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On the Efficiency of the Coincidence Search in Gravitational Wave Experiments

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We discuss the problem of the detection of gravitational waves (gw) signals with small energy signal to noise ratio (SNR). We consider coincidence experiments between data processed by optimum filters matched to delta-like bursts. It is shown, by calculation and by simulation, that, because of the noise, the "event" lists produced by the same signals on different detectors, using the same filters, overlap only partially — about 30 percent for SNR close to the threshold used for defining the events. Furthermore, because of the noise, the correlation of the event energy between identical detectors is weak and cannot be used as a strong discriminator against noise in coincidence search, even for SNR = 10 or more.

1. INTRODUCTION

In a coincidence experiment with gravitational wave (gw) antennas one deals with measurements having small signal to noise ratios (SNR) and the search for coincidences, with very few gw signals expected, is indeed, for this reason, very difficult because many small signals can be lost in the noise.

The procedure usually adopted is to apply optimum filters to the data, in order to extract the small signals as well as possible. An arbitrary threshold is then applied and the "event" is defined when the filtered signal exceeds the chosen threshold.

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Each detector provides a list of events for comparison with lists of events from other detectors. The final step, in the coincidence search, is to count how many events fall within a given time window and to compare this number with the number of events obtained accidentally, that is when the event occurrence times are changed in a random way or by a simple time shifting. This comparison gives the probability that the found coincidences might have occurred by chance.

The above statistical analysis is complicated by the small SNR. The purpose of this note is to study how the small SNR affects the search for coincidences between two or more identical gw detectors using the same filter to process the signals, and possibly to find the best strategy to adopt as regards the choice of the threshold.

We are well aware that this is a simplified representation of the actual experimental situation, as one should also consider the effects of using different filters (in particular as regards their sampling and optimum times), the presence of the non-gaussian disturbances that often show up in the tails of the data distributions, and the different sampling times of the detectors as well as possible differences of orientation between the detectors. We found, however, that the dispersion of the signal amplitude due to the noise is a dominant effect, and this result has to be taken into consideration when searching for coincidences between gravitational wave detectors.

2. THE EFFECT OF THE NOISE ON THE OBSERVED SIGNALS

We report here on the effect of the noise on the observation in the most simple way. We consider a resonant antenna whose signal is processed by a lock-in amplifier, which extracts the Fourier components at one resonance frequency of the detector, producing two signals in quadrature. To each one of these signals we apply the optimum filter, finally obtaining the two components

$$x(t) = n(t) + s(t), \qquad y(t) = m(t),$$

where $n(t)$ and $m(t)$ indicate the noise and $s(t)$ the signal due to gw bursts. The signal, in general, appears in both the x and y component, but, for the purpose of this paper and for simplicity we only consider one component. The noise is indicated by two different symbols, $n(t)$ and $m(t)$, to stress the fact the two noise processes in the two components are, in general, independent.

Following the usual procedure we compute the quantity

$$\rho^2 = x^2 + y^2$$

which represents the optimum estimate of the energy innovation, that is, the variation of the energy status of the detector as due, for example, to a gw excitation. The mean value of ρ^2, in the absence of signals, is called the effective temperature T_{eff} of the detector when expressed in units of Kelvin. In the presence of signals of given amplitude s (representing the response of the detector to a short burst), the corresponding estimate ρ^2 of the energy innovation is not s^2 but, due to the effect of the noise, a random variable with a noncentral χ^2 distribution probability with two degrees of freedom [1,2],

$$f(\rho^2, s^2) = \frac{1}{T_{\text{eff}}} \exp\left(- \frac{\rho^2 + s^2}{T_{\text{eff}}} \right) I_0\left(\frac{2\sqrt{\rho^2 s^2}}{T_{\text{eff}}} \right), \tag{1}$$

where I_0 is the modified Bessel function of order zero.

