large frequency bandwidth (attainable with a good transducer followed by a very low noise electronic amplifier) for burst detection. It is less important for the stochastic measurement. For monochromatic waves the sensitivity is independent on the bandwidth, but a larger bandwidth allows the exploration of a larger frequency region.

There are five cryogenic bars in operation (Allegro[3], Auriga[4] , Explorer[5] Nautilus[6] and Niobe[7]). They have at present roughly the same experimental sensitivity given below.

Niobe, made with niobium, has resonance frequency of 700 Hz, the other ones, with aluminum, have resonance frequency near 900 Hz. The above minimum values for monochromatic waves and for the quantity Ω have been estimated by considering one year of integration time (for Ω we suppose to use the cross-correlation of two antennas). These experimental quantities are in agreement with eq. (1) for the values $T \approx 2.6$ K and $Q \approx 1.5\ 10^6$ (i.e.:Explorer) or with $T \approx 0.16$ K and $Q \approx 10^5$ (i.e.:Nautilus).

Table 1. Sensitivity of the resonant detectors in operation

resonance frequency [Hz]	$\sqrt{S_h}$ at resonance $1/\sqrt{Hz}$	frequency bandwidth Δf [Hz]	minimum h for 1ms bursts	minimum h for monochromatic waves	minimum Ω
900 700	$7\ 10^{-22}$	0.5 - 1	$4 - 6\ 10^{-19}$	$2\ 10^{-25}$	0.1

The burst sensitivity for all bars can be increased by improving the transducer and associated electronics. It has been estimated[8)] that a factor of 50 be within the technical possibility. In addition to improving the bandwidth, Auriga and Nautilus can improve their spectral sensitivity by making full use of their capability to go down in temperature, $T_e \approx 100$ mK. At present the major difficulty is due to excess noise, sometimes of unknown origin, and work is in progress for eliminating this noise. For a bandwidth of 50 Hz and a spectral sensitivity corresponding to $T_e = 100$ mK and $Q=10^7$ (see next section) the target sensitivity for Auriga and Nautilus is then

Table 2. Target sensitivity for Auriga and Nautilus

$\sqrt{S_h}$ at resonance $1/\sqrt{Hz}$	frequency bandwidth Δf [Hz]	minimum h for 1ms bursts	minimum h for monochromatic waves	minimum Ω
$6\ 10^{-23}$	50	$3\ 10^{-21}$	$2\ 10^{-26}$	10^{-4}

The search for signals due to GW bursts is done after the raw data have been filtered with optimum filter algorithms[9,10,11)]. These algorithms may have various expressions but they all have in common an optimum integration time that is roughly the inverse of the detector bandwidth and all give approximately the same value of T_{eff} (all algorithms being optimal filters for short bursts).

3 – MAJOR PROBLEMS ENCOUNTERED FOR THE RESONANT DETECTORS

The design and construction of resonant bar cryogenic detectors has started in 1970. To reach an operational phase it has taken much longer than expected. This happened because the research program was rather optimistic and the work started when many necessary techniques were not developed yet.

Finally in 1990 Explorer, installed at CERN, was the first cryogenic antenna to enter in a steady operation, joined soon at various times by the other four antennas.

We would like here to present and briefly discuss the major issues which we have dealt with during the almost thirty year of work, so to help the design and construction of new resonant detectors.

a) Unexpected noise. This is perhaps the most difficult problem to solve. We have found experimentally that the noise distribution does not follows the behavior calculated under the assumption that the only present noise is the thermal one and the electronic one from the transducer amplifier. Additional noise of mechanical and electromagnetic nature enters in the apparatus from the laboratory environment. Therefore it is important to improve the mechanical as well the electric shields. The design of the mechanical filters is today much simpler, due to the availability of powerful computer utilities. Also the electromagnetic shields can take advantage from special materials and diagnostic instrumentation.

b) The quality factor Q. It is important to have detectors with high quality factors. The original decision to cool the antenna was due to the need to reduce the thermal noise. But an unexpected bonus came with it. It was found that at low temperature the quality factor of various materials increases by orders of magnitude[12)]. Q values over one billion were found[13)]. However it was soon also found that the practical Q is that obtained after the electromechanical transducer is mounted on the antenna. This Q includes also the electrical losses due to the transducer, which can be very large[14)]. For all experiments in operation the loaded Q today is less than 10 million. In some cases is as low as a few hundred thousands. We believe that a detector with an operating Q of the order of 10 million is already a good achievement and so we shall consider in our calculations the value $Q=10^7$ for future resonant cryogenic detectors.

