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The CP Conserving Contribution to $K_L \to \pi^0 \nu\bar{\nu}$ in the Standard Model

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Abstract

The rare decay $K_L \to \pi^0 \nu\bar{\nu}$ is dominated by direct CP violation and can be computed with extraordinarily high precision. In principle, also a CP-conserving contribution to this process can arise within the Standard Model. We clarify the structure of the CP-conserving mechanism, analysing both its short-distance and long-distance components. It is pointed out that the calculation of the CP-conserving amplitude, although sensitive in part to non-perturbative physics, is quite well under control. The resulting CP-conserving contribution to the rate for $K_L \to \pi^0 \nu\bar{\nu}$ turns out to be very strongly suppressed because of several factors, which we discuss in detail.

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1 Introduction

The rare decay mode $K_L \to \pi^0\nu\bar{\nu}$ provides one of the most promising opportunities in flavour physics. It proceeds through a loop-induced flavour-changing neutral current (FCNC) transition and therefore probes Standard Model (SM) dynamics at the quantum level. In particular, $K_L \to \pi^0\nu\bar{\nu}$ offers a unique possibility to test the mechanism of CP violation [1]. The decisive virtue of $K_L \to \pi^0\nu\bar{\nu}$ is the exceptional degree to which the theoretical analysis of this decay is under control, with theoretical uncertainties at the level of a few per cent at most. The basic reason for this favourable situation is the absence of contributions from virtual photons, resulting in a power-like ($\sim m_i^2/M_W^2$, $i = u, c, t$) GIM cancellation pattern of the FCNC amplitude. Reliably calculable contributions from high-mass intermediate states ($m_t \gg m_c \gg \Lambda_{QCD}$) are therefore systematically enhanced over long-distance effects. This short-distance dominance is further reinforced in $K_L \to \pi^0\nu\bar{\nu}$, compared to the related mode $K^+ \to \pi^+\nu\bar{\nu}$, by the large CP-violating phase associated with the top loops.

Although the detection of $K_L \to \pi^0\nu\bar{\nu}$ is very challenging, because of a very small branching fraction ($\sim 3 \cdot 10^{-11}$ within the SM) and a difficult signature, considerable interest exists around the world in studying this decay experimentally and important steps toward this goal have already been undertaken. An experiment with the sensitivity to measure $B(K_L \to \pi^0\nu\bar{\nu})$ at the SM level has been proposed at Brookhaven (BNL-E926) [2]. The KAMI collaboration at Fermilab has published an Expression of Interest for such a measurement in the Main Injector era [3] and plans to search for this decay with similar sensitivities also exist at KEK in Japan [4]. Finally, the potential of KLOE at DAΦNE (the Frascati Φ-Factory) to search for $K_L \to \pi^0\nu\bar{\nu}$, with a smaller but still interesting sensitivity and on a short time scale, has recently been emphasized in [5].

Let us briefly summarize the status of the theory of $K_L \to \pi^0\nu\bar{\nu}$. The relevant low-energy effective Hamiltonian that describes the short-distance FCNC interaction inducing $K_L \to \pi^0\nu\bar{\nu}$ can be written as

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\Theta_W} \lambda_t X_0(x_t)\ (\bar{s}d)_{V-A} (\bar{\nu}_l \nu_l)_{V-A} + \text{h.c.}\ , \tag{1}$$

where $\lambda_i = V_{is}^* V_{id}$, $x_i = m_i^2/M_W^2$ and

$$X_0(x) = \frac{x}{8}\left[\frac{x+2}{x-1} + \frac{3x-6}{(x-1)^2}\ln x\right]\ . \tag{2}$$