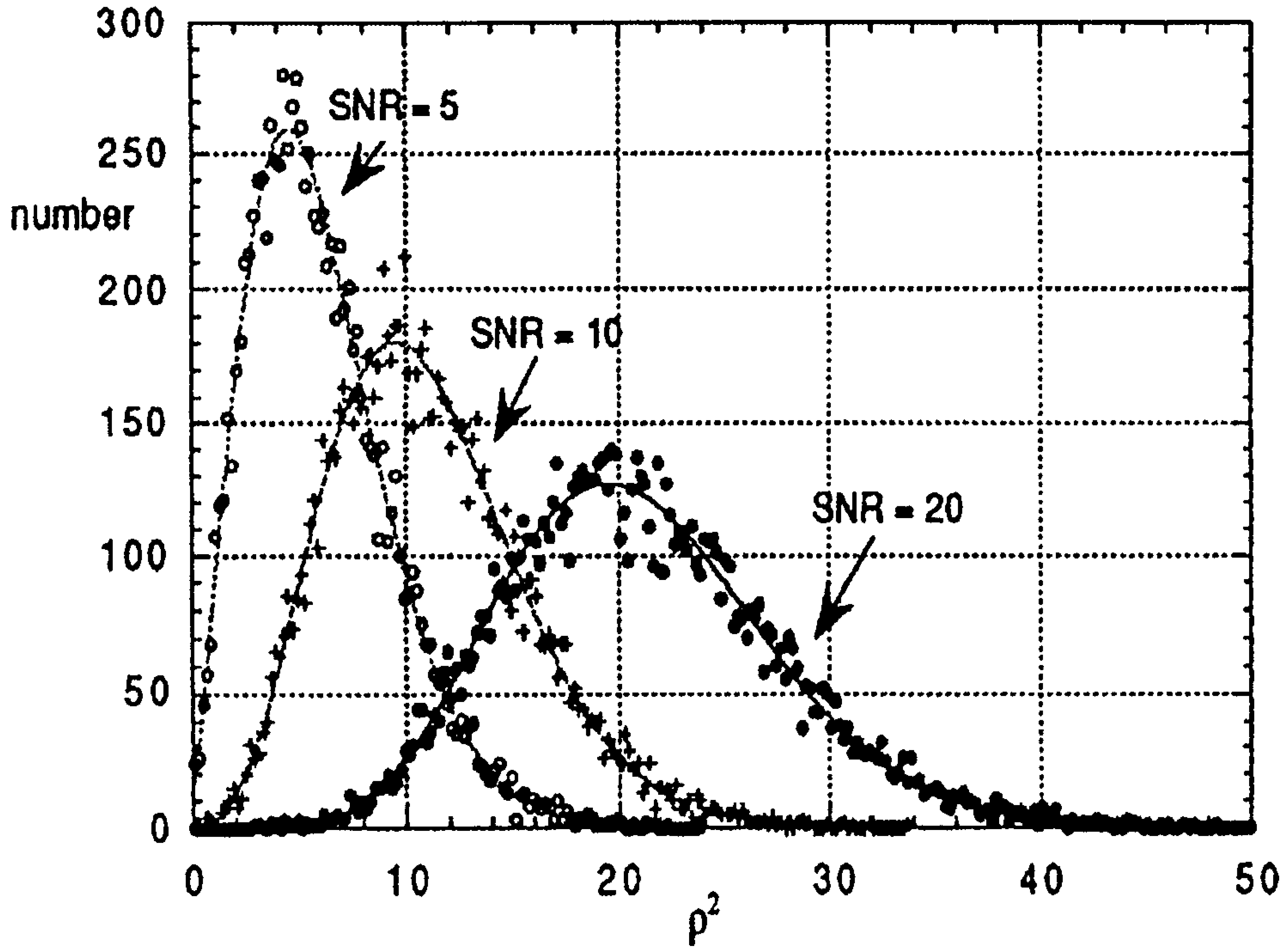

Figure 1. Histogram of ρ^2 for SNR = 5,10,20 and the expected distribution as obtained by eq. (1). We notice that, due to the noise, ρ^2 covers rather wide intervals in spite of the relatively large SNR. The relative dispersion of ρ^2 increases for decreasing values of the SNR (the ratio between s^2 and T_{eff}) and is large even for relatively high values of SNR.

Table 1. Average μ, standard deviation σ and their ratio for signals distributed according to eq. (1), with various SNR.

SNR	μ	σ	σ/μ
5	6	3.32	0.558
10	11	4.58	0.417
15	16	5.57	0.348
20	21	6.40	0.308
25	26	7.14	0.275

This is shown in Table 1 and in Fig. 1, where we have also reported the results of a simulation performed according to the above model (in the simulation $T_{\text{eff}} = 1$).

