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Nucleon time-like Form Factors below the $N\overline{N}$ threshold

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Abstract

The nucleon magnetic form factors in the unphysical region, i.e. for time-like Q^2 but below the $N\overline{N}$ threshold, have been obtained by means of dispersion relations in a model independent way, without any bias towards expected resonances. Space-like and time-like data have been employed along with a regularization unfolding method to solve the integral equation. Remarkably, resonance structures with peaks for the $\rho(770)$ and $\rho'(1600)$ and a structure near the $N\overline{N}$ threshold are automatically generated. The obtained ρ has a much larger width, whose significance is explored. No evidence is found for a peak at the Φ mass, in spite of some expectation if there is a sizeable polarized $s\overline{s}$ content in the vector current of the nucleon.

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Introduction

Nucleon spectral functions and electromagnetic form factors (FF) play a fundamental role in our understanding of the hadronic dynamics. Hence, for over 30 years, many attempts have been made through dispersion relations (DR) [1–3,8,9,12] to unravel the spectral functions and the FF in the time-like region, in particular for unphysical Q^2, below the $N\overline{N}$ threshold. In principle, spectral functions and FF in the time-like region may be computed via DR from the measurements of the space-like FF only [12]. However this is an ill-posed mathematical question, because the answer depends in an unstable way on the input data and an impossible accuracy is needed to get a unique, stable solution [12,13]. Up to now the FF evaluations in the time-like region have been done assuming a model, mostly VDM [4], unitarized VDM [5] and in the framework [6] of the Skyrme model [7]. The lack of data on $e^+e^- \to N\overline{N}$ had prevented much progress in this endeavor.

Recently, measurements of the Proton time-like magnetic FF have been done on a large Q^2 interval [14,15] and, even more recently, data concerning the Neutron time-like FF have also become available [16]. These data turn out to be quite different from QCD expectations [17,18]. In particular, it has been shown that the Neutron time-like magnetic FF measurements are twice the prediction of a dispersive approach [9], assuming PQCD asymptotic behaviours and a reasonable model for unphysical Q^2. Different conclusions have been achieved on the basis of a unitarized VDM[10]. Yet the discrepancy is large enough to prompt further measurements of the neutron time-like FF [9] and new proposals towards this purpose are under way [19]. Pending future experiments, let us explore some implications of the available experimental data, through a consideration of several open questions:

- A sizeable, polarized, $s\overline{s}$ content in the nucleon has been suggested long time ago and has been resumed to interpret sum rules violations in deep inelastic scattering of polarized leptons on polarized nucleons [20], an $s\overline{s}$ content is also suggested by the nucleon sigma term and other measurements [21]: the evidence or not of a sizeable Φ peak in the nucleon magnetic FF in the unphysical region, obtained in a model independent way, would be a very direct check of a polarized $s\overline{s}$ content in the vector current;

- At high Q^2, the neutron time-like magnetic FF is found to be larger than that for the proton, while one had expected it to be - as in the space-like region - $\sim$ 1/2 of the proton magnetic FF [17,18]. This indicates some non trivial dynamical activity in between;

- At high Q^2, the size of the time-like proton magnetic FF itself is somewhat unexpected, since it is twice the value of its space-like FF counterpart at the same $|Q^2|$. We note that PQCD [22] and analyticity [23] predict both to be asymptotically the same;

- Below threshold, there are indications for narrow structures in the total $\sigma(e^+e^- \rightarrow$ hadrons) cross section [24], suggested also by the proton and the neutron FF. These may be related to similar effects in $\bar{p}d$ annihilation in odd and even C channels [25]) and suggest a close investigation of the FF just below threshold (in principle, such is feasible experimentally, through a very high statistics analysis of the reaction $p\overline{p} \rightarrow e^+e^-\pi^0$ [26]);

- FF phases are needed to interpret anomalies in J/Ψ $\rightarrow$hadrons [27].

In the present work, we shall assume that the available data are indeed reliable and we shall use them as input to evaluate the FF for unphysical Q^2. To make our results as much model independent as possible, *we do not bias our analysis towards expected resonances*. Instead, we let resonance structures and phases arise directly from the solution of the DR. In contrast with the past, presently the nucleon FF are unknown in a limited interval only. Thus, continuity through the interval limits can be implemented. Hence the concerns mentioned above about a stable evaluation may be relieved, being rather an interpolation than an analytical continuation. Unfortunately, present accuracy is still not sufficient to provide a unique solution. Until even better time-like data become available, we seek a solution under an additional smoothness hypothesis, implementing a regularization method described below.

1 Solving DR by means of a regularization method

In order to get a form factor $G(Q^2)$ for $0 < Q^2 < 4M_N^2$ (Q^2 defined to be positive in the time-like region) DR for $\log G(Q^2)$ are considered [1]. The quantity $\log G(Q^2)$, just as $G(Q^2)$, is analytic on the first sheet of the complex Q^2 plane, with the same cuts on the real positive axis and with additional poles where the FF has zeros. In the following, we shall assume the absence of zeros (in FF) on the first sheet. This hypothesis will be discussed later.

The DR relates the space-like $\log G(Q^2)$ to the time-like $\log |G(Q^2)|$. After making a subtraction at $Q^2 = 0$, we have [1]:

$$\log G(Q^2) = \frac{Q^2\sqrt{Q_0^2 - Q^2}}{\pi} \int_{Q_0^2}^{\infty} \frac{\log |G(t)|}{t(t - Q^2)\sqrt{t - Q_0^2}} dt \tag{1}$$

where $Q_0^2 = 4m_\pi^2$.