The charm-quark contribution, sizeable in the charged mode $K^+ \to \pi^+\nu\bar{\nu}$, is completely negligible for $K_L \to \pi^0\nu\bar{\nu}$ and was omitted from (1). The Hamiltonian (1), with the one-loop function $X_0(x)$ first calculated in [6], provides a good starting point for the calculation of $K_L \to \pi^0 \nu\bar{\nu}$ in the Standard Model. Several important refinements have subsequently been added to the theoretical analysis of this decay. The dominant uncertainty of the lowest-order prediction can be eliminated by including NLO QCD effects [7]. The hadronic matrix elements $\langle\pi|(\bar{s}d)_V|K\rangle$ are known from the leading semileptonic decay $K^+ \to \pi^0 e^+ \nu$ using isospin symmetry. Corrections due to small isospin-breaking effects have been computed in [8]. Finally, the impact of higher-order electroweak effects ($\sim G_F^2 m_t^4$ in the amplitude) has been studied in [9].

Overall the theoretical uncertainties in the $K_L \to \pi^0 \nu\bar{\nu}$ branching fraction are thus under control to better than $\pm 3\%$, assuming that potential long-distance effects can be neglected. Such effects, which are not included in the description provided by (1), have been estimated in [10] and were indeed found to be safely negligible.

The dominant short-distance mechanism for $K_L \to \pi^0 \nu\bar{\nu}$ based on (1) violates CP symmetry as a consequence of the CP-transformation properties of $K_L \approx K^0_{CP-odd}$, π^0 and the hadronic $(V - A)$ transition current $\lambda_t (\bar{s}d)_{V-A} + \lambda_t^* (\bar{d}s)_{V-A}$. These imply (in standard CKM phase conventions)

$$A(K_L \to \pi^0 \nu\bar{\nu}) \sim \mathrm{Im}\lambda_t \langle\pi^0|(\bar{s}d)_{V-A}|K^0\rangle \; , \tag{3}$$

which would be zero in the limit of CP conservation. By contrast, the long-distance effects studied in [10] survive in this limit. In this respect, CP violation in $K_L \to \pi^0 \nu\bar{\nu}$ differs from the case of $K_L \to \pi\pi$, where the transition itself is forbidden by CP invariance.

The purpose of this letter is to present a systematic discussion of the CP-conserving contribution to $K_L \to \pi^0 \nu\bar{\nu}$ in the Standard Model. This question is of interest not only for estimating theoretical uncertainties from long-distance dynamics, but also as a matter of principle, in view of the role of $K_L \to \pi^0 \nu\bar{\nu}$ as a CP-violation "standard". In some New Physics scenarios the CP-conserving contributions to $K_L \to \pi^0 \nu\bar{\nu}$ can be important [11,12]. Thus also from this perspective it is interesting to quantify the CP-conserving effect that, in principle, exists in the Standard Model itself.

The present analysis confirms the estimate of [10] that the CP-conserving contribution to $K_L \to \pi^0 \nu\bar{\nu}$ in the Standard Model is very small. We differ, however, from [10] in our general approach and include in our discussion in particular the short-distance contribution to the CP-conserving amplitude, which has not been considered before.

In Sections 2 and 3 we analyse, respectively, the short-distance and the long-distance mechanism of the CP-conserving $K_L \to \pi^0 \nu\bar{\nu}$ amplitude. We conclude in Section 4.

2 The short-distance part of the CP-conserving amplitude

In the limit of exact CP symmetry, the leading term in the operator product expansion (OPE) for $s \to d\nu\bar{\nu}$ transitions (1) gives a vanishing contribution to the $K_L \to \pi^0\nu\bar{\nu}$ amplitude. More explicitly, in this limit the matrix element of the hadronic $\Delta S = 1$ transition current is given by

$$\langle\pi^0(p)|(\bar{s}d)_{V-A} + (\bar{d}s)_{V-A}|K_L(k)\rangle \ , \tag{4}$$

where the CKM parameters, chosen to be real, have been factored out. Using the CP-transformation properties ($\tilde{k}_\mu \equiv k^\mu$)

$$CP|K_L(k)\rangle = -|K_L(\tilde{k})\rangle \ , \qquad CP|\pi^0(p)\rangle = -|\pi^0(\tilde{p})\rangle \ , \tag{5}$$

$$CP(\bar{s}d)^\mu_{V-A}(CP)^{-1} = -(\bar{d}s)_{V-A,\mu} \ , \tag{6}$$

the matrix element (4) is seen to be zero.