c) The matching of the electromechanical transducer to the bar or sphere. The GW interacts with the antenna leaving in it only a very tiny amount of energy. Thus it is important that this energy be extracted as much as possible for measurement. Then the requirements for the transducer are that its mechanical part be well matched to the antenna, and its electrical part well matched to the electronic amplifier. In the past this has posed severe problems and it is worth to invest a good effort in this area.

4. SENSITIVITY OF A LARGE SPHERICAL DETECTOR

In order to further increase the sensitivity it has been proposed to construct new resonant detectors of much larger mass[15,16,17,18,19)]. The best geometry for an heavy detector is the spherical one, because a sphere has the largest possible mass for a given occupied space and because a spherical detector can be instrumented with transducers installed in various locations on its surface, allowing the best detection of a GW with any direction and polarization status. Among various proposals an aluminum sphere with a diameter of 3 m, having the mass M=38 ton and operating at T_e=20 mK has been considered. To estimate the sensitivity for this detector we make use of the above formulas and obtain the sensitivity given in table 3. We have assumed that the detector operates near the quantum limit, that is with $T_{eff} \approx 10^{-7}$ K.

Table 3. Target sensitivity for a 38 ton resonant spherical detector at SNR=1

$\sqrt{S_h}$ at resonance $1/\sqrt{\mathrm{Hz}}$	frequency bandwidth Δf [Hz]	minimum h for 1ms bursts	minimum h for monochromatic waves	minimum Ω
$6\ 10^{-24}$	50	$3\ 10^{-22}$	$2\ 10^{-27}$	10^{-6}

Furthermore we must consider that the sphere is sensitive to GW with any incoming direction and degree of polarization. Among the other proposals a sphere made with a different material for a mass greater than 100 ton has been also considered[20)].

Before concluding this section we have to make the following consideration that applies to all GW detectors. When we state that the burst sensitivity is $h \approx 3\ 10^{-22}$ at SNR=1 we implicitly admit that at this level many small signals, due to noise, are present. In practice when searching for coincidences between two antennas one has to take into consideration the accidental coincidences. It can be shown that for a number of accidentals very small, say 3 per year, one has to move up the energy threshold by at least one order of magnitude. This point is discussed in the following section.

5. EFFECTS OF COSMIC RAYS ON A RESONANT DETECTOR

One problem with a large mass detector arises from the effects of cosmic rays. Calculations have shown that[21)], when the detector is sensitive enough, a number of impulsive signals should be observed, due to interaction of the various components of cosmic rays with the material of the detector. In fig.1 we show the results of calculations made by Mazzitelli and Papa[22)] for an aluminum sphere with 3 m diameter.

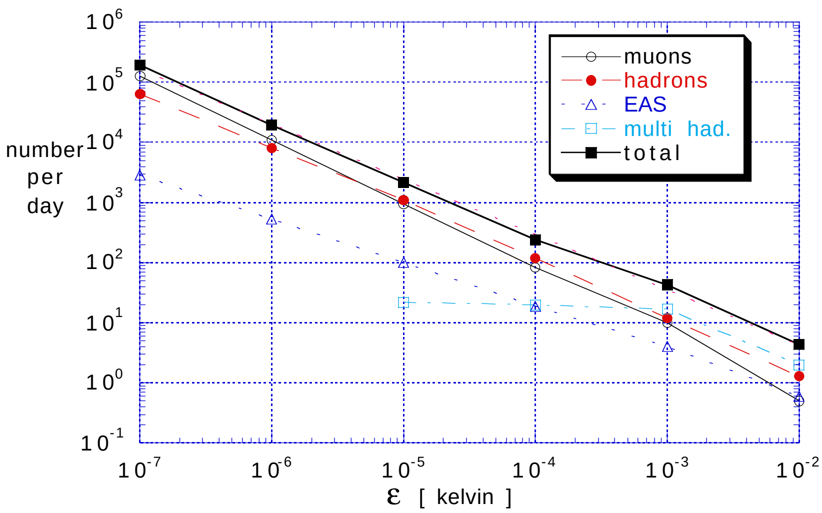

Fig. 1 – Integral number of signals per day due to the various components of cosmic rays versus the energy released in the detector.

