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D^* Production from e^+e^- to ep Collisions in NLO QCD

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Abstract

Fragmentation functions for D mesons, based on the convolution of a perturbative part, related to the heavy quark perturbative showering, and a non-perturbative model for its hadronization into the meson, are used to describe D^* production in e^+e^- and ep collisions. The non-perturbative part is determined by fitting the e^+e^- data taken by ARGUS and OPAL at 10.6 and 91.2 GeV respectively. When fitting with a non perturbative Peterson fragmentation function and using next-to-leading evolution for the perturbative part, we find an ϵ parameter significantly different from the one commonly used, which is instead found with a leading order fit. The use of this new value is shown to increase considerably the cross section for D^* production at HERA, suggesting a possible reconciliation between the next-to-leading order theoretical predictions and the experimental data.

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1 Introduction

The study of fragmentation functions (FF) for heavy quarks has recently attracted an increased interest due to the large amount of data accumulated at LEP and HERA. From the theoretical side predictions have been obtained by combining perturbative QCD - which allows to resum large logarithms with a resulting milder renormalization/factorization scale sensitivity - with a non perturbative component which describes the hadronization of the heavy quark into the meson, after the perturbative cascade.

In e^+e^- annihilation an analysis along these lines was performed by Colangelo and Nason [1] up to LEP energies, for both charm and beauty mesons. Due to the presence of the $c(b)$ component only, their results were not applicable to the production of heavy mesons in hadronic collisions, where the gluon-gluon and quark-gluon scattering play an important role. Then in a previous analysis [2] a set of NLO fragmentation functions for D , D^* mesons was given, including the gluon term as well, and predictions for large transverse momentum production cross sections were also provided.

The aim of the present analysis is to reconsider the situation of charmed meson fragmentation functions both in e^+e^- annihilation and in photoproduction, where new data have been obtained at HERA.

On the perturbative side, we consider the full set of perturbative fragmentation functions (PFF's) and their mixing in the evolution. This is important as the OPAL data do indeed show a rise in the small x region, due to the gluon splitting, which is absent in the ARGUS data. In addition, by parametrizing the non-perturbative component by different forms and fitting e^+e^- data, we study the variation of the non perturbative parameters, in particular for the Peterson form [3], as related to the accompanying approximation, leading (LO) or next-to-leading order (NLO), used in the perturbative component. We find indeed that a NLO evolution favours a much smaller value of the ϵ parameter in the non-perturbative Peterson FF than given in the literature. In turn this also helps reconciling the recent HERA data with the theoretical predictions. When however a LO evolution only is considered, as in many of the parton shower Monte Carlo codes used in the experimental analyses, the "conventional" value for ϵ is recovered. This result can be understood by noting that the effect of parton showering, which is larger in a NLO analysis, softens the distribution of the partons, acting qualitatively as a non perturbative FF, which can henceforth behave more softly. Therefore the value of ϵ used in the phenomenological analyses must be closely related to the level of the approximation followed in the perturbative QCD evolution.

This paper is structured as follows: in Section 2 we recall the theoretical framework, partly already introduced in [2], on which this work is based. Section 3 presents the results of fits to ARGUS and OPAL data in e^+e^- collisions. Section 4 makes use of the non perturbative parameters previously determined to give predictions for D^* photoproduction in ep collisions at HERA. Our conclusions are then given in Section 5.

2 Theoretical Framework

We have already introduced in Ref. [2] the theoretical framework for evaluating D mesons cross section within a fragmentation approach. In that paper, the following ansatz for the fragmentation function (FF) of a parton i into a meson D was made:

$$D_i^D(x, \mu) = D_i^c(x, \mu) \otimes D_{np}^D(x). \quad (1)$$

In this equation, $D_i^c(x, \mu)$ is the perturbative fragmentation function (PFF) for a massless parton to fragment, via a perturbative QCD cascade, into the massive charm quark c . $D_{np}^D(x)$ is instead a non-perturbative fragmentation function, describing the transition from the heavy quark to the meson. Finally, the symbol \otimes indicates convolution, i.e.

$$f(x) \otimes g(x) \equiv \int_x^1 \frac{dz}{z} f(z)g(x/z). \quad (2)$$

The formalism of PFF's has been introduced a few years ago [4], and will not be given here in detail. We just recall that it allows to extract from perturbative QCD (pQCD) the initial state conditions for the PFF's at a scale μ_0 of the order of the heavy quark mass m (and we will take $\mu_0 = m$):

$$D_c^c(x, \mu_0) = \delta(1-x) + \frac{\alpha_s(\mu_0)C_F}{2\pi} \left[\frac{1+x^2}{1-x} \left(\log \frac{\mu_0^2}{m^2} - 2 \log(1-x) - 1 \right) \right]_+ \quad (3)$$

$$D_g^c(x, \mu_0) = \frac{\alpha_s(\mu_0)T_F}{2\pi} (x^2 + (1-x)^2) \log \frac{\mu_0^2}{m^2} \quad (4)$$

$$D_{q,\bar{q},\bar{c}}^c(x, \mu_0) = 0 \quad (5)$$

where c represents here the heavy quark and g and q the gluon and light quarks respectively. Moreover, $C_F = 4/3$ and $T_F = 1/2$.

The PFF's, evolved up to any scale μ via the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations, can be used to evaluate heavy quark cross sections in the large transverse momentum (p_T) region (i.e. $p_T \gg m$) by convoluting them with cross section kernels for massless partons [5, 6, 7], subtracted in the modified minimal subtraction (\overline{MS}) scheme, where the heavy quark is also treated as a massless active flavour and therefore also appears in the parton distribution functions of the colliding hadrons. This has been done in Ref. [8] for $p\bar{p}$, in Ref. [9] for γp and finally in Ref. [10] for $\gamma\gamma$ collisions. In all cases it has been shown how the results agree with the full massive ones (Refs. [11], [12] and [13] respectively) in an intermediate p_T region (say from twice to four times the mass of the heavy quark). For larger p_T they are more reliable (and hence have a smaller scale dependence) because the large logarithms originating from gluon emission and gluon splitting are resummed by the evolution of the PFF's (see Ref. [8] for a more complete discussion on this point).

The fragmentation functions of eq. (1) will be also evolved with the DGLAP equations. It is to be noted that in doing so we assume the evolution to be entirely perturbative in character: we evolve the full FF's (1) as we would the PFF's only. The non-perturbative part of the overall FF's is kept fixed and determined at a given experiment.

Indeed, the non-perturbative part of the FF's cannot be predicted by pQCD. In fact, the process through which a heavy quark binds to a light one to form the meson involves exchanges of gluons with momenta of order Λ_{QCD} or smaller, and is therefore intrinsically non-perturbative. However, a few features of this function can be determined. In contrast to light quark hadronization, this FF is hard [14] because the meson retains a larger fraction of the heavy quark initial momentum. Moreover, one expects the non-perturbative FF to be squeezed towards $x = 1$ linearly in the mass of the heavy quark. This statement is proved in [15] under the hypothesis of softness of the hadronization process and in the infinite mass limit (see also [16] for a discussion on this point).

In the following we will employ two different functional forms for the non-perturbative part of the fragmentation function.

The first one is dictated mainly by its simplicity, and is given by

$$D_{np}(x; \alpha, \beta) = A(1-x)^\alpha x^\beta \quad (6)$$

with

$$\frac{1}{A} = \int_0^1 (1-x)^\alpha x^\beta dx = B(\beta+1, \alpha+1), \quad (7)$$

$B(x, y)$ being the Euler Beta function. This functional form had already been employed in [1] for fits to e^+e^- data and was also used in our previous paper on charmed meson FF's [2]. It is flexible enough to describe the data and has the advantage of an easily calculable Mellin transform, given by

$$\begin{aligned} D_{np}(N; \alpha, \beta) &\equiv \int_0^1 dx x^{N-1} D_{np}(x; \alpha, \beta) = \\ &= \frac{B(\beta+N, \alpha+1)}{B(\beta+1, \alpha+1)} = \frac{\Gamma(\beta+N)\Gamma(\alpha+\beta+2)}{\Gamma(\beta+1)\Gamma(\alpha+\beta+N+1)}, \end{aligned} \quad (8)$$

with $\Gamma(x)$ being the Euler Gamma function.

