CP VIOLATION in the K-SYSTEM

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Abstract

CP violation in the K system is pedagogically reviewed. We discuss its manifestations in the neutral K meson systems, in rare K meson decays and in decays of charged K mesons. Results from classical experiments, and perspectives for upcoming experiments are included. We also briefly discuss the possibility of CPT tests.

1. Introduction

1.1. K mesons and strangeness

K mesons were discovered in 1944 in the cosmic radiation[1] and have been responsible for many new ideas in particle physics. K mesons led to the the concept of strangeness and flavor mixing in the weak interactions. Parity violation was first observed through the θ–τ decay modes of K mesons. All these concepts are fundamental parts of the standard model, SM.

The strange properties, of K mesons and certain other particles, the hyperons, led to the introduction of a new quantum number, the strangeness, S.[2] Strangeness is conserved in strong interactions while first order weak interaction can induce transitions in which strangeness is changed by one unit. Today we describe these properties in terms of quarks with different "flavors", first suggested in 1964 independently by Gell-Mann and Zweig.[3]

The "normal particles" are bound states of $q\bar{q}$, the mesons, or of qqq, baryons, where

$$q = \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \text{up} \\ \text{down} \end{pmatrix}.$$

K's, hyperons and hypernuclei contain a strange quark, s:

$$K^0 = d\bar{s} \qquad \overline{K}^0 = \bar{d}s$$
$$K^+ = u\bar{s} \qquad K^- = \bar{u}s$$
$$S = +1 \qquad S = -1$$

Assigning negative strangeness to the strange quark s is totally arbitrary but maintains today the original assignment of positive strangeness for K^0, K^+ and negative for the Λ and Σ hyperons and for $\overline{K}^0$ and K^-. An important consequence of the fact that K mesons carry strangeness, a new additive quantum number, is that the neutral K and anti neutral K meson are distinct particles:

$$C|K^0\rangle = |\overline{K}^0\rangle, \quad S|K^0\rangle = |K^0\rangle, \quad S|\overline{K}^0\rangle = -|\overline{K}^0\rangle$$

An apocryphal story says that upon hearing of this hypothesis, Fermi challenged Gell-Mann to devise an experiment which shows an observable difference between the K^0 and the $\overline{K}^0$. Today we know that it is trivial to do so. For example, the process $p\bar{p} \to \pi^- K^+ \overline{K}^0$, produces $\overline{K}^0$'s which interact with matter in a totally different manner than the K^0's produced in $p\bar{p} \to \pi^+ K^- K^0$.

Since the fifties K mesons have been produced at accelerators, first amongst them was the Cosmotron.

1.2. Mass and CP eigenstates

While the strong interactions conserve strangeness, the weak interactions do not. In fact, not only do they violate S with $\Delta S = 1$, they also violate charge conjugation, C, and parity, P, though except for a very peculiar case as we shall see, not the combined

2

CP symmetry. We assume for now that CP is a symmetry of the world. We define an arbitrary, unmeasurable phase by:

$$CP|K^0\rangle = |\overline{K}^0\rangle$$

Then the physical mass eigenstates are:[4]

$$
\begin{aligned}
|K_1\rangle &\equiv \frac{|K^0\rangle + |\overline{K}^0\rangle}{\sqrt{2}} \\
|K_2\rangle &\equiv \frac{|K^0\rangle - |\overline{K}^0\rangle}{\sqrt{2}},
\end{aligned}
\tag{1}
$$

where K_1^0 has $CP=+1$ and K_2^0 has $CP=-1$. While K^0 and $\overline{K}^0$ are degenerate states, as required by CPT invariance, the weak interactions, which induces to second order $K^0 \leftrightarrow \overline{K}^0$ transitions, induces a small mass difference between K_1^0 and K_2^0. We expect $\Delta m \sim$, , since the decay is a first order process, but we must take the square of the appropriate matrix element while the mass difference is just the second order matrix element.

1.3. K_1^0 and K_2^0 lifetimes and mass difference

K-mesons have numerous decay modes. For neutral K's one of the principal decay modes is into two or three pions. The relevant properties of the neutral two and three pion systems with zero total angular momentum are given below.

1. $\pi^+\pi^-$, $\pi^0\pi^0$: $P = +1$, $C = +1$, $CP = +1$.
2. $\pi^+\pi^-\pi^0$: $P = -1$, $C = (-1)^l$, $CP = \pm 1$, where l is the angular momentum of the charged pions in their center of mass. States with $l > 0$ are suppressed by the angular momentum barrier.
3. $\pi^0\pi^0\pi^0$: $P = -1$, $C = +1$, $CP = -1$. Bose statistics requires that l for any pion pair be even in this case.

If the total Hamiltonian conserves CP, i.e. $[H, CP] = 0$, the decays of the K_1's and K_2's must conserve CP. Thus the K_1's with $CP = 1$, must decay into two pions (and three pions in an $l = 1$ state, surmounting an angular momentum barrier), while the K_2's with $CP = -1$, must decay into three pion final states. Since the energy available in the two pion decay mode is approximately 220 MeV, while that for the three pion decay mode is only about 90 MeV, the lifetime of the K_1 is much much shorter than that of the K_2.

The first verification of the above consideration was obtained as early as 1956 by Ledermann et al.[5] who observed that, in fact, neutral K mesons were still present at times much larger than the then accepted value of the neutral K lifetime, which was in fact the K_1^0 lifetime. Today we know, ignoring for the moment CP violation,

$$
\begin{aligned}
,{}_1 &= (0.892 \pm 0.002) \times 10^{-10}\ \mathrm{s}^{-1} \\
,{}_2 &= (1.72 \pm 0.02\times)10^{-3} \times ,{}_1 \\
\Delta m = m(K_2) - m(K_1) &= (0.477 \pm 0.003) \times ,{}_1.
\end{aligned}
\tag{2}
$$

3

1.4. Strangeness oscillations

The mass eigenstates K_1^0 and K_2^0 evolve in vacuum and in their rest frame according to

$$i\frac{\mathrm{d}}{\mathrm{d}t}\Psi = H\Psi = \mathcal{M}\psi, \qquad (3)$$

where the complex mass $\mathcal{M}_{1,2} = m_{1,2} - i\Gamma_{1,2}/2$, with $\Gamma = 1/\tau$. The state evolution is therefore given by:

$$|K_{1,2},\ t\rangle = |K_{1,2},\ t=0\rangle e^{-i(m_{1,2}-\Gamma_{1,2}/2)t}$$

If the initial state has definite strangeness, say it is a K^0 as from the production process $\pi^- p \rightarrow K^0\Lambda^0$, it must first be rewritten in terms of the mass eigenstates K_1^0 and K_2^0 which then evolve in time as above. Since the K_1^0 and K_2^0 amplitudes change differently in time, the pure $S{=}1$ state at $t{=}0$ acquires an $S{=}-1$ component at $t > 0$. Using equations (1) the wave function at time t is:

$$\Psi(t) = \sqrt{1/2}\big[e^{(im_1-\Gamma_1/2)t}|K_1\rangle + e^{(im_2-\Gamma_1/2)t}|K_2\rangle\big]$$
$$= 1/2\big[\big(e^{i(m_1-\Gamma_1/2)t} + e^{i(m_2-\Gamma_2/2)t}\big)|K^0\rangle + \big(e^{i(m_1-\Gamma_1/2)t} - e^{i(m_2-\Gamma_2/2)t}\big)|\overline{K}^0\rangle\big].$$

The intensity of K^0 ($\overline{K}^0$) at time t is given by:

$$I(K^0\ (\overline{K}^0), t) = |\langle K^0\ (\overline{K}^0)|\Psi(t)\rangle|^2 = \frac{1}{4}\big[e^{-t\Gamma_1} + e^{-t\Gamma_2} + (-)2e^{-t(\Gamma_1+\Gamma_2)/2}\big]\cos\Delta m\,t$$

which exhibits oscillations whose frequency depends on the mass difference, see fig. 1.
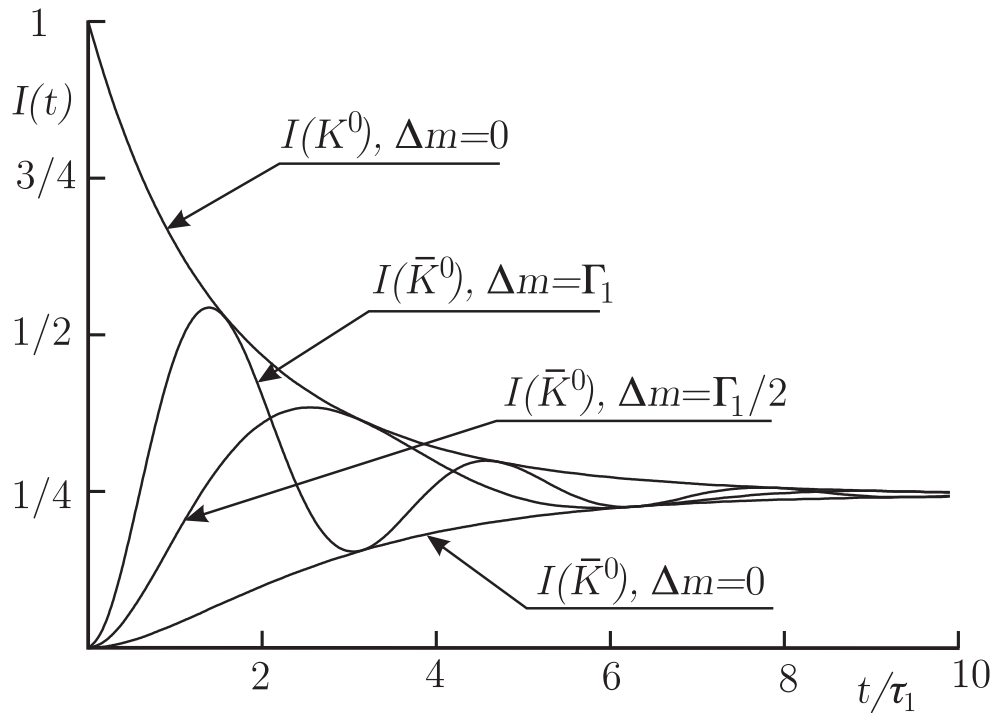

Figure 1. Evolution in time of a pure $S = 1$ state at time $t = 0$.

The appearance of $\overline{K}^0$'s from an initially pure K^0 beam can be detected by the production of hyperons, according to the reactions

$$\overline{K}^0 p \to \pi^+ \Lambda^0, \quad \to \pi^+ \Sigma^+, \quad \to \pi^0 \Sigma^+,$$
$$\overline{K}^0 n \to \pi^0 \Lambda^0, \quad \to \pi^0 \Sigma^0, \quad \to \pi^- \Sigma^-.$$

and the K_L-K_S mass difference can be obtained from the oscillation frequency.

1.5. Regeneration

Another interesting, and extremely useful phenomenon, is that it is possible to regenerate K_1's by placing a piece of material in the path of a K_2 beam. Let's take our standard reaction,

$$\pi^- p \to K^0 \Lambda^0,$$

the initial state wave function of the K^0's is

$$\Psi(t = 0) \equiv |K^0\rangle = \frac{|K_1\rangle + |K_2\rangle}{\sqrt{2}}.$$

Note that it is composed equally of K_1's and K_2's. The K_1 component decays away quickly via the two pion decay modes, leaving a virtually pure K_2 beam. This K_2 beam has equal K^0 and $\overline{K}^0$ components, which interact differently in matter, for example, the K^0's undergo elastic scattering, charge exchange etc. whereas the $\overline{K}^0$'s can in addition produce hyperons via strangeness conserving transitions. Thus we have emerging, from a target material placed in front of the K_2 beam, see fig. 2, an apparent rebirth of K_1's!
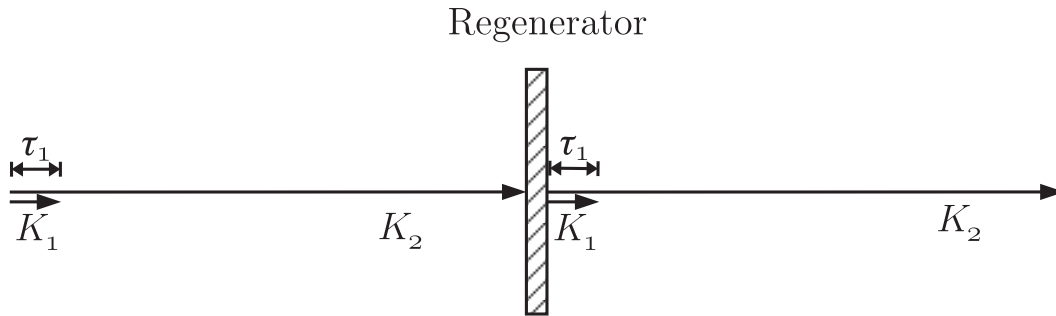
Regenerator

Figure 2. K_1^0 regeneration.