This figure gives a quantitative feeling of the effect of the noise on the signal. The nominal SNR is given by s^2 in units of the ρ^2 average when only the noise is present ($T_{\text{eff}} = 1$). Similar results have been experimentally obtained by the Stanford [3] and Louisiana [4] groups.

3. COINCIDENCE SEARCH

It is evident that if a gw burst, with energy close to the threshold THR chosen for selecting the events, impinges on two different gw antennas, there is a high chance that no coincidence is observed. This occurs when, because of the noise, the resulting ρ^2, after optimum filtering, happens to be below the threshold in one or in both antennas. Our goal here is to discuss this point on a quantitative basis, in order to investigate the possible best strategy for coincidence search. We simulate two identical antennas. Each time we construct the quantity ρ^2 for an applied signal $s(t)$ (whose square we call SNR and is expressed in units of the gaussian variance), and verify the condition $\rho^2 \geq \text{THR}$ for two subsequent determination of ρ^2, one for each antenna. The result is shown in Fig. 2 for three THR values together with the theoretical curves obtained by eq. (2),

$$n_c(s^2) = N \int_{\text{THR}}^{\infty} f(\rho^2, s^2) d\rho^2 \int_{\text{THR}}^{\infty} f(\rho^2, s^2) d\rho^2. \qquad (2)$$

We notice that for SNR = THR the coincidence detection efficiency is of the order of $\epsilon = 30\%$ for all the THR values. If we normalize SNR to THR we obtain the plots of Fig. 3.

We also notice that in order to have a detection efficiency close to unity we have to choose a threshold of about one half the energy of the

