



LABORATORI NAZIONALI DI FRASCATI

SIS – Pubblicazioni

LNF-97/003 (P)

24 Gennaio 1997

gr-**qc/9701065**

Consistency of Orthodox Gravity¹

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Abstract

A recent proposal for quantizing gravity is investigated for self consistency. The existence of a fixed-point all-order solution is found, corresponding to a consistent quantum gravity. A criterion to unify couplings is suggested, by invoking an application of our argument to more complex systems.

Submitted to *Physics Letters*

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1 Introduction

The unification of gravity and the standard model describing strong and electroweak interactions, thirty years after the formulation of the latter, has yet to be achieved and is a formidable task. Conceptual progress was made by introducing an effective theory for processes at low energy (typically less than the Planck mass [1]). However, a fundamental theory of gravity is still lacking. Our ignorance is parametrized by the renormalized counter terms of the corresponding effective Lagrangian. At very low energy, the theory is provided by general relativity. The natural question to address is then the following: what about quantum corrections? Donoghue tackled this issue [2] and we repeated his calculation of the first quantum correction to the Newtonian potential [3].

The validity of the abovementioned low-energy results holds, irrespectively of any definite proposal for the complete quantum theory of gravity. In [4, 5] an approach to quantizing gravity with a modified renormalization scheme was proposed, taking a massive scalar field interacting with gravity as an example. In this Letter we take up the issue of self consistency for the candidate theory to achieve the perturbative quantization of gravity. For the sake of the simplicity, we focus on the theory of self-interacting massive scalar fields coupled to quantum gravity.

We plan this Letter in the following way.

We start out in Section 2 by describing the essential features of the perturbative quantization of gravity. We also investigate the structure of the zero points of the beta functions, in the mass independent scheme, proving that a consistent quantum description of gravity exists to all loop orders. We then discuss in Section 3 the application of our treatment of quantum gravity and particle field theories, to include a nontrivial fixed point with a residual cosmological constant, as well the possibility of accommodating a gauge symmetry group in the matter sector. In Section 4 we briefly consider some relevant aspects of our analysis, concerning triviality. We finish this Letter in Section 5 with a summary and offering our conclusive remarks.

2 Orthodox Gravity

A recent proposal for the perturbative quantization of gravity [4, 5] suggests the quantization of an extended Lagrangian for gravity, such that the counter terms are all accommodated within the Lagrangian and renormalization is formally achieved. This extended starting Lagrangian is constrained by symmetry to be:

$$L_0 = \sqrt{-g_0} \left(\begin{aligned} & -2\Lambda_0 + R_0 + \frac{1}{2}p_0^2 + \frac{1}{2}m_0^2\phi_0^2 + \frac{1}{4!}\phi_0^4\lambda_0(\phi_0^2) + p_0^2\phi_0^2\kappa_0(\phi_0^2) + R_0\phi_0^2\gamma_0(\phi_0^2) \\ & + p_0^4a_0(p_0^2, \phi_0^2) + R_0p_0^2b_0(p_0^2, \phi_0^2) + R_0^2c_0(p_0^2, \phi_0^2) + R_{0\mu\nu}R_0^{\mu\nu}d_0(p_0^2, \phi_0^2) + \dots \end{aligned} \right) \quad (1)$$

(using units where $16\pi G = 1$, $c = 1$)

where p_0^2 is shorthand for $g_0^{\mu\nu}\partial_\mu\phi_0\partial_\nu\phi_0$ and not the independent variable of Hamiltonian mechanics. $\lambda_0, \kappa_0, \gamma_0, a_0, b_0, c_0, d_0 \dots$ are arbitrary analytic functions, and the second line carries all the higher derivative terms. Strictly this is formal in having neglected gauge fixing and the resulting presence of ghost particles. Quantum anomalies arise from a conflict between symmetries, where only one can be maintained [6]. For this reason no such trouble is anticipated here.

At this point some remarks are in order, to clarify why there is no trouble with a gravitational anomaly in our case. We are a bit cavalier on this point, but anomalies do not just turn up to break a symmetry (one can always fix things, so that the symmetry is restored). The trouble arises when there are two symmetries (say the conformal one also). Then one can restore any symmetry, *except* the fixing is different for each symmetry, and the two fixings tend to be in conflict. One could, for example, get rid of the conformal anomaly in massless gravity, but at the price of a worse anomaly. All this is detailed in Mann's review [6].