However, this functional form has no immediate physical motivation. A successful description of e^+e^- data could be not enough to ensure the correctness of the predicted cross sections in, say, ep production evaluated with the same non-perturbative FF, since higher moments could play an important role. Indeed, in e^+e^- collisions it is the mean scaled energy, i.e. $\int dz z D(z)$ - or the second moment when talking Mellin transforms language - the most important observable. Different FF's could therefore agree on this second moment but then have different higher moments which could lead to different prediction in other kinds of reactions.

We have therefore chosen to employ also a different non-perturbative fragmentation, based on a physical model: the so called Peterson form [3]. It is derived by considering the transition amplitude for a fast moving heavy quark Q to fragment into $(Q\bar{q}) + q$, q being a light quark. It reads

$$D_{np}(x; \epsilon) = \frac{A}{x [1 - 1/x - \epsilon/(1-x)]^2}, \quad (9)$$

with the normalization factor A now given by

$$\frac{1}{A} = \frac{(\epsilon^2 - 6\epsilon + 4)}{(4 - \epsilon)\sqrt{4\epsilon - \epsilon^2}} \left\{ \arctan \frac{\epsilon}{\sqrt{4\epsilon - \epsilon^2}} + \arctan \frac{2 - \epsilon}{\sqrt{4\epsilon - \epsilon^2}} \right\} + \frac{1}{2} \ln \epsilon + \frac{1}{4 - \epsilon}. \quad (10)$$

From the derivation one finds that the ϵ parameter is related to the heavy quark mass by $\epsilon \simeq \Lambda^2/m^2$, where Λ stands for a hadronic scale. Since the average scaled energy goes like $\langle x \rangle = 1 - \sqrt{\epsilon}$, we see it respects the prediction of scaling linearly with the heavy quark mass.

While this form of non-perturbative fragmentation function is certainly more physical and the order of magnitude of its unknown parameter can be estimated from first principles, it has however the drawback of a much more complicated Mellin transform. The full expression is given in the Appendix of Ref. [17], and will not be repeated here.

3 Production in e^+e^- collisions

According to QCD factorization theorems, the cross section for the production of a hadron H in the e^+e^- process

$$e^+e^- \rightarrow \gamma, Z \rightarrow H X, \quad (11)$$

at a center-of-mass energy $Q = \sqrt{s}$, can be written as

$$\frac{d\sigma^H}{dx} = \sum_i \int_x^1 \frac{dz}{z} C_i(z, \alpha_s(\mu), Q, \mu) D_i^H\left(\frac{x}{z}, \mu\right) \equiv \sum_i C_i(z, \alpha_s(\mu), Q, \mu) \otimes D_i^H(z, \mu), \quad (12)$$

x being the energy fraction of the produced hadron, $x = 2E/Q$. The functions $C_i(z, \alpha_s(\mu), Q, \mu)$ are the so called coefficient functions, which describe the hard part of the scattering process and can be calculated in perturbation theory as series expansions in the strong coupling $\alpha_s(\mu)$. Explicit expressions up to NLO for all the coefficient functions we need can be found, for instance, in Ref. [18]. Since we take the partons in the hard scattering to be massless, collinear singularities appear, and these are subtracted in the \overline{MS} scheme and reabsorbed into the fragmentation functions. μ is the factorization scale at which this subtraction is performed, which in this case we have for simplicity taken equal to the renormalization scale. The sum is to run on all the partons which can be considered massless in the coefficient functions. Since in general mass terms of the form of powers of m/Q will appear, we see that already at $Q = 10$ GeV the charm can to a good approximation be taken as massless. The same will be true also for the bottom quark at $Q = 91$ GeV, whereas its production should instead be strongly suppressed at the lower energy. We will therefore include four and five active flavours respectively at these two center-of-mass energies.

When dealing with light hadrons the fragmentation functions can only be determined by comparison with experiment. Since in our case the hadron in question is instead the heavy meson D^* , we can make use of our ansatz of eq. (1), and fit to the experimental data only the non-perturbative part of the FF's.

We start by trying to fit the non-perturbative FF to experimental data for $D^{*\pm}$ production taken by ARGUS [19] and OPAL [20] at 10.6 GeV and 91.2 GeV respectively. The cross section is evaluated by means of the formula in eq.(12), the fragmentation functions are given by the initial conditions reported in the previous section, evolved up to the desired scale with the DGLAP equations to next-to-leading (NLO) order and convoluted with the non-perturbative component.

	α	β	$\chi^2/\text{d.o.f}$
With Sudakov resummation			
ARGUS, Ref. [1]	0.4	4.6	
ARGUS	0.51 ± 0.37	4.9 ± 1.7	0.70
OPAL	0.30 ± 0.21	4.5 ± 1.5	1.26
Without Sudakov resummation			
ARGUS	1.0 ± 0.6	6.7 ± 2.3	0.86
OPAL	0.9 ± 0.3	6.4 ± 1.9	1.32

Table 1: Results for the fitting of α and β in $(1-x)^\alpha x^\beta$ to ARGUS and OPAL data. Evolution is performed to NLO and with $\Lambda_5 = 200$ MeV and $\mu_0 = m$.

3.1 Fits with $(1-x)^\alpha x^\beta$

We first perform fits with the “simple” form $(1-x)^\alpha x^\beta$. Similar fits had already been performed a few years ago in Ref. [1]. In that paper only the non-singlet component of the FF’s had been taken into account, a valid approximation at the low energy of 10.6 GeV. When going to higher energy, on the other hand, the mixing with the gluons through the evolution will become more and more important. The OPAL data do indeed show a rise in the small x region, due to gluon splitting and absent in the ARGUS data. We have therefore included the full set of FF’s and mixings in the evolution.

As a first step, we have refitted the same ARGUS data already considered in Ref. [1]. We have taken $\Lambda_5 = 200$ MeV and included in the PFF’s the resummation of Sudakov terms in the $x \simeq 1$ region, as described in [4] and consistently with [1]. A normalization factor is always fitted along with the parameters determining the shape of the non-perturbative FF. The results are shown in the upper part of Table 1.

They can be seen to be consistent with those obtained in Ref. [1]. It is also worth mentioning that the last point in the ARGUS data has not been included in our fit. In that region non-perturbative effects become very large, spoiling the evaluation of the perturbative part of the FF’s: the PFF’s evolved to NLO become indeed negative in the large x region. We have therefore preferred not to include that point in the fit.

We have also presented along with the fits to ARGUS the results of a similar fit to OPAL data. Also in this case a few points have been excluded from the fit: the last one, where again large non-perturbative effects set it, and the first three ones, where the rise due to gluon splitting is observed. Since unaccounted for threshold effects may play an important role here, and the theoretical curve cannot be made to describe the data very well, we have preferred to avoid biasing the fitted parameters and therefore excluded this region altogether.

The main result is the consistency of the two sets of parameters: the same values which fit the ARGUS data also describe the OPAL data, taken at a center-of-mass energy almost one order of magnitude larger. This finding lends support to our initial hypothesis of scale independence of the non-perturbative part of the fragmentation functions.