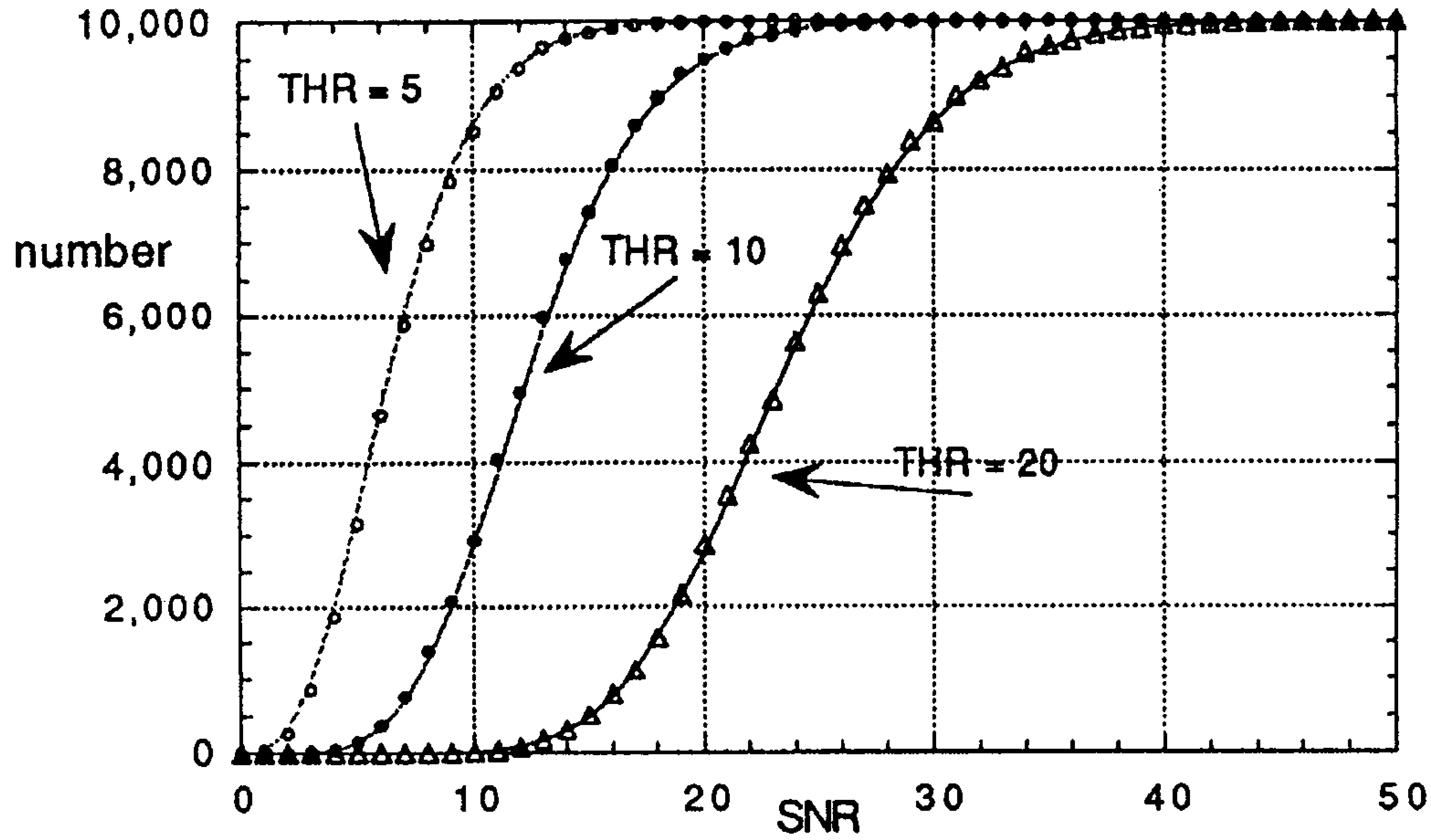

Figure 2. Number of coincidences with 10,000 trials and the eq. (2) versus s^2 = SNR, for the thresholds THR = 5,10, 20.