Once $\log|G(Q^2)|$ has been determined, the FF phase $\delta(Q^2)$ for time-like Q^2, is given by:

$$\delta(Q^2) = -\frac{Q^2\sqrt{Q^2 - Q_0^2}}{\pi}\,\mathrm{Pr}\int_{Q_0^2}^{\infty}\frac{\log|G(t)|}{t(t-Q^2)\sqrt{t-Q_0^2}}dt \qquad (2)$$

By splitting the integral in eq.(1) into two parts, $\int_{Q_0^2}^{\infty} = \int_{Q_0^2}^{Q_1^2} + \int_{Q_1^2}^{\infty}$, where $Q_1^2 = 4M_N^2$ is the upper boundary of the unphysical region, one can separate the unphysical region, in which the FF are unknown, from the experimentally accessible region. In this way, an integral equation of the first kind, linear in the unknown $\log|G|$, can be derived from the DR:

$$\log G(Q^2) - I(Q^2) = \frac{Q^2\sqrt{Q_0^2 - Q^2}}{\pi}\int_{Q_0^2}^{Q_1^2}\frac{\log|G(t)|}{t(t-Q^2)\sqrt{t-Q_0^2}}dt \qquad (3)$$

where

$$I(Q^2) = \frac{Q^2\sqrt{Q_0^2 - Q^2}}{\pi}\int_{Q_1^2}^{\infty}\frac{\log|G(t)|}{t(t-Q^2)\sqrt{t-Q_0^2}}dt \qquad (4)$$

This integral is a known quantity that can be calculated directly from the experimental data in the time-like region with some recipe to extrapolate them to very high Q^2.

In order to avoid intabilities in solving eq.(3), a regularization technique exploiting smoothness has to be applied [13]. These techniques have been mostly applied in the unfolding of a spectrum affected by a finite resolution, to avoid instability usually met in solving first kind integral equations. The procedure adopted here is described in the following.

- $I(Q^2)$ has been calculated by fitting the time-like data to a rational, smooth, function having the expected asymptotic behaviour. The subtraction at $Q^2 = 0$, as usual, helps in making the results less dependent on the asymptotic extrapolation.

- A special treatment is adopted near the $N\overline{N}$ threshold: the upper boundary of the unphysical region has been shifted to $Q_2^2 = Q_1^2 + \Delta$, with $\Delta \simeq 0.5$ GeV^2, in order to avoid instabilities that can possibly be originated by the steep threshold behaviour of the nucleon FF. A new DR is then considered for the region (Q_1^2, Q_2^2) as described in sect.3.

- Continuity of the function through the upper boundary of the unphysical region is imposed. At the lower boundary, $4m_\pi^2$, only a very mild continuity is demanded, respect to the FF calculated by means of eq. (1), to allow any steep variation.

- A regularization is finally introduced by requiring the total curvature of the FF in the unphysical region, $r = \int_{Q_0^2}^{Q_2^2} \left(\frac{d^2|G(t)|}{dt^2} \right)^2 dt$, to be limited. Instead of the second derivative of $\log|G|$, as in the standard procedure[13], the second derivative of $|G|$ has been chosen for this purpose. The reason being that fluctuations in $|G|$ are important only when $|G|$ is large, while $\log|G|$ fluctuations would be important also when $|G|$ is small.

To solve eq.(3), a linearization method has been used: the integrals have been transformed into sums over $M = 50$ suitable subintervals in Q^2, with their widths increasing with Q^2. This is tantamount to a further smoothness hypothesis, effectively integrating over any structure, whose half width is narrower than a minimum of about 50 MeV.

The integral over the j^{th} subinterval has been approximated by

$$F_j \int_{T_j}^{T_{j+1}} \frac{dt}{t(t-Q^2)\sqrt{t-Q_0^2}} \tag{5}$$

where $F_j = \log|G[(T_{j+1}+T_j)/2]|$ is the function calculated in the middle of the subinterval with boundaries at T_j and T_{j+1}.

The integral equation (3) is then solved by minimizing the quantity:

$$R_{tot} = \sum_{i=1}^{L} \left\{ \sum_{j=1}^{M} F_j \frac{Q_i^2\sqrt{Q_0^2 - Q_i^2}}{\pi} \int_{T_j}^{T_{j+1}} \frac{dt}{t(t-Q_i^2)\sqrt{t-Q_0^2}} + I(Q_i^2) - \log(G(Q_i^2)) \right\}^2 +$$

$$+\tau^6 r(e^{F_j}) \tag{6}$$

where the F_j's are free parameters and Q_i^2, with $i = 1, ..., L$, correspond to experimental points available in the space-like region.

The dumping parameter τ has to be set experimentally. It will not respond to sharp structures if it is set too large, while unstable solutions will found using too small a value.

The uncertainties in the solution of eqs.(3) and (2), due to the experimental errors, have been evaluated by simulating new sets of space-like and time-like data according to the quoted errors and solving the DR for each simulated data set.

2 Test of the regularization method

To test the whole procedure and also to get a suitable range for the τ parameter, we selected the pion FF. In the time-like region, this FF is known up to the J/Ψ mass and at higher Q^2 is extrapolated according to first order PQCD prescriptions [22]. Pion space-like FF has been reobtained in two ways: (*i*) according to eq.(1), from the time-like data