A non-vanishing CP-conserving contribution, although forbidden by (1), can however arise at higher orders in the OPE. The leading effect of this type comes from the W-box diagram with intermediate charm (up) quarks depicted in Fig. 1. Matching this amplitude onto an effective Hamiltonian leads to the following, CP-conserving interaction term of dimension 8

$$\mathcal{H}_{CPC} = -\frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\Theta_W}\lambda_c \ln\frac{m_c}{\mu}\,\frac{1}{M_W^2}T_{\alpha\mu}\bar{\nu}(\overleftarrow{\partial^\alpha} - \partial^\alpha)\gamma^\mu(1-\gamma_5)\nu \ , \tag{7}$$

$$T_{\alpha\mu} = \bar{s}\overleftarrow{D}_\alpha\gamma_\mu(1-\gamma_5)d - \bar{d}\gamma_\mu(1-\gamma_5)D_\alpha s \ . \tag{8}$$

In this case we have $CP\,(T_{\alpha\mu})\,(CP)^{-1} = +T^{\alpha\mu}$, using the same CP conventions as above, and $\langle\pi^0|T_{\alpha\mu}|K_L\rangle$ is in general non-zero. Note that the relative minus sign in (8) results from the neutrino current in (7) being antisymmetric ($\sim (q_1 - q_2)^\alpha$) in the neutrino and antineutrino momenta (q_1 and q_2 respectively), and from the hermiticity of $\mathcal{H}_{CPC}$.

In order to obtain the charm contribution to $\mathcal{H}_{CPC}$, shown in (7), we have expanded the diagram of Fig. 1 in powers of external momenta divided by the charm-quark mass m_c (after contracting the W-boson lines). Only the leading term, of second order in momenta and independent of m_c up to a logarithm, has been retained. A similar expression would hold for the up-quark contribution if also $m_u \gg \Lambda_{QCD}$. Then a finite coefficient would result from the GIM cancellation and the logarithms would combine to $\ln(m_c/m_u)$. In reality m_u is small and the up-quark contribution has to be treated non-perturbatively. We will come back to this point below. In the meantime we have included in (7) only the charm part and kept the explicit dependence on a renormalization scale μ, which is to be

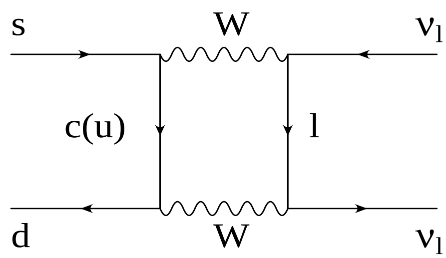

Figure 1: Box diagram that generates the leading short-distance CP-conserving contribution to $K_L \to \pi^0 \nu\bar{\nu}$.

cancelled by the up-quark sector. For an order-of-magnitude estimate, $\mu \gtrsim \Lambda_{QCD}$. We neglect lepton masses throughout this paper.

Besides the expression shown in (7), which is antisymmetric in the neutrino momenta, the graph of Fig. 1 also yields terms symmetric in q_1 and q_2. These contributions cannot induce $K_L \to \pi^0 \nu\bar{\nu}$ in the limit of CP conservation. A similar comment holds for the Z-penguin diagram, which is entirely symmetric in q_1 and q_2. In principle also intermediate top quarks could yield an interaction analogous to (7), but this contribution is strongly suppressed by small CKM couplings.