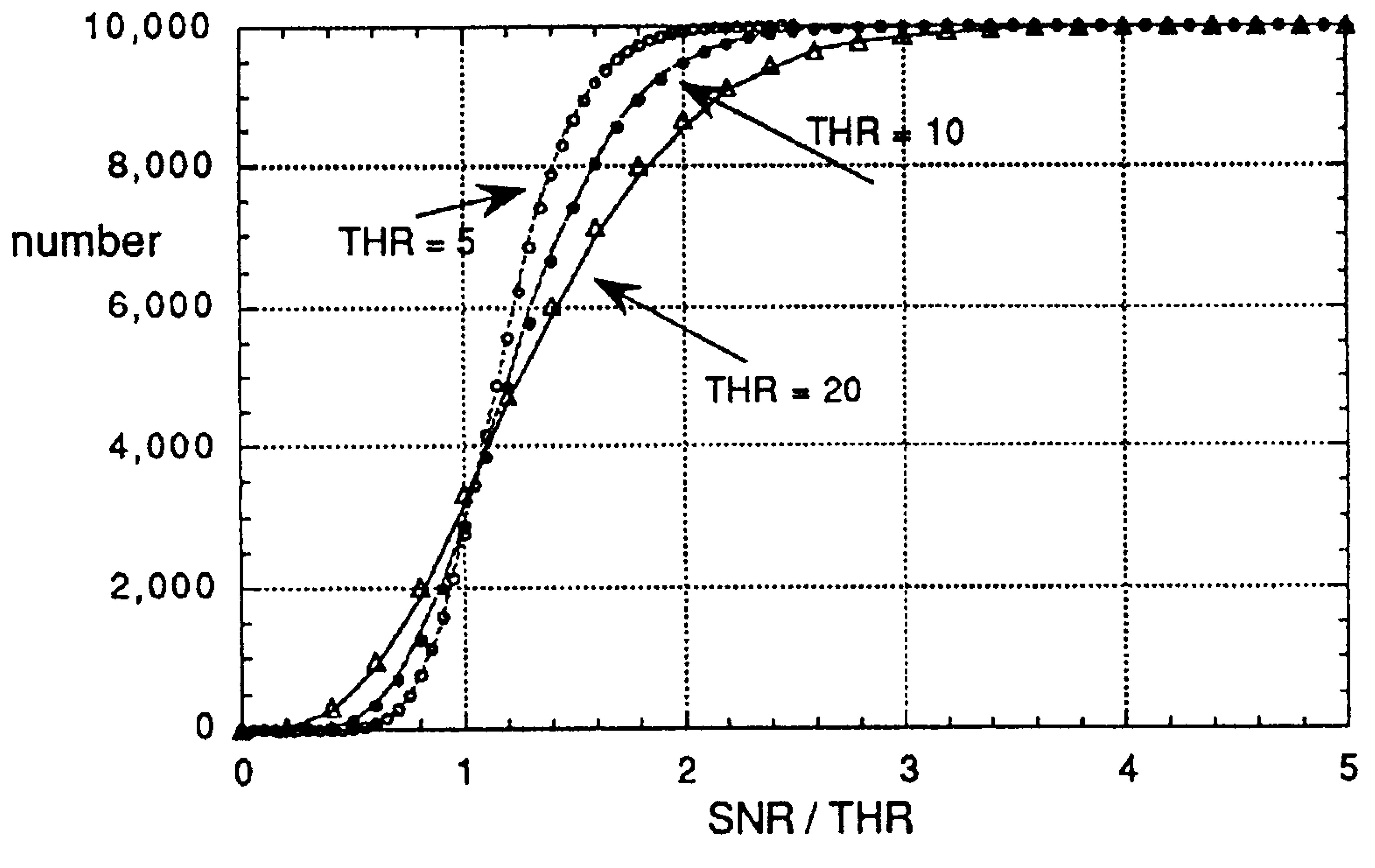

Figure 3. The data of Fig. 2 plotted versus the normalized SNR.

signals.[4]

The above result is valid for the coincidences between two different antennas that have independent noise but is does not take into account an additional coincidence loss due to time delays introduced by the noise.

[4] On the other hand we have the interesting (though low-probability) case where we detect coincidences even for SNR < THR. This happens when the noise adds to the signals in a coherent way.

However it can be applied also to a single antenna when using two different optimum filters that have nearly independent noise. For example, this is the case when the two different algorithms process data with very different sampling rates, say 1 ms and 100 ms. In this case we expect that the two filters produce event lists that overlap only partially.[5]

In the Rome group overlapping rates ranging from a few percent to twenty or thirty percent were found [5]. Making use of the result shown in Fig. 3 we justify the small percentage if we assume that, in such a case, most of the signals (which we consider local disturbances) have a SNR of the order of or below the threshold THR.

It has sometimes been suggested that only those events should be considered which are obtained for one antenna from the coincidences obtained using two different filters, such that the noise in one filter can be considered nearly independent from the noise in the other filter (both matched to a delta signal). Then one would search for coincidences with a similar list of events from a second antenna.[6] In this case for SNR = THR the detection efficiency is reduced to 30% $\times$ 30%, that is about 9%.

It has also been proposed, as a test to verify if the coincidences between the events of two antennas (assumed to have the same cross-section) are really due to the same gravitational wave bursts, that the coincident events be required to have approximately the same energy. It is clear that, because of the noise, this might not be so.

We tested this requirement with the following procedure. We have considered, in two identical simulated antennas, equal signals with SNR $\geq$ THR $= 5$, more precisely signals with SNR$i = i$ ($i = 5, 6, \ldots, 50$), 20 signals for each value of i (a total of 920 signals). As usual, we found the coincidences by requiring that, for each pair of events, both $\rho^2 \geq$ THR. We have then calculated the correlation coefficient between the energies of the observed pairs of events in various cases as shown in Tables 2 and 3.

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[5] Note, in addition, that in this case a signal with duration larger than the sampling time of one detector and smaller than that of the other one will be taken as a delta function by the latter but not by the former. The consequence is that the energy estimates will be different.

[6] We assume that both filters have the same noise, otherwise it is obvious that only the filter with the lower noise should be used.

Table 2. The last column indicates the probability [2] (in percent) that the observed correlation be due to chance. The number of coincidences is smaller than the number of applied signals because of the threshold, THR = 5.

signal energy interval	number of signals	number of coincidences	energy correlation coefficient	probability (%)
5–50	920	868	0.73	≈ 0
5–15	220	168	0.28	0.3
5–10	120	73	0.14	24

Table 3. We arbitrarily consider 12 randomly selected coincidences in the given energy interval, as a realistic and very optimistic example of one gw burst per month, in one year of operation. The last column indicates the probability [2] (in percent) that the observed correlation be due to chance.

signal energy interval	number of coincidences	energy correlation coefficient	probability (%)
5–10	12	0.011	97
5–15	12	0.18	60
5–20	12	0.38	23
5–30	12	0.67	2
15–30	12	0.37	24

In Figs. 4 and 5 we show two examples of energy correlation graphs, obtained from the above simulations, both with a very poor correlation.

