Effective Gravity and OSp(N,4) Invariant Matter

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Abstract

We re-examine the $OSp(N,4)$ invariant interacting model of massless chiral and gauge superfields, whose superconformal invariance was instrumental, both in proving the all-order no-renormalization of the mass and chiral self-interaction lagrangians, and in determining the linear superfield renormalization needed. We show that the renormalization of the gravitational action modifies only the cosmological term, without affecting higher-order tensors. This could explain why the effect of the cosmological constant is shadowed by the effects of newtonian gravity.

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1 Introduction

The introduction of locally supersymmetric theories [1] was motivated by the wish for a unified description, encompassing the theories of elementary particles and gravity. The resulting supergravity is not renormalizable, but its large symmetry provides powerful constraints, such as the vanishing of the lepton anomalous magnetic moment, as required by the supersymmetry of the theory [2]. The finiteness of the fermionic (and therefore also the bosonic) contribution to it, which can be traced back to an effective chiral symmetry in the gravitino sector [3], has been checked in [4,3] and, more recently in [5], making use of supersymmetry preserving regularization schemes.

The quantum corrections to the low-energy limit of a theory coupling gravity to scalar fields have been computed [7]. The equivalence principle has been invoked, in order to reduce the terms with an arbitrary number of derivatives in the effective theory [8].[1]

$OSp(N,4)$ invariant models in the fixed four-dimensional background of anti-de Sitter space (AdS_4) occur as both ground state solutions of gauged extended supergravity theories (see [11] and references therein) and vacuum configurations for superstrings [12]. The $OSp(N,4)$ invariant generalization of the Wess-Zumino model [13] is the simplest one. We recall that the Wess-Zumino model with softly broken supersymmetry in de Sitter space plays a role in the Affleck-Dine mechanism [14] for baryogenesis, in contrast to the maximal symmetry of AdS_4 which grants the existence of global supersymmetry. This mechanism is effective for supersymmetric grand unified theories, as the quantum corrections do not affect the flat directions in the superpotential, owing to the no-renormalization theorem [15]. Very recently, the one-loop effective potential along a flat direction in this model has been calculated [16]. In [17] the one-loop effective action was computed for scalar-QED, taking into account the large-scale configurations that change the topology. Also, the question of the dynamics of a superstring propagating in AdS_4, with the $OSp(1,4)$ supersymmetry group, deserves further study, within a geometrical framework, especially in connection with the underlying algebraic structure of the W-algebra extension of two-dimensional conformal symmetry (see, e.g. [18-20]).

The renormalization procedure for $OSp(N,4)$ invariant theories breaks the naturality implied by the no-renormalization theorem and so allows *all* classically invariant counterterms to appear in the divergent structure of the quantum effective action [21-33]. This breakdown of the no-renormalization theorem in AdS_4 forces us to introduce a linear superfield in the effective action, for the purpose of renormalization. An important point to recall is that the corresponding modification of the classical potential does not induce the breaking of supersymmetry invariance [24,11]. On the other hand, the fact that the superspace integral that gets renormalized is mathematically of the same type, as the integrals of the quadratic and cubic superfield terms in the action, suggests that a similar violation of the theorem might occur at higher loops, for the mass and interaction lagrangians of the model.

Superfield techniques and a non-standard perturbative approach in the effects of the curvature allowed us [32,33], to fully address the questions formulated above. The important fact that the reflective boundary conditions do not play any role in the calculation of the counterterms [31,26] enabled us to implement a perturbative treatment of the effects of the background space. The superconformal invariance of the model is discussed in Refs.

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[1]This principle constrains the spin-1 and spin-0 partners of the graviton in the $N = 2,8$ supergravity multiplets [9,10].

[32,33], where the superfield formulation of the model with interacting chiral and real gauge superfields in AdS_4 is considered, following the line suggested in Ref. [34]. This perturbative analysis, although completely independent from the standard DeWitt-Schwinger expansion, allowed us

- to recover [32] the results of Refs. [21-24], for the one-loop renormalization of the chiral self-interacting model, and

- to prove [33] that there is no renormalization of the mass nor the cubic interaction action, to all orders in perturbation theory.