We show in fig. 1 the number of signals per day, having energy equal or greater than ε, which are expected to be seen by a 308-ton spherical detector, due to cosmic rays. The energy is expressed in kelvin units for better comparison with the noise that is usually expressed in kelvin. We notice that near the quantum limit, $T_{eff} = \varepsilon \approx 10^{-7}$ K, there will be more than 10^5 signals/day due to cosmic rays.

We go now to estimate the effect of the cosmic rays on the measurement of the various types of GW.

5.1 Burst detection

Let us start by considering the effect of cosmic rays on a simulated coincidence experiment made with two equal M=38 ton spherical detectors. The raw data be filtered with optimum filter for bursts detection and the sampling time be 20 ms equal to the optimum integration time corresponding to a detector bandwidth of 50 Hz. The detector operate near the quantum limit, that is with $T_{eff} \approx 2\ 10^{-7}$ K. In the case of well behaved noise the number of samples of one detector, after filtering, which have energy greater than E is given by

$$N(E) = N_o\, e^{-\frac{E}{T_{eff}}} \tag{8}$$

where $N_o = t_m/20$ ms (t_m is the total measuring time).

The number of accidental coincidences with energy greater than E, for the two identical detectors, is

$$N(E) = N_o\, e^{-\frac{E}{T_{eff}/2}} \tag{9}$$

To these accidental coincidences we must add the accidental coincidences due to cosmic rays.

In fig.2 we have considered a simulated coincidence experiment between two 38 ton spherical detectors located in various laboratories at sea-level and underground and operating at the quantum limit. We show the calculated number of accidental coincidences per day due to the background, including the cosmic ray signals.

The first important result from this figure is that the number of accidentals at very low energies is not influenced by the cosmic rays which begin to enter in the game at energies one order of magnitude larger than the noise of the detector. We have considered some special cases:

a) If the two detectors are both located at sea-level the number of accidental coincidences exceeds the arbitrary threshold of 3/year already at energy $E \approx 2\ 10^{-4}$ K (corresponding to a pulse sensitivity $h \approx 1.4\ 10^{-20}$).
b) A veto system reducing the cosmic ray contribution to 80%[23)] allows to go down, at 3/year, to $E = 2\ 10^{-5}$ K ($h \approx 4\ 10^{-21}$).
c) With one underground detector and one at sea-level with a cosmic ray veto system we can reach $E = 10^{-6}$ K($h \approx 1\ 10^{-21}$).

No much more is gained by having both detectors underground. Thus the real sensitivity for bursts detection without the use of an underground laboratory turns out to be $h \approx 4\ 10^{-21}$. We notice that if we drop the requirement to be able to observe just 3 events/year and go down to one event/day the sensitivity improves to $T_{eff} = 2\ 10^{-6}$ K, corresponding to $h \approx 1.4\ 10^{-21}$.

We now estimate the time span the detector would not be operative if a veto from the cosmic ray sensor is applied. For an optimum integration time of 20 ms a veto for a duration of 3x20 ms=60 ms for each cosmic ray pulse is appropriate. Then from fig. 1 we notice that a c.r. veto at a threshold of 10^{-5} K will make the detector inoperative for 1000 times 60 ms, that is for 60 s per day, and at a threshold of 10^{-6} K the detector is inoperative for 20,000 times 60 ms, that is for 1200 s per day.