Virtually all past and present experiments, with the exception of a couple which will be mentioned explicitly, use this method to obtain a source of K_1^0's (or K_S's, as we shall see later). Denoting the amplitudes for K^0 and $\overline{K}^0$ scattering on nuclei by f and $\bar{f}$ respectively, the scattered amplitude for an initial K_2^0 state is given by:

$$\sqrt{1/2}(f|K^0\rangle - \bar{f}|\overline{K}^0\rangle) = \frac{f + \bar{f}}{2\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) + \frac{f - \bar{f}}{2\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$
$$= 1/2(f + \bar{f})|K_2\rangle + 1/2(f - \bar{f})|K_1\rangle.$$

The so called regeneration amplitude for $K_2^0 \to K_1^0$, f_{21} is given by $1/2(f - \bar{f})$ which of course would be 0 if $f = \bar{f}$.

Another important property of regeneration is that when the K_1^0 is produced at non-zero angle to the incident K_2^0 beam, regeneration on different nuclei in a regenerator is incoherent, while at zero degree the amplitudes from different nuclei add up coherently. The intensity for coherent regeneration depends on the K_1^0, K_2^0 mass difference. Precision mass measurements have been performed by measuring the ratio of coherent to diffraction regeneration. The interference of K_1^0 waves from two or more generators has also allowed us to determine that the K_2^0 meson is heavier than the K_1^0 meson. This perhaps could be expected but is nice to have it measured.

K_1^0 and K_2^0 amplitudes after regeneration are coherent and can interfere if CP is violated.

2. CP Violation in Two Pion Decay Modes

2.1. Discovery

For some years after the discovery that C and P are violated in the weak interactions, it was thought that CP might still be conserved. CP violation was discovered in '64,[6] through the observation of the unexpected decay $K_2^0 \rightarrow \pi^+\pi^-$. This beautiful experiment is conceptually very simple, see fig. 3.
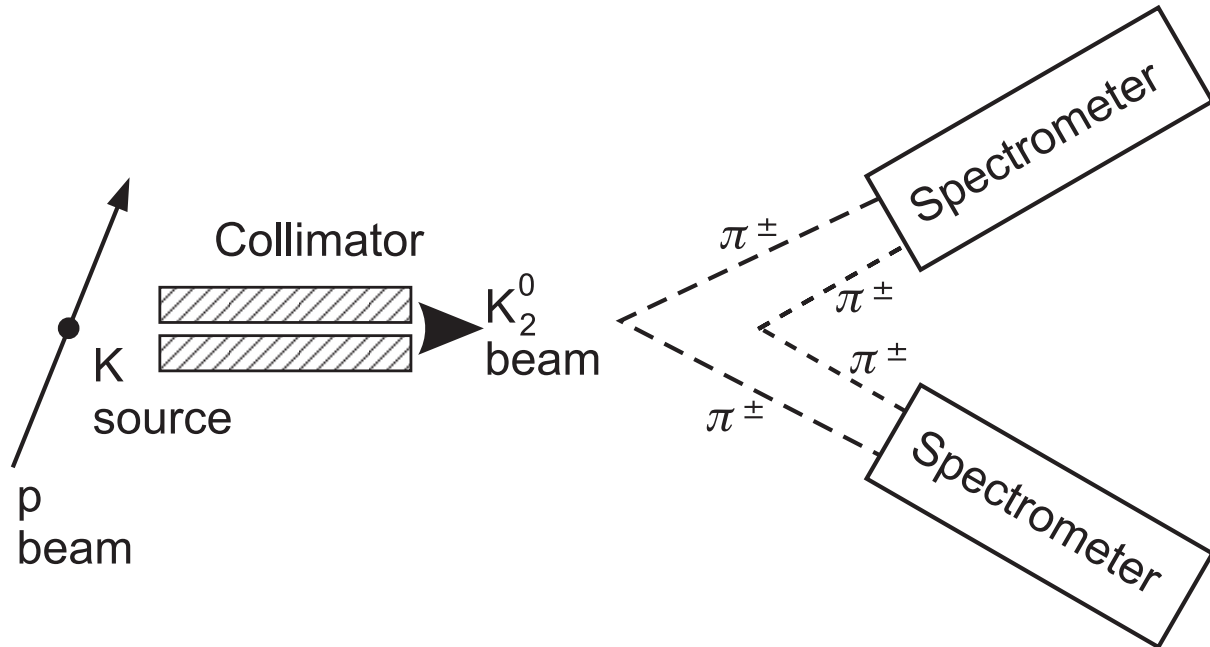

Figure 3. The setup of the experiment of Christenson *et al.*.

Let a K_2^0 beam pass through a long collimator and decay in an empty space (actually a big helium bag) in front of two spectrometers. The decay products are viewed by spark chambers and scintillator hodoscopes in the spectrometers placed on either side of the beam. Two pion decay modes are distinguished from three pion and leptonics decay modes by the reconstructed invariant mass $M_{\pi\pi}$, and the direction θ of their resultant momentum vector relative to the beam. In the mass interval between 490 MeV and 510 MeV, 50 events were

found which were exactly collinear with the beam ($\cos\theta > 0.999$), which demonstrated for the first time that K_2^0's decayed into two pions, with a branching ratio of the order $1/10^{-3}$, thus CP is shown to be violated! The CP violating decay $K_L \to \pi^0\pi^0$ has also been observed.

2.2. Neutral K Decays with CP Violation

Since CP is violated in K decays, the mass eigenstates are no more CP eigenstates and can be written, assuming CPT invariance, as:

$$|K_S\rangle = \frac{|K_1^0\rangle + \epsilon|K_2^0\rangle}{\sqrt{1+|\epsilon|^2}}$$
$$|K_L\rangle = \frac{|K_2^0\rangle + \epsilon|K_1^0\rangle}{\sqrt{1+|\epsilon|^2}}$$

(4)

with $|\epsilon| = (2.259 \pm 0.018) \times 10^{-3}$ from experiment. Note that the K_S and K_L states are not orthogonal states, contrary to the case of K_1^0 and K_2^0. Equation (3) can be rewritten, to lowest order, as:

$$\frac{\mathrm{d}}{\mathrm{dt}}|K_{S,L}\rangle = -i\mathcal{M}_{S,L}|K_{S,L}\rangle, \quad \mathcal{M}_{S,L} = M_{S,L} - i\Gamma_{S,L}/2$$

and the values of masses and decay widths given in eq. (2) belong to K_S and K_L rather than to K_1^0 and K_2^0.

Since 1964 we have been left with an unresolved problem: is CP violated directly in K^0 decays, i.e. is the $|\Delta S|=1$ amplitude $\langle\pi\pi|K_2\rangle \neq 0$ or the only manifestation of $\cancel{CP}$ is to introduce a small impurity of K_1 in the K_L state, via $K^0 \leftrightarrow \overline{K}^0$, $|\Delta S|=2$ transitions? With the standard definitions, using the phase choice of Wu and Yang,[7] the two pion decay amplitude ratios η's can be written as

$$\frac{\langle\pi^+\pi^-|K_L\rangle}{\langle\pi^+\pi^-|K_S\rangle} = \eta_{+-} = \epsilon - 2\epsilon'$$
$$\frac{\langle\pi^0\pi^0|K_L\rangle}{\langle\pi^0\pi^0|K_S\rangle} = \eta_{00} = \epsilon + \epsilon'$$

where ϵ is defined above and ϵ' is essentially

$$\frac{A(K_2^0 \to \pi\pi)}{A(K_1^0 \to \pi\pi)}.$$

The question above is then the same as: is $\epsilon' \neq 0$?

Since 1964, experiments searching for a difference in η_{+-} and η_{00} have been going on. If $\eta_{+-} \neq \eta_{00}$ the ratios of branching ratios for $K_{L,S} \to \pi^+\pi^-$ and $\pi^0\pi^0$ are different. Most experiments measure the quantity $\mathcal{R}$, the so called double ratio of the four rates for $K_{L,S} \to \pi^0\pi^0, \pi^+\pi^-$, which is related, to lowest order in ϵ and ϵ', to ϵ and ϵ' by

$$\mathcal{R} \equiv \frac{\Gamma(K_L \to \pi^0\pi^0)/\Gamma(K_S \to \pi^0\pi^0)}{\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_S \to \pi^+\pi^-)} \equiv \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = 1 - 6\Re(\epsilon'/\epsilon).$$

Observation of $\mathcal{R} \neq 0$ is proof that $\Re(\epsilon'/\epsilon) \neq 0$ and therefore of "direct" CP violation, i.e.

that the amplitude for $|\Delta S|=1$, CP violating transitions $A(K_2 \to 2\pi) \neq 0$.

All present observations of CP violation, $\cancel{CP}$ for short, $i.e.$ the decays $K_L \to 2\pi$, $\pi^+\pi^-\gamma$ and the charge asymmetries in $K_{\ell 3}$ decays are examples of so called "indirect" violation, due to $|\Delta S|=2$ $K^0 \leftrightarrow \overline{K}^0$ transitions introducing a small CP impurity in the mass eigenstates K_S and K_L. Because of the smallness of ϵ', most formula given above for K_1^0 and K_2^0 remain valid substituting $K_1^0 \to K_S$ and $K_2^0 \to K_L$.

There is no new information on direct $\cancel{CP}$ from the last round of precision experiments. One of the two, NA31, was performed at CERN and reported a tantalizing positive result.[8] NA31 alternated K_S and K_L data taking by the insertion of a K_S target every other run, while the experimental apparatus collected both charged and neutral two pion decay modes simultaneously. The other experiment, E731, was done at Fermilab and reported an essentially null result.[9] E731 had a fixed thick K_S target in front of one of the two parallel K_L beams which entered the detector which, however, collected alternately the neutral and charged two pion decay modes.

Therefore, at present we are confronted with the following experimental situation:

$$\Re(\epsilon'/\epsilon) = (23 \pm 6.5) \times 10^{-4}$$
$$\Re(\epsilon'/\epsilon) = (7.4 \pm 5.9) \times 10^{-4}$$

Taking the Particle Data Group's[10] average at face value, we could say that the confidence level that $0 < \Re(\epsilon'/\epsilon) < 3 \times 10^{-3}$ is 94%. We will come back to what is being done to overcome this problem.

3. CP violation in other modes

3.1. Semileptonic decays

K-mesons also decay semileptonically, into a hadron, with charge Q and strangeness zero, and a pair of lepton-neutrino. These decays at quark levels are due to the elementary processes

$$s \to W^- u \to \ell^- \bar{\nu} u$$
$$\bar{s} \to W^+ \bar{u} \to \ell^+ \nu \bar{u}$$

which for the physical K-mesons correspond to the decays

$$K^0 \to \pi^- \ell^+ \nu, \ \Delta S = -1, \ \Delta Q = -1$$
$$\overline{K}^0 \to \pi^+ \ell^- \bar{\nu}, \ \Delta S = +1, \ \Delta Q = +1$$
$$\overline{K}^0 \to \pi^- \ell^+ \nu, \ \Delta S = +1, \ \Delta Q = -1$$
$$K^0 \to \pi^+ \ell^- \bar{\nu}, \ \Delta S = -1, \ \Delta Q = +1$$

Therefore K^0 should decay only to ℓ^+ and $\overline{K}^0$ to ℓ^-. This is commonly referred to as the $\Delta S = \Delta Q$ rule, experimentally established in the very early days of strange particle studies.

The leptonic asymmetry

$$\mathcal{A}_\ell = \frac{\ell^- - \ell^+}{\ell^- + \ell^+}$$

in K_L decays should therefore be $2\Re\epsilon \sim \sqrt{2}\epsilon$. The measured value of $\mathcal{A}_\ell$ for K_L decays is $(0.327\pm0.012)\%$, in good agreement with the above expectation, a proof that CP violation is in the mass term.

In a situation where the neutral K-mesons are produced in a strangeness tagged state as in

$$p + \bar{p} \rightarrow K + K^\pm + \pi^\mp$$

the charge of the charged kaon (pion) defines the strangeness of the neutral K. In the leptonic kaon decay, assuming the $\Delta S = \Delta Q$ rule, the lepton charge defines the strangeness of the neutral K.

3.2. CP violation in K_S decays

So far CP violation has only been seen in K_L decays ($K_L \rightarrow \pi\pi$ and semileptonic decays). At a ϕ-factory such as DAΦNE, where $\mathcal{O}(10^{10})$ K_S/y are produced, one can look for $K_S \rightarrow \pi^0\pi^0\pi^0$, the counterpart to $K_L \rightarrow \pi\pi$. The branching ratio for this process is proportional to $\epsilon + \epsilon'_{000}$ where ϵ'_{000} is a quantity similar to ϵ', signalling direct CP violation. It is not as suppressed as the normal ϵ', perhaps a factor of twenty less. Nonetheless, as the expected BR is 2×10^{-9}, the whole signal will be at the 30 event level, and therefore there is here only the possibility to see the CP impurity of K_S, never observed before, not direct CP violation. The current limit on this BR is 3.7×10^{-5}. Finally the leptonic asymmetry $\mathcal{A}_\ell(K_S)$ in K_S decays has never been measured. The expected value is 3.2×10^{-4} and at DAΦNE it can be measured to an accuracy of $\sim 2.5\times10^{-4}$. Again this would be only a measurement of ϵ, not ϵ', but the observation for the first time of CP violation in two new channels of K_S decay would be nonetheless of considerable interest.