Hence the no-renormalization theorem holding in flat space-time, is also valid in AdS_4 background, except for the linear superfield renormalization, already present at the one-loop level. The proof was easily extended to the model with gauge interactions [33].

We organize the present work as follows. We begin in section 2 with recalling the superfield formulation of the AdS_4 Wess-Zumino model. In section 3 we consider the interacting model of chiral and gauge superfields. The superconformal invariance of the massless model allows us to implement an expansion in the curvature effects, in terms of the interaction vertices of the quantum model. In chapter 4 we present the renormalization of the gravitational action, based on the use of $OSp(N,4)$ superfield techniques, and propose some interpretation for the shadowing of the effect of the cosmological constant by the effects of newtonian gravity. We draw our conclusions in section 5.

2 The superfield formulation

We describe in this section a superspace approach [34,32] to the $OSp(N,4)$ invariant theory of a supergravity multiplet coupled to the scalar multiplet of the Wess-Zumino model. We recall that, in the covariant approach to supergravity, the covariant derivatives ∇_A can be determined in terms of the prepotential H^m and the density compensator Ψ for the superconformal transformations. Here the superspace coordinates are denoted as follows:

$$z^A = (x^a, \theta^\alpha, \overline{\theta}^{\dot{\alpha}}) \quad . \tag{2.1}$$

A convenient way of breaking the superconformal invariance is to introduce a covariantly chiral tensor-type compensator

$$\overline{\nabla}_{\dot{\alpha}}\Phi = 0 \quad . \tag{2.2}$$

The choice of a gauge for the Φ compensator fixes a relation between the Ψ and H^m superfields. Then, the supermultiplet is described in terms of H^m and a flat space-time chiral superfield ϕ (in the chiral representation) and the only gauge freedom left is that of super-Poincaré transformations.

The transformation rules for the superdeterminant E made out of the supervierbein E_A^M

$$E^{-1} \quad \to \quad E^{-1}exp(iK) \quad , \tag{2.3}$$

$$K \quad = \quad K^M i D_M + K_\alpha^\beta i M_\beta^\alpha + K_{\dot{\alpha}}^{\dot{\beta}} i \overline{M}_{\dot{\beta}}^{\dot{\alpha}} \tag{2.4}$$

and for the ϕ field

$$\phi^3 \to \phi^3 exp(i\Lambda_{ch}) \quad , \quad \Lambda_{ch} = \Lambda^m i\partial_m + \Lambda^\mu i D_\mu \tag{2.5}$$

2

allow us to write invariant actions for a real scalar superfield L_{gen}

$$S_1 = \int d^4x d^4\theta E^{-1} L_{gen} \qquad (2.6)$$

$$S_2 = \int d^4x d^2\theta \phi^3 L_{chiral} \quad . \qquad (2.7)$$

In particular, the supergravity action is obtained by setting

$$L_{gen} = -\frac{3}{\kappa^2} \quad . \qquad (2.8)$$

One can, of course, introduce also a cosmological term

$$L_{chiral} = \alpha \frac{1}{\kappa^2} \quad . \qquad (2.9)$$

In order to choose AdS_4 as a background space for the matter model, we set H to zero and introduce the background only through the compensator ϕ. Its equation of motion can be obtained from the action for supergravity with the cosmological term

$$S = -\frac{3}{\kappa^2} \int d^4x d^4\theta E^{-1} + (\alpha \frac{1}{\kappa^2} \int d^4x d^2\theta \phi^3 + h.c.) \quad . \qquad (2.10)$$

In the present case we have, after the choice of the gauge $\phi = 1$, $\Psi = \phi^{-1}\overline{\phi}^{1/2}$ and $E^{-1} = \overline{\phi}\phi$. The equation of motion for the ϕ field concides with that of the chiral superfield describing the massless Wess-Zumino model

$$\overline{D}^2\overline{\phi} = \alpha\phi^2 \quad . \qquad (2.11)$$

The solution to this equation which behaves regularly at infinity reads

$$\phi = \frac{1}{1 - \alpha\overline{\alpha}x^2/4} - \frac{\overline{\alpha}\theta^2}{(1 - \alpha\overline{\alpha}x^2/4)^2} \quad . \qquad (2.12)$$