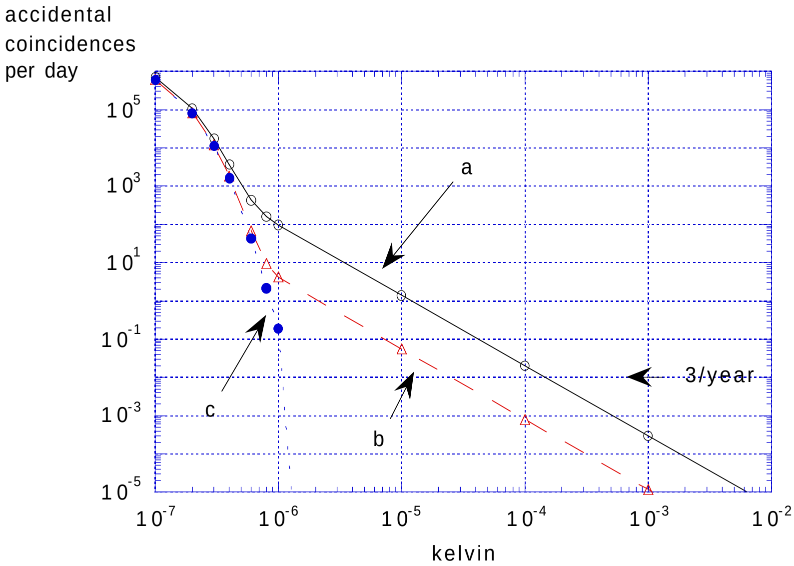

Fig. 2 – Number of accidental coincidences per day with two quantum limited 38 ton resonant detectors, located in various places: a) both detectors at sea-level, b)both detectors at sea-level with 80% cosmic ray veto systems, c) one detector at sea-level with c.r. veto and the other one in an underground laboratory

5.2 Stochastic and monochromatic GW detection

We are going now to consider the sensitivity for stochastic background and monochromatic waves. The situation here is completely different, as the most important quantity is the power spectrum $S_h(f)$.The effect of the cosmic rays here is to increase the value of $S_h(f)$, the minimum power spectrum of GW that can be detected.

We calculate this increase in the following way.

From the previous calculations of the effect of cosmic rays on a spherical detector, we can model the effect of the cosmic rays as an additional brownian noise, a series of pulses randomly distributed in time that should produce a temperature increase of the resonant mode of the detector-oscillator. For calculating this increase of temperature we consider the number $n(\varepsilon)$ of pulses per day due to cosmic rays that deliver an energy greater than ε per each pulse. This is obtained from fig.1 using a least square fit with power law:

$$n(\varepsilon) = 0.063\ \varepsilon^{-0.92} \tag{9}$$

The average energy of the oscillator at the equilibrium, limiting ourselves here to the energy delivered by cosmic rays, is obtained by equating this input energy to the losses of the oscillator.

For calculating the total energy delivered per day by the cosmic rays we perform the following integral

$$\Phi = \int_{o}^{E_{max}} \frac{d\,n(\varepsilon)}{d\varepsilon}\ \varepsilon\, d\varepsilon = 0.8\ (E_{max})^{0.08} \tag{10}$$

where E_{max} is chosen at such a level that the cosmic ray effect is negligible or it can be taken care for. If we take E_{max} =1 K, by extrapolation of the calculation shown in fig.1 we find that we expect only one c.r. event in 16 days, delivering an energy greater than 1 K. We believe that if this happens it can be easily spotted and two or three hours of data that include such an event can be vetoed.

The energy loss per second by an oscillator with resonant angular frequency ω and quality factor Q is

$$\frac{dE}{dt} = \frac{E\omega}{Q} \qquad (11)$$

The equilibrium average energy of the oscillator is obtained by equating the input energy due to cosmic rays to the losses

$$\frac{E\omega}{Q}\; 86400 = 0.8\;(E_{max})^{0.08} \qquad (12)$$

where 86400 is the number of seconds in one day. Taking E_{max} =1 K we obtain E = 0.019 K. This means that the effect of cosmic rays is equivalent to increase the thermodynamic temperature of the detector by about 20 mK.

If the choice of E_{max} = 0.1 K is made (just one event in two days delivering an energy greater than 0.1 K) then the equivalent increase of the temperature turns out to be 15 mK, showing, as expected, that the greatest contribution comes from the low energy region. It is clear that this heating effect cannot be eliminated by a c.r. veto system.

Our conclusion it that the cosmic ray at sea-level degrade the power spectrum sensitivity of the resonant spherical detector by the factor (20 mK+T)/T, that is a factor of two if the detector is cooled at T = 20 mK. As a consequence, in the worst case of two detectors at sea-level, the minimum value of h that can be measured for monochromatic waves increases by the factor of √2, and the minimum Ω value for a stochastic background measurement increases by the factor of two.

6. CONCLUSIONS

From the above considerations we conclude that, for a two-detector experiment, only one detector needs to be installed in an underground laboratory, gaining not more than a factor of four in burst sensitivity with respect to two sea-level detectors equipped with veto systems.