3.3. CP violation in charged K decays

Evidence for direct CP violation can be also be obtained from the decays of charged K mesons. CP invariance requires equality of the partial rates for $K^\pm \rightarrow \pi^\pm\pi^+\pi^-$ ($\tau^\pm$) and for $K^\pm \rightarrow \pi^\pm\pi^0\pi^0$ ($\tau'^\pm$). With the luminosities obtainable at DAΦNE one can improve the present rate asymmetry measurements by two orders of magnitude. There, one could also observe differences in the Dalitz plot distributions for K^+ and K^- decays in both the τ and τ' modes and reach sensitivities of $\sim 10^{-4}$. Finally, differences in rates in the radiative two pion decays of $K^\pm$, $K^\pm \rightarrow \pi^\pm\pi^0\gamma$, are also proof of direct CP violation.

4. CP violation at a ϕ-factory

4.1. $e^+e^- \rightarrow \phi$, $\phi \rightarrow K^0\bar{K}^0$

The cross section for production of a bound $q\bar{q}$ pair of mass M and total width , with

$J^{PC} = 1^{--}$ *i.e.* a so vector meson V in $e^+ e^-$ annihilation (see fig. 4) is given by:

$$\sigma_{q\bar{q},\mathrm{res}} = \frac{12\pi}{s} \frac{\Gamma_{ee} \Gamma M^2}{(M^2 - s)^2 + M^2 \Gamma^2} = \frac{12\pi}{s} B_{ee} B_{q\bar{q}} \frac{M^2 \Gamma^2}{(M^2 - s)^2 + M^2 \Gamma^2}$$
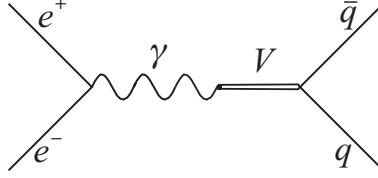

Figure 4. Amplitude for production of a bound $q\bar{q}$ pair.

The ϕ meson is a $s\bar{s}$ $^3S_1$ bound state with $J^{PC}=1^{--}$ same as a photon. The cross section for its production in $e^+ e^-$ annihilations at 1020 MeV is

$$\sigma_{s\bar{s}}(s = (1.02)^2 \text{ GeV}^2) \sim \frac{12\pi}{s} B_{ee} = 36.2 \times (1.37/4430) = 0.011 \text{ GeV}^{-2} \sim 4000 \text{ nb}$$

The Frascati ϕ–factory, DAΦNE, will have a luminosity $\mathcal{L} = 10^{33}$ cm^{-2} s$^{-1} = 1$ nb^{-1}s^{-1}. Integrating over one year, taken as 10^7 s or one third of a calendar year, we find

$$\int_{1 \text{ y}} \mathcal{L} \mathrm{d}t = 10^7 \text{ nb}^{-1},$$

corresponding to the production at DAΦNE of $\sim 4000 \times 10^7 = 4 \times 10^{10}$ ϕ mesons per year or approximately 1.3×10^{10} K^0, $\overline{K}^0$ pairs, a large number indeed. One of the advantages of studying K mesons at a ϕ–factory is that they are produced in a well defined quantum and kinematical state. Neutral K mesons are produced in collinear pairs, with a momentum of about 110 MeV/c, thus detection of one K gives the direction of the other. In addition in the reaction $e^+ e^- \to$ "γ" $\to \phi \to K^0 \overline{K}^0$, $C(K^0 \overline{K}^0) = C(\phi) = C(\gamma) = -1$. Let $|i\rangle = |K\overline{K}, \ t = 0, \ C = -1\rangle$, then:

$$|i\rangle = \frac{|K^0, \mathbf{p}\rangle |\overline{K}^0, -\mathbf{p}\rangle - |\overline{K}^0, \mathbf{p}\rangle |K^0, -\mathbf{p}\rangle}{\sqrt{2}}$$

From eq. (4) the relations between K_S, K_L and K^0, $\overline{K}^0$, to lowest order in ϵ, are:

$$|K_S (K_L)\rangle = \frac{(1 + \epsilon)|K^0\rangle + (-)(1 - \epsilon)|\overline{K}^0\rangle}{\sqrt{2}}, \qquad |K^0 (\overline{K}^0)\rangle = \frac{|K_S\rangle + (-)|K_L\rangle}{(1 + (-)\epsilon)\sqrt{2}}$$

from which

$$|i\rangle = \frac{1}{\sqrt{2}} \left(|K_S, -\mathbf{p}\rangle |K_L, \mathbf{p}\rangle - |K_S, \mathbf{p}\rangle |K_L, -\mathbf{p}\rangle\right)$$

so that the neutral kaon pair produced in $e^+ e^-$ annihilations is a pure K^0, $\overline{K}^0$ as well as a pure K_S, K_L for *all times,* in vacuum. This is valid to all orders in ϵ and also for $\cancel{CPT}$.

4.2. Correlations in K_S, K_L decays

To obtain the amplitude for decay of $K(\mathbf{p})$ into a final state f_1 at time t_1 and of $K(-\mathbf{p})$ to f_2 at time t_2, see the diagram in fig. 5, we time evolve the initial state in our usual way:

$$|\, t_1,\ \mathbf{p};\ t_2,\ -\mathbf{p}\,\rangle = \frac{1 + |\epsilon^2|}{(1 - \epsilon^2)\sqrt{2}} \times$$

$$\left(|\, K_S(-\mathbf{p})\,\rangle |\, K_L(\mathbf{p})\,\rangle e^{-i(\mathcal{M}_S t_2 + \mathcal{M}_L t_1)} \ - |\, K_S(\mathbf{p})\,\rangle |\, K_L(-\mathbf{p})\,\rangle e^{-i(\mathcal{M}_S t_1 + \mathcal{M}_L t_2)} \right)$$
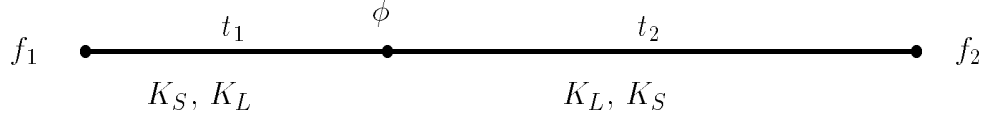

Fig. 5. $\phi \to K_L$, $K_S \to f_1$, f_2

where $\mathcal{M}_{S,L} = M_{S,L} - i\,,_{S,L}/2$ are the complex K_S, K_L masses. In terms of the previously mentioned ratios $\eta_i = \langle f_i |\, K_L\,\rangle / \langle f_i |\, K_S\,\rangle$ and defining $\Delta t = t_2 - t_1$, $t = t_1 + t_2$, $\Delta \mathcal{M} = \mathcal{M}_L - \mathcal{M}_S$ and $\mathcal{M} = \mathcal{M}_L + \mathcal{M}_S$ we get the amplitude for decay to states 1 and 2:

$$A(f_1, f_2, t_1, t_2) = \langle f_1 |\, K_S\,\rangle \langle f_2 |\, K_S\,\rangle e^{-i\mathcal{M}t/2} \left(\eta_1 e^{i\Delta\mathcal{M}\Delta t/2} - \eta_2 e^{-i\Delta\mathcal{M}\Delta t/2} \right)/\sqrt{2}. \quad (5)$$

This implies $A(e^+ e^- \to \phi \to K^0 \overline{K}{}^0 \to f_1 f_2) = 0$ for $t_1 = t_2$ and $f_1 = f_2$ (Bose statistics). For $t_1 = t_2$, $f_1 = \pi^+ \pi^-$ and $f_2 = \pi^0 \pi^0$ instead, $A \propto \eta_{+-} - \eta_{00} = 3 \times \epsilon'$ which suggests a (unrealistic) way to measure ϵ'. The intensity for decay to final states f_1 and f_2 at times t_1 and t_2 obtained taking the modulus squared of eq. (5) depends on magnitudes and arguments of η_1 and η_2 as well as on $,_{L,S}$ and ΔM. The intensity is given by

$$I(f_1, f_2, t_1, t_2) = |\langle f_1 |\, K_S\,\rangle|^2 |\langle f_2 |\, K_S\,\rangle|^2 e^{-\Gamma_S t/2} \times$$

$$\left(|\eta_1|^2 e^{\Gamma_S \Delta t/2} + |\eta_2|^2 e^{-\Gamma_S \Delta t/2} - 2|\eta_1||\eta_2| \cos(\Delta m\, t + \phi_1 - \phi_2) \right)$$

where we have everywhere neglected $,_L$ with respect to $,_S$. Thus the study of the decay of K pairs at a ϕ–factory offers the unique possibility of observing interference patterns in time, or space, in the intensity observed at two different points in space. This fact is the source of endless excitement and frustration to some people. Rather than studying the intensity above, which is a function of two times or distances, it is more convenient to consider the once integrated distribution. In particular one can integrate the intensity over all times t_1 and t_2 for fixed time difference $\Delta t = t_1 - t_2$, to obtain the intensity as a function of Δt. Performing the integrations yields, for $\Delta t > 0$,

$$I(f_1,\ f_2;\ \Delta t) = \frac{1}{2,} |\langle f_1 |\, K_S\,\rangle \langle f_2 |\, K_S\,\rangle|^2 \times$$

$$\left(|\eta_1|^2 e^{-\Gamma_L \Delta t} + |\eta_2|^2 e^{-\Gamma_S \Delta t} - 2|\eta_1||\eta_2| e^{-\Gamma \Delta t/2} \cos(\Delta m \Delta t + \phi_1 - \phi_2) \right)$$

and a similar expression is obtained for $\Delta t < 0$.

The interference pattern is quite different according to the choice of f_1 and f_2 as illustrated in figs. 6-8.
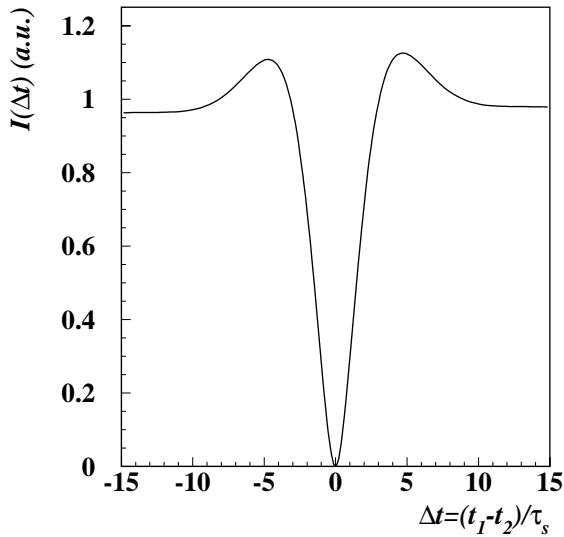

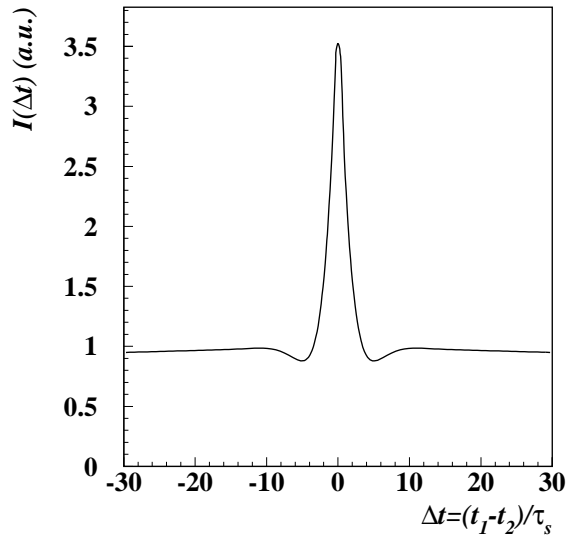**Fig. 6.** Interference for $f_1 = \pi^+\pi^-, f_2 = \pi^0\pi^0$.

Fig. 7. Interference for $f_1 = \ell^-, f_2 = \ell^+$.

One can thus perform a whole spectrum of precision "kaon-interferometry" experiments at DAΦNE by measuring the above decay intensity distributions for appropriate choices of the final states f_1, f_2. Four examples are listed below.

1. With $f_1 = f_2$ one measures , $_S$, , $_L$ and Δm, since all phases cancel. Rates can be measured with a $\times 10$ improvement in accuracy and Δm to $\sim \times 2$.

2. With $f_1 = \pi^+\pi^-$, $f_2 = \pi^0\pi^0$, one measures $\Re(\epsilon'/\epsilon)$ at large time differences, and $\Im(\epsilon'/\epsilon)$ for $|\Delta t| \leq 5\tau_s$. Fig. 6 shows the interference pattern for this case. The strong destructive interference at zero time difference is due to the antisymmetry of the initial KK state, decay amplitude phases being identical.

3. With $f_1 = \pi^+\ell^-\nu$ and $f_2 = \pi^-\ell^+\nu$, one can measure the CPT–violation parameter δ, see our discussion later concerning tests of CPT. Again the real part of δ is measured at large time differences and the imaginary part for $|\Delta t| \leq 10\tau_s$. Fig. 7 shows the interference pattern. The destructive interference at zero time difference becomes positive since the amplitude for $K^0 \to \ell^+$ has opposite sign to that for $\overline{K}^0 \to \ell^-$ thus making the overall amplitude symmetric.