Then, by applying this solution to the construction of invariant actions of the general and chiral type, we can formulate different supersymmetric matter models in the given background. We recall the expression of the covariant derivatives in terms of the ϕ field

$$\overline{\nabla}_{\dot\alpha} = \phi^{-1}\overline{\phi}^{1/2}\overline{D}_{\dot\alpha} \quad , \quad \nabla_\alpha = \overline{\phi}^{-1}\phi^{1/2}D_\alpha \quad . \qquad (2.13)$$

In the rest of this section we focus on the theory of a self-interacting chiral field η, i.e. the analogue in AdS_4 of the flat space-time Wess-Zumino model. Its action is readily obtained with the help of the two prescriptions given above

$$S(\eta,\overline{\eta}) = \int d^4x d^4\theta E^{-1}\overline{\eta}\eta + (\int d^4x d^2\theta\phi^3(m\frac{1}{2}\eta^2 + \lambda\frac{1}{6}\eta^3) + h.c.) \quad . \qquad (2.14)$$

This model possesses a partial superconformal invariance, which has been exploited [32], in order to treat perturbatively the effects of the background curvature, when carrying out the renormalization procedure that yields its quantum effective action. For this task, the functional integral approach proved useful. We define, as usual, the generating functional

$$Z(J) = \int \mathcal{D}\eta\mathcal{D}\overline{\eta}exp[S(\eta,\overline{\eta})]exp(\int d^4x d^2\theta\phi^3 J\eta + h.c.) \quad , \qquad (2.15)$$

but consider the mass term

$$\left(m\frac{1}{2}\int d^4x d^2\theta \phi^3 J\eta^2 + h.c.\right) \tag{2.16}$$

as an interaction term, when splitting the action into the free-field part and the interaction

$$S_0 = \int d^4x d^4\theta E^{-1}\bar{\eta}\eta \tag{2.17}$$

$$S_{int} = \left(m\frac{1}{2}\int d^4x d^2\theta \phi^3\eta^2 \lambda\frac{1}{6}\int d^4x d^2\theta \phi^3\eta^3 + h.c.\right) \quad . \tag{2.18}$$

The usefulness of this splitting comes from the fact that in the equivalent expression of the generating functional

$$Z(J) = exp[S_{int}(\frac{\delta}{\delta J}, \frac{\delta}{\delta \bar{J}})] \int \mathcal{D}\eta \mathcal{D}\bar{\eta}\, exp(S_0) exp(\int d^4x d^2\theta \phi^3 J\eta + h.c.) \tag{2.19}$$

all the dependence on the ϕ field in the free-field functional integral can be removed by carrying out a superconformal transformation, in accordance with the η, J canonical weights

$$\hat{\eta} = \phi\eta \quad , \quad \hat{J} = \phi^2 J \quad . \tag{2.20}$$

This points in the direction that the quantum model can be described in the most natural form in terms of the transformed fields, i.e. defining the effective action in terms of the hatted fields $\hat{\eta}$ and $\hat{\bar{\eta}}$. Hence we derive the Feynman rules from the generating functional

$$\begin{aligned}
\hat{Z}(\hat{J}) &= Z(J) = sdet\left(\frac{\delta\eta}{\delta\hat{\eta}}\right) sdet\left(\frac{\delta\bar{\eta}}{\delta\hat{\bar{\eta}}}\right) exp[m\frac{1}{2}\int d^4x d^2\theta \phi^7 (\frac{\delta}{\delta\hat{J}})^2 + h.c.] \\
&\times exp[(\lambda\frac{1}{6}\int d^4x d^2\theta \phi^9 (\frac{\delta}{\delta\hat{J}})^3 + h.c.] \\
&\times \int \mathcal{D}\hat{\eta}\mathcal{D}\hat{\bar{\eta}}\, exp(\int d^4x d^4\theta \hat{\eta}\hat{\bar{\eta}}) exp(\int d^4x d^2\theta \hat{J}\hat{\eta} + h.c.)
\end{aligned} \tag{2.21}$$

Notice that the Jacobian factors contain *all* ϕ dependence and can contribute to the vacuum-to-vacuum amplitudes, yielding a conformal anomaly in the model [32].