Other fits with this “simple” non-perturbative FF have been performed, this time excluding

	$\Lambda_5 = 100 \text{ MeV}$	$\Lambda_5 = 200 \text{ MeV}$	$\Lambda_5 = 300 \text{ MeV}$
Next-to-leading order evolution			
ARGUS	.031 (1.09)	.019 (1.27)	.011 \pm .003 (1.53)
OPAL	.033 \pm .005 (1.25)	.015 \pm .002 (1.54)	.008 \pm .001 (1.72)
Leading order evolution			
ARGUS	.07 (1.65)	.055 (2.1)	.036 (2.72)
OPAL	.10 \pm .01 (2.02)	.08 \pm .01 (2.48)	.06 \pm .01 (2.98)

Table 2: Results for the fitting of the ϵ parameter of the Peterson FF to ARGUS and OPAL data, for three different values of Λ_5 and with next-to leading order coefficient functions and NLO or LO evolution of the PFF's. Sudakov resummation is not included explicitly, and is therefore effectly reabsorbed into the non-perturbative FF. The number between the round brackets is the χ^2 per degree of freedom of each fit.

the resummation of Sudakov terms. The reason for this is that when making convolutions of the PFF's with the Sudakov included in the x space (rather than in Mellin moments space as we do now) the integration convergence is much more difficult. We have therefore chosen to incorporate the effect of the Sudakov resummation into the non-perturbative part, with the results given in the lower part of Table 1. Once more, full consistency is found between the fits to ARGUS and to OPAL data. The results of these two fits are shown in figure 1.

3.2 Fits with the Peterson form

Fits to the same ARGUS and OPAL data have also been performed using the Peterson form (9) as the non-perturbative part of the FF's. The fit is in this case a two- rather than a three-parameter one, namely the normalization and the ϵ parameter only. Using NLO evolution and coefficient functions, but again no Sudakov resummation, and three different values for Λ_5 , we have found the results displayed in Table 2, while the curves resulting from these fits, for the choice $\Lambda_5 = 200 \text{ MeV}$, are shown in figure 1.

It is to be noted that the fitter was not able, in a few instances, to produce realistic errors when fitting ARGUS data, due to numerical inaccuracies resulting from the inverse Mellin transform of the Peterson FF. However, taking the error in the corresponding fit to OPAL data as an indication, we see that also in this case the two fits are consistent, pointing to a scale independence of the non-perturbative part of the fragmentation functions.

The most striking feature of these fits is however the discrepancy between their results and the value commonly used for the parameter ϵ when describing c quarks fragmentation to D^* mesons. It is indeed found in the literature (see, for instance, Ref. [21]), and has been used in recent phenomenological papers [22, 17], the value $\epsilon = 0.06$. The fitted values (except for the one at $\Lambda_5 = 100 \text{ MeV}$) also appear to be at variance with the result found by the OPAL Collaboration [20] as a fit to their own data, $\epsilon_{\text{OPAL}} = 0.035 \pm 0.007 \pm 0.006$.

This discrepancy should however not come as a surprise if one considers carefully how ϵ so far has been extracted from experimental data. Experiments usually report the energy or

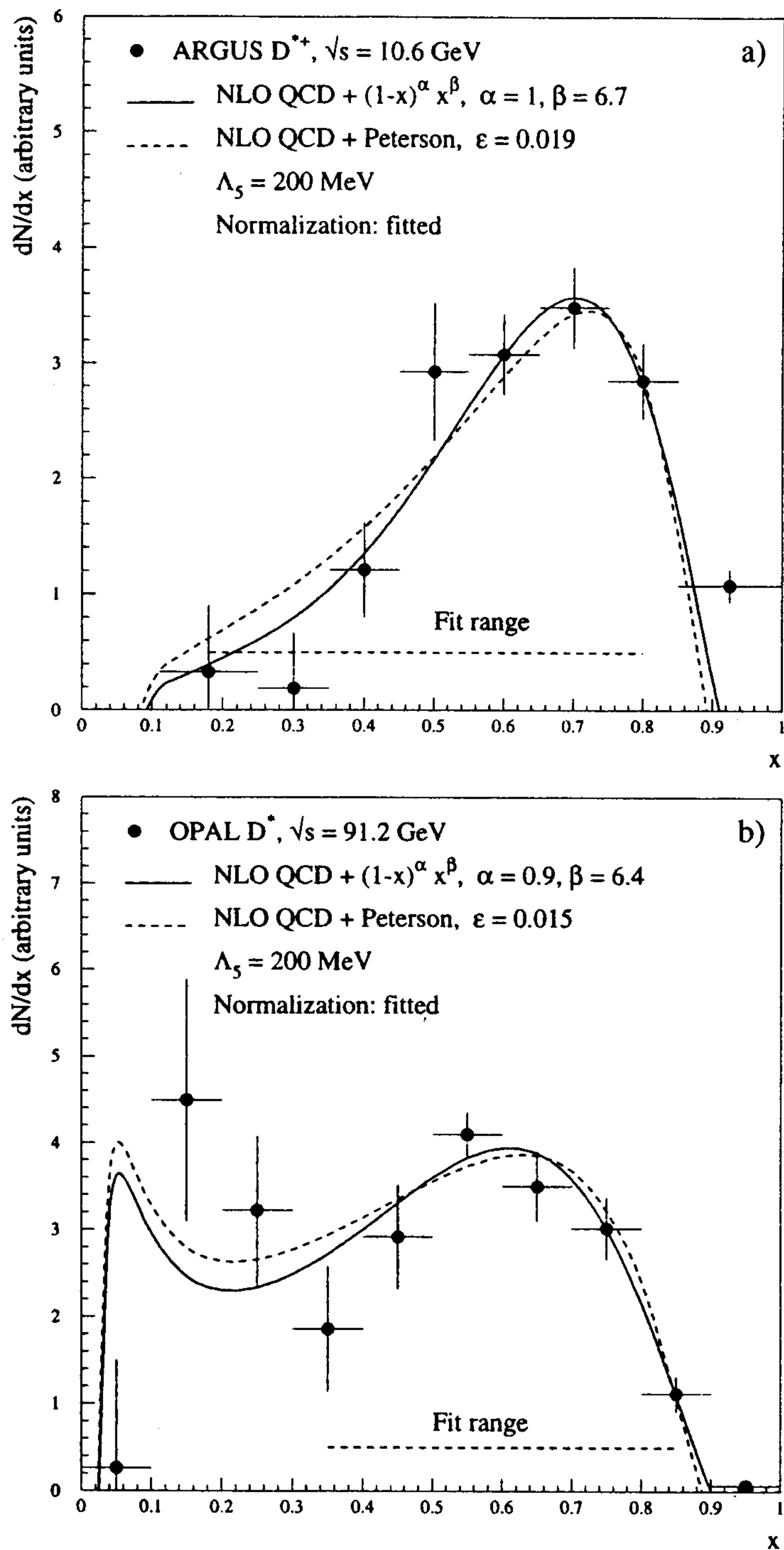


Figure 1: Distributions of D^* mesons as measured by the ARGUS and OPAL experiments, together with the theoretical curves fitted to the same data with the $(1-x)^\alpha x^\beta$ (full line) and the Peterson (dashed line) non-perturbative fragmentation functions.

momentum fraction (x_E or x_p) of the observed hadron with respect to the beam energy. On the other hand the momentum fraction which appears as the argument of the non-perturbative FF

is rather the fraction with respect to the fragmenting quark momentum, usually denoted by z (see for instance [21] for a discussion on this point). These two fractions are not coincident, due to radiation processes which lower the energy of the quark before it fragments into the hadron. In order to deconvolute these effects one usually runs a Monte Carlo simulation of the collision process at hand, including both the parton showers and the subsequent hadronization of the partons into the observable hadrons. The latter can be parametrized in the Monte Carlo by the same Peterson fragmentation function we have been using, and the value of ϵ which best describes the data can be extracted. But what can be different in our approach is of course the perturbative QCD part, namely the parton shower. This showering softens the distribution of the partons, producing an effect qualitatively similar to that of the non-perturbative FF. On the quantitative level, the amount of softening (and hence the value of ϵ) required by the non-perturbative FF to describe the data is related to the amount of softening already performed at the perturbative level. Monte Carlo's simulations so far only perform a leading order description of the showering, and can hence differ from our NLO evolution.