4. For $f_1 = 2\pi$, $f_2 = \pi^+\ell^-\nu$ or $\pi^-\ell^+\nu$, small time differences yield Δm, $|\eta_{\pi\pi}|$ and $\phi_{\pi\pi}$, while at large time differences, the asymmetry in K_L semileptonic decays provides tests of T and CPT. The *vacuum regeneration* interference is shown in fig. 8.
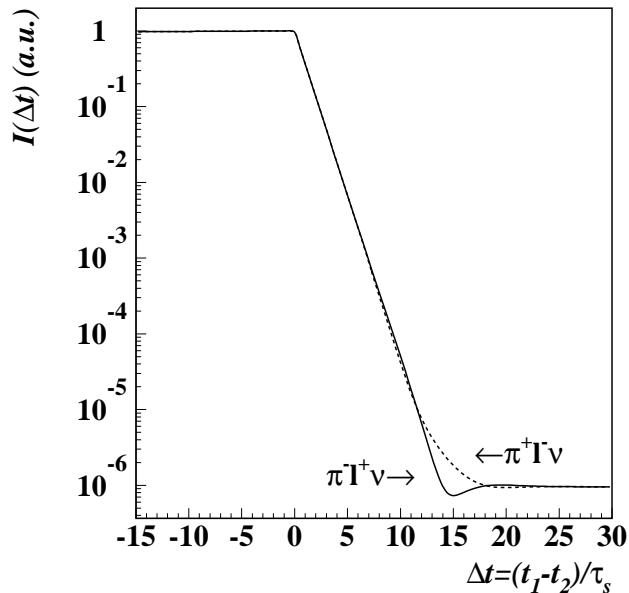
12

Fig. 8. Interference pattern for $f_1 = 2\pi, f_2 = \ell^{\pm}$.

5. CP violation in the standard model

The Standard Model has a *natural* place for CP violation (Cabibbo, Kobayashi-Maskawa, Maiani). A phase can be introduced in the unitary matrix **V** which mixes the quarks

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

but the theory does not predict the magnitude of the effect. The constraint that the mixing matrix be unitary corresponds to the desire of having a universal weak interaction. Our present knowledge of the magnitude of the V_{ij} elements is given below.

$$\begin{pmatrix} 0.9745 - 0.9757 & 0.219 - 0.224 & 0.002 - 0.005 \\ 0.218 - 0.224 & 0.9736 - 0.9750 & 0.036 - 0.047 \\ 0.004 - 0.014 & 0.034 - 0.046 & 0.9989 - .9993 \end{pmatrix}$$

The diagonal elements are close but definitely not equal to unity. If such were the case there could be no CP violation.

However, if the violation of CP which results in $\epsilon \neq 0$ is explained in this way then, in general, we expect $\epsilon' \neq 0$. For *technical* reasons, it is difficult to compute the value of ϵ'. Predictions are $\epsilon'/\epsilon \leq 10^{-3}$, but cancellations can occur, depending on the value of the top mass and the values of appropriate matrix elements, mostly connected with understanding the light hadron structure.

13

A fundamental task of experimental physics today is the determination of the four parameters of the CKM mixing matrix, including the phase which results in $C\!\!\!/P$. A knowledge of all parameters is required to confront experiments. Rather, many experiments are necessary to complete our knowledge of the parameters and prove the uniqueness of the model or maybe finally break beyond it. As it happens rare K decays can be crucial to this task. We will therefore discuss the following topics: recent measurements of K_S, K_L parameters and searches for symmetry violations; new rare K decay results; other searches for $C\!\!\!/P$ and $T\!\!\!/$ as well as present limits on $C\!\!\!/P\!\!\!/T\!\!\!/$. We will also briefly describe perspectives for developments in the near future.

To this end it is convenient to parameterize the mixing matrix above in a way which reflects more immediately our present knowledge of the value of some of the elements and has the CP violating phase appearing in only two off-diagonal elements. The Wolfenstein[11] *approximate* parameterization of the mixing matrix expanded up to λ^3 is

$$
\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}.
$$

λ=0.2215±0.0015 is the Cabibbo angle, a real number describing mixing of s and d quarks. A, also real, is close to one $A{\sim}0.84{\pm}0.06$ and $|\rho - i\eta|{\sim}0.3$. CP violation occurs only if η does not vanish. η and ρ are not really known. Several constraints on η and ρ can however be obtained from the values of measured parameters. The value of ϵ can be calculated from the ΔS=2 amplitude of fig. 9, the so called box diagram. At the quark level the calculations is straightforward, but complications arise in estimating the correct matrix element between K^0 and $\overline{K}^0$ states. Apart from the uncertainties in these estimates ϵ depends on η and ρ as:

$$
|\epsilon| = a\eta + b\eta\rho
$$

which is a hyperbola in the η, ρ plane whose central value is shown in figure 16. The calculation of ϵ' is more complicated. There are three ΔS=1 amplitudes that contribute to $K{\rightarrow}\pi\pi$ decays:

$$
A(s \rightarrow u\bar{u}d) \propto U_{us}U^*_{ud} \sim \lambda \tag{6}
$$

$$
A(s \rightarrow c\bar{c}d) \propto U_{cs}U^*_{cd} \sim -\lambda + i\eta A^2\lambda^5 \tag{7}
$$

$$
A(s \rightarrow t\bar{t}d) \propto U_{ts}U^*_{td} \sim -A^2\lambda^5(1 - \rho + i\eta) \tag{8}
$$

where the amplitude (6) correspond to the natural way for computing $K{\rightarrow}\pi\pi$ in the standard model and the amplitudes (7), (8) account for direct $C\!\!\!/P$. If the latter amplitudes were zero there would be no direct CP violation in the standard model. The flavor changing neutral current (FCNC) diagram of fig. 10 called, for no good reason in the world, the penguin diagram, contributes to the amplitudes (7), (8). The calculation of the hadronic matrix elements is even more difficult because there is a cancellation between the electroweak (γ,

Z) and the gluonic penguins, for m_t around 200 GeV, close to the now known top mass. Estimates of $\Re(\epsilon'/\epsilon)$ range from few$\times 10^{-3}$ to 10^{-4}.
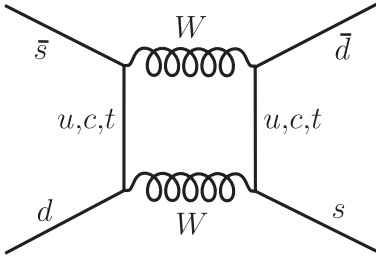

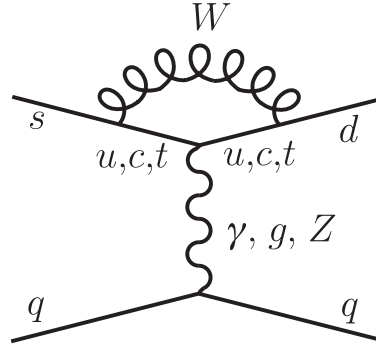**Fig. 9.** Box diagram for $K^0 \to \overline{K}^0$.

Fig. 10. "Penguin" FCNC diagram.

6. New Measurements of the Neutral Kaon Properties

6.1. CPLEAR

The CPLEAR experiment[12] studies neutral K mesons produced in equal numbers in proton-antiproton annihilations at rest:

$$p\bar{p} \to K^-\pi^+ K^0 \quad BR=2\times 10^{-3}$$
$$\to K^+\pi^- \overline{K}^0 \quad BR=2\times 10^{-3}$$

The charge of $K^\pm(\pi^\pm)$ tags the strangeness S of the neutral K at $t=0$. They have recently presented several new results[13,14] from studying $\pi^+\pi^-$, $\pi^+\pi^-\pi^0$ and $\pi^\pm\ell^\mp\bar{\nu}(\nu)$ final states. Their measurement of the K_L–K_S mass difference Δm is independent of the value of ϕ_{+-}, unlike in most other experiments. They have improved limits on the possible violation of the $\Delta S = \Delta Q$ rule, quantified by the amplitude's ratio $x = A(\Delta S = -\Delta Q)/A(\Delta S = \Delta Q)$, without assuming CPT invariance. A direct test of CPT invariance has also been obtained. The data require small corrections for background asymmetry $\sim 1\%$, differences in tagging efficiency, $\varepsilon(K^+\pi^-)-\varepsilon(K^-\pi^+)\sim 10^{-3}$ and in detection, $\varepsilon(\pi^+e^-)-\varepsilon(\pi^-e^+)\sim 3\times 10^{-3}$. They also correct for some regeneration in the detector.

6.1.1 $K^0(\overline{K}^0) \to e^+(e^-)$

Of particular interest are the study of the decays $K^0(\overline{K}^0) \to e^+(e^-)$. One can define the four decay intensities:

$$\left.\begin{array}{l} I^+(t) \text{ for } K^0 \to e^+ \\ \overline{I}^-(t) \text{ for } \overline{K}^0 \to e^- \end{array}\right\} \Delta S = 0 \qquad \left.\begin{array}{l} \overline{I}^+(t) \text{ for } \overline{K}^0 \to e^+ \\ I^-(t) \text{ for } K^0 \to e^- \end{array}\right\} |\Delta S| = 2$$

where $\Delta S = 0, 2$ mean that the strangeness of the decaying K is the same as it was at $t=0$ or has changed by 2, because of $K^0 \leftrightarrow \overline{K}^0$ transitions. One can then define four

15

asymmetries:

$$A_1(t) = \frac{I^+(t) + \overline{I}^-(t) - (\overline{I}^+(t) + I^-(t))}{I^+(t) + \overline{I}^-(t) + \overline{I}^+(t) + I^-(t)}$$

$$A_2(t) = \frac{\overline{I}^-(t) + \overline{I}^+(t) - (I^+(t) + I^-(t))}{\overline{I}^-(t) + \overline{I}^+(t) + I^+(t) + I^-(t)}$$

$$A_T(t) = \frac{\overline{I}^+(t) - I^-(t)}{\overline{I}^+(t) + I^-(t)}, \quad A_{CPT}(t) = \frac{\overline{I}^-(t) - I^+(t)}{\overline{I}^-(t) + I^+(t)}$$

From the time dependence of A_1 they obtain: $\Delta m = (0.5274 \pm 0.0029 \pm 0.0005) \times 10^{10}$ s^{-1}, a result which is independent of ϕ_{+-} and $\Re x = (12.4 \pm 11.9 \pm 6.9) \times 10^{-3}$, without assuming CPT. From A_2 and assuming CPT they obtain $\Im x = (4.8 \pm 4.3) \times 10^{-3}$, a result $\sim$5 times more stringent than the PDG94 world average. A_T gives a direct measurement of T violation. Assuming CPT, the expected value for A_T is 6.52×10^{-3}. The CPLEAR result is $A_T = (6.3 \pm 2.1 \pm 1.8) \times 10^{-3}$. From a study of the CPT violating asymmetry, $A_{CPT}(t)$, they obtain $\Re \delta_{CPT} = (0.07 \pm 0.53 \pm 0.45) \times 10^{-3}$. We will come back later to the definition of δ_{CPT}, which we will simply call δ.
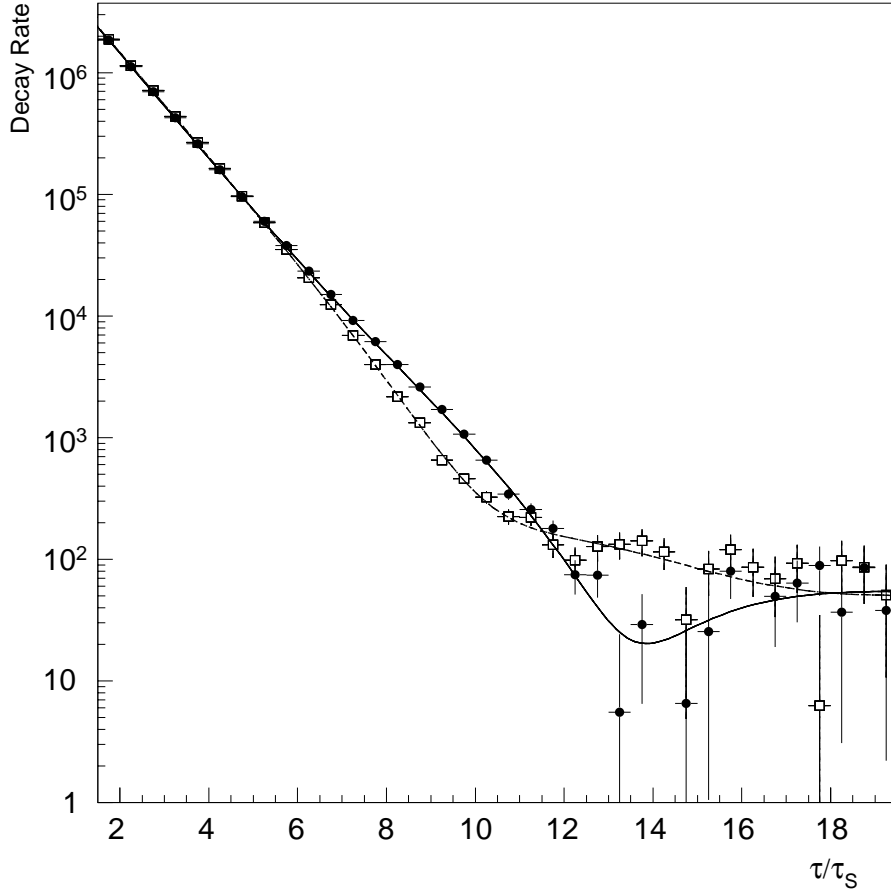

Fig. 11. Decay distributions for K^0 and $\overline{K}^0$.