It seems thus very reasonable the idea by Astone, Lobo and Schutz [24)] to install a resonant detector in the proximity of a large interferometric antenna that is being constructed, perhaps at a distance of a few km. An important advantage is that the interferometer is insensitive to cosmic rays. Therefore it can be, ideally, considered equivalent to a perfectly shielded sphere. Thus, assuming that the interferometer might reach a sensitivity of the order of a quantum limited large sphere, one should consider curve c) of fig.2, which shows a sensitivity of $T_{eff} \approx 10^{-6}$ K (h ≈ 1 10^{-21}) for 3 accidentals/year.

In this way one would have powerful systems of pairs of GW antennas. Both antennas in each pair having similar sensitivity but realized with different techniques, complementing each other. The resonant detector being very sensitive in the kHz frequency region and the interferometer covering in particular, with large sensitivity, the frequencies below 1 kHz.

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REFERENCES

(1) P.Astone, G.V.Pallottino, G.Pizzella, Class. Quantum Grav. **14** (1997) 2019-2030
(2) J.S.Bendat and A.G.Piersol, "Measurement and analysis of random data" John Wiley & Sons,, New York, 1966.
(3) E. Mauceli et al., Phys.Rev. D, **54**, 1264 (1996).
(4) M.Cerdonio et al., First Edoardo Amaldi Conference on Gravitational wave Experiments, Frascati, 14-17 June 1994.
(5) P. Astone *et al.*, Phys. Rev. D. **47**, 362 (1993).
(6) P. Astone et al, Astroparticle Physics, **7** (1997) 231-243.
(7) I.S.Heng, D.G.Blair, E.N.Ivanov, M.E.Tobar, Physics Letters **A 218** (1996) 190-196.
(8) M.Bassan and G.Pizzella, "Sensitivity of a capacitive transducer for resonant gravitational wave antennas", Internal Report, LNF-95/064, Frascati (1995).
(9) P.Astone, P.Bonifazi, G.V.Pallottino, G.Pizzella; Il Nuovo Cimento 17,713 (1994)
(10) P.Astone, C.Buttiglione, S.Frasca, G.V.Pallottino, G.Pizzella, Il Nuovo Cimento **20**,9,1997
(11) P.Astone, G.V.Pallottino, G.Pizzella; Journal of General Relativity and Gravitation, **30** (1998)105-114
(12) P. Carelli, A. Foco, U. Giovanardi, I. Modena, D. Bramanti, G. Pizzella; Cryogenics, p.406, July 1975.
(13) Kh.S.Bagdasarov, V.B.Braginsky, P.I.Zubietov; Letters to JETP **39**, 26 January 1977.
(14) S.P.Boughn et al., Phys. Rev. Letters 38, 454 (1977).
(15) R.Forward, Journal of General Relativty and Gravitation **2**, 199 (1971).
(16) W.W. Johnson and S.M. Merkowitz , Phys. Rev. Letters **70**, 2367 (1993).
(17) E. Coccia, "Spherical Gravitational Wave Detectors"; Les Rencontres de Physique de la Vallee d'Aoste: Results and Perspectives in Particles Physics". La Thuille, March 1994.
(18) E.Coccia, J.A. Lobo and J.A. Ortega, Phys. Rev. **D 52**, 3735 (1995).
(19) S.M. Merkowitz and W.W. Johnson Phys. Rev. **D 51**, 2546 (1995).
(20) G. Frossati "A 100 ton 10 mK Gravitational Wave Antenna"; Proceedings of the First Intern. Workshop OMNI 1, 26-31 May 1996, Sao Jose dos Campos (Brazil), World Scientific 1997.
(21) E. Amaldi, G. Pizzella, Il Nuovo Cimento **9C**, 612 (1986).
(22) G.Mazzitelli, M.A.Papa, "Interaction of high energy muons and hadrons with a large aluminum spherical resonant detector"; Proceedings of the First Intern. Workshop OMNI 1, 26-31 May 1996, Sao Jose dos Campos (Brazil), World Scientific 1997.
(23) F.Ronga, It is possible to completely shield the hadronic and the EAS components by means of a thick (2 meters) layer of concrete. Private communication (1996).
(24) P.Astone, J.A.Lobo, B.F.Schutz, Class.Quantum Grav. **11** (1994) 2093-2112s.