Therefore there is not a “unique” and “true” value for the parameter ϵ , but only a value closely interconnected with the details of the description of the pQCD part of the problem. For instance, a higher value of Λ_5 results in a larger α_s , and hence in more parton showering. This softens even more the perturbative part of the FF, and consequently less softening will be required from the non-perturbative part. The results in Table 2 show that this is indeed the case, a smaller value of ϵ corresponding to a harder Peterson FF.

A double check that the different description of the perturbative part can indeed be responsible for the different ϵ can be done by rerunning our fits with a leading order evolution, in such a way to mimick as closely as possible the Monte Carlo description of the process. The results are displayed in Table 2, and can be seen to be indeed much closer to the commonly used value of 0.06. The tendency to a discrepancy between ARGUS and OPAL fits could actually be an indication of the inadequacy of a leading order description of the scale violations taking place from 10 to 90 GeV. All this should however not be taken literally, as many other details might be included in the leading order Monte Carlo description of the perturbative showering and be missing or differently treated here.

A further check of the modification of the value for ϵ when going from a leading to a next-to-leading description of the perturbative parton shower can be obtained in the following way, to be taken as a kind of toy-model.

Consider a distribution for the energy variable x , like the ones given by ARGUS and OPAL and plotted in fig. 1. Thinking of them as described by the convolution of a perturbative and a non perturbative fragmentation function, the average value of x , call it $\langle x \rangle_{exp}$, can be written as a product of the average values of the perturbative and the non-perturbative FF's, i.e.

$$\langle x \rangle_{exp} = \langle x \rangle_{pert} \langle x \rangle_{np}. \quad (13)$$

If we now assume that both a leading and a next-to-leading description of the perturbative part can describe the data, provided they are matched by the appropriate non-perturbative FF (i.e., the appropriate value of ϵ is chosen), we can write

$$\langle x \rangle_{exp} = \langle x \rangle_{pert}^{LO} \langle x \rangle_{np}^{LO} = \langle x \rangle_{pert}^{NLO} \langle x \rangle_{np}^{NLO}, \quad (14)$$

which leads us to

$$\langle x \rangle_{np}^{NLO} = \frac{\langle x \rangle_{pert}^{LO}}{\langle x \rangle_{pert}^{NLO}} \langle x \rangle_{np}^{LO}. \quad (15)$$

In this equation $\langle x \rangle_{pert}$ refers to the second Mellin moment of the perturbative fragmentation function D_c^c , while the $\langle x \rangle_{np}$ can be calculated from the Peterson FF, like $\langle x \rangle_{np} = \int x D(x; \epsilon) dx$. The suffixes “LO” and “NLO” on the perturbative parts mean that a leading or next-to-leading evolution kernel has been included before taking the average. The non-perturbative part is considered to be adjusted to fit the data together with the given perturbative term.

The perturbative fragmentation function returns the following averages when evolved with $\Lambda_5 = 200$ MeV:

	$\langle x \rangle_{pert}^{LO}$	$\langle x \rangle_{pert}^{NLO}$	$\langle x \rangle_{pert}^{LO} / \langle x \rangle_{pert}^{NLO}$
10.6 GeV	.75	.65	1.15
91.2 GeV	.64	.56	1.14

We can clearly see from this table how the NLO description does indeed soften the perturbative FF more than the LO one, producing a lower value for the average energy.

Assuming $\epsilon = 0.06$ to be the right value to describe the data when a leading order perturbative description is used, we get $\langle x \rangle_{np}^{LO} = 0.67$ and hence, from eq. (15), $\langle x \rangle_{np}^{NLO} = 0.77$. Upon inspection we see this average value for the Peterson FF corresponds to $\epsilon = 0.016$, i.e. a value fully compatible with the ones returned by the fits.

Before closing this Section on the fits, we wish to point out once more that there is not a “best candidate” value for ϵ , but only a value of ϵ more suited to match the description of the perturbative showering one is actually employing. Surely enough, if the QCD description is at NLO a harder ϵ , like our $\epsilon = 0.015$, should be used rather than the larger (and softer) $\epsilon = 0.06$, since part of the softening is now already included through more perturbative gluon emission.

4 Production in ep collisions

The use of fragmentation functions for heavy quarks to evaluate NLO cross sections for charm photoproduction has already been considered in Ref. [9].

In this paper we use exactly the same formalism to evaluate cross sections for D^* production, by complementing the PFF’s used in the previous work with a non-perturbative component as described by eq. (1) and according to Ref. [2].

The γp cross section reads, schematically,

$$d\sigma_{\gamma p} = \int F_{i/p} d\hat{\sigma}_{\gamma i \rightarrow k} D_k^D + \int F_{i/p} F_{j/\gamma} d\hat{\sigma}_{ij \rightarrow k} D_k^D. \quad (16)$$

In this expression $F_{i/p}$ and $F_{j/\gamma}$ are the parton distribution functions (pdf’s) for the proton and the photon, since the so called direct and resolved component are both included. Unless otherwise stated, we will make use of the MRS-G [23] and ACFGP [24] sets respectively. The $\hat{\sigma}$ ’s are the kernel cross sections (= coefficient functions) for massless parton production [5, 6] and D_k^D is the meson fragmentation function of eq. (1). We will use in this FF the non-perturbative

parameters fitted in the previous section to e^+e^- data and, since the non-perturbative FF's are normalized to one, we include the branching ratio $BR(c \rightarrow D^{*+}) = BR(\bar{c} \rightarrow D^{*-}) = 0.26$ [20]. This produces an absolute, parameter free, prediction, to be directly compared with the experimental data.

We also convolute our γp cross sections with the Weizsäcker-Williams flux factor,

$$\sigma_{ep}(s) = \int_{y_{min}}^{y_{max}} dy f_{\gamma/e}(y) \sigma_{\gamma p}(ys) \quad (17)$$

with

$$f_{\gamma/e}(y) = \frac{\alpha}{2\pi} \left[\frac{1 + (1-y)^2}{y} \ln \frac{Q_{max}^2}{Q_{min}^2} + 2m_e^2 y \left(\frac{1}{Q_{max}^2} - \frac{1}{Q_{min}^2} \right) \right], \quad (18)$$

where $y = E_\gamma/E_e$, $Q_{min}^2 = m_e^2 y/(1-y)$ and m_e is the electron mass, to mimick as closely as possible the experimental setup. For comparisons with ZEUS data we will adopt $Q_{max}^2 = 4 \text{ GeV}^2$ and $y_{min} = 0.147$, $y_{max} = 0.869$, according to [26]. Moreover, we will present cross sections in the pseudorapidity range $-1.5 < \eta < 1$ and in the p_T range $3 < p_T < 12 \text{ GeV}$.

As already stressed in Ref. [2], it is important to point out how this low p_T boundary casts doubts on the validity of an approach based on the use of massless cross section kernels, and which had originally been devised for the resummation of large logarithms in the large p_T region. In principle, one is missing terms of order m/p_T , and the errors may therefore be large when $p_T \simeq m$. Only a comparison with a full massive calculation can finally assess whether the results are meaningful enough. Such a comparison will be presented in fig. 2.

A description of D^* photoproduction in ep collisions similar to ours has recently been given in [17]. When including the Peterson FF these authors tackle the problem from an apparently different point of view, by evolving directly this non-perturbative FF and inserting instead the initial conditions (3), (4) and (5) for the heavy quark PFF's into the coefficient functions for γp to massless parton scattering. One can however easily see the two approaches are equivalent at the perturbative level. The Appendix does indeed show how they should only differ by uncontrollable higher order terms and, other than this, in the interpretation of the various components.