16

6.1.2 $\pi^+\pi^-$ Final State

From an analysis of 1.6×10^7 $\pi^+\pi^-$ decays of K^0 and $\overline{K}^0$ they determine $|\eta_{+-}| = (2.312 \pm 0.043 \pm 0.03 \pm 0.011_{\tau_S}) \times 10^{-3}$ and $\phi_{+-} = 42.6° \pm 0.9° \pm 0.6° \pm 0.9°_{\Delta m}$. The *errors* in the values quoted reflect uncertainties in the knowledge of the K_S lifetime and the K_S–K_L mass difference, respectively. Fig. 11 shows the decay intensities of K^0 and $\overline{K}^0$, while fig. 12 is a plot of the time dependent asymmetry $A_{+-} = \left(I(\overline{K}^0 \to \pi^+\pi^-) - \alpha I(K^0 \to \pi^+\pi^-)\right)/\left(I(\overline{K}^0 \to \pi^+\pi^-) + \alpha I(K^0 \to \pi^+\pi^-)\right)$. Most systematics cancel in the ratio and the residual difference in efficiencies for K^0 and $\overline{K}^0$ decays is determined from a fit to the same data: $\alpha = 0.9989 \pm 0.0006$.
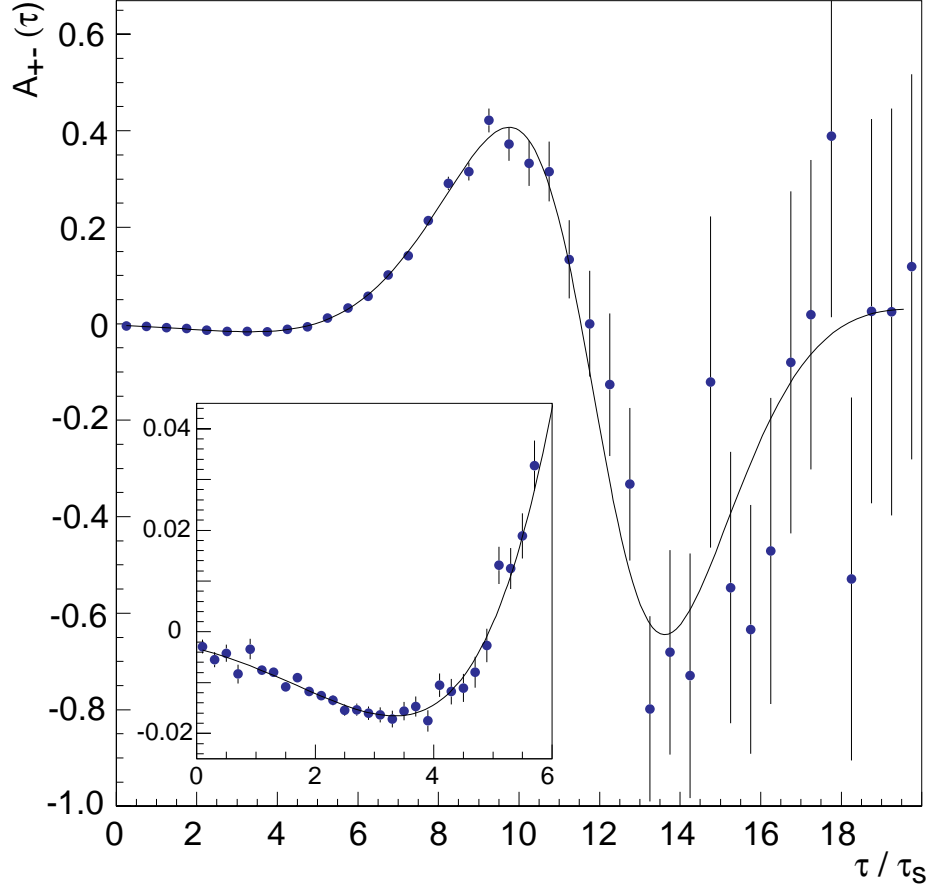
Fig. 12. Difference of decay distributions for K^0 and $\overline{K}^0$.

6.1.3 $\pi^+\pi^-\pi^0$ Final States

Studies of K^0-$\overline{K}^0 \to \pi^+\pi^-\pi^0$ decays give the results $\Re\eta_{+-0} = (-4 \pm 17 \pm 3) \times 10^{-3}$ and $\Im\eta_{+-0} = (-16 \pm 20 \pm 8) \times 10^{-3}$, where $\eta_{+-0} = A(K_L \to \pi^+\pi^-\pi^0)/A(K_S \to \pi^+\pi^-\pi^0)$. By setting $\Re\eta_{+-0} = \Re\eta_{+-}$ they obtain $\Im(\eta_{+-0} = (-11 \pm 14 \pm 8) \times 10^{-3}$. These results are significantly more precise than any previous ones.

For completeness the results by E621 at FNAL for $K \to \pi^+\pi^-\pi^0$ must be mentioned.[15]

In this experiment the CP conserving amplitude $A(K_S \to \pi^+\pi^-\pi^0)$ is measured, obtaining

$$|\rho_{\pi^+\pi^-\pi^0}| = \left| \frac{A(K_S \to \pi^+\pi^-\pi^0, I = 2)}{A(K_L \to \pi^+\pi^-\pi^0)} \right| = 0.035^{+0.019}_{-0.011} \pm 0.004$$

$$\phi_\rho = -59° \pm 48°$$

$$BR(K_S \to \pi^+\pi^-\pi^0) = (3.9^{+0.54+0.8}_{-1.8-0.7}) \times 10^{-7}$$

$$\Im(\eta_{+-0}) = -0.015 \pm 0.017 \pm 0.025, \quad \text{assuming } \Re(\eta_{+-0}) = \Re(\epsilon).$$

6.2. E773 at FNAL

E773 is a modified E731 setup, with a downstream regenerator added. New results have been obtained on Δm, τ_S, $\phi_{00} - \phi_{+-}$ and ϕ_{+-} from a study of $K \to \pi^+\pi^-$, $\pi^0\pi^0$ decays.[16]

6.2.4 Two Pion Final States

This study of $K \to \pi\pi$ is a classic experiment where one beats the amplitude $A(K_L \to \pi\pi]_i) = \eta_i A(K_S \to \pi\pi)$ with the coherently regenerated $K_S \to \pi\pi$ amplitude $\rho A(K_S \to \pi\pi)$, resulting in the decay intensity

$$I(t) = |\rho|^2 e^{-\Gamma_S t} + |\eta|^2 e^{-\Gamma_L t} +$$
$$2|\rho||\eta| e^{-\Gamma t} \cos(\Delta m t + \phi_\rho - \phi_{+-})$$

Measurements of the time dependence of I for the $\pi^+\pi^-$ final state yields , $_S$, , $_L$, Δm and ϕ_{+-}. They give the following results: $\tau_S = (0.8941 \pm 0.0014 \pm 0.009) \times 10^{-10}$ s setting $\phi_{+-} = \phi_{SW} = \tan^{-1} 2\Delta m/\Delta$, and floating Δm; $\Delta m = (0.5297 \pm 0.0030 \pm 0.0022) \times 10^{10}$ s^{-1} using for τ_S the PDG94 value, leaving ϕ_{+-} free in the fit; $\phi_{+-} = 43.53° \pm 0.58° \pm 0.40°$, using for τ_S the PDG94 value and for the mass difference the combined values of E731 and E773, $\Delta m = (0.5282 \pm 0.0030) \times 10^{10}$ s^{-1}. Including the uncertainties on Δm and τ_S and the correlations in their measurements they finally quote $\phi_{+-} = 43.53° \pm 0.97°$

From a simultaneous fit to the $\pi^+\pi^-$ and $\pi^0\pi^0$ data they obtain $\Delta\phi = \phi_{00} - \phi_{+-} = 0.62° \pm 0.71° \pm 0.75°$, which combined with the E731 result gives $\Delta\phi = -0.3° \pm 0.88°$.

6.2.5 $K \to \pi^+\pi^-\gamma$

From a study of $\pi^+\pi^-\gamma$ final states $|\eta_{+-\gamma}|$ and $\phi_{+-\gamma}$ are obtained. The time dependence of the this decay, like that for two pion case, allows extraction of the corresponding parameters $|\eta_{+-\gamma}|$ and $\phi_{+-\gamma}$. The elegant point of this measurement is that because interference is observed (which vanishes between orthogonal states) one truly measures the ratio

$$\eta_{+-\gamma} = \frac{A(K_L \to \pi^+\pi^-\gamma, \ \cancel{CP})}{A(K_S \to \pi^+\pi^-\gamma, \ CP \text{ OK })}$$

which is dominated by E1, inner bremsstrahlung transitions. Thus, again, one is measuring the CP impurity of K_L. Direct CP could contribute via E1, direct photon emission K_L decays, but it is not observed within the sensitivity of the measurement.

The results obtained are:[17] $|\eta_{+-\gamma}| = (2.362 \pm 0.064 \pm 0.04) \times 10^{-3}$ and $\phi_{+-\gamma} = 43.6° \pm 3.4° \pm 1.9°$. Comparison with $|\eta_{+-}| \sim |\epsilon| \sim 2.3$, $\phi_{+-} \sim 43°$ gives excellent agreement.

This implies that the decay is dominated by radiative contributions and that all one sees is the CP impurity of the K states.

6.3. Combining Results for Δm and ϕ_{+-} from Different Experiments

The CPLEAR collaboration[18] has performed an analysis for obtaining the best value for Δm and ϕ_{+-}, taking properly into account the fact that different experiments have different correlations between the two variables. The data[13,14,16,19−25] with their correlations are shown in fig. 13.
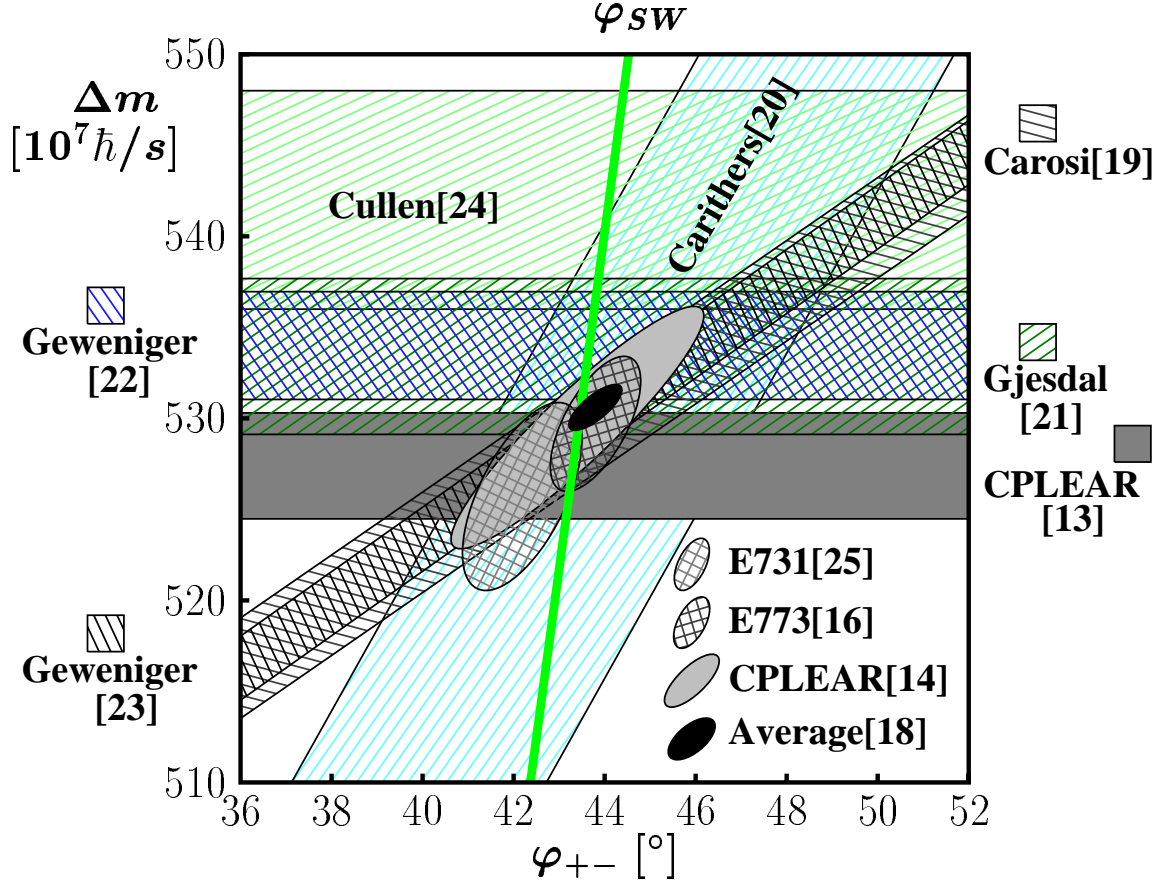
Fig. 13. A compilation of Δm and ϕ_{+-} results, from ref. 18.

A maximum likelihood analysis of all data gives $\Delta m = (530.6 \pm 1.3) \times 10^7$ s^{-1} and $\phi_{+-} = 43.75° \pm 0.6°$. ϕ_{+-} is very close to the *superweak phase* $\phi_{\mathrm{SW}} = 43.44° \pm 0.09°$.