Parametrically, the amplitude arising from (7) is suppressed with respect to the leading CP-violating $K_L \to \pi^0 \nu\bar{\nu}$ amplitude by a factor

$$\delta^{SD}_{CPC} = \frac{m_K^2}{M_W^2} \frac{\lambda_c \ln(m_c/\Lambda_{QCD})}{\mathrm{Im}\lambda_t X_0(x_t)} \sim \mathcal{O}(10\%) \ . \tag{9}$$

Note that the very strong suppression due to the smallness of m_K^2/M_W^2 is at least partially compensated by a large enhancement factor $\lambda_c/\mathrm{Im}\lambda_t$, reflecting the CP-conserving nature of the mechanism under consideration. Since the CP-conserving amplitude is antisymmetric in the neutrino momenta, there is no interference with the leading CP-violating contribution in the integrated rate. Therefore the CP-conserving part is simply to be added in rate (rather than amplitude) to the CP-violating one. From the naive order-of-magnitude estimate given above, it would thus seem that an effect in the per cent range could still be possible. We will see in the following that its actual size is in fact considerably smaller.

To evaluate the amplitude generated by the dimension-8 operator in (7), one has to consider hadronic matrix elements of the form $\langle \pi^0 | \bar{s} \overleftarrow{D}_\alpha \gamma_\mu (1-\gamma_5) d | K^0 \rangle$, involving quark currents with QCD covariant derivatives. We will next determine these matrix elements to leading order in chiral perturbation theory [13]. In this framework the lowest-order realization of the QCD Lagrangian in the chiral limit, as a function of light pseudoscalar fields and external sources, is given by

$$\mathcal{L}_S^{(2)} = \frac{f^2}{8}\mathrm{tr}\left\{\bigtriangledown^\mu U \bigtriangledown_\mu U^\dagger + \chi U^\dagger + U\chi^\dagger\right\} \ , \tag{10}$$

where U, transforming as $U \to g_R U g_L^\dagger$ under $SU(3)_R \otimes SU(3)_L$ chiral rotations, can be written as $U = \exp(2i\Phi/f)$ with

$$\Phi = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{bmatrix} . \tag{11}$$

Here $f = 132\,\mathrm{MeV}$ is the pion decay constant; χ, transforming as U, is the external scalar source; and $\bigtriangledown_\mu = \partial_\mu U - ir_\mu U + iUl_\mu$ is the $SU(3)_R \otimes SU(3)_L$ covariant derivative, which depends on the vector sources r_μ and l_μ (see e.g. [14] for conventions and a review on the subject). As usual, to incorporate mass terms we shall replace

$$\chi \to 2BM \ , \qquad\qquad M = \mathrm{diag}(m_u, m_d, m_s) \ , \tag{12}$$

where B is a real constant that can be expressed in terms of quark and meson masses (e.g. to lowest order $m^2_{K^0} = B(m_s + m_d)$).

The derivatives of $\mathcal{L}_S^{(2)}$ with respect to the external sources allow us to compute the lowest-order matrix elements of the corresponding quark currents. For instance, this leads to

$$\bar{q}_i\gamma_\mu(1-\gamma_5)q_j = i\frac{f^2}{2}(\partial_\mu U^\dagger U)_{ji} + O(p^3) \ , \tag{13}$$

$$\bar{q}_i(1-\gamma_5)q_j = -\frac{f^2}{2}BU^\dagger_{ji} + O(p^2) \ , \tag{14}$$

where the above equalities have to be understood as identities of the corresponding matrix elements. For the derivative operators we are interested in here, the lowest non-vanishing order is $\mathcal{O}(p^2)$ and this statement alone would lead to the m_K^2 factor in the naive estimate (9). To be more specific, chiral symmetry implies the following general representation at $\mathcal{O}(p^2)$

$$\begin{aligned} \bar{q}_i \overleftarrow{D}_\alpha \gamma_\mu(1-\gamma_5)q_j = \ & i[g_{\alpha\mu}(c_0\chi^\dagger U + \bar{c}_0 U^\dagger\chi) + c_1\partial_\alpha U^\dagger\partial_\mu U + c_2\partial_\mu U^\dagger \partial_\alpha U \\ & + c_3\partial_\alpha\partial_\mu U^\dagger U - ic_4\varepsilon_{\alpha\mu\beta\nu}\partial^\beta U^\dagger\partial^\nu U]_{ji} \ . \end{aligned} \tag{15}$$