We notice, in addition, that the situation in the real case is worse, as we have to consider that, even if some coincidence due to gw do exist, accidental coincidences will also be present. The worsening of the correlation due to a number N_a of accidental coincidences which add to the number N_c of coincidences due to gw, can be approximately estimated with the formula

$$r_{\mathrm{c+a}} = r_\mathrm{c} \frac{N_\mathrm{c}}{N_\mathrm{c} + N_\mathrm{a}},$$

where r_c is the correlation coefficient for the N_c signals alone and $r_{\mathrm{c+a}}$ the coefficient when we add the N_a accidentals.

For example, in the case of Fig. 5, adding 12 accidentals to the 12 coincidences we obtain a correlation coefficient of 0.19, for a probability of 39% that the correlation among the event energies is due to chance. Note that if we found a total number of $N_\mathrm{c} + N_\mathrm{a}$ coincidences over a background of N_a, the Poisson probability to have such a coincidence excess was very

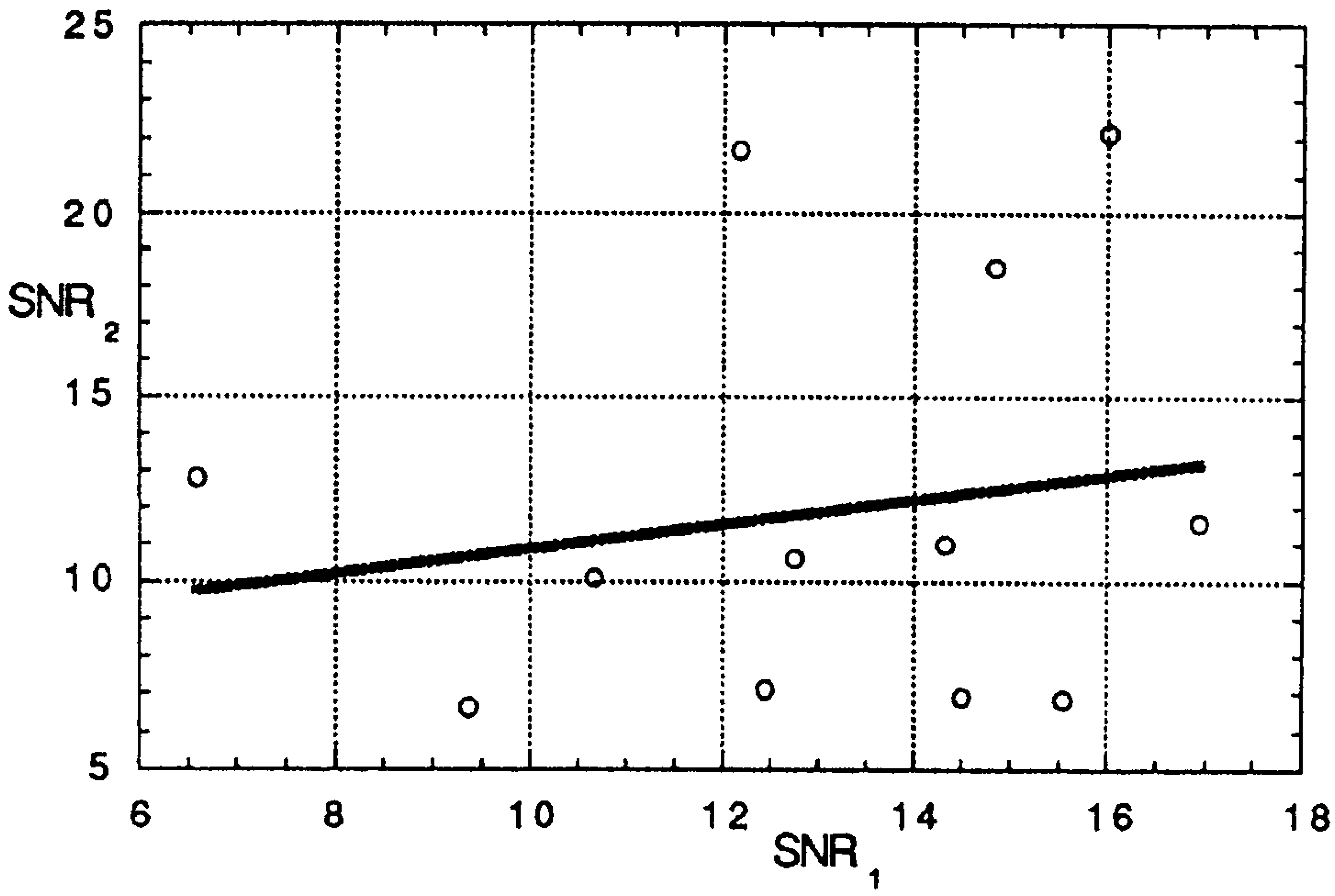

Fig. 4. Correlation among the simulated 12 coincidences between two identical antennas, with event energies in the range 5–15. The correlation coefficient is 0.18. The probability that the correlation be accidental is about 60%.

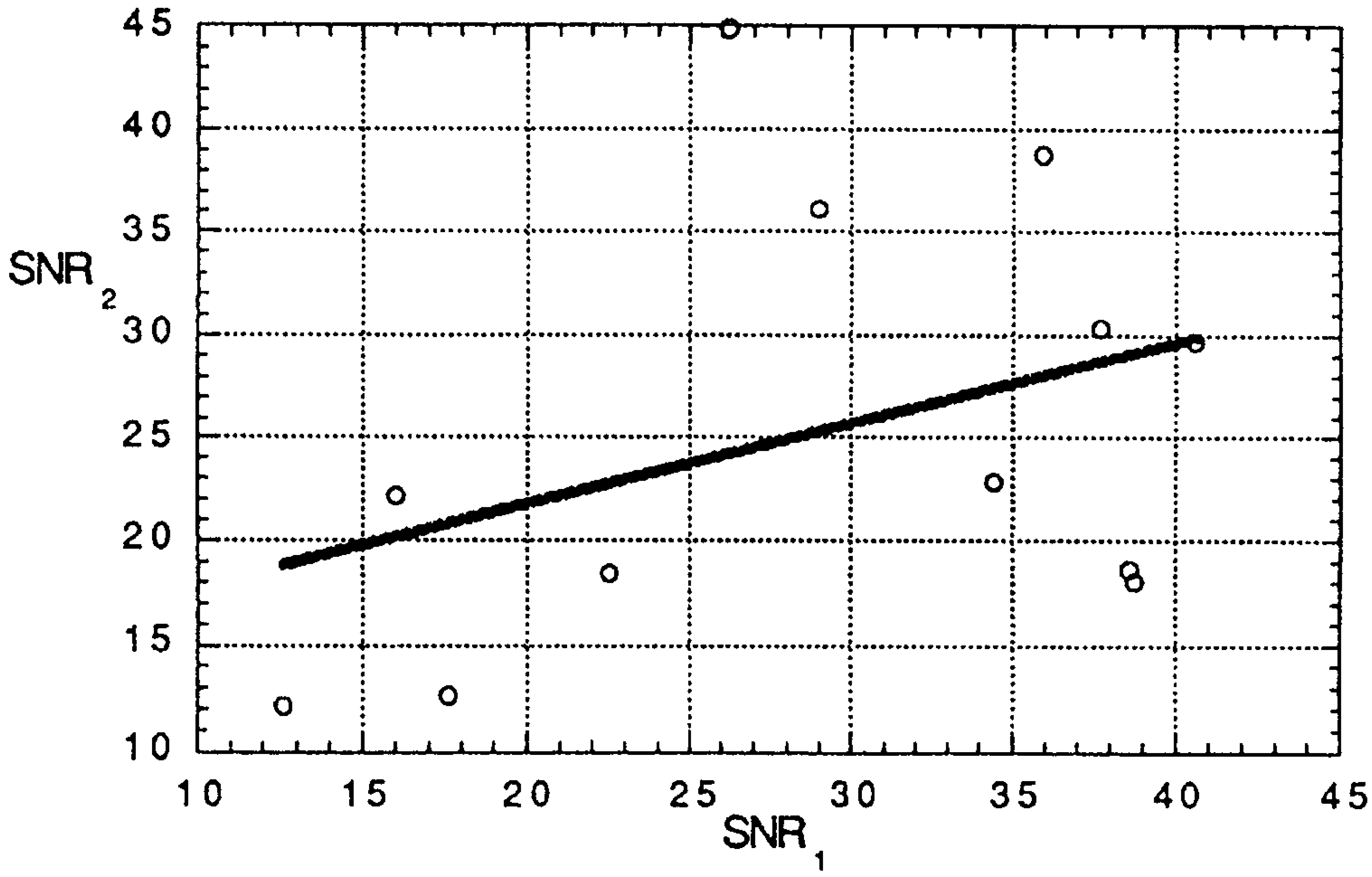

Fig. 5. Correlation among the simulated 12 coincidences between two identical antennas, with event energies in the range 15–30. The correlation coefficient is 0.373. The probability that the correlation be accidental is about 24%.

good, $p = 0.0004$, but, in spite of that, no clear correlation among the energies shows up.