7. Tests of CPT Invariance

In field theory, CPT invariance is a consequence of quantum mechanics and Lorentz invariance. Experimental evidence that CPT invariance might be violated would therefore invalidate our belief in either or both quantum mechanics and Lorentz invariance. We might not be so ready to abandon them, although recent ideas,[26] such as distortions of the metric at the Planck mass scale or the loss of coherence due to the properties of black holes might

make the acceptance somewhat more palatable. Very sensitive tests of CPT invariance, or lack thereof, can be carried out investigating the neutral K system at a ϕ–factory. One should not however forget other possibilities.[27]

7.1. CPT at a ϕ–factory

A ϕ–factory such as DAΦNE can produce of the order of 10^{10} neutral K pairs per year which allow study of CP, T and CPT invariance. An advantage of ϕ-factories, in this respect, is that the K pair is produced in a well defined quantum state, allowing more refined tests than otherwise. K mesons are produced via the reaction $e^+e^- \rightarrow \text{``}\gamma\text{''} \rightarrow \phi \rightarrow K^0\overline{K}^0$ in a $C = -1$ state. Therefore the two kaons are in a pure K^0, $\overline{K}^0$ or K_S, K_L state with a $K_L K_L$ or $K_S K_S$ impurity of $\ll 10^{-5}$. In addition only at a ϕ–factory it is possible to obtain a pure K_S beam using the observation of a decay at long time as a tag for the presence of a K_S. In general, CPT requires

$$M_{11} - M_{22} = M(K^0) - M(\overline{K}^0) = 0$$

and in the following we discuss present experimental limits on $(M(K^0) - M(\overline{K}^0))/\langle M \rangle$ and possible future improvements.

7.2. Neutral K decays without assuming CPT

One of the problems in dealing with the neutral K system is the large number of parameters which are necessary for its description.[28] Moreover different authors use different notations. For consistency we will redefine, following Maiani's analysis[28] but with some different symbols, all the relevant parameters which will be used below. To lowest order in "ϵ" we write the K_S and K_L states as

$$|K_S\rangle = [(1 + \epsilon_S)|K^0\rangle + (1 - \epsilon_S)|\overline{K}^0\rangle]/\sqrt{2}$$
$$|K_L\rangle = [(1 + \epsilon_L)|K^0\rangle + (1 - \epsilon_L)|\overline{K}^0\rangle]/\sqrt{2}$$

and define the parameters $\tilde{\epsilon}$ and δ through the identities

$$\epsilon_S \equiv \tilde{\epsilon} + \delta \qquad \epsilon_L \equiv \tilde{\epsilon} - \delta.$$

Following the usual convention, we introduce the ratios of the amplitudes for K decay to a final state f_i, $\eta_i = A(K_L \rightarrow f_i)/A(K_S \rightarrow f_i)$, and define the parameters ϵ and ϵ' with the identities

$$\eta_{+-} \equiv \epsilon + \epsilon' \qquad \eta_{00} \equiv \epsilon - 2\epsilon' \tag{9}$$

From Eq. (9), ϵ is given, in terms of the measurable amplitude ratios η, by:

$$\epsilon = (2\eta_{+-} + \eta_{00})/3$$
$$\text{Arg}(\epsilon) = \phi_{+-} + (\phi_{+-} - \phi_{00})/3.$$

To treat consistently the possibility of CPT violation we write the most general decay

amplitudes for $K \to 2\pi$ as

$$A(K^0 \to 2\pi, \ I) \equiv \sqrt{3/2}(A_I + B_I)$$
$$A(\overline{K}^0 \to 2\pi, \ I) \equiv \sqrt{3/2}(A_I^* - B_I^*), \tag{10}$$

where I is the isospin of the 2 pion state and we use the Wu and Yang phase convention *i.e.* A_0 real and positive. The symmetry properties of the A and B amplitudes in Eq. (10) are given below:

$$
\begin{array}{ccccc}
 & \Re A & \Im A & \Re B & \Im B \\
CP & + & - & - & + \\
T & + & - & + & - \\
CPT & + & + & - & - \\
\end{array}
$$

If CPT invariance is valid we have $\delta = 0$, $\epsilon = \tilde{\epsilon}$, $B_I = 0$, otherwise, including "direct" CPT in the decay amplitude to two pions:

$$\epsilon = \tilde{\epsilon} - (\delta - \frac{\Re B_0}{A_0}) \qquad \mathrm{Arg}(\tilde{\epsilon}) = \phi_{SW} \equiv \tan^{-1} \frac{2\Delta m}{\Delta,}. \tag{11}$$

From unitarity,[28] with the most reasonable assumption that $,_{11} - ,_{22} \ll ,_S$ for all channels but 2 pions with $I=0$, it follows that $\delta - \Re B_0/A_0$ is orthogonal to ϵ, see fig. 14, from which:

$$\mathrm{Arg}\left(\delta - \frac{\Re B_0}{A_0}\right) = \phi_{SW} \pm 90°$$
$$\left| \delta - \frac{\Re B_0}{A_0} \right| = |\epsilon| \times |\phi_{SW} - \mathrm{Arg}(\epsilon)|. \tag{12}$$

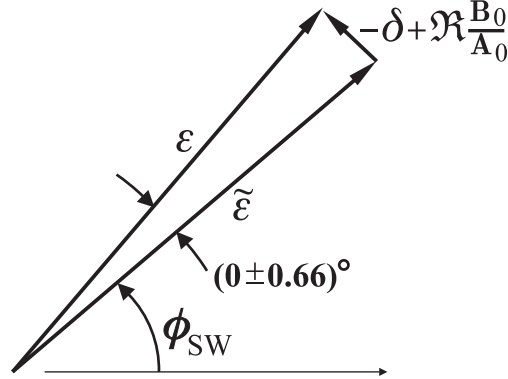**Fig. 14**. Diagram of the complex quantities in Eq. (11).

7.3. Experimental Data

In table 1 we have collected the data relevant to K decays, as known today, as well as the values of some derived quantities according to the definitions above. From the values in the table

$$\left| \delta - \frac{\Re B_0}{A_0} \right| = 2.282 \times 10^{-3} \times (0 \pm 0.66°)$$
$$= (0 \pm 2.6) \times 10^{-5}. \tag{13}$$

If there is no CPT violation in the semileptonic decay amplitudes, the leptonic asymmetry for K_L decays, $\mathcal{A}_L^\ell = (,{}_{L,S}^{\ell^+} - ,{}_{L,S}^{\ell^-})/(,{}_{L,S}^{\ell^+} + ,{}_{L,S}^{\ell^-})$, can be used together with this result for determining δ and ultimately put limits on the CPT violating quantity $M(K^0) - M(\overline{K}^0)$. Under this assumption one has

$$\mathcal{A}_L^\ell = 2\Re(\tilde{\epsilon} - \delta)$$

$$\Re\delta = \frac{2\Re\tilde{\epsilon} - \mathcal{A}_L^\ell}{2} = \frac{2|\tilde{\epsilon}|\cos\phi_{SW} - \mathcal{A}_L^\ell}{2}$$

$$= (1.6 \pm 6) \times 10^{-5}.$$

Table 1: Parameters of the neutral K mesons.

	Parameter	Value
a	Δm	$(0.534 \pm 0.0014) \times 10^{10}$ s^{-1}
a	$,{}_S$	$(1.1202 \pm 0.0010) \times 10^{10}$ s^{-1}
b	ϕ_{SW}	$43.63 \pm 0.08°$
a	ϕ_{+-}	$43.7 \pm 0.6°$
a	$\phi_{00} - \phi_{+-}$	$-0.2 \pm 0.8°$
a	$|\eta_{+-}|$	$(2.285 \pm 0.019) \times 10^{-3}$
a	$|\eta_{00}|$	$(2.275 \pm 0.019) \times 10^{-3}$
b	$|\epsilon|$	$(2.282 \pm 0.014) \times 10^{-3}$
a	$\text{Arg}(\epsilon)$	$43.63 \pm 0.66°$
a	$\mathcal{A}_L^\ell$	$(3.27 \pm 0.12) \times 10^{-3}$

a From reference 29.
b Derived using definitions in the text.

Using the magnitude and phase of $\delta - \Re B_0/A_0$ from eqs. (12) and (13) we find:

$$\Im\delta = (0 \pm 1.9) \times 10^{-5}$$

$$\Re B_0/A_0 = (-3 \pm 6) \times 10^{-5}.$$

From

$$|M(K^0) - M(\overline{K}^0)| = |,{}_S - ,{}_L| \, |\Re\delta\tan\phi_{SW} - \Im\delta|$$

it follows that

$$\frac{|M(K^0) - M(\overline{K}^0)|}{\langle M(K)\rangle} = (0.2 \pm 0.9) \times 10^{-18},$$

the uncertainty in this result being due mostly to the error on $\mathcal{A}_L^\ell$. Note that if there were no CPT violation in the two pion decay amplitudes, the limit on the mass difference would

be:

$$\frac{|M(K^0) - M(\overline{K}^0)|}{\langle M(K) \rangle} = (0.0 \pm 0.3) \times 10^{-18}.$$

The complete relation between the various parameters in the text is illustrated in fig. 15.
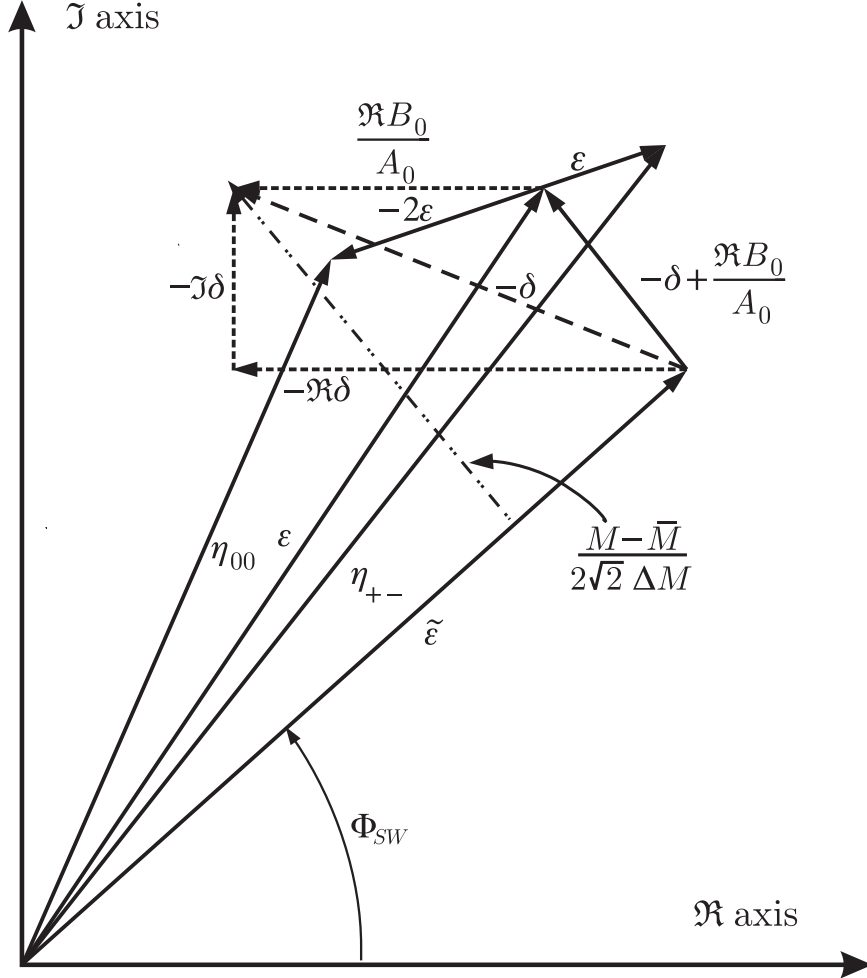
Fig. 15. Diagram of all the quantities discussed in the text.