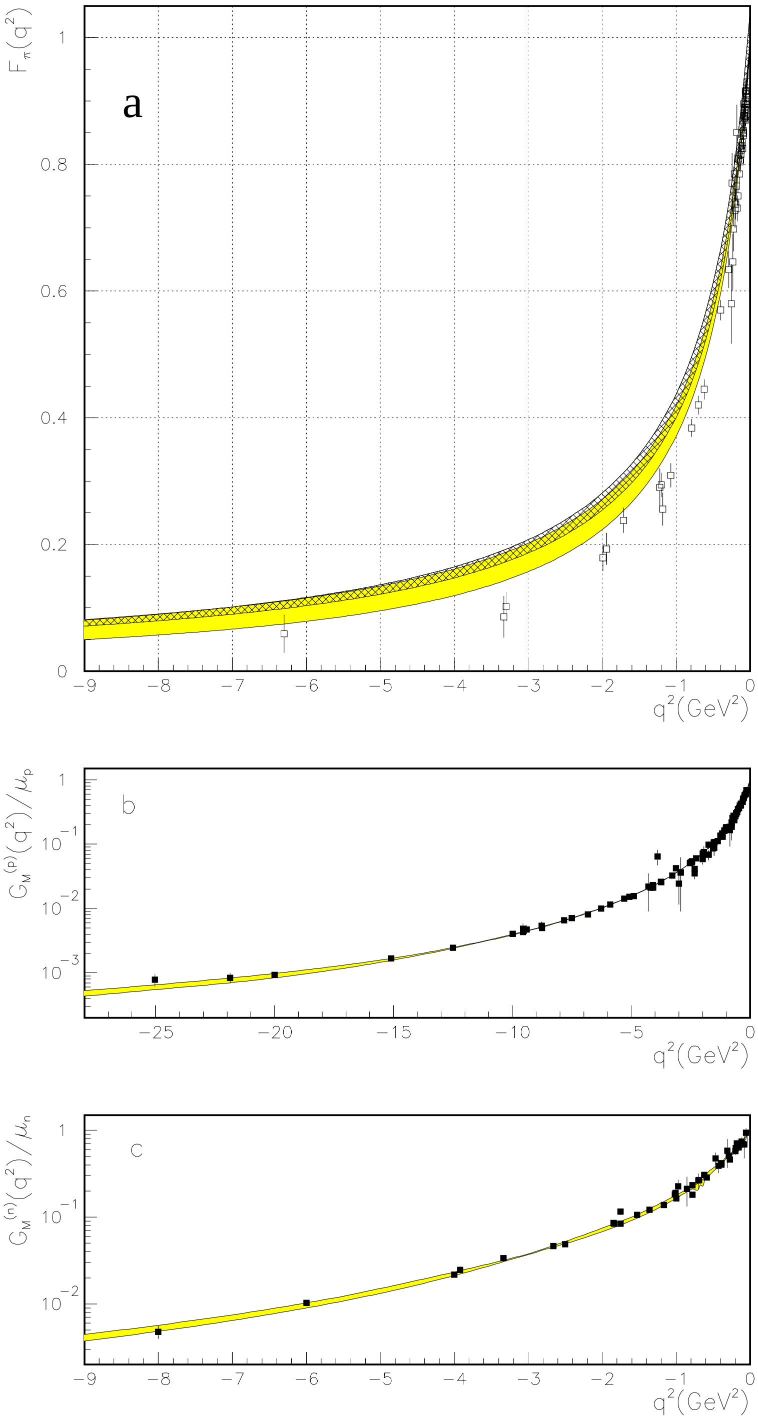

Figure 1: (a) Space-like pion (a), proton magnetic (b) and neutron magnetic (c) form factor from DR applied to time-like data. In (a) the results with and without (dashed area) subtraction are compared.

and (*ii*) according to the unsubtracted DR. The latter is in fair agreement with present space-like data, as shown in fig.1a. Their difference is most likely due to the uncertainty in the data above the ρ and in the asymptotic extrapolation, as indicated by the effect of including a subtraction. This also provides a check that there are no relevant zeros in the isovector FF. This check would have been airtight were pion space-like FF data for high Q^2 not obtained through extrapolations of the pion electroproduction data.

In order to fix the τ parameter, the space-like dashed area, achieved by means of eq.(1), and the time-like dashed area, obtained fitting the data above $Q_2^2 = 4.0\ GeV^2$, have been used as input in eq.(6) to retrieve the time-like pion FF in the region between Q_0^2 and Q_2^2. It turns out that $\tau \sim M_\pi$ is a suitable value to recover quite satisfactorily the ρ peak, the ρ width and also the dip at 1.6 GeV, as shown in fig.2. It is worthwhile stressing that, in solving the DR, steep structures like the ρ and even the interfence pattern beyond the ρ are well retrieved from smooth inputs, once these inputs are built from these structures.

The phase of the pion FF, from the solution of eq.(2), is shown in fig.3. Its expected asymptotic value of 180 degrees is already reached above ~ 2 GeV [27].

3 The proton time-like magnetic FF

Some comments are in order about the hypotheses governing the extraction of the proton FF from cross section measurements. It has been assumed that at threshold there is only one FF, because $G_M(4M_N^2) = G_E(4M_N^2)$, assuming analyticity for electric and magnetic FF as well as for the Pauli and Dirac FF or that exactly at threshold there is only an S wave. Data are consistent with this hypothesis. Furthermore, at high Q^2 the contribution of G_E to the total cross section is dumped by a factor $4M^2/Q^2$. In conclusion, in the whole range explored, what is actually measured is very likely to be G_M.

G_M^p seems to reach its expected asymptotic behaviour $1/Q^4$ quite precociously, but it is a factor of 2 higher than G_M^p at the same space-like $|Q^2|$, while asymptotically they should be equal [23]. Therefore, an asymptotic extrapolation done according to PQCD may be suspect. Yet it has been checked that all the achieved results are quite insensitive to the details of this extrapolation.

Very near threshold, the data show a steep variation [15], beyond Coulomb enhancement (which has already been corrected in the data). In the following, this steep rise has been assumed to affect the FF in a limited Q^2 region, below and above the threshold. This is the reason for choosing $Q_2^2 = 4M_N^2 + \Delta$ as upper limit in eq.(3). G_M and the first two derivatives are supposed to be continuous functions through this upper limit.