Note that the further possible term $U^\dagger\partial_\alpha\partial_\mu U$ satisfies the identity

$$U^\dagger\partial_\alpha\partial_\mu U \equiv -\partial_\alpha\partial_\mu U^\dagger U - \partial_\alpha U^\dagger\partial_\mu U - \partial_\mu U^\dagger\partial_\alpha U \tag{16}$$

and is not independent of those already present in (15).

CP invariance of the strong interactions implies that all coefficients c_0, $\bar{c}_0$, c_1, ..., c_4 are real (we employ the CP convention $CP\,\Phi_{ij} = -\Phi_{ji}$). Adding to (15) its Hermitian conjugate yields an expression for $\partial_\alpha(\bar{q}_i\gamma_\mu(1-\gamma_5)q_j)$ in terms of the c_i. This can be compared with the derivative of the $V-A$ current from (13). Taking into account (16), we obtain

$$c_0 = \bar{c}_0\,, \qquad c_2 - c_1 = \frac{f^2}{4} \qquad \text{and} \qquad c_3 = \frac{f^2}{4}\,. \tag{17}$$

Additional constraints follow from the quark equations of motion

$$\bar{q}_i \overleftarrow{\not{D}}(1-\gamma_5)q_j = i\bar{q}_i m_i(1-\gamma_5)q_j = -i\frac{f^2}{4}(U^\dagger\chi)_{ji}\,, \tag{18}$$

or equivalently from the equations of motion of the U field

$$(\partial^2 U^\dagger U - U^\dagger\partial^2 U)_{ji} = (\chi^\dagger U - U^\dagger\chi)_{ji}\,, \qquad (i \neq j)\,. \tag{19}$$

Comparing (18) with (15) for $\mu = \alpha$ and using

$$\begin{aligned} \partial^2 U^\dagger U &\equiv \frac{1}{2}(\partial^2 U^\dagger U + U^\dagger\partial^2 U) + \frac{1}{2}(\partial^2 U^\dagger U - U^\dagger\partial^2 U) &(20)\\ &= -\partial U^\dagger\partial U + \frac{1}{2}(\chi^\dagger U - U^\dagger\chi)\,, &(21)\end{aligned}$$

one finds

$$c_0 = -\frac{1}{8}c_3 \qquad \text{and} \qquad c_1 + c_2 = \frac{f^2}{4}\,. \tag{22}$$

Together with (17) we then have

$$\begin{aligned} \bar{q}_i\overleftarrow{D}_\alpha\gamma_\mu(1-\gamma_5)q_j &= i\frac{f^2}{4}[\partial_\mu U^\dagger\partial_\alpha U + \partial_\alpha\partial_\mu U^\dagger U - \frac{1}{8}g_{\alpha\mu}(\chi^\dagger U + U^\dagger\chi)]_{ji}\\ &\quad + c_4\varepsilon_{\alpha\mu\beta\nu}(\partial^\beta U^\dagger\partial^\nu U)_{ji}\,. \end{aligned} \tag{23}$$

Recalling that in our convention $CP|K\rangle = -|\bar{K}\rangle$ and $|K_L\rangle = (|K\rangle + |\bar{K}\rangle)/\sqrt{2}$, we finally get

$$\langle\pi^0(p)|T_{\alpha\mu}|K_L(k)\rangle = -\frac{i}{2}\left[(k-p)_\alpha(k+p)_\mu + \frac{1}{4}m_K^2 g_{\alpha\mu}\right]\,, \tag{24}$$

where the undetermined coefficient c_4 has dropped out.