Therefore the approach introduced in Ref. [2] and now discussed here in detail, and the one successively used in [17] should give similar results. We compare them in fig. 2. It shows the curve extracted from Ref. [17] (wide-dotted line) and our results, for the same value of $\epsilon = 0.06$. No agreement is found, however, neither (full line) with what will be our standard choice of renormalization/factorization scales ($\mu = \mu_R = \mu_F = \xi m_T = \xi \sqrt{m^2 + p_T^2}$ with $\xi = 1$, $\mu_0 = m$, and $\Lambda_5 = 200 \text{ MeV}$) nor (dashed line) when we make the same choice as Ref. [17], taking $\mu_R = m_T$, $\mu_F = 2m_T$, $\mu_0 = 2m$, GRV-G HO [25] as the photon pdf's set.

Spurious higher order terms could be responsible for the discrepancy. If one does indeed check fig. 2 of Ref. [17], by comparing curves C and D a difference similar to the one found above between the wide-dotted and the dashed line can be seen. This large difference could therefore be due to the moving of the initial condition terms for the fragmentation function to the kernel cross sections for massless parton scattering (see Appendix). Curve D of Ref. [17] has been made following our standard PFF formalism, and by comparing it with our results we have indeed found agreement.

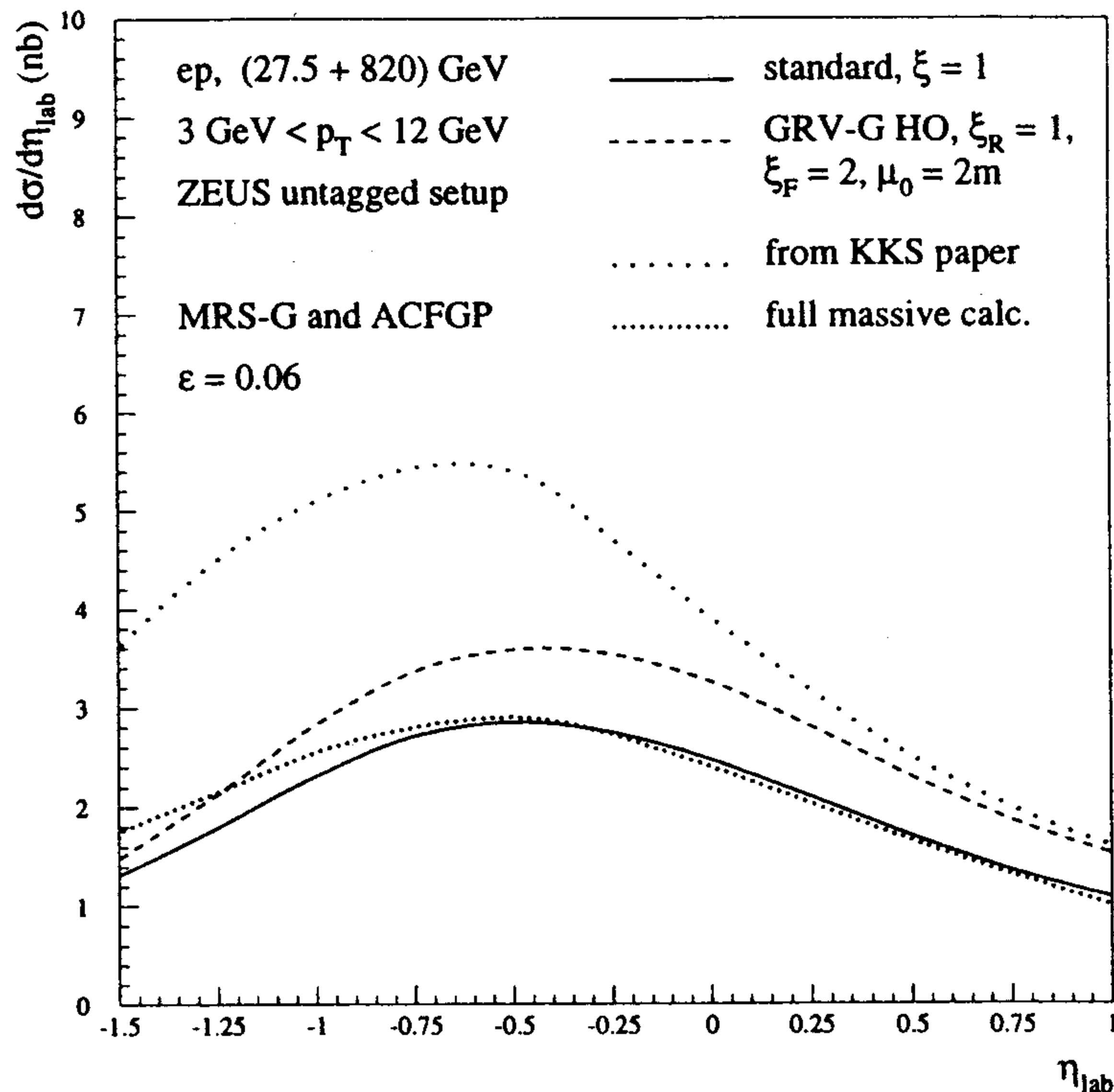


Figure 2: Comparison of our results with those of Ref. [17] (KKS) and with the full NLO massive calculation of Ref. [22]. The GRV-G HO photon parton distribution functions set is employed for all the curves except for the “standard” one (full line). ξ_R and ξ_F refer to the ratios of the renormalization and factorization scales to the transverse mass m_T respectively.

It is worth noting that the spurious terms contain large Sudakov logarithms of the form $\log(1-x)$, and could indeed be not negligible. Since we fitted e^+e^- data with the same overall fragmentation function we are now using here, we believe the large effect of these terms - if present - to be effectively absorbed into the fixed non-perturbative component. Hence it should not spoil a reliable evaluation of photoproduction cross sections.

Also shown on the same plot is the result of the full NLO massive calculation by [22] (close-dotted line), itself convoluted with a Peterson FF with $\epsilon = 0.06$ too, as taken from [26]. Good agreement with our result is found, especially when making our standard choice of scales. Such a successful comparison could probably not have been expected beforehand, given the missing m/p_T terms, but a posteriori it can perhaps be considered a check of our results, being the massive result the benchmark at these low p_T values. The agreement will also allow us to extrapolate to the massive calculation the effect of varying the value of ϵ .

4.1 Comparison with experiment

We now compare our results with experimental photoproduction data obtained at HERA by ZEUS [26] and H1 [27] Collaborations.

