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A modification of the 10D superparticle action inspired by the Gupta-Bleuler quantization scheme

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Abstract

We reconsider the issue of the existence of a complex structure in the Gupta-Bleuler quantization scheme. We prove an existence theorem for the complex structure associated with the $d = 10$ Casalbuoni-Brink-Schwarz superparticle, based on an explicitly constructed Lagrangian that allows a holomorphic-antiholomorphic splitting of the fermionic constraints consistent with the vanishing of all first class constraints on the physical states.

As it is well known, the puzzle of the covariant quantization of the superparticle, superstring models can be viewed as the problem of mixed first and second class fermionic constraints in the Hamiltonian formalism [1–3]. One of the interesting approaches to treat the second class constraints is the Gupta-Bleuler-type quantization scheme [4–7] which, for the case at hand, reduces to the construction of a specific complex structure J on a phase space of the models [4]. The latter provides a holomorphic-antiholomorphic splitting of the mixed constraints which proved to yield a successful covariant quantization of the 4D superparticle [5].

A recipe how to construct such a J in arbitrary space-time dimensions has been proposed in the recent work [10]. The strategy adopted then was to decompose the tensor J into irreducible representations (irreps) of the Lorentz group and then reduce the equations for determining J to those for the irreps. The explicit solution in $d = 10$ has been found [10]

$$J_{ab} = \frac{1}{\alpha}(A_a B_b - A_b B_a), \tag{1a}$$

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[4] For simplicity, in what follows we shall discuss the superparticle case only. A discussion of questions related to the covariant quantization of the Green-Schwarz superstring can be found e.g. in [8,9]

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$$\alpha = \pm\sqrt{A^2B^2 - (AB)^2}, \tag{1b}$$

$$(Ap) = 0, \quad (Bp) = 0, \tag{1c}$$

requiring the extension of the original phase space $(x^n, p_n), (\theta^\alpha, p_{\theta\alpha})$ through the new vector variables A^n, B^n. Generally, such an extension can easily be realized by introducing two pairs of canonically conjugate variables $(A^n, p_{An}), (B^n, p_{Bn})$ subject to the first class constraints

$$p_{An} = 0, \quad p_{Bn} = 0, \tag{2}$$

and treating the Eqs. (1c) as gauge fixing conditions for some of the constraints (2). However, as the first class constraints remaining in Eq. (2) do not commute with the complex structure, in passing to a quantum description the vanishing of these constraints on physical states would be incompatible with the vanishing of the holomorphic constraints on those states.

In this brief note we suggest a way to cure this inconsistency. The idea is to completely fix the gauge freedom in the sector (A, p_A), (B, p_B) by introducing further auxiliary variables. If the first class constraints from the sector of the new variables turn out to commute with A and B, the complete description is self-consistent.

The action to be examined reads

$$S = \int d\tau \frac{1}{2e}\left(\dot{x}^n - i\theta\Gamma^n\dot{\theta} - \omega_1 A^n - \omega_2 B^n - \mu_i \Lambda_i^n\right)^2 - \rho_1(A^2 - 1) - \rho_2(B^2 - 1) - \nu_{1i}(A\Lambda_i)$$
$$- \nu_{2i}(B\Lambda_i) - \Phi_{ij}(\Lambda_i\Lambda_j + \Delta_{ij}) - \sum_{i=1}^{8}\mu_i, \tag{3}$$

where

$$\Delta_{ij} \equiv \begin{cases} 0, & i = j \\ 1, & i \neq j \end{cases} \quad i, j = 1, \ldots, 8.$$

Here the summation over repeated indices is understood. As compared to the Casalbuoni-Brink–Schwarz model [11] one finds a set of auxiliary variables $(A^n, B^n, \Lambda^n{}_i, \mu_i, \nu_{1i}, \nu_{2i}, \Phi_{ij}, \omega_1, \omega_2, \rho_1, \rho_2)$, with Φ_{ij} being symmetric.