Interestingly enough, the matrix element (24) gives zero when multiplied with the leptonic current from (7). We then conclude that $\langle\pi^0\nu\bar{\nu}|\mathcal{H}_{CPC}|K_L\rangle$ vanishes to leading order in chiral perturbation theory and the CP-conserving $K_L \to \pi^0\nu\bar{\nu}$ transition from $\mathcal{H}_{CPC}$ therefore receives an additional $\mathcal{O}(m_K^2/(8\pi^2 f^2))$ suppression.

To obtain a quantitative estimate we may write

$$\langle\pi^0(p)|T_{\alpha\mu}|K_L(k)\rangle = i\frac{a_\chi}{2}(k+p)_\alpha(k+p)_\mu \;, \qquad a_\chi = \mathcal{O}\left(\frac{m_K^2}{8\pi^2 f^2}\right) \sim 20\% \;. \tag{25}$$

Introducing the kinematical variables

$$y_{1,2} = \frac{2k\cdot q_{1,2}}{m_K^2}\;, \qquad y = \frac{y_1-y_2}{2}\;, \tag{26}$$

$$u = \frac{(q_1+q_2)^2}{m_K^2} \quad \text{and} \quad z = \frac{m_\pi^2}{m_K^2} \tag{27}$$

(with q_1 (q_2) the (anti)neutrino momentum), this implies

$$\left|A(K_L\to\pi^0\nu\bar\nu)^{SD}_{CPC}\right| = \left|y\; a_\chi\; \delta^{SD}_{CPC}\; A(K_L\to\pi^0\nu\bar\nu)_{CPV}\right|. \tag{28}$$

Including the phase-space integrations we obtain for the decay rates

$$\Gamma(K_L\to\pi^0\nu\bar\nu)^{SD}_{CPC} = \Gamma(K_L\to\pi^0\nu\bar\nu)_{CPV}\; |a_\chi|^2\left|\delta^{SD}_{CPC}\right|^2 R_{kin}\;, \tag{29}$$

where ($\lambda(a,b,c) = a^2+b^2+c^2-2ab-2ac-2bc$)

$$R_{kin} = \frac{\int \mathrm{d}\Gamma_{\pi^0\nu\bar\nu}\,[\lambda(1,u,z)-4y^2]y^2}{\int \mathrm{d}\Gamma_{\pi^0\nu\bar\nu}\,[\lambda(1,u,z)-4y^2]} = \frac{1}{30}\frac{g(z)}{f(z)} \;\simeq\; 0.03 \tag{30}$$

and

$$g(z) \;=\; 1-9z+45z^2-45z^4+9z^5-z^6+60z^3\ln z\;, \tag{31}$$

$$f(z) \;=\; 1-8z+8z^3-z^4-12z^2\ln z\;. \tag{32}$$

We thus find that the CP-conserving rate $\Gamma(K_L\to\pi^0\nu\bar\nu)^{SD}_{CPC}$ is further suppressed by phase space, in addition to the parametric effect from δ^{SD}_{CPC} and the chiral suppression described by a_χ. Numerically the three suppression factors on the r.h.s. of (29) give $\sim 4\cdot 10^{-2}\times 10^{-2}\times 3\cdot 10^{-2} = 10^{-5}$. We therefore conclude that $\Gamma(K_L\to\pi^0\nu\bar\nu)^{SD}_{CPC}$ is safely negligible, by a comfortably large margin.