The calculation of the free-field functional integral can be carried out in complete analogy to the flat space-time limit [34], yielding the equations for the vacuum expectation values

$$\Box < T\hat{\eta}(x)\hat{\bar{\eta}}(x') > = i\bar{\hat{D}}^2 D^2 \delta^4(x-x')\delta^4(\theta-\theta') \quad , \quad \Box < T\hat{\eta}(x)\hat{\eta}(x') > = 0 \quad . \tag{2.22}$$

The task of solving these equations in the flat conformal projection space is not straightforward, as one must take the solutions of the homogenous problem in such a way, as to enforce the boundary conditions needed, in order to preserve supersymmetry for the scalar and spinor propagators in AdS_4 [31]. This task has been accomplished in [32], where the solutions of (2.22) are given, with the appropriate boundary conditions built in

$$< T\hat{\eta}(x')\hat{\bar{\eta}}(0) > = \frac{1}{4\pi^2}\bar{\hat{D}}^2 D^2 \delta^4(\theta-\theta')\frac{1}{(x')^2} \quad , \quad < T\hat{\eta}(x')\hat{\eta}(0) > = \frac{1}{16\pi^2}(|\alpha|^2 + 2|\alpha|^3\theta'\theta) \quad . \tag{2.23}$$

One can read from the generating functional also the contribution of the quadratic vertex, naively, as follows:

$$\frac{1}{2}m\phi^7 + h.c. \quad . \tag{2.24}$$

The naive contribution of the cubic vertex reads

$$\frac{1}{6}\lambda\phi^9 + h.c. \quad .$$ (2.25)

However, taking into account the covariant functional derivative [34,32]

$$\frac{\delta\hat{J}(z)}{\delta\hat{J}(z')} = \frac{\phi^2(z)}{\phi^2(z')}\frac{\delta J(z)}{\delta J(z')} = \frac{1}{\phi^3(z)}\overline{D}^2\delta^8(z-z') \quad ,$$ (2.26)

one effectively decreases the power of the ϕ field, so that in the calculation of any derivative of the generating functional there are no ϕ's at every cubic vertex (in $D=4$), whereas the quadratic vertex appears with a factor ϕ. This reflects the deviation of the theory from a superconformal one, owing to the introduction of a mass term. Finally, we notice that the usual flat space-time D-algebra remains intact and we can consider the Feynman rules, for our purposes, as equivalent to those for the flat Wess-Zumino model with the inclusion of the quadratic vertex

$$\frac{1}{2}m\phi + h.c. \quad .$$ (2.27)

A remarkable feature of the above rescaling, which effectively removes ϕ from the super-conformal invariant part of the superfield action (with the caveat of possible anomalous contributions [32]), is that it takes automatically into consideration the need to resort to some perturbative approach in the effects of the curvature of the background space, leading us naturally to introduce the above implicit expansion in the compensator ϕ.

3   Interacting chiral and real gauge superfields

The above superspace approach can be extended in a straightforward fashion to the case of interacting chiral and real gauge superfields. This system is described (in the gauge-chiral representation) by the action [34]

$$S(\eta, \overline{\eta}, V) = \int d^4x d^4\theta E^{-1}\overline{\eta}_j[exp(V)]^j{}_i\eta^i$$
$$+ [\int d^4x d^2\theta\phi^3 W^2 + \int d^4x d^2\theta\phi^3(m\frac{1}{2}\eta^2 + \lambda\frac{1}{6}\eta^3) + h.c.] \quad ,$$ (3.1)

with $V^i_j = V^A(T_A)^i{}_j$, and where $(T_A)^i{}_j$ is a matrix representation of the generators of the gauge group that leaves this action invariant. Comparing with (2.14) it is easy to see that the new terms are symmetric under a superconformal transformation. Therefore, we can complement the transformation (2.20) with the following one:

$$\hat{W}_\alpha = \phi^{3/2}W_\alpha \quad , \quad \hat{J}_V = \phi\overline{\phi}J_V \quad .$$ (3.2)

This reduces the $m=0$ part of the action (3.1) to its flat space-time form.