Once a FF G_0 has been determined from eq.(6), another DR is considered in the

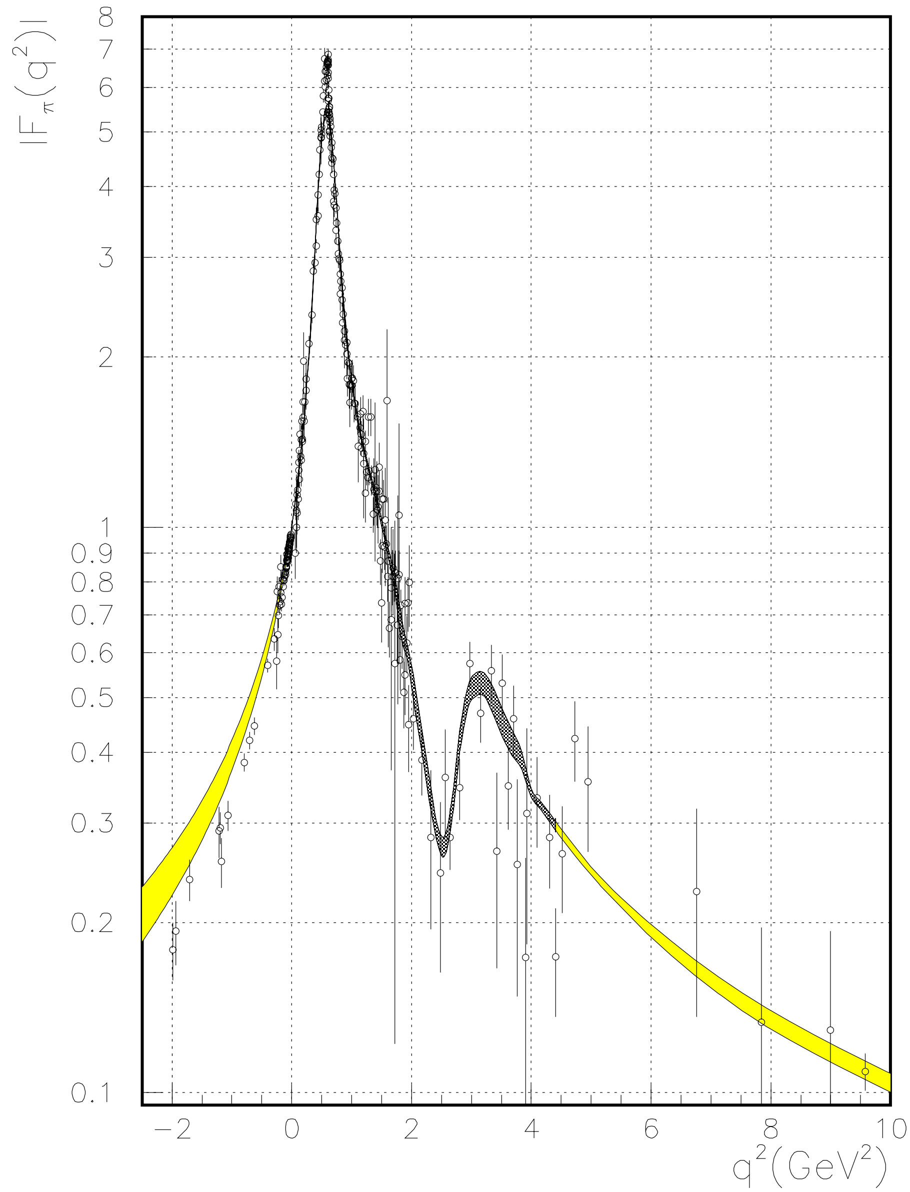

Figure 2: Pion form factor. The black shaded area is the solution of eq.(6), the gray shaded area is the input of the equation.

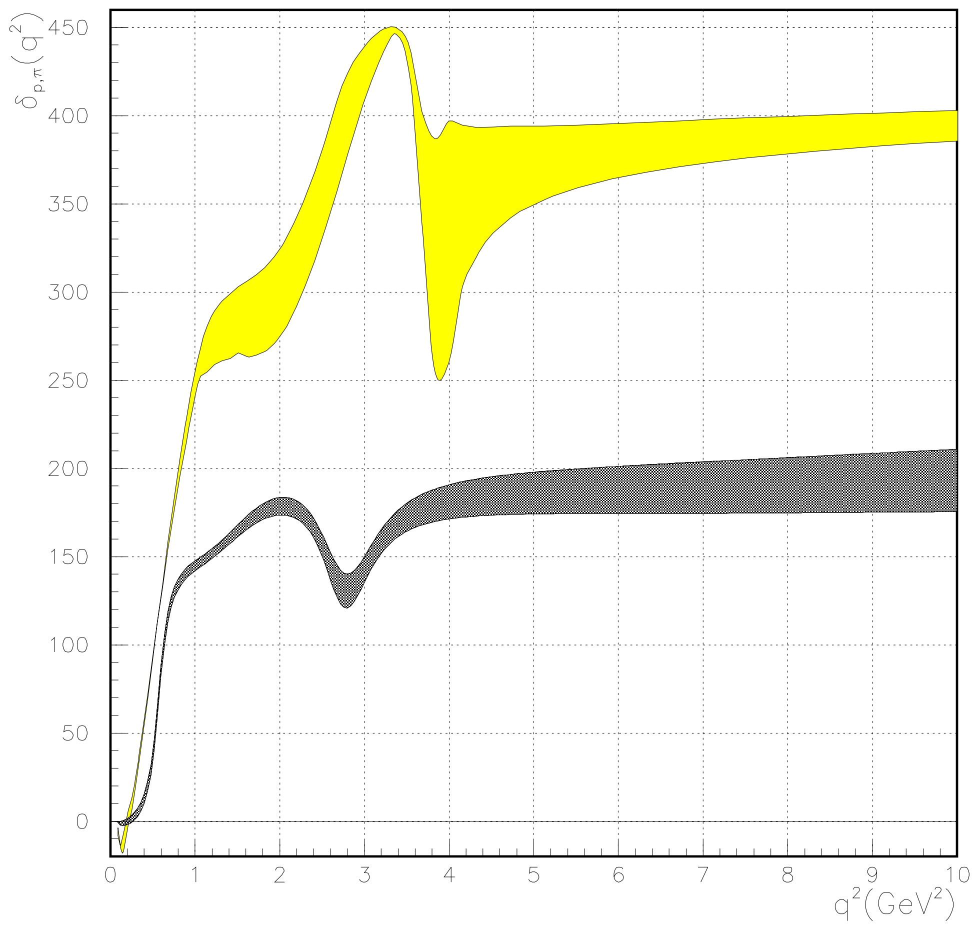

Figure 3: Phase of pion (black shaded) and proton magnetic (gray shaded) form factor according to eq.(2).