7.4. CPT Violation in $A(K \to \ell^{\pm}\pi\nu)$

To describe consistently K decays without assuming CPT, we must allow fo $\cancel{C}\cancel{P}\cancel{T}$ in semileptonic decays. We therefore introduce four complex amplitudes a, b, c, and d, in terms of which we write

$$A(K^0 \to \ell^+) = a + b$$
$$A(\overline{K}^0 \to \ell^-) = a^* - b^*$$
$$A(K^0 \to \ell^-) = c + d$$
$$A(\overline{K}^0 \to \ell^+) = c^* - d^*,$$

where the c and d amplitudes are for $\Delta S = -\Delta Q$ transitions. Their symmetry properties

are displayed below:

$$
\begin{array}{ccccc}
 & \Re(a,\,c) & \Im(a,\,c) & \Re(b,\,d) & \Im(b,\,d) \\
CP & + & - & - & + \\
T & + & - & + & - \\
CPT & + & + & - & - \quad.
\end{array}
$$

To first order in δ, $\Im a$, b, c and d the leptonic asymmetries are

$$
\mathcal{A}_L^\ell = 2(\Re\tilde{\epsilon} - \Re\delta + \frac{\Re b}{\Re a} + \frac{\Re d}{\Re a})
$$

$$
\mathcal{A}_S^\ell = 2(\Re\tilde{\epsilon} + \Re\delta + \frac{\Re b}{\Re a} - \frac{\Re d}{\Re a})
$$

which implies that K_S, K_L experiments cannot disentangle $\cancel{CPT}$ from violation of the $\Delta S = \Delta Q$ rule. The validity or otherwise of the rule can be checked by studying the decays of strangeness tagged K^0 and $\overline{K}^0$ states, if the tagging is done using strong interactions and not semileptonic decays. This was successfully done by CPLEAR, unfortunately with relatively limited statistics. In the standard model, it is very hard to imagine how $\Delta S = -\Delta Q$ transitions can be induced at the level of 10^{-5}. Assuming $c=d=0$, the remaining $\cancel{CPT}$ term cancel in the difference of the leptonic asymmetries and we obtain

$$
\mathcal{A}_S^\ell - \mathcal{A}_L^\ell = 4\Re\delta.
$$

At DAΦNE, it should be possible to reach an accuracy of $\sim 2.5\times 10^{-4}$ for $\mathcal{A}_S^\ell$ (we assume a tagging efficiency of order 75%, see section 9.3, which means a measurement of $\Re\delta$ to an accuracy $\sim 0.6\times 10^{-4}$, almost ten times better than CPLEAR, albeit with the assumption of the validity of the $\Delta S = \Delta Q$ rule. The rule itself can also be checked to good accuracy at DAΦNE by using strangeness tagged neutral K mesons produced by charge exchange of $K^\pm$ which are produced even more copiously than neutral K's. The ratio , $_L^\ell / , _S^\ell = 1+4\Re(c/a)$ can put limits on the CPT conserving part of the $\Delta S = -\Delta Q$ amplitude, to about the same sensitivity as above. Time ordered asymmetries[30] $(\ell^+\ell^- - \ell^-\ell^+)/(\ell^+\ell^- + \ell^-\ell^+)$, where $+-$ means that the positive lepton appears earlier than the negative and viceversa for $-+$, are also sensitive to the various CPT odd terms at small and large time differences.

The CPLEAR limit for the mass difference does not assume $\Delta S = \Delta Q$ and uses their own new limits on $\Im x$ and $\Im\eta_{+-0}$. The limit on $\cancel{CPT}$ is only slightly weaker[31]

$$
\frac{|m_{K^0} - m_{\overline{K}_0}|}{m_{K^0}} < 2.2 \times 10^{-18}
$$

8. Rare K Decays

Rare K decays offer several interesting possibilities, which could ultimately open a window beyond the standard model. They allow the determination of the CKM matrix parameters, as for instance from the $\cancel{CP}$ decay $K_L \to \pi^0 \nu\bar{\nu}$, as well as from the CP conserving one $K^+ \to \pi^+ \nu\bar{\nu}$. They also permit the verification of conservation laws which are not strictly required in the standard model, for instance by searching for $K^0 \to \mu e$ decays.

The connection between measurements of neutral K properties and branching ratios and the ρ and η parameters of the Wolfenstein parameterization of the CKM matrix, is shown *schematically* in fig. 16.

Fig. 16. Constraints on η and ρ from measurements of ϵ, ϵ', rare decays and B meson properties.

In general the situation valid for the more abundant K decays, *i.e.* that the $\cancel{C}\cancel{P}|_{\text{direct}}$ decays have much smaller rates then the $\cancel{C}\cancel{P}|_{\text{indirect}}$ ones, can be reversed for very rare decays. In addition, while the evaluation of ϵ' is particularly unsatisfactory because of the uncertainties in the calculation of the hadronic matrix elements, this is not the case for some rare decays. A classifications of measurable quantities according to increasing uncertainties in the calculation of the hadronic matrix elements is given by Buras[32] as: 1. $BR(K_L \to \pi^0 \nu \bar{\nu})$, 2. $BR(K^+ \to \pi^+ \nu \bar{\nu})$, 3. $BR(K_L \to \pi^0 e^+ e^-)$, ϵ_K, and 4. ϵ'_K, $BR(K_L \to \mu \bar{\mu}]_{\text{SD}})$, where SD stands for *short distance* contributions. The observation $\epsilon' \neq 0$ remains a unique proof of direct $\cancel{C}\cancel{P}$. Measurements of 1 through 3, plus present knowledge, over determine the CKM matrix. Rare K decay experiments are not easy however, just like measuring $\Re(\epsilon'/\epsilon)$ has turned out to be difficult. Typical expectations for some of the interesting decays are:

$$BR(K_L \to \pi^0 e^+ e^-, \cancel{C}\cancel{P}]_{dir}) \sim (5 \pm 2) \times 10^{-12}$$

$$BR(K_L \to \pi^0 \nu \bar{\nu}) \sim (3 \pm 1.2) \times 10^{-11}$$

$$BR(K^+ \to \pi^+ \nu \bar{\nu}) \sim (1 \pm .4) \times 10^{-10}$$

The most extensive program in this field has been ongoing for a long time at BNL and large statistics have been collected recently and are under analysis. Sensitivities of the order of 10^{-11} will be reached, although $10^{-(12 \text{ or } 13)}$ are really necessary. Experiments with high energy kaon beams have been making excellent progress toward observing rare decays. We will discuss new results from E799-I[33−39] (E731 without regenerators) and NA31.[40−46]

The results obtained by the two experiments are summarized in the tables below.

Table 3. E799-I Rare K-decays Results.

Reaction	Events	BR or limit	Ref.
$K_L \to \pi^0 \nu \bar{\nu}$		$< 5.8 \times 10^{-5}$	33
$K_L \to e^+ e^- e^+ e^-$	27	$(4.0 \pm 0.8 \pm 0.3) \times 10^{-8}$	34
$K_L \to \pi^0 \pi^0 \gamma)$		$< 2.3 \times 10^{-4}$	35
$K_L \to e^+ e^- \gamma \gamma$, $E_\gamma > 5$ MeV		$(6.5 \pm 1.2 \pm 0.6) \times 10^{-7}$	36
$K_L \to \mu^+ \mu^- \gamma$	207	$(3.23 \pm 0.23 \pm 0.19) \times 10^{-7}$	37
$K_L \to \pi^0 \mu^\pm e^\mp$		$< 6.4 \times 10^{-9}$	38
$K_L \to e^+ e^- \mu^+ \mu^-$	1	$(2.9^{+6.7}_{-2.4}) \times 10^{-9}$	39

Table 4. NA31 Rare K-decays Results.

Reaction	Events	BR or limit	Ref.
$K_S \to \pi^0 e^+ e^-$	0	$< 1.1 \times 10^{-6}$	40
$K_L \to \pi^0 \pi^0 \gamma$	3	$< 5.6 \times 10^{-6}$	41
$K_L \to e^+ e^- e^+ e^-$	8	$(10.4 \pm 3.7 \pm 1.1) \times 10^{-8}$	42
$K_L \to \pi^0 \pi^0 \pi^0$		0.211 ± 0.003	43
	, $(K_L \to 3\pi^0)/$, $(K_L \to \pi^+ \pi^- \pi^0) = 1.611 \pm 0.037$		43
	, $(K_L \to 3\pi^0)/$, $(K_L \to \pi e \nu) = 0.545 \pm 0.01$		43
$K_L \to \pi^0 \gamma \gamma$	57	$(1.7 \pm 0.3) \times 10^{-6}$	44
$K_L \to e^+ e^- \gamma$	2000	$(9.1 \pm 0.3 \pm 0.5) \times 10^{-6}$	44
$K_L \to 3\gamma$		$< 2.4 \times 10^{-7}$	45
$K_S \to \gamma \gamma$	16	$(2.4 \pm 0.9) \times 10^{-6}$	46

These new results do not yet determine ρ and η. They do however confirm the feasibility of such a program.

8.1. Search for $K^+ \to \pi^+ \nu \bar{\nu}$

This decay, CP allowed, is best for determining V_{td}. At present there is no information, other than E787-BNL's limit $BR < 2.4 \times 10^{-9}$.[47] The new E787[48] detector, which has found 12 events of $K \to \pi \mu^+ \mu^-$, $BR \sim 10^{-8}$, has collected data for a total of 2.55×10^{12} stopped kaons, ~ 7 times the previous statistics. This corresponds to about two $K^+ \to \pi^+ \nu \bar{\nu}$ event. At least 100 are necessary for a first V_{td} measurements.

26

8.2. $K \to \gamma\gamma$

Direct $\not{C}\not{P}$ is possible in this channel. Defining the two photon states, where L and R refer to the photon polarizations,

$$| + \rangle = (| LL \rangle + | RR \rangle)/\sqrt{2}$$
$$| - \rangle = (| LL \rangle - | RR \rangle)/\sqrt{2}$$

we have four possibilities for $K_L, K_S \to \gamma\gamma$, given below, with the expected BR's:

	$\| + \rangle$	$\| - \rangle$
K_L	7×10^9, $\not{C}\not{P}$	6×10^{-4}
K_S	2×10^{-6}	5×10^{-12}, $\not{C}\not{P}$

The $\not{C}\not{P}$ channels can be isolated by measuring the γ polarization, using Dalitz conversion. The present results confirm expectations on the CP conserving channels. Both E799-I and NA31 have detected $K_L \to e^+ e^- e^+ e^-$ decays, 27 and 8 events respectively, finding $BR = (3.9 \pm 0.8, 10 \pm 4) \times 10^{-8}$ to be compared with the expectation $(3.4 \pm 0.2) \times 10^{-8}$. They also have determined that $CP | K_2 \rangle = -| K_2 \rangle$. NA31 has also observed 69 $K \to \gamma\gamma$ events, of which 52 are from K_L and one is background. From this they derive $BR(K_S \to \gamma\gamma) = (2.4 \pm 0.9) \times 10^{-6}$. These results are in agreement with expectations, still one needs sensitivities of 10^{-12}.

8.3. $K \to \mu^+ \mu^-$

Second order weak amplitudes give contributions which depend on ρ, with $BR|_{\mathrm{SD}} \sim 10^{-9}$. Measurements of the muon polarization are necessary. One however needs to confirm the calculations for $K \to \gamma\gamma \to \mu^+ \mu^-$, which can confuse the signal. The following results are relevant

1. NA31 with 2000 $K_L \to e^+ e^- \gamma$ events finds $BR = (9.1 \pm 0.3 \pm 0.5) \times 10^{-6}$. The BR depends on the $K\gamma^*\gamma$ form factor, with contributions from vector meson dominance and the $KK^*\gamma$ coupling, $f(q^2) = f_{VMD} + \alpha_{K^*} f_{KK^*\gamma}$. The measured BR corresponds to $\alpha_{K^*} = -0.27 \pm 0.1$.

2. E799-I observes 207 $K_L \to \mu^+ \mu^- \gamma$ events, giving $BR = (3.23 \pm 0.23 \pm 0.19) \times 10^{-7}$ and $\alpha_{K^*} = 0.13^{+0.21}_{-0.35}$

3. E799-I has found one $K_L \to e^+ e^- \mu^+ \mu^-$ event,[39] on the basis of which they estimate the branching ratio as $BR = (2.9^{+6.7}_{-2.4}) \times 10^{-9}$. Expectations are 2.3×10^{-9}, from VMD and 8×10^{-10} for $f(q^2) =$const. Previous limits were $BR < 4.9 \times 10^{-6}$.

At BNL the experiment E871[49] has completed collecting data which should allow the observation of one event for a BR of 10^{-12}.

8.4. $K_L \to \pi^0 e^+ e^-$

The direct $\not{C}\not{P}$ BR is expected to be $\sim 5 \times 10^{-12}$. There are however three contributions to the rate plus a potentially dangerous background.

1. $K_2 \to \pi^0 \gamma \gamma \to \pi^0 e^+ e^-$, a CP allowed transition.

2. $K_L \to \pi^0 e^+ e^-$, from the K_L CP impurity ($\epsilon | K_1 \rangle$).

3. Direct $\not{C}\not{P}$ from short distance, second order weak contributions, via $s \to d + Z$, the signal of interest.

4. Background from $K_L \to \gamma \gamma^* \to e^+ e^- \gamma \to e^+ e^- \gamma \gamma$, with a photon from final state radiation.

The relevant experimental results are:

1. NA31 finds 57 $K_L \to \pi^0 \gamma \gamma$ events corresponding to $BR=(1.6\pm0.3)\times10^{-6}$, equivalent to $BR(K_L \to \pi^0 e^+ e^-) = 5\times10^{-13}$

2. NA31 finds no $K_S \to \pi^0 e^+ e^-$ events or $BR<1.1\times10^{-6}$, from which $BR(K_L \to \pi^0 e^+ e^-)$ $\sim |\epsilon|^2(, _S/, _L)BR(K_S) < 3.2 \times 10^{-9}$, which is not quite good enough yet.

3. 799-I finds 58 $K_L \to e^+ e^- \gamma \gamma$ events, $BR=(6.5\pm1.2\pm0.6)\times10^{-7}$.

The background from point 3 above will not be dangerous for the new proposed experiments (KTEV and NA48), because of the superior resolution of their new electromagnetic calorimeter. The observation of direct $\not{C}\not{P}$ contributions to $K_L \to \pi^0 e^+ e^-$ should be convincing when the necessary sensitivity is reached.

8.5. $K_L \to \pi^0 \nu \bar{\nu}$

This process is a pure direct $\not{C}\not{P}$ signal. The present limits are far from the goal. The sensitivities claimed for E799-II and at KEK are around 10^{-9}. Another factor of 100 improvement is necessary.