The price for having achieved 'formal' renormalization, is that the theory (with its infinite number of arbitrary renormalized parameters) is devoid of predictive content. The failure to quantize has been rephrased from a problem of non-renormalizability to one of non-predictability.

Despite this, after renormalization we are led to:

$$L = \sqrt{-g} \left(\begin{array}{l} -2\Lambda + R + \frac{1}{2}p^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\phi^4\lambda(\phi^2) + p^2\phi^2\kappa(\phi^2) + R\phi^2\gamma(\phi^2) \\ + p^4a(p^2, \phi^2) + Rp^2b(p^2, \phi^2) + R^2c(p^2, \phi^2) + R_{\mu\nu}R^{\mu\nu}d(p^2, \phi^2) + \dots \end{array} \right) \quad (2)$$

However, there remain physical criteria to pin down some of these arbitrary factors. Since in general the higher derivative terms lead to acausal classical behavior, their renormalized coefficient can be put down to zero on physical grounds. This still leaves the three arbitrary functions: $\lambda(\phi^2)$, $\kappa(\phi^2)$ and $\gamma(\phi^2)$, associated with the terms ϕ^4 , $p^2\phi^2$, and $R\phi^2$ respectively. The last may be abandoned on the grounds of defying the equivalence principle. To see this, begin by considering the first term of the Taylor expansion, namely $R\phi^2$; this has the form of a mass term, hence one would be able to make local measurements of mass to determine the curvature, and so contradict the equivalence principle (charged particles, with their non-local fields, have this term present with a fixed coefficient).⁴ The same line of reasoning applies to the remaining terms, $R\phi^4$, $R\phi^6$, ... etc.

This leaves us the two remaining infinite families of ambiguities with the terms $\phi^4\lambda(\phi^2)$ and $p^2\phi^2\kappa(\phi^2)$. In the limit of flat space in 3+1 dimensions this will reduce to a renormalized theory in the traditional sense if $\lambda(\phi^2) = \text{constant}$, and $\kappa(\phi^2) = 0$. So one is led to proposing that the physical parameters should be:

$$\begin{aligned} \Lambda = \kappa(\phi^2) = \gamma(\phi^2) &= 0 \\ a(p^2, \phi^2) = b(p^2, \phi^2) = c(p^2, \phi^2) = d(p^2, \phi^2) &= \dots = 0 \\ \lambda(\phi^2) = \lambda &= \text{scalar particle self coupling constant} \\ m &= \text{mass of the scalar particle} \end{aligned} \quad (3)$$

and so the renormalized theory of quantum gravity for a scalar field will have the Einstein form:

$$L = \sqrt{-g} \left(R + \frac{1}{2}p^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4 \right) \quad (4)$$

⁴Tests of the equivalence principle and bounds on the deviations from Newton's inverse square law constrain also the antigravity fields advocated in $N = 2, 8$ supergravity [7, 8]. For a discussion of possible violations of the equivalence principle, see also [9].

One might now worry about the renormalization group pulling the coupling constants around, and this is the very issue we wish to address in this work.

Since we are interested only that the zeroed couplings remain so, we shall name them as external couplings, in so much as they belong to terms outside the final renormalized Lagrangian in eq. (4). The finite number remaining will naturally take up the designation of internal couplings.

Now we note some essential properties of beta functions, defined as coefficients of the renormalization group equations. The standard definition for the beta function of the scalar self-coupling reads

$$\beta_\lambda \equiv \mu \frac{\partial \lambda}{\partial \mu} \tag{5}$$

with entirely analogous expressions holding for the beta functions of the remaining coupling constants in the theory.

When calculated perturbatively, they are in general homogeneous functions of the couplings, with no constant term. More critically, it is possible to adopt a mass independent scheme, and in this case the beta functions take the form:

$$\beta_a = f_a(\Lambda, \lambda, a, b, c, \dots) \quad \text{such that} \quad f_a(0, 0, 0, 0, 0, \dots) = 0 \tag{6}$$

Similarly for the beta functions of the remaining external couplings, b, c, d, \dots . In this way one can see that so long as one imposes the additional condition that $\lambda = 0$, the external couplings once put to zero will stay there. For a more complex theory than $\lambda\phi^4$ one would anticipate this to become a unification criterion between the internal couplings.