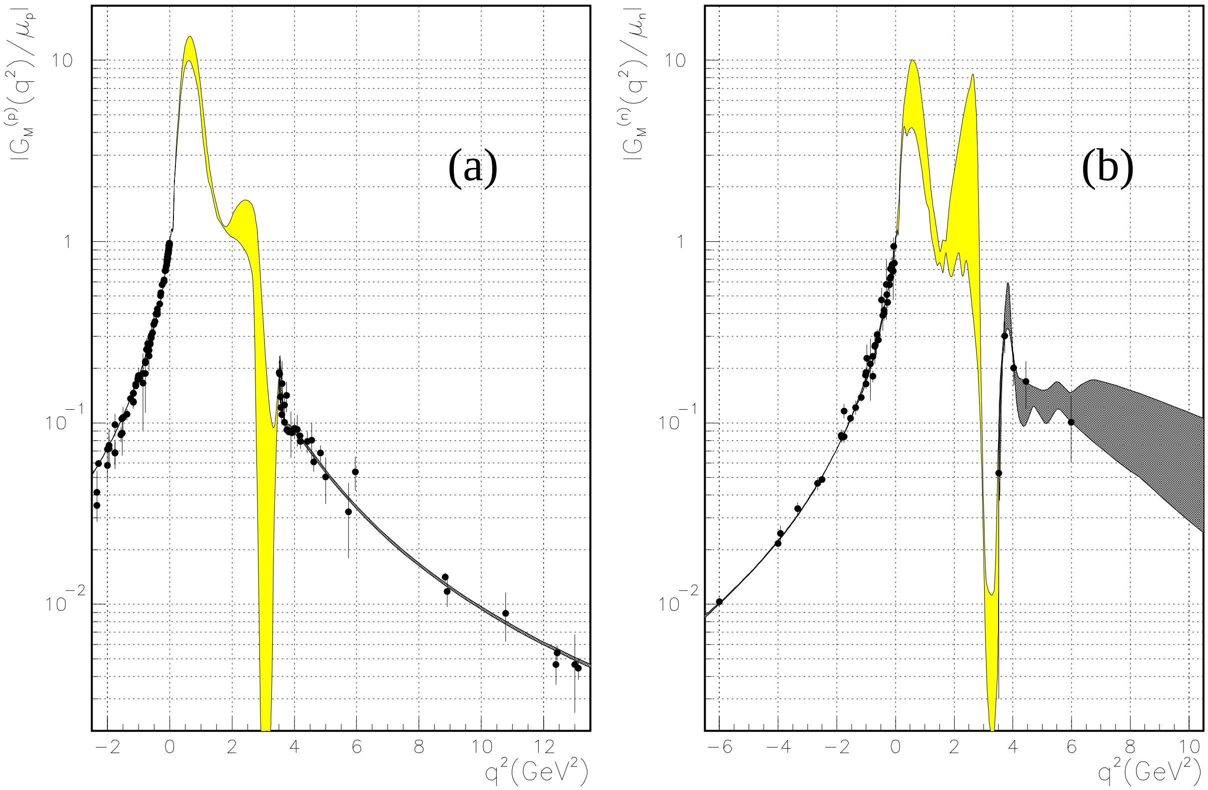

Figure 4: Proton (a) and neutron (b) magnetic form factor according to eq.(6)

interval $[Q_1^2 - \Delta, Q_1^2 + \Delta]$:

$$\frac{Q^2}{\pi}\int_{Q_1^2-\Delta}^{Q_1^2} \frac{\log|G_1(t)|}{t(t-Q^2)\sqrt{t-Q_0^2}}dt + \frac{Q^2}{\pi}\int_{Q_1^2}^{Q_1^2+\Delta} \frac{\log|G_1(t)|}{t(t-Q^2)\sqrt{t-Q_0^2}}dt = 0 \qquad (7)$$

where G_1 is determinated from the relation: $G_M = G_0 G_1$ in this interval and $G_M = G_0$ outside. Finally, the proton magnetic FF in the unphysical region as obtained by our procedure is reported in fig.4a.

The most striking feature of fig.4a is the evidence for two resonances, not built in a priori, at $M \sim 770 \ \ MeV$ and $M \sim 1600 \ \ MeV$. It is most satisfying to “deduce” the presence, through the first, of $\rho + \omega$ and, through the second, of $\rho' + \omega'$ exactly as expected. On the other hand, the width of the bump at the ρ mass is $\sim 350 \ \ MeV$ to be compared to $\Gamma_\rho \sim 150 \ \ MeV$. Previous, old, analyses of the nucleon FF had already found a similar discrepancy [2].

The anomalous width, mainly related to the real part, turns out to be independent of the choice of the τ parameter within an order of magnitude. It cannot be due to the bin width, whose contribution is added quadratically and is relatively small in the ρ case. On