9. Future

Three new experiments: NA48[50] in CERN, KTEV[51] at FNAL and KLOE[52] at LNF, are under construction and will begin taking data in '97 – '98, with the primary aim to reach an ultimate error in $\Re(\epsilon'/\epsilon)$ of $\mathcal{O}(10^{-4})$. The sophistication of these experiments takes advantage of our experience of two decades of fixed target and $e^+ e^-$ collider physics. Fundamental in KLOE is the possibility of continuous self-calibration while running, via processes like Bhabha scattering and charged K decays.

9.1. NA48

The layout of the NA48 experiment, with its main components is shown in fig. 17.

A new feature of NA48, with respect to its predecessor NA31, is that K_L and K_S beams simultaneously illuminate the detector, by the very clever use of a bent crystal to deflect a portion of the incident proton beam. This deflected beam is brought to a K_S production target located close to the detector, reducing systematic errors due to different dead times when detecting $\pi^+ \pi^-$ or $\pi^0 \pi^0$ K decays.

The NA48 Detector

μ-veto counters

Hadron calorimeter

Liquid krypton
calorimeter

Fe

Hodoscope

Anti counter

Wire chamber 4

Wire chamber 3

Anti counter

Magnet

Wire chamber 2

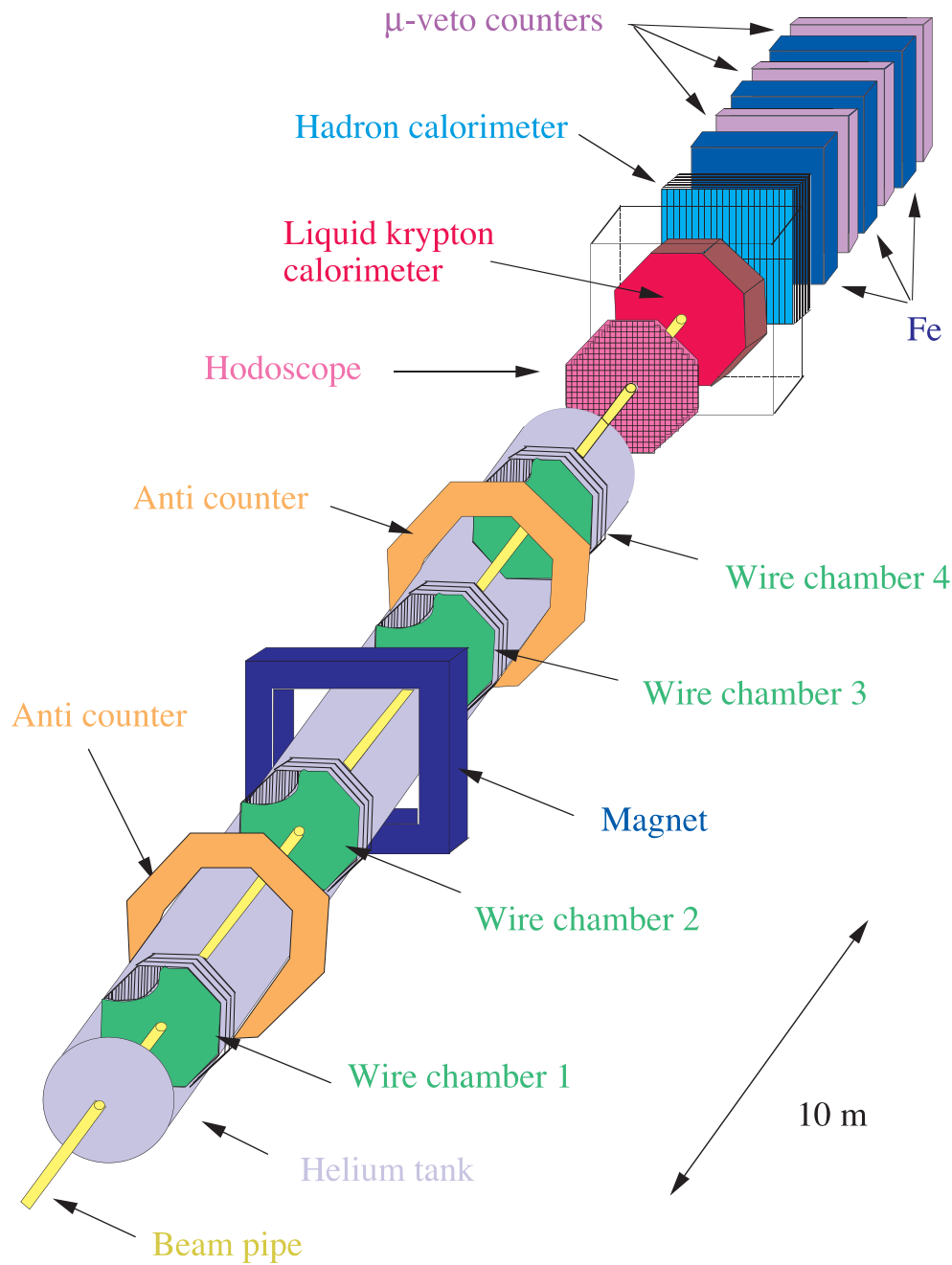Wire chamber 1

10 m

Helium tank

Beam pipe

Fig. 17. The NA48 experiment at CERN.

The superior resolution of the liquid krypton calorimeter further improves the definition of the fiducial regions and improves rejection of $3\pi^0$'s decays. A magnetic spectrometer has also been added in order to improve resolution and background rejection for the $K^0 \rightarrow \pi^+ \pi^-$ decays.

Fig. 18. Plan view of the KTEV experiment at FNAL.

The KTEV experiment retains the basic principle of E731, with several significant improvements, the most important being the use of CsI crystals for the electromagnetic calorimeter. This results in better energy resolution which is important for background rejection in the $\pi^0\pi^0$ channel as well as in the search for rare K decays.

The KLOE detector,[53] designed by the KLOE collaboration and under construction by the collaboration at the Laboratori Nazionali di Frascati, is shown in cross section in fig. 19. The KLOE detector looks very much like a collider detector and will be in fact operated at the DAΦNE collider under construction at the Laboratori Nazionali di Frascati, LNF. At DAΦNE K-meson are produced in pairs at rest in the laboratory, via the reaction $e^+e^- \to \phi \to 2K$. ∼5000 ϕ-mesons are produced per second at a total energy of W=1020 MeV and full DAΦNE luminosity.

The main motivation behind the whole KLOE venture is the observation of direct CP violation from a measurement of $\Re(\epsilon'/\epsilon)$ to a sensitivity of 10^{-4}. The first requirement for achieving such accuracy is to be able to collect enough statistics, which in turn requires studying of the order of few$\times 10^{10}$ K_L decays. The dimensions of the detector are dictated by one parameter, the mean free path for decay of K_L's which is about 3.4 m.
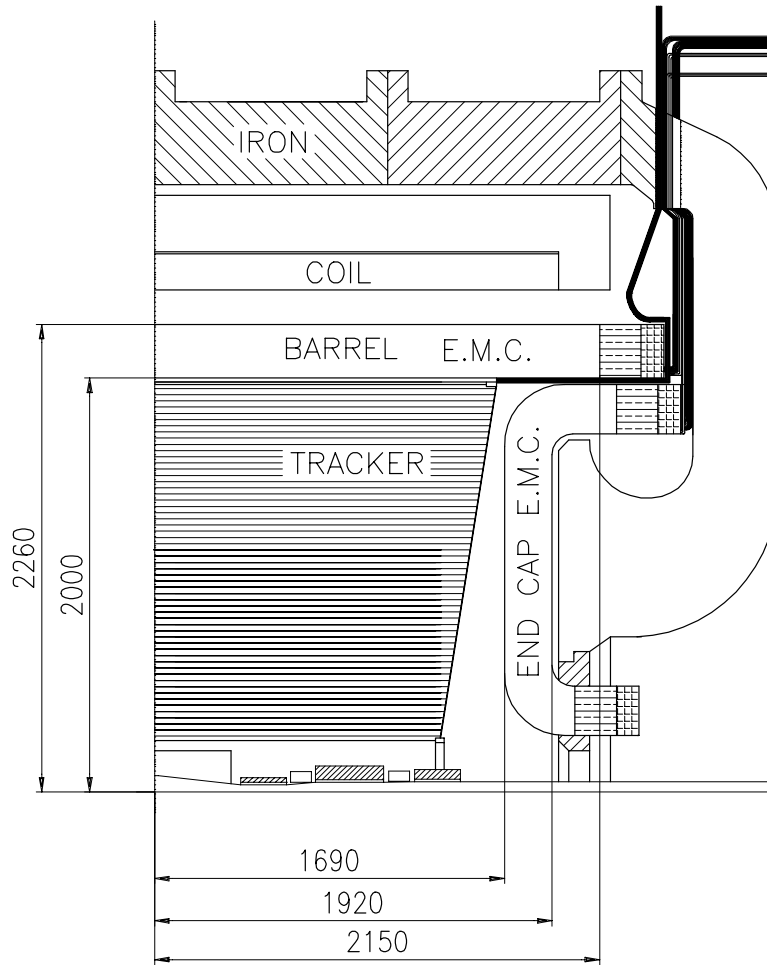

Fig. 19. Cross section of the KLOE detector at DAΦNE.

The detectors consists of a 2 m radius drift chamber, employing helium rather than

argon, to control multiple scattering at energies below 500 MeV and to minimize regeneration. The chambers has 13,000 W sense wires plus 39,000 Al field wires. The chamber is surrounded by a sampling electromagnetic calorimeter consisting of 0.5 mm Pb foils and 1 mm diameter scintillating fibers. The calorimeter resolution in energy is $\sigma(E)/E$=4.7% at 1 GeV and timing resolution is $\sigma(t)$=55 ps, also at 1 GeV.

At full DAΦNE luminosity, $\mathcal{L}$=10^{33} cm^{-2} s^{-1}, KLOE will collect almost 2000 K_S, K_L decays per second. Measurements of the leptonic decays mentioned in section 3.1 is possible with KLOE because of the large statistics and the tagged K_S beam unique to a ϕ–factory. The two neutral K mesons are produced in a pure C-odd quantum state. This implies that, to a very high level of accuracy, the final state is always $K_S K_L - K_L K_S$ or $K^0 \overline{K}^0 - \overline{K}^0 K^0$. Tagging of K_S, K_L, K^0, $\overline{K}^0$ is therefore possible. The produced K mesons are monochromatic, with $\beta \sim 0.2$. This allows measurement of the flight path of neutral K's by time of flight. A pure K_S beam of about 10^{10} per year is a unique possibility at DAΦNE at full luminosity. A very high K_S tagging efficiency, $\sim 75\%$, can be achieved in KLOE by detecting K_L interactions in the calorimeter, in addition to K_L decays in the tracking volume.

Finally because of the well defined quantum state, spectacular interference effects are observable,[54,55] allowing a totally different way of measuring $\Re(\epsilon'/\epsilon)$, in addition to the classical method of the double ratio $\mathcal{R}$.

10. Conclusions

Ultimately three independent measurements performed with very different techniques should be able to determine whether $\Re(\epsilon'/\epsilon) \neq 0$, as long as $\Re(\epsilon'/\epsilon) \sim$few$\times 10^{-4}$. Each experiment has additional by-products of interest in kaon physics. From KTEV and NA48, more precise values of ϕ_{+-} and $\Delta\phi$ will be obtained. KTEV expects to reach an error of 0.5° in the experimental determination of ϕ_f or ϕ_ρ using semileptonic decays. NA48 can measure ϕ_{+-} by oscillations of the decay rate behind their production targets, if $n(K^0) \neq n(\overline{K}^0)$. The strong correlation between Δm and ϕ_{+-} does not change. However all errors will be smaller. Likewise other parameters relevant to testing CPT invariance will be measured to higher accuracy, e.g. the charge asymmetry A_L in semileptonic decays. In this respect the uniqueness of DAΦNE is that of providing a tagged, pure K_S beam which allows KLOE to measure the charge asymmetry $\mathcal{A}_S^\ell$ in leptonic decays of K_S-mesons to an accuracy of a few$\times 10^{-4}$. The value of , $_L$ is becoming relevant in the analysis of the K^0–$\overline{K}^0$, K_S–K_L system. This is a measurement which KLOE can perform, improving the accuracy by $\sim \times 15$.

Concerning rare decays the number of events collected by KTEV and NA48 should increase by a factor of 100, corresponding to putting limits of few$\times 10^{-11}$ on unobserved decays and an improvement of a factor ten in the measurable rates. The statistics available at DAΦNE for K_L decays cannot compete with that of KTEV and NA48. However the tagged K_S beam will allow us to improve the measurements of rare K_S decays by three orders of magnitude.

One last open question is a better test of the $\Delta S = \Delta Q$ rule. This is not possible with

the K^0-$\overline{K}^0$ state produced at DAΦNE (without invoking CPT) nor with high energy K beams. K's tagged via strong interactions are required to test the rule. The copious K^+K^- production at DAΦNE provides tagged $K^+(K^-)$ beams which, via charge exchange, results in strangeness tagged $K^0(\overline{K}^0)$'s, much in the same way it is done in CPLEAR. CPLEAR has collected tens of million events, KLOE can do at least a factor of ten better.

11. Acknowledgements

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