The beta functions are in general scheme dependent (here we adopted for the mass independent scheme for simplicity). However, fortunately, the zero points are scheme independent [10].

3 Discussion: towards an all-order solution?

Clearly, the cosmological constant scales in our renormalization scheme, hence one might worry that the condition $\lambda = \Lambda = 0$ needs to be explicitly maintained. For consistency of the argument one must prove that this be a fixed point of the renormalization group equations (and also investigate whether it has an attractive or repulsive nature).

We limited ourselves to setting the cosmological constant to zero even though it is *not* an external coupling. For the same reason that the *external* couplings stay to zero, so will the two *internal* couplings (i.e. Λ and λ). This is proved by the argument in the previous Section. The only difference in the treatment of Λ , with respect to the remaining couplings is that we did not *have* to set Λ to zero, rather we did so only for the sake of simplicity. However, once having done so, we can apply the same argument as for the remaining zeroed couplings, in order to keep its value to zero.

If one left both λ and Λ values open (as one should) then there might well be another solution beyond the trivial one $\lambda = \Lambda = 0$. This would illustrate the unification scheme we referred to.

Seeking a complete answer to the above question would also involve calculating and working loop order by loop order. The advantage of the *trivial* fixed point we proposed (with no real unification) is that the argument is valid to all orders. As a matter of fact, the idea in the present Letter has been to show Orthodox Gravity is consistent, before opening up to heavy fire by seeking a unification criterion. In other words, there does exist at least one easy quantum gravity, namely that with non self-interacting scalars and zero cosmological constant, and that it exists to all loop orders.

On the one hand, if one goes after the unification idea (letting Λ off zero might be enough), one must set out on loop calculations and open ones study up for the criticism that maybe the whole thing fails at some loop order. On the other hand, maybe there is an all loop argument for self consistency. In this case one should keep in mind our simple, alas perhaps restrictive solution. This is obviously a matter for future investigations. For our present purposes it is quite something that even a toy quantum gravity lives. Toy in the sense that the matter particles are scalar ones, although the gravity part is the full Einstein's Lagrangian in 4 dimensions.

For further developments, one may want to start by relaxing the constraint on the cosmological constant, or one may want to bring in a gauge symmetry. Besides obvious calculational difficulties, it is also not certain that there exists a non-trivial fixed point if one relaxes the condition on the cosmological constant, i.e. one may find that $\lambda = \Lambda = 0$ is the only way. This would not exclude a non trivial relation between the couplings, when we have more around. In this way, one might be forced at some point to bring in gauge groups.

We cannot see yet at this point how to maintain the all loop so far obtained. In this sense we feel the results we have written down so far might be worth keeping distinct (the unification scheme, if any exists, might only converge loop by loop to a limit). If one likes to see an example to illustrate the unification criterion in the presence of more couplings one might address two questions:

- what is the case for gauge couplings, e.g. for the Einstein-Maxwell system of Gravity plus QED?
- What happens when the symmetry is spontaneously broken, e.g. for $SO(N)$ with N ϕ -fields? What happens then to the beta functions and the fixed points?

We hope to come back to these points in the near future.

4 Triviality of $\lambda\phi^4$

Recalling the big discussion taking place in current literature on the triviality of $\lambda\phi^4$ in 4 dimensions, we are naturally led to address the following question: has our work any bearing on that issue? We are in 4 dimensions, and could just as well take the limit of zero gravity. We would then still be compelled to turn λ trivial.

When one turns off gravity the cosmological constant becomes irrelevant in the Lagrangian, so one would argue that the fact we set Λ to zero is not a special case. In other words the triviality argument is general.

A word of warning. Drawing any definite conclusion about the phenomenological consequences of these remarks would be completely premature, since the $\lambda\phi^4$ theory is not really physics. Addressing these issues will become viable, after including gauge fields in our analysis.