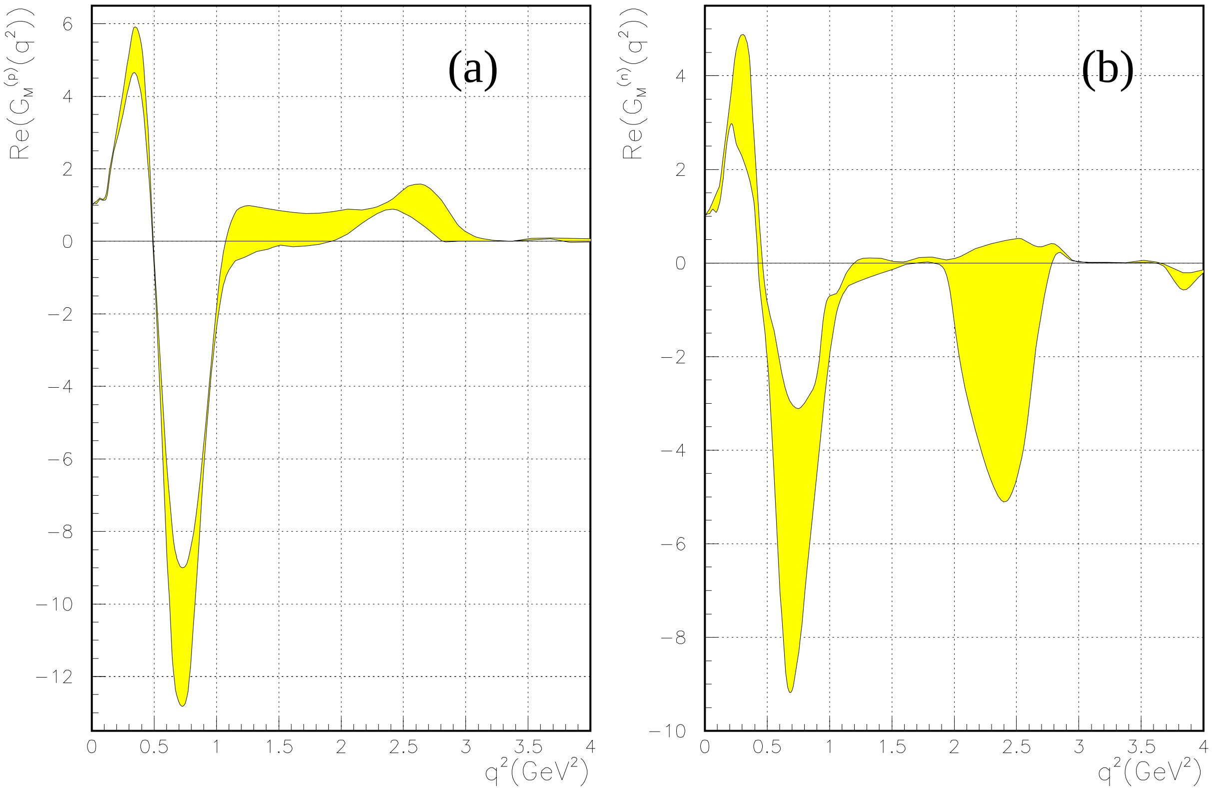

Figure 5: Real part of the proton (a) and the neutron (b) magnetic form factor according to eq.(6) and (2).

the other hand, as mentioned previously, the ρ width was recovered in the case of the pion FF.

Concerning the strange, polarized, content of the nucleon there is no evidence of a bump at the Φ mass, even if integrated on the bin width. If indeed the strange content of the nucleon is $\int dQ^2(|G_M^{s\bar{s}}|/|G_M|)^2 \sim 0.15 \div 0.2$, it should be quite visible, concentrated mainly in the Φ mass bin. However, to make a more quantitative statement the anomalous ρ width should be understood.

In fig.3, the phase of the proton magnetic FF is shown, the real part and the spectral function are shown in fig.5a and fig.6a. Above $\sim$ 2 GeV the phase is $\sim$ 390 degrees to be compared to the expected asymptotic value of 360 degrees [27].

In fig.1b the proton space-like magnetic FF data are compared with the expectation from the solution of the DR on $\log G(Q^2)$. The hypothesis there are no zeros on the physical Q^2 sheet may be questioned. Yet (once the imaginary part has been achieved) a remarkable, non trivial, test has been performed: space-like FF data and the calculation by means of DR involving this imaginary part are in good agreement, at least at low Q^2

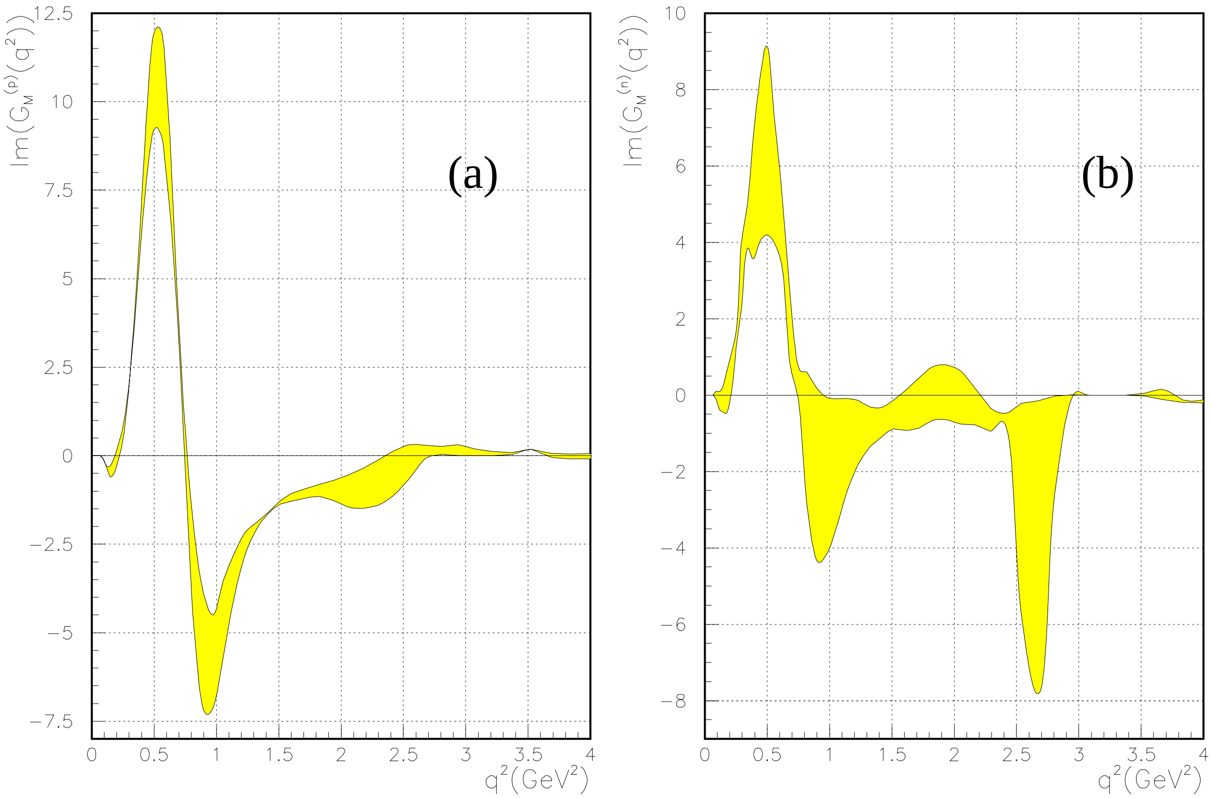

Figure 6: Imaginary part of the proton (a) and the neutron (b) magnetic form factor according to eq.(6) and (2)