3 The long-distance part of the CP-conserving amplitude

Chiral perturbation theory provides a reliable framework also to estimate the long-distance CP-conserving contribution to $K_L\to\pi^0\nu\bar\nu$ generated by the up-quark exchange in Fig. 1. The lowest-order contributions correspond to the charged π and K exchange in Fig. 2, added to an appropriate local counterterm. Since the lowest-order coupling of

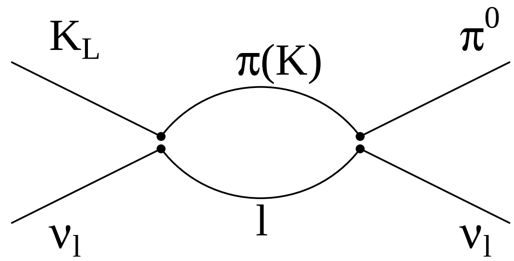

Figure 2: Long-distance diagram that contributes to the CP-conserving $K_L \to \pi^0 \nu\bar{\nu}$ amplitude.

the pseudoscalar mesons with the leptonic currents can be derived from $\mathcal{L}_S^{(2)}$, the loop amplitude is completely determined. The result is logarithmically divergent and is given by

$$A(K_L \to \pi^0 \nu\bar{\nu})^{LD}_{CPC} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \Theta_W} \lambda_u \, \frac{1}{M_W^2} \, H_\mu \bar{\nu}\gamma^\mu (1-\gamma_5)\nu \, , \tag{33}$$

where

$$H_\mu = H_\mu^\pi + H_\mu^K \, , \tag{34}$$

$$\begin{aligned} H_\mu^\pi &= \frac{1}{2} k_\mu \, m_K^2 y \left[\ln \frac{\mu^2}{m_K^2} + 2 + \right. \\ &\quad \left. + \frac{(1-y_1)\ln(1-y_1) - (1-y_2)\ln(1-y_2)}{y_1 - y_2} + i\pi \right] \, , \end{aligned} \tag{35}$$

$$H_\mu^K = \frac{1}{4} k_\mu \, m_K^2 y \left[\ln \frac{\mu^2}{m_K^2} + 2 + \frac{1}{y_1 - y_2} \left(\frac{y_1^2 \ln y_1}{1-y_1} - \frac{y_2^2 \ln y_2}{1-y_2} \right) \right] . \tag{36}$$

Here we have used dimensional regularization and subtracted the divergence according to the $\overline{MS}$ prescription. The imaginary part is due to the absorptive contribution of the pion. We are neglecting the pion mass in the loop integration. The μ dependence in (35) and (36) has to be cancelled by the counterterm, whose finite contribution is however not know. As a matter of fact, adding the counterterm to the loop result would just produce the effect of fixing μ in (35) and (36) to some typical hadronic scale.

In principle, looking at the quark-level result, one could think that the μ dependence of (35) and (36) should be partially related to the scale dependence of the charm contribution in (7). However, the vanishing of $\langle \pi^0 \nu\bar{\nu} | \mathcal{H}_{CPC} | K_L \rangle$ to the lowest order implies that there is no direct matching between the two contributions at this level: the ultraviolet cut-off of the leading long-distance contribution is more likely to be a hadronic scale below the charm mass.

An indirect confirmation of the above statement is obtained by estimating higher-order contributions in the framework of vector-meson dominance. In this limit the point-like form factors of the vector currents in Fig. 2 are replaced by vector-meson propagators ($1 \to M_V^2/(M_V^2 - q^2)$) and as a result the loop amplitude becomes finite. Note that other potentially divergent contributions generated by vector or axial-vector exchange vanish to leading order, as can be checked explicitly using the lowest-order chiral Lagrangian of [15]. Thus the main effect of vector resonances is just to provide a natural cut-off for the loop amplitude of Fig. 2. Expanding the full result thus obtained in $1/M_V$ and neglecting terms suppressed by powers of m_K^2/M_V^2, one recovers an amplitude of the form (35) and (36), with

$$\ln \frac{\mu^2}{m_K^2} \to \ln \frac{M_V^2}{m_K^2} - \frac{3}{2} \,. \tag{37}$$

It is convenient to further expand the result in the kinematical variables y and u, which yields

$$H_\mu = \frac{1}{2} k_\mu \, m_K^2 y \left[\frac{3}{2} \ln \frac{M_V^2}{m_K^2} + \frac{1}{4} - \frac{1}{2} \ln 2 + i\pi + \mathcal{O}\left(u, y^2\right) \right] \,. \tag{38}$$

Comparing (38) with (35) and (36), it is seen that the common vector-meson mass $M_V \simeq 800$ MeV (we can safely neglect the small $SU(3)$-breaking effects in the vector-meson sector) provides the ultraviolet cut-off for the lowest-order calculation. The expansion in powers of the kinematical variables is well justified not only by the size of the higher-order terms but, especially, because the phase-space integration strongly suppresses their contribution to the total rate.