Nevertheless some rather sketchy conclusion is perhaps worth anticipating. We know that the standard model has an unstable scalar (symmetry breaking) sector - induced by the quantum corrections to the classical Higgs potential - and requires the introduction of a cutoff M . The beta functions, which have been computed to the two-loop level in [11], dictate the evolution of λ , as a function of M , according to the (coupled) differential equations

$$\frac{d\lambda(t)}{dt} = \beta_\lambda(\lambda, g_i) \quad \text{and} \quad \frac{dg_i^2(t)}{dt} = \beta_i(\lambda, g_i) \quad (7)$$

where $t = \ln(M/\mu)$ and g_i denote collectively the gauge couplings of the $SU(3) \times SU(2) \times U(1)$ symmetry group of the standard model, as well as the Yukawa coupling of the t-quark.

A recent review of the Higgs sector [12] indicates a very narrow range for the mass of the Higgs boson, i.e. $136 \text{ GeV} \leq m_H \leq 174 \text{ GeV}$, taking a 175 GeV pole mass for the t-quark and a value of the strong interaction coupling constant at the mass of the Z gauge boson $\alpha_S(m_Z) = 0.118$. This range is obtained assuming $0 \leq \lambda(M) \leq 10$ at the Planck scale. It is extremely suggestive that setting $\lambda(M)$ to zero would yield a Higgs mass of about 130–140 GeV (depending on the values set for the t-quark mass and $\alpha_S(m_Z)$ [13, 14]), i.e. a definitely predicted value. The latter, although far beyond the capabilities of LEP 200, could be discovered at LHC and is in quite tempting disagreement with expectations of the fashionable minimal supersymmetric standard model [14]. This is so, because our simple analysis assumes the standard model with a very high cutoff, and hence no new physics (particles) up to 10^{15} GeV.

The authors of [15] also assume the condition $\lambda(M) = 0$ at the Planck mass, and hence predict $m_H = 135 \pm 9 \text{ GeV}$. Their arguments, however, are different from those presented in this Letter. Indeed, in addition to requiring the validity of the standard model up to the Planck mass (as we also do) - implying that supersymmetry is not allowed -, they invoke the existence of a yet unspecified mechanism, in order to explain their principle on the coexistence of two degenerate phases for the Higgs potential of the standard model.

Having clarified the importance of the scale evolution of λ , one can ask: where to set it to zero? The answer is that for us it does not matter, since λ does not run, i.e. it is zero at *all* scales. This is part of the magic due to the coupling unification criterion we are suggesting.

In principle one must set λ to zero at a low scale, but since it stays zero, it will remain zero

also at the Planck scale, so we get a prediction on m_H as a pretty bonus. Given that we can release the choice of scale, the theory becomes *more* predictive. It is very devoid of tweaking parameters, and we consider this as a good feature of the idea of a consistent orthodox gravity. The nice thing about the non-running coupling constants is that they remove scale dependence. In other words, our only point is that one needs not choose a scale. In our opinion this is physics!

5 Conclusions

We can summarize our investigation by stressing that we end up with a finite theory, whose predictive power relies upon the fewness of its arbitrary constants. Irrespectively of the different assumptions needed, in order to set to zero many of the arbitrary factors originally present, we looked closely at the consequences of the beta functions of the couplings tuned to zero. In order to ensure that the beta functions of the zeroed couplings in a general theory vanish - and hence the couplings put to zero at low energy remain there and do not reappear around the Planck scale - we expect that certain couplings, besides the zeroed ones, be related. Thus, the fixed point of the renormalization group equation spells out a consistency condition that could be the basis of a unification scheme, whose implementation for the internal couplings might be realizeable only beyond the perturbative formulation. Despite the present lack of experimental input to guide us in developing our candidate for quantum gravity, and data to test the latter against, we showed that at least in our simple case the theory exists to all loop orders.

In conclusion, we believe that a finite well-behaved theory including gauge couplings related to internal symmetries deserves further investigation, in order to test the criterion of couplings unification we suggested in the present Letter. We stress that the non-vanishing internal couplings continue to run, i.e. our proposal does *not* imply losing the renormalization group.

We would like to finish with mentioning one last remark. In Section 3 we mentioned a potentially worrisome point, i.e. the stability of the zero point of the beta function (where we mention its attractive or repulsive nature). Our personal mind is that we really do not think of it as an issue. It is the pencil balancing on its tip. Our point of view is that we are talking about a purely mathematical concern, and there is no perturbation, and thus the zero point

stays so. Hence this work has a comforting strength to it, i.e. minimal assumptions, maximal predictivity.

Acknowledgement

We are pleased to thank G. Isidori for drawing ref. [15] to our attention.

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