Consider the model (3) in the Hamiltonian formalism. Introducing momenta $(p_e, p^n, p_{\theta\alpha}, p_A{}^n, p_B{}^n, p_{\Lambda ni}, p_{\mu i}, p_{\nu_1 i}, p_{\nu_2 i}, p_{\Phi ij}, p_{\omega_1}, p_{\omega_2}, p_{\rho_1}, p_{\rho_2})$ canonically conjugate to the configuration space variables one has a set of primary constraints

$$p_e = 0, \quad p_\theta + i\theta\Gamma^n p_n = 0, \tag{4a}$$

$$p_A^n = 0, \quad p_B^n = 0, \quad p_{\Lambda ni} = 0, \tag{4b}$$

$$p_{\mu i} = 0, \quad p_{\nu_1 i} = 0, \quad p_{\nu_2 i} = 0, \tag{4c}$$

$$p_{\omega_1} = 0, \quad p_{\omega_2} = 0, \quad p_{\rho_1} = 0, \tag{4d}$$

$$p_{\rho_2} = 0, \quad p_{\Phi ij} = 0, \tag{4e}$$

and the relation to eliminate $\dot{x}^n$

$$\dot{x}_n = ep_n + i\theta\Gamma_n\dot{\theta} + \omega_1 A_n + \omega_2 B_n + \mu_i\Lambda_{ni}. \tag{5}$$

The canonical Hamiltonian is

$$H = (p_\theta + i\theta\Gamma^n p_n)\lambda_\theta + p_e\lambda_e + p_A\lambda_A + p_B\lambda_B + p_{\Lambda i}\lambda_{\Lambda i} + p_{\mu i}\lambda_{\mu i} + p_{\nu_1 i}\lambda_{\nu_1 i} + p_{\nu_2 i}\lambda_{\nu_2 i} + p_{\Phi ij}\lambda_{\Phi ij}$$
$$+ p_{\omega_1}\lambda_{\omega_1} + p_{\omega_2}\lambda_{\omega_2} + p_{\rho_1}\lambda_{\rho_1} + p_{\rho_2}\lambda_{\rho_2} + e\frac{p^2}{2} + \omega_1(pA) + \omega_2(pB) + \rho_1(A^2 - 1) + \rho_2(B^2 - 1)$$

$$+\nu_{1i}(A\Lambda_i)+\nu_{2i}(B\Lambda_i)+\Phi_{ij}(\Lambda_i\Lambda_j+\Delta_{ij})+\mu_1((p\Lambda_1)+1)+\mu_2((p\Lambda_2)+1)$$
$$+\ldots+\mu_8((p\Lambda_8)+1), \tag{6}$$

where the λ's denote Lagrange multipliers corresponding to the primary constraints.

The consistency conditions for the primary constraints imply the secondary ones [5]

$$p^2=0,\quad p\Lambda_i+1=0,\quad \Lambda_i\Lambda_j+\Delta_{ij}=0, \tag{7a}$$
$$pA=0,\quad A^2-1=0,\quad A\Lambda_i=0, \tag{7b}$$
$$pB=0,\quad B^2-1=0,\quad B\Lambda_i=0, \tag{7c}$$
$$\omega_1 p^n+2\rho_1 A^n+\nu_{1i}\Lambda^n{}_i=0, \tag{7d}$$
$$\omega_2 p^n+2\rho_2 B^n+\nu_{2i}\Lambda^n{}_i=0, \tag{7e}$$
$$\nu_{1i}A^n+\nu_{2i}B^n+\mu_i p^n+2\Phi_{ij}\Lambda^n{}_j=0, \tag{7f}$$

and determine half of the λ_θ

$$\Gamma^n p_n\lambda_\theta=0. \tag{8}$$

Consider now Eq. (7d). Multiplying it by A^n and taking into account Eq. (7b) one gets

$$\rho_1=0. \tag{9}$$

Subsequent multiplication of the remaining equation $\omega_1 p^n+\nu_{1i}\Lambda^n{}_i=0$ by $p^n,\Lambda^n{}_i$ reduces it to a system of linear homogeneous equations which has the trivial solution

$$\omega_1=0,\quad \nu_{1i}=0, \tag{10}$$

since the matrix

$$\begin{pmatrix} p^2 & p\Lambda_i \\ p\Lambda_j & \Lambda_i\Lambda_j \end{pmatrix},$$

is nondegenerate on the constraint surface (7a)–(7f). In the same spirit Eqs. (7e), (7f) simplify to

$$\omega_2=0,\quad \rho_2=0,\quad \nu_{2i}=0,\quad \mu_i=0,\quad \Phi_{ij}=0. \tag{11}$$