4. COINCIDENCE SEARCH STRATEGY

The number of coincidences has to be compared with the number of coincidences we expect accidentally. This last quantity depends on the noise and on the coincidence window. If the noise is gaussian of the type we have considered here, the number of accidentals increases exponentially with decreasing energy. We expect that also the number of the possible gw signals diminishes with increasing energy, although many scenarios can be imagined.

The simplest one is to imagine that the gw bursts to be detected have a given SNR. The question is what threshold is convenient to apply for the definition of event. From Fig. 3 we could deduce that a convenient threshold is THR = SNR/2, providing a detection efficiency close to unity, with the number of accidentals now proportional to $\exp(-\mathrm{THR})\exp(-\mathrm{THR}) = \exp(-2\mathrm{THR}) = \exp(-\mathrm{SNR})$. Instead if we take THR = SNR we have a detection efficiency of about 30%, but the number of accidentals is much smaller, being proportional to $\exp(-2\mathrm{SNR})$. This second choice is, of course, in general more convenient.

Suppose now that the gw bursts have an integral energy distribution of the exponential type, like

$$N_{\mathrm{s}} = N_{\mathrm{gw}} \exp(-\gamma \mathrm{SNR}),$$

where γ depends on the gw source. For simplicity we consider that all of them are detected for SNR $\geq$ THR. The number of accidentals will be

$$N_{\mathrm{acc}} = N_0 \exp(-2 \cdot \mathrm{THR}).$$

The probability we can detect an excess of coincidences can be roughly obtained from the critical ratio

$$\mathrm{CR} = \frac{N_{\mathrm{s}}}{\sqrt{N_{\mathrm{acc}}}} = \frac{N_{\mathrm{gw}}}{\sqrt{N_0}} \exp[-(\gamma - 1) \cdot \mathrm{THR}].$$

Thus, if $\gamma < 1$ it is convenient to use a larger THR. If $\gamma > 1$ it is better to use a smaller THR.

As far as comparing the energies of the coincidence events, this can be done but it should not be considered of primary importance for deciding whether a possible coincidence excess is indeed caused by gravitational waves (or any other common cause) acting on the two antennas, unless the event energies of the coincidences are sufficiently large and spread over a wide energy range.

6. CONCLUSIONS

The strategy for a coincidence search can be outlined in the simple case that gw signals are expected with a given SNR. In such a case it is convenient to set the threshold as high as possible, THR = SNR, in order to reduce the number of accidentals, in spite of the diminished detection efficiency for the gw signals. The main conclusions we have reached here can be summarized in the following:

a) it is important to understand that different algorithms applied to the same experimental data might generate event lists that overlap only partially. From Fig. 3 we have deduced that, in proximity of the threshold, an overlapping rate of about $\epsilon = 30\%$ for real signals should be expected, independently of the value of the threshold THR chosen for defining the events. Since the events are generated also by the noise (i.e., Brownian and electronic noise), in total we expect a number of events

$$N = N_{\text{noise}} \exp(-\text{THR}) + N_{\text{signal}}$$

and a number of coincidence events of

$$N_{\text{coincidence}} = N_{\text{noise}} \exp[-2 \cdot \text{THR}] + N_{\text{signal}}\, \epsilon,$$

where N_{noise} is equal to the time of measurement divided the coincidence window. Thus the overlapping event rate might be very small.

b) while some coincidences due to possible gw bursts are lost, if the SNR is just below the threshold a few percent of the bursts, because of the noise effect, might give coincidences.

c) finally, we have seen that not much credit can be given to a possible correlation among the event energies, at the present stage of gw research when, even in the most optimistic case, few coincidences are expected, and those with energy close to the threshold.

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