Using the approximate expression (38) the integration over the $\pi^0 \nu \bar{\nu}$ Dalitz plot can be done analytically. Normalizing the CP-conserving amplitude to the leading CP-violating term, generated by (1), we can write

$$\left| A(K_L \to \pi^0 \nu \bar{\nu})_{CPC}^{LD} \right| = \left| y \, \delta_{CPC}^{LD} \, A(K_L \to \pi^0 \nu \bar{\nu})_{CPV} \right| , \tag{39}$$

where, analogously to (9), we have defined

$$\delta_{CPC}^{LD} = \frac{m_K^2}{4 M_W^2} \frac{\lambda_u}{\mathrm{Im} \lambda_t X_0(x_t)} \left| \frac{3}{2} \ln \frac{M_V^2}{m_K^2} + \frac{1}{4} - \frac{1}{2} \ln 2 + i\pi \right| \;\approx\; 0.04 \,. \tag{40}$$

Taking into account the phase-space integration, the CP-conserving rate can be written as

$$\Gamma(K_L \to \pi^0 \nu \bar{\nu})_{CPC}^{LD} = \Gamma(K_L \to \pi^0 \nu \bar{\nu})_{CPV} \, \left| \delta_{CPC}^{LD} \right|^2 R_{kin} \,. \tag{41}$$

The total suppression factor on the r.h.s. of (41) is thus estimated to be $\sim 2 \cdot 10^{-3} \times 3 \cdot 10^{-2} = 6 \cdot 10^{-5}$.

4 Conclusions

Even if CP were exactly conserved, the decay $K_L \to \pi^0 \nu \bar{\nu}$ could in principle proceed within the Standard Model. We have presented a detailed analysis of both the short-distance and the long-distance contributions to the CP-conserving rate. We have shown that these contributions are suppressed by more than four orders of magnitude in comparison to the leading, direct CP-violating branching ratio of about $3 \cdot 10^{-11}$. Several reasons are responsible for this very strong suppression: first, the CP-conserving amplitude, which is parametrically $\mathcal{O}(10\%)$ of the direct CP-violating one, does not interfere with the latter, but is added simply in rate. In addition, there is a substantial suppression factor (~ 0.03) from phase space. Finally, the short-distance part of the CP-conserving amplitude is also chirally suppressed.

The hierarchy of the direct CP-violating, indirect CP-violating [16] and CP-conserving contributions to $B(K_L \to \pi^0 \nu \bar{\nu})$ in the Standard Model is thus $1 \;:\; 10^{-2} \;:\; \lesssim 10^{-4}$. Note, however, that in the absence of direct CP violation, as in a superweak model [17], the $K_L \to \pi^0 \nu \bar{\nu}$ branching fraction due to indirect CP violation alone would only be $\sim 2 \cdot 10^{-15}$, comparable to $B(K_L \to \pi^0 \nu \bar{\nu})_{CPC}$. The above hierarchy of contributions can be contrasted with the case of $K_L \to \pi^0 e^+ e^-$, where all three mechanisms are of roughly comparable magnitude [18].

We have also found that the calculation of the CP-conserving $K_L \to \pi^0 \nu \bar{\nu}$ rate is quite well under control, although the precise value depends on long-distance dynamics that is hard to quantify in detail. The extremely strong suppression of such contributions highlights the theoretically clean nature of $K_L \to \pi^0 \nu \bar{\nu}$ in a striking manner.

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