The preservation in time of the secondary constraints (7a)–(7c), (9)–(11) determine some of the Lagrange multipliers

$$p\lambda_A=0,\quad A\lambda_A=0,\quad \Lambda_i\lambda_A+A\lambda_{\Lambda i}=0 \tag{12a}$$
$$p\lambda_B=0,\quad B\lambda_B=0,\quad \Lambda_i\lambda_B+B\lambda_{\Lambda i}=0 \tag{12b}$$
$$p\lambda_{\Lambda i}=0,\quad \Lambda_i\lambda_{\Lambda j}+\Lambda_j\lambda_{\Lambda i}=0, \tag{12c}$$
$$\lambda_{\rho_1}=0,\quad \lambda_{\omega_1}=0,\quad \lambda_{\nu_1 i}=0, \tag{12d}$$
$$\lambda_{\rho_2}=0,\quad \lambda_{\omega_2}=0,\quad \lambda_{\nu_2 i}=0, \tag{12e}$$
$$\lambda_{\mu i}=0,\quad \lambda_{\Phi ij}=0, \tag{12f}$$

and no tertiary constraints appear.

[5] We define the Poisson brackets of the variables $(\Lambda,p_\Lambda),(\Phi,p_\Phi)$ in the form $\{\Lambda^n{}_i,p_{\Lambda mj}\}=\delta^n{}_m\delta_{ij}$, $\{\Phi_{ij},p_{\Phi ks}\}=\frac{1}{2}(\delta_{ik}\delta_{js}+\delta_{is}\delta_{jk})$.

Taking into account Eqs. (4), (9)–(11) one concludes that the variables $(p_1,p_{p_1}),(p_2,p_{p_2}),(\mu_i,p_{\mu i})$, $(\nu_{1i},p_{\nu_1 i}),(\nu_{2i},p_{\nu_2 i}),(\Phi_{ij},p_{\Phi ij}),(\omega_1,p_{\omega_1}),(\omega_2,p_{\omega_2})$ are unphysical and can be omitted after introducing the associated Dirac bracket. Thus, the only nontrivial constraints to be analyzed are those from Eqs. (7a)–(7c), together with the corresponding momenta (4b).

Let us now return to (7). The constraints (7b) together with the corresponding momentum $p_{A_n}=0$ are second class. In a full agreement with this, Eq. (12a) involving the associated Lagrange multiplier λ_A can be solved explicitly. Actually, since the vectors $p^n, A^n, \Lambda^n{}_i$ satisfying (7) are linearly independent for any fixed value compatible with (7), the matrix

$$\begin{pmatrix} p_0 & \cdots & p_9 \\ A_0 & \cdots & A_9 \\ \Lambda_{0i} & \cdots & \Lambda_{9i} \end{pmatrix}, \tag{13}$$

is invertible on the constraint surface. The latter fact implies that the system of linear inhomogeneous Eqs. (12a) has a unique solution for any fixed value of $p^n, A^n, \Lambda^n{}_i$. Analogously, the constraints (7c) and $p_{B_n}=0$ are second class and Eq. (12b) uniquely determines λ_B.

Thus, it remains to discuss the constraints (7a) and the corresponding momentum $p_{\Lambda_{ni}}=0$. In order to extract the first class constraints contained in $p_{\Lambda_{ni}}=0$, it suffices to construct operators projecting onto subspaces orthogonal to (A^n,B^n) and (p^n,Λ_{ni}) respectively. The explicit form of the projectors is

$$\Pi^m_{(A,B)n}=\delta^m_n+\frac{(AB)}{1-(AB)^2}A^mB_n+\frac{(AB)}{1-(AB)^2}B^mA_n-\frac{1}{1-(AB)^2}A^mA_n-\frac{1}{1-(AB)^2}B^mB_n, \tag{14}$$