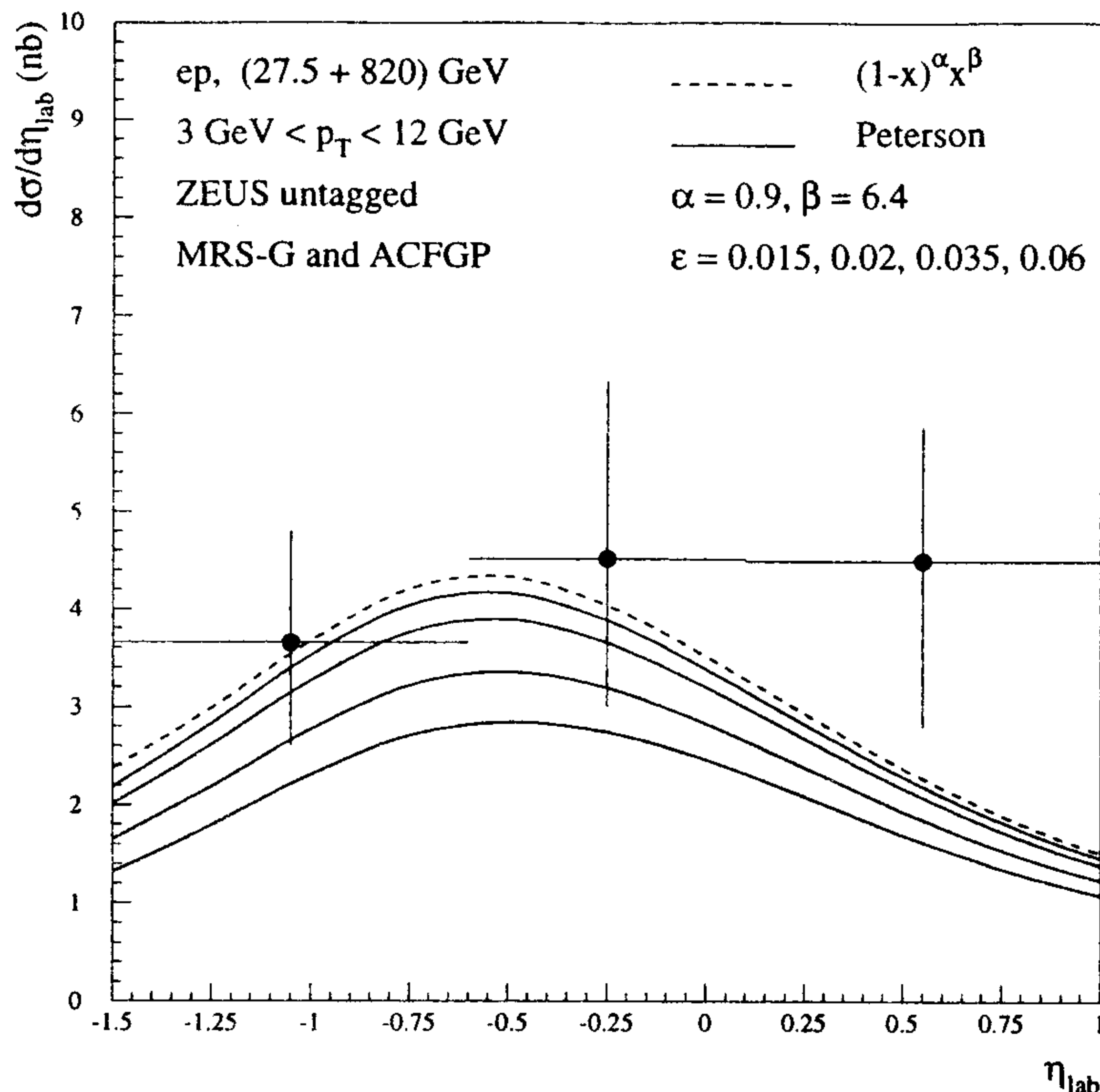


Figure 3: Pseudorapidity distribution of D^* as measured by the ZEUS experiment and theoretical predictions for different values of ϵ in the Peterson fragmentation function (full line) and with the $(1-x)^\alpha x^\beta$ FF (dashed line).

We first plot, in fig. 3, the pseudorapidity distributions, integrated over the p_T , obtained with the Peterson FF with different values of ϵ . These results have been obtained with the pdf's set MRS-G for the proton, and ACFGP for the photon in the resolved component. For the renormalization/factorization scales we made the standard choice $\mu = \mu_R = \mu_F = m_T$ and taken $\mu_0 = m$ as the starting value for the evolution of the FF's. Λ_5 is taken equal to 200 MeV.

As expected, the use of a smaller ϵ hardens the non-perturbative FF and hence enhances the cross section, since the partonic kernels fall rapidly with increasing p_T . The cross section obtained with $\epsilon = 0.015$ is 50% larger than that with $\epsilon = 0.06$, and while the latter seems to fall short of describing the ZEUS data, the former does a good job, at least in the first two bins. But we emphasize here once more how a full assessment of the reliability of these results needs a comparison with the full massive calculation, rather than with the experimental data, which however need to be improved in precision.

For comparison, the cross section obtained with the simple FF, $(1-x)^\alpha x^\beta$, with $\alpha = 0.9$ and $\beta = 6.4$, is also shown (dashed line) in fig. 3. These values for α and β fit the OPAL data from e^+e^- collisions like $\epsilon = 0.015$ does, see Section 3, and the photoproduction cross sections are indeed also in good agreement. This on one side shows how in this case there is little dependency on the precise shape of the non-perturbative fragmentation function. On the other side, it strengthens our trust of the cross section with the Peterson, much harder to evaluate due to the numerical difficulties related to the inverse Mellin transform.

	σ (nb)
$\alpha = 0.9, \beta = 6.4$	8.0
$\epsilon = 0.015$	7.7
$\epsilon = 0.02, \xi = 0.5$	9.6
$\epsilon = 0.02$	7.2
$\epsilon = 0.02, \xi = 2$	6.2
$\epsilon = 0.035$	6.2
$\epsilon = 0.06$	5.3

Table 3: Predictions for the total cross sections in the ZEUS untagged setup, $3 < p_T < 12$ GeV and $-1.5 < \eta < 1$.

The total cross sections, obtained by integrating the curves in fig. 3 over the pseudorapidity, are also shown in Table 3. They are to be compared with the experimental result from ZEUS [26] $\sigma = 10.6 \pm 1.7(\text{stat.}) \pm 1.3^{+0.6}_{-0.5}(\text{syst.})$ nb. Notice that the 17% increase found going from $\epsilon = 0.06$ to 0.035 is in good agreement with the 15% estimated in [26] using the massive calculation.

To get a feeling of the stability of our results we plot in fig. 4a the results obtained for the pseudorapidity distribution with different choices of renormalization/factorization scales and with the conservative value $\epsilon = 0.02$. While the central curve is obtained with $\mu = m_T$, the two others are produced with $\mu = \xi m_T$, with $\xi = 0.5$ and 2. The variation is not negligible, especially in the lower scale direction, but we should bear in mind that at such a low scale we are at the border of the applicability of perturbative QCD. Also shown on this plot are the results obtained with two other photon pdf's sets, namely GRV-G H0 and AFG [28]. The variations are smaller than those given by varying the scales.

By comparing with the experimental results we can see that we can get a fairly good description of the data already with a central choice of scales.

A similar comparison is also made, in fig. 4b, with the p_T distribution obtained by the ZEUS Collaboration. The curves, obtained with $\epsilon = 0.02$, seem to offer a fair description of the data.

Finally, we want to present a comparison of the results of our approach with more sets of data. We now use H1 results, both in the tagged and the untagged experimental setup. These we reproduce by taking in the Weizsäcker-Williams convolution $Q_{max}^2 = 0.01$ GeV², $y_{min} = 0.28$, $y_{max} = 0.65$ and $Q_{max}^2 = 4$ GeV², $y_{min} = 0.1$, $y_{max} = 0.8$ respectively, according to Ref. [27]. Fig. 5 shows the results for the rapidity distributions¹, obtained with the Peterson FF with $\epsilon = 0.02$ and the standard choice of scales, i.e. $\xi = 1$.

The total cross sections for these curves, integrated within the $2.5 < p_T < 10$ GeV and $-1.5 < y < 1$ range, read 4.2 nb and 14.4 nb for the tagged and the untagged sample respectively, to be compared with the experimental results $4.9 \pm 0.7 \pm 0.74_{0.59}$ nb and $20.2 \pm 3.3 \pm 4.0_{3.6}$ nb.

¹H1 presents its experimental results as a function of the rapidity rather than of the pseudorapidity. Our approach, in that it deals with massless partons in the kernel cross sections, cannot distinguish between the two. The two quantities become of course identical in the large p_T region, and at $p_T = 2.5$ GeV already only differ by about 10%.