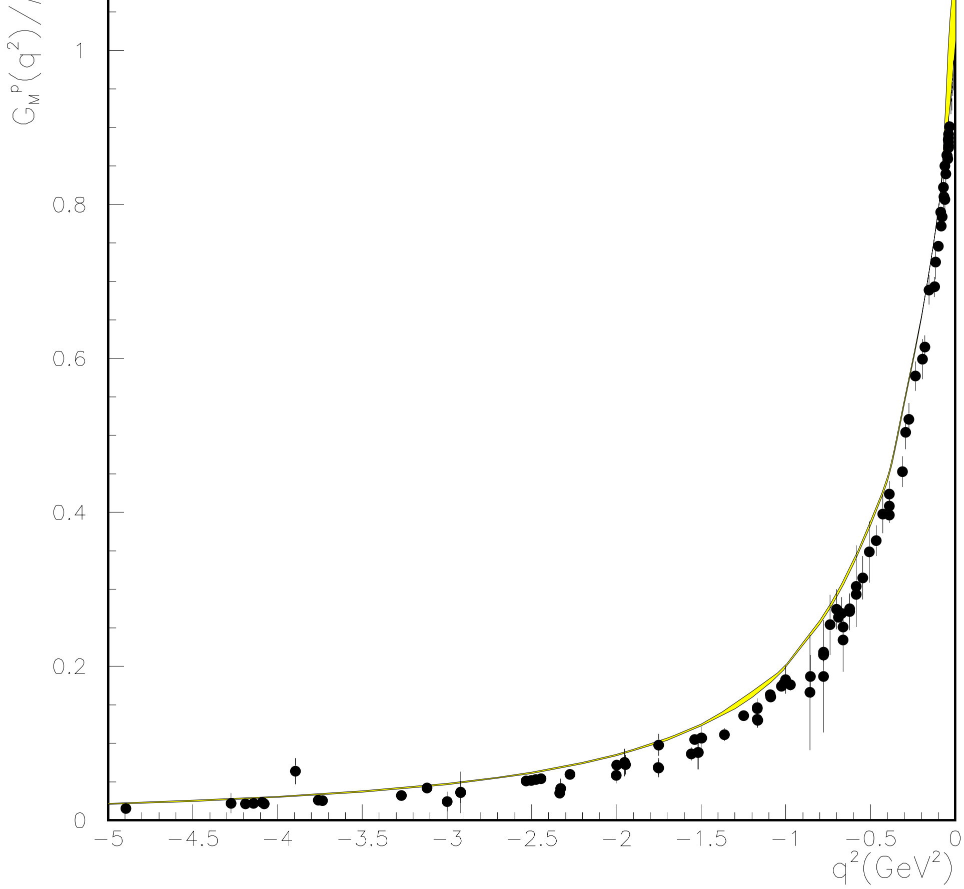

Figure 7: Space-like proton form factor re-obtained from the calculated imaginary part.

(fig.7), as expected by the way if there is no zero. Imposing full agreement does not produce significant changes. Of course a conspiracy by a suitable set of zeros, restoring the ρ width, cannot be excluded. In the early days of VDM [28] a zero in the FF was related to the naked ρ mass. Zeros were foreseen in some versions of the Veneziano model [29], but once widths are introduced in the model zeros should be shifted to the unphysical sheet, as it is for the poles.

4 The nucleon isovector time-like magnetic FF

To obtain G_M^V , the nucleon isovector FF, and G_M^S , the isoscalar one, the neutron FF has to be considered as well. As mentioned before, the neutron time-like FF have been measured through only one experiment. The neutron magnetic FF has been derived [16] under the hypothesis that the neutron electric FF in the time-like region is much smaller just as it is for space-like region. In fact data are consistent with an anisotropic angular distribution. The DM2 measurement [30] of the Λ FF leads to results in very good agreement with FENICE, assuming U-spin invariance [31] and a Λ electric FF negligible too. The relationship $G_M(4M_n^2) = G_E(4M_n^2)$ should imply that, just at threshold, also the neutron magnetic FF vanishes. This assumption, relevant very near threshold only, has been considered in the following. In fig.1c the neutron space-like magnetic FF data are compared with the expectation from the solution of the DR on $\log G(Q^2)$. The neutron magnetic FF in the unphysical region as obtained by our procedure is reported in fig.4b.

The hypothesis that the FENICE data are wrong by a factor of ~ 2 has been simulated and the results are that the height of the ρ' resonance for the neutron is higher than the height of the ρ' resonance for the proton. Therefore, the apparent anomaly in the FENICE data (neutron FF not smaller than the proton FF) would still be there, but shifted to another energy range.

G_M^V and G_M^S are derived from the aforementioned proton and neutron FF and the imaginary part of G_M^V is shown in fig.8a. $|G_M^V|$ at the peak and its imaginary part are in rough agreement with the expectation, valid up to $Q^2 \sim 0.8\ GeV^2$, from the extended unitarity relation, using pion FF data and analytic continuation of πN scattering amplitude [8,9], which is also shown in fig.8a.

The different width of the ρ found for the nucleon with respect to the pion is shown in fig.9.

With a view to expose any possible common patterns, in the near vicinity of the threshold, a suitable linear combination of $(G_M^V)^2$ and $(G_M^S)^2$ is satisfactorily compared to the various measurements of the total $\sigma(e^+e^- \to$hadrons) cross section[24], taking into account the Q^2 bin width.

In fig.8b the imaginary part of G_M^S is shown. There is a peak at the ω mass , whose half width is compatible with the bin width and, remarkably, the imaginary part of G_M^S becomes different from zero at higher Q^2 than G_M^V , as expected.

There are predictions also for G_M^S . For instance, chiral perturbation theory suggests that the imaginary part of G_M^S is small up to $Q^2 \sim 0.5\ GeV^2$ [32]. However, given that G_M^S comes from a difference and also given its sensitivity to the bin width, the isoscalar sector is more affected by the regularization procedure and thus demands further work. In particular, promising results have been obtained by approximating the equation (1) in the unphysical region by means of orthogonal polinomials.

The fact that the total areas of the imaginary parts are equal to zero is in agreement with the superconvergence expectation.