$$\Pi^m_{(A,B)n}A^n\approx 0,\quad \Pi^m_{(A,B)n}B^n\approx 0,\quad \Pi^m_{(A,B)n}p^n\approx p^m,\quad \Pi^m_{(A,B)n}\Lambda^n_i\approx\Lambda^m_i, \tag{15}$$

$$\Pi^m_{(p,\Lambda)n}=\delta^m_n-\frac{(p\Lambda_j)\nabla^{ji}p^m\Lambda_{ni}}{(p\Lambda)\nabla(p\Lambda)}-\frac{(p\Lambda_j)\nabla^{ji}\Lambda^m_ip_n}{(p\Lambda)\nabla(p\Lambda)}$$
$$+\nabla^{ji}\Lambda^m_i\Lambda_{nj}-\frac{(p\Lambda_k)\nabla^{ki}\Lambda^m_i(p\Lambda_s)\nabla^{sj}\Lambda_{nj}}{(p\Lambda)\nabla(p\Lambda)}-\frac{p^mp^n}{(p\Lambda)\nabla(p\Lambda)}, \tag{16}$$

$$\Pi^m_{(p,\Lambda)n}p^n\approx 0,\quad \Pi^m_{(p,\Lambda)n}\Lambda^n_i\approx 0,\quad \Pi^m_{(p,\Lambda)n}A^n\approx A^m,\quad \Pi^m_{(p,\Lambda)n}B^n\approx B^m, \tag{17}$$

where ∇ is the inverse matrix to Δ, $\nabla_{ij}\Delta_{jk}=\delta_{ik}$ and $\approx$ means weak equality. Note also that $(p\Lambda)\nabla(p\Lambda)$ $\approx\frac{8}{7}\neq 0$. In the presence of the projectors the first class constraints can be written in the form

$$\tilde{p}^m_\Lambda\equiv\Pi^m_{(p,\Lambda)n}\Pi^n_{(A,B)k}p^k_{\Lambda i}=0. \tag{18}$$

At the next stage, one needs to construct the Dirac bracket associated with all the second class constraints of the problem, which will look like

$$\{M,N\}_D=\{M,N\}+\{M,pA\}\ldots\{p_{A_n},N\}+\{M,A^2-1\}\ldots\{p_{A_n},N\}+\{M,A\Lambda_i\}\ldots\{p_{A_n},N\}$$
$$+\{M,pB\}\ldots\{p_{B_n},N\}+\{M,B^2-1\}\ldots\{p_{B_n},N\}+\{M,B\Lambda_i\}\ldots\{p_{B_n},N\}$$
$$-(-1)^{\epsilon(M)\epsilon(N)}(M\leftrightarrow N)+\text{terms not involving } p_A,p_B, \tag{19}$$

where $\ldots$ denotes some specific functions and $\{M,N\}$ is the usual Poisson bracket. As it is seen, under this bracket A^n,B^m commute both with each other and with the first class constraints $\tilde{p}^m_\Lambda=0$ from the sector of additional variables. This implies that the subsequent split of the fermionic constraints $p_\theta+i\theta\Gamma^np_n=0$ into holomorphic and antiholomorphic sets will be consistent with the vanishing of the first class constraints $\tilde{p}^m_\Lambda=0$ on physical states. This was the problem to solve.

Thus, in this letter we have reconsidered the complex structure in the Gupta-Bleuler quantization scheme, introducing a gauge fixing procedure based on the addition of a set of auxiliary variables, which makes the vanishing of the first class constraints on physical states compatible with the holomorphic-antiholomorphic splitting of the fermionic constraints. We have built explicitly the corresponding Lagrangian formulation.

Since this Lagrangian looks like a monster, we have little hope to be really able to quantize a model on the basis of this scheme. However, our understanding is that the Lagrangian above can be viewed as the existence theorem for the complex structure associated with the 10D Casalbuoni-Brink-Schwarz model.

Although the approach proposed here proved to be too complicated, we expect that the technique will be efficient when applied to theories possessing a constraint like $(Ap) = 0$, with A *a dynamical variable.* One of the possible applications seems to be the particle in anti-de Sitter space and this work is in progress now.

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