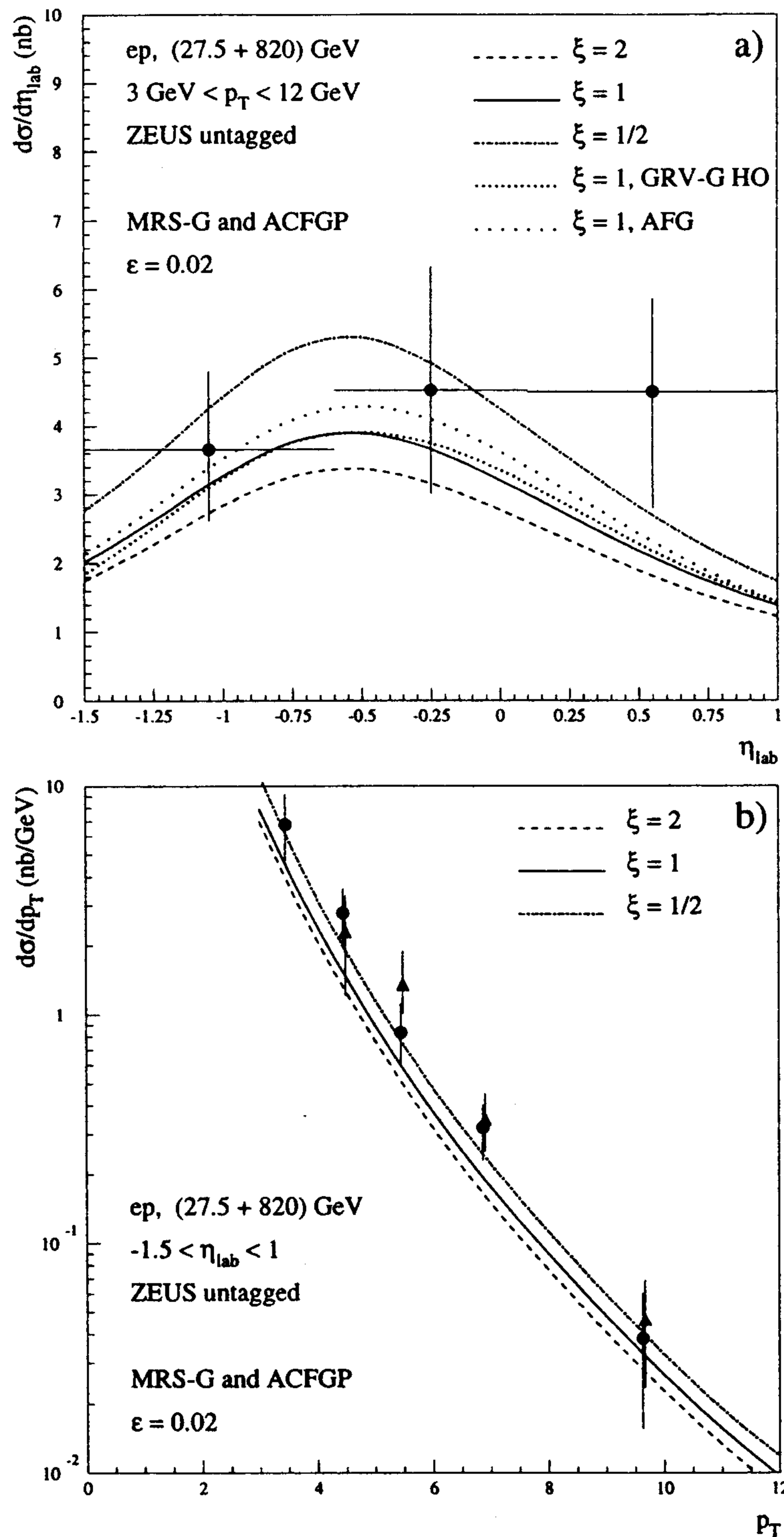


Figure 4: Comparison with pseudorapidity (a) and transverse momentum (b) experimental distributions from ZEUS [26], and effect of variation of renormalization and factorization scales, as $\mu = \xi m_T$, and of the photon pdf's sets.

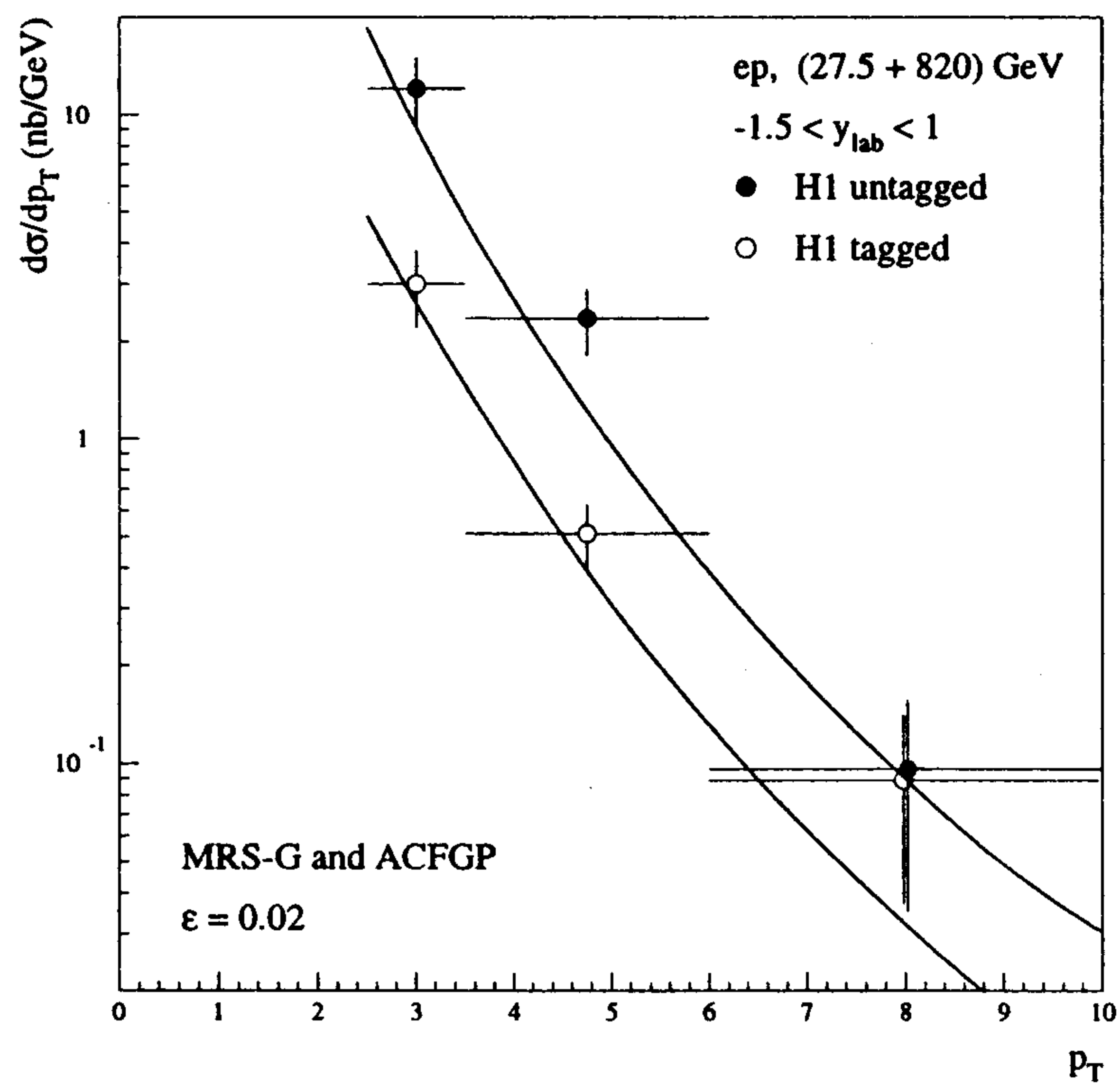
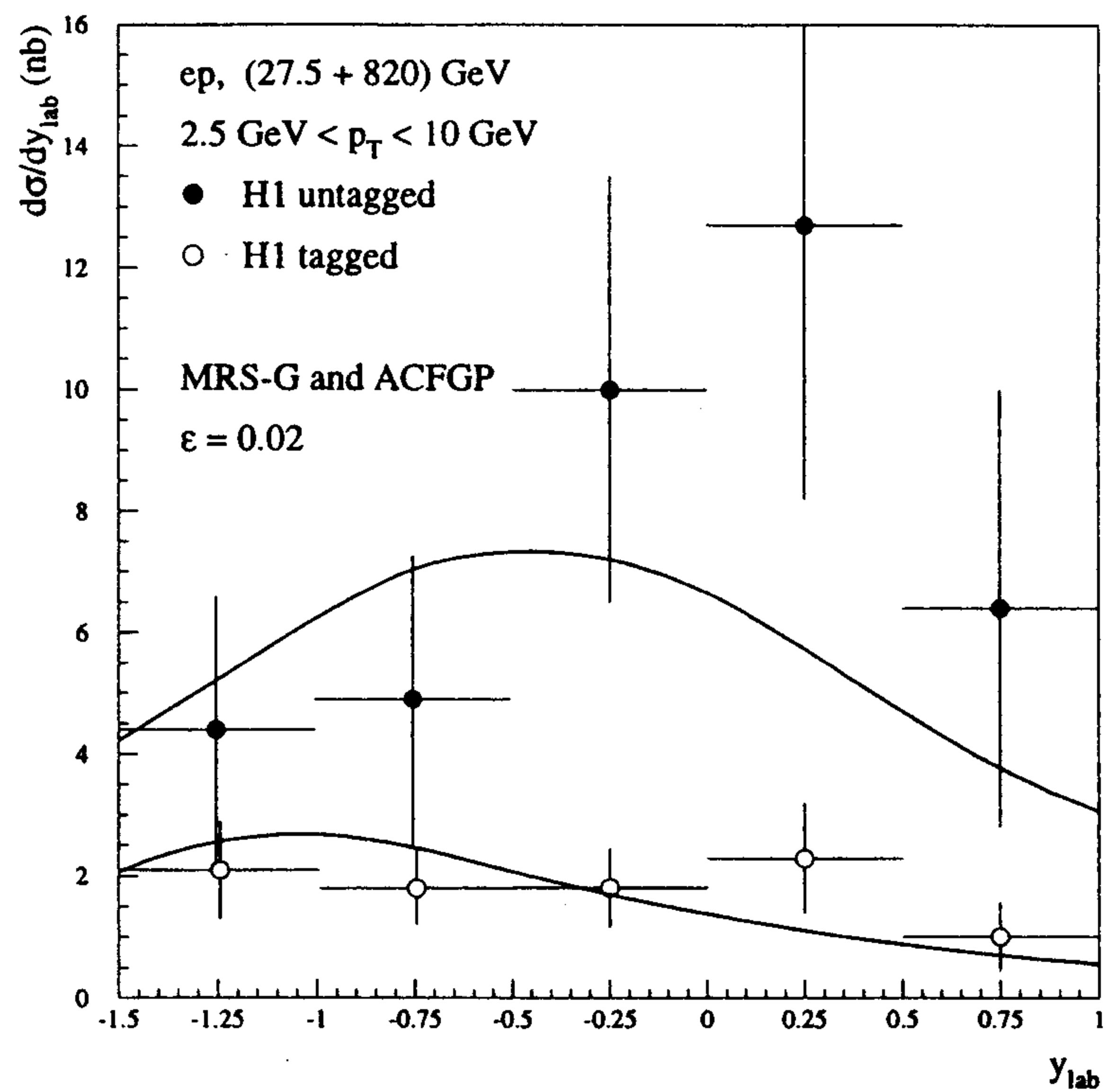