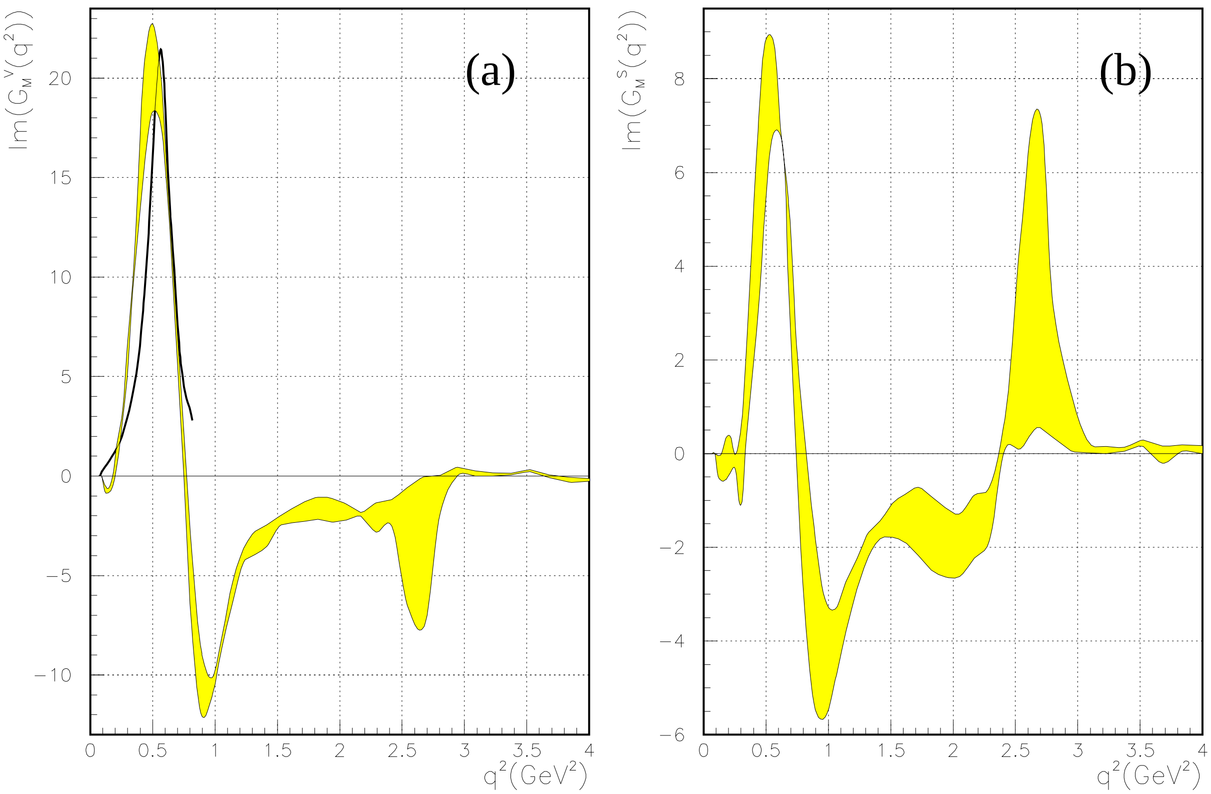

Figure 8: Imaginary part of the nucleon magnetic isovector (a) and isoscalar (b) form factor. Expectation from unitarity relation is also shown in (a).

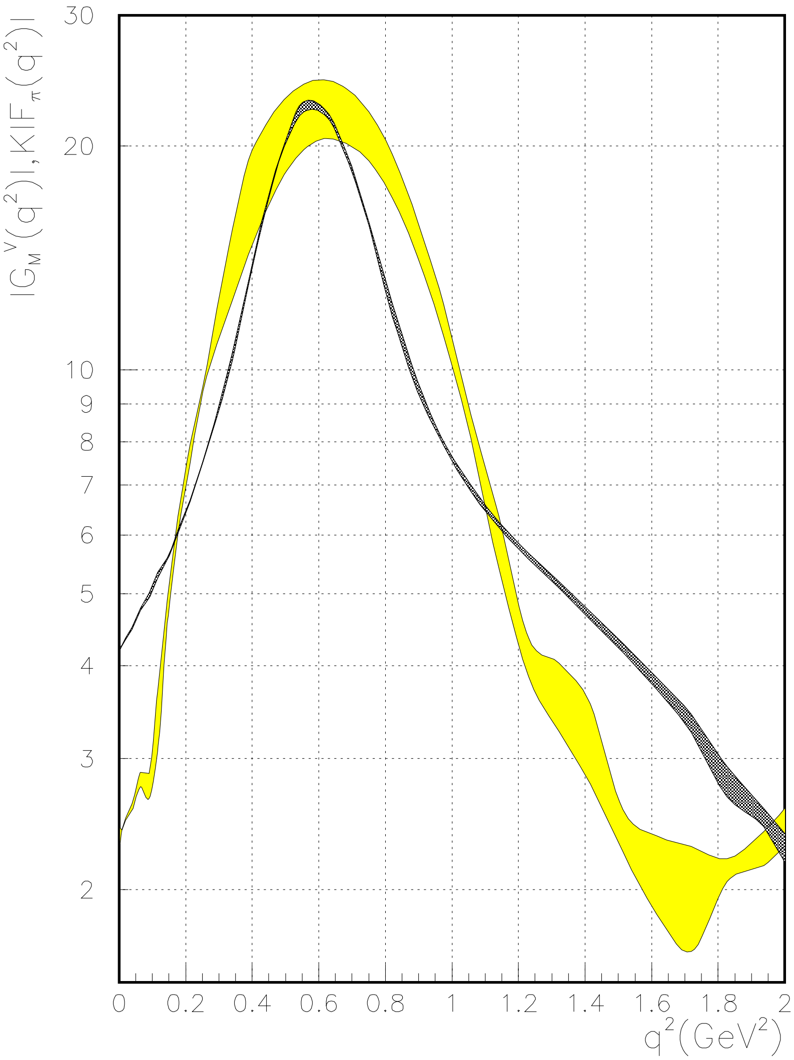

Figure 9: Comparison of the isovector nucleon form factor (gray shaded area) and the pion form factor (black shaded area) in the ρ mass region.

Conclusions

The nucleon time-like magnetic FF in the unphysical region has been obtained in an almost model independent way by means of a DR for $\log |G(Q^2)|$, using space-like and time-like data together with a regularization method.

Resonances have been found consistent with the $\rho(770)$ and $\rho'(1600)$ masses. However, a very large ρ width is obtained. This result reminds models in which mesons are different from baryons. No evidence has been found for a sizeable Φ contribution, contrary to the expectation, if there is indeed a large polarized strange content in the nucleon. This work is in progress, also to understand the sources of the discrepancies between our conclusions and other dispersion analyses as well as evaluations by means of the unitarized VDM.

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