Figure 5: Comparison of our results with the experimental data from the H1 Collaboration, Ref. [27].

A quite good agreement can be seen, especially for the tagged sample.

5 Conclusions

In this paper we have applied the technique of fragmentation functions for heavy mesons to D^* production in e^+e^- and ep collisions.

These fragmentation functions are made of a perturbative part, which we evolve with next-to-leading accuracy, and a non-perturbative one, which we fit to e^+e^- data taken by ARGUS and OPAL and subsequently use to predict photoproduction cross sections, to be compared with data by H1 and ZEUS.

When fitting e^+e^- data with a Peterson non-perturbative form we find values for the ϵ parameter sensibly different from the commonly accepted value 0.06. A central value for our fits, when using NLO evolution, is $\epsilon = 0.02$. This hardens the non-perturbative fragmentation function, and increases the photoproduction cross section, bringing it in better agreement with the data.

Our photoproduction results are found in good agreement with the NLO full massive ones, which are reliable at the low values of p_T probed by the experiments and can be taken as a benchmark for comparisons. Convoluting them with a Peterson with a lower ϵ will also increase the cross section, again producing a better agreement with the data. Slightly less conservative choices than those made here for the renormalization/factorization scales, the photon parton distribution functions set, the $c \rightarrow D^*$ branching ratio and the value of ϵ could easily make the agreement even better.

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A Appendix

In this Appendix we show how the approaches of Refs. [2] and [17] are identical at the perturbative level.

Consider a cross section for producing a heavy quark of mass m at the large scale Q , $\sigma(Q, m)$, given by the convolution of a coefficient function $C(Q, \mu)$ and a perturbative fragmentation function $D(\mu, m)$. In the Mellin moments space we write this as a product:

$$\sigma(Q, m) = C(Q, \mu)D(\mu, m), \quad (19)$$

and μ is the factorization scale. Since $D(\mu, m)$ is the fragmentation function evolved up to the scale μ , we can write it in terms of an initial condition at a scale μ_0 as

$$D(\mu, m) = E(\mu, \mu_0)D(\mu_0, m)D_{np}. \quad (20)$$

The factor $E(\mu, \mu_0)$ is the so called evolution kernel, and we have now also included a non-perturbative term D_{np} , for instance the Peterson FF, according to eq. (1). Indeed, to think it to multiply the perturbative initial condition or the evolved PFF is absolutely identical, since $D(\mu)$ is in both cases simply a product of three terms.

Putting together the two equations we have

$$\sigma(Q, m) = C(Q, \mu)E(\mu, \mu_0)D(\mu_0, m)D_{np}, \quad (21)$$

which is for instance the way we write our e^+e^- cross section in Mellin space, the one in x -space to be found by numerical inverse Mellin transform.

If we now consider that both the coefficient functions (see for instance Ref. [18]) and the initial conditions of the PFF's (see eqs. (3), (4) and (5)) can be calculated as series expansions in α_s , like

$$C(Q, \mu) = 1 + \alpha_s(\mu)c(Q, \mu) \quad \text{and} \quad D(\mu_0, m) = 1 + \alpha_s(\mu_0)d(\mu_0, m), \quad (22)$$

inserting these expressions into eq.(21) and rearranging it, up to uncontrollable $O(\alpha_s^2)$ terms we can write

$$\sigma(Q, m) = \left(1 + \alpha_s(\mu)c(Q, \mu) + \alpha_s(\mu_0)d(\mu_0, m)\right)E(\mu, \mu_0)D_{np}. \quad (23)$$

This is (with the exception of $\alpha_s(\mu_0)$ which they take $\alpha_s(\mu)$ instead) the form employed in Ref. [17] when $E(\mu, \mu_0)D_{np}$ is considered as an ‘‘evolved’’ non-perturbative FF, and with the $d(\mu_0, m)$ functions changing the coefficient function’s scheme. If one takes $\mu_0 = \mu$ the new coefficient function will be close to the cross section for massive quark production, containing the logarithmic terms $\log(Q/m)$. Indeed, the $d(\mu_0, m)$ functions had been determined in [4] exactly this way, but going the opposite way round, i.e. evaluating the full massive $\sigma(Q, m)$, extracting from it the coefficient function $c(Q, \mu)$ in the \overline{MS} scheme, and defining the remaining piece to be the initial state condition of the fragmentation function.

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