The definition of $\hat{W}$ in terms of the familiar derivatives in flat background reads

$$\hat{W}_\alpha = i\overline{D}^2 D_\alpha V \quad .$$ (3.3)

The covariantization of this expression in the Yang-Mills chiral representation

$$W_\alpha = i(\overline{\nabla}^2 + \alpha)exp(-V)\nabla_\alpha exp(V)$$ (3.4)

5

gives W_α in terms of the background covariant derivatives (2.13). In writing the transformation (3.2), the relation holding in the background chiral representation

$$\phi^3(\overline{\nabla}^2 + \alpha)f = \overline{D}^2\phi\overline{\phi}f \tag{3.5}$$

has to be taken into account.

In (3.2) J_V denotes the source in the term

$$\int d^4x d^4\theta \phi\overline{\phi}V J_V \quad . \tag{3.6}$$

After the superconformal transformation, the gauge-fixing procedure can be carried out, along the line of the flat background theory. The resulting gauge propagator reads (in the Fermi-Feynman gauge, with $\xi = 1$)

$$<VV>_0 = -\frac{1}{p^2}\delta^4(\theta - \theta') \quad . \tag{3.7}$$

In splitting the action we have now in the interaction part the additional term

$$\int d^4x d^4\theta \hat{\overline{\eta}}[exp(V) - 1]\hat{\eta} = \int d^4x d^4\theta \hat{\overline{\eta}} \sum_{m=1}^{\infty} \frac{1}{m!} V^m \hat{\eta} \quad . \tag{3.8}$$

The corresponding modifications introduced in the generating functional $\hat{Z}(\hat{J})$ involve the operator [33]

$$exp[\int d^4x d^4\theta \phi^3\overline{\phi}^3 \frac{\delta}{\delta\hat{\overline{J}}} \sum_{m=1}^{\infty} \frac{1}{m!}(\phi\overline{\phi}\frac{\delta}{\delta\hat{J}_V})^m \frac{\delta}{\delta\hat{J}}] \quad . \tag{3.9}$$

Recalling (2.26), together with the covariant functional derivative [34,33].[2]

$$\frac{\delta\hat{J}_V(z)}{\delta\hat{J}_V(z')} = \frac{\phi(z)\overline{\phi}(z)}{\phi(z')\overline{\phi}(z')} \frac{\delta J_V(z)}{\delta J_V(z')} = \frac{1}{\phi(z)\overline{\phi}(z)}\delta^8(z - z') \quad , \tag{3.10}$$

we conclude that there are no modifications of this vertex, with respect to the corresponding one in the flat background theory. It is worthwhile to notice that, as a consequence of superconformal invariance, the ghost propagators and vertices of the flat space-time theory remain intact in AdS_4. The conclusion is that, for our purposes, we can handle the quantum system of $N = 1$ super Yang-Mills coupled to matter scalar superfields in AdS_4 in a way similar to the corresponding theory in a flat background, with the only difference of the additional quadratic vertex (2.27) present among the Feynman rules.

4 The renormalization of the gravitational action

Herewith we describe our main result, in an attempt to explain the shadowing of the cosmological constant. We build upon our previous work [32,33] and carry out the renormalization of the gravitational action induced by $OSp(N,4)$ invariant matter multiplets in curved space. Here great care is needed, as every sign at each step is crucial.

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[2]Here we are correcting a minor typographical error in the intermediate passage of Eq. (21) of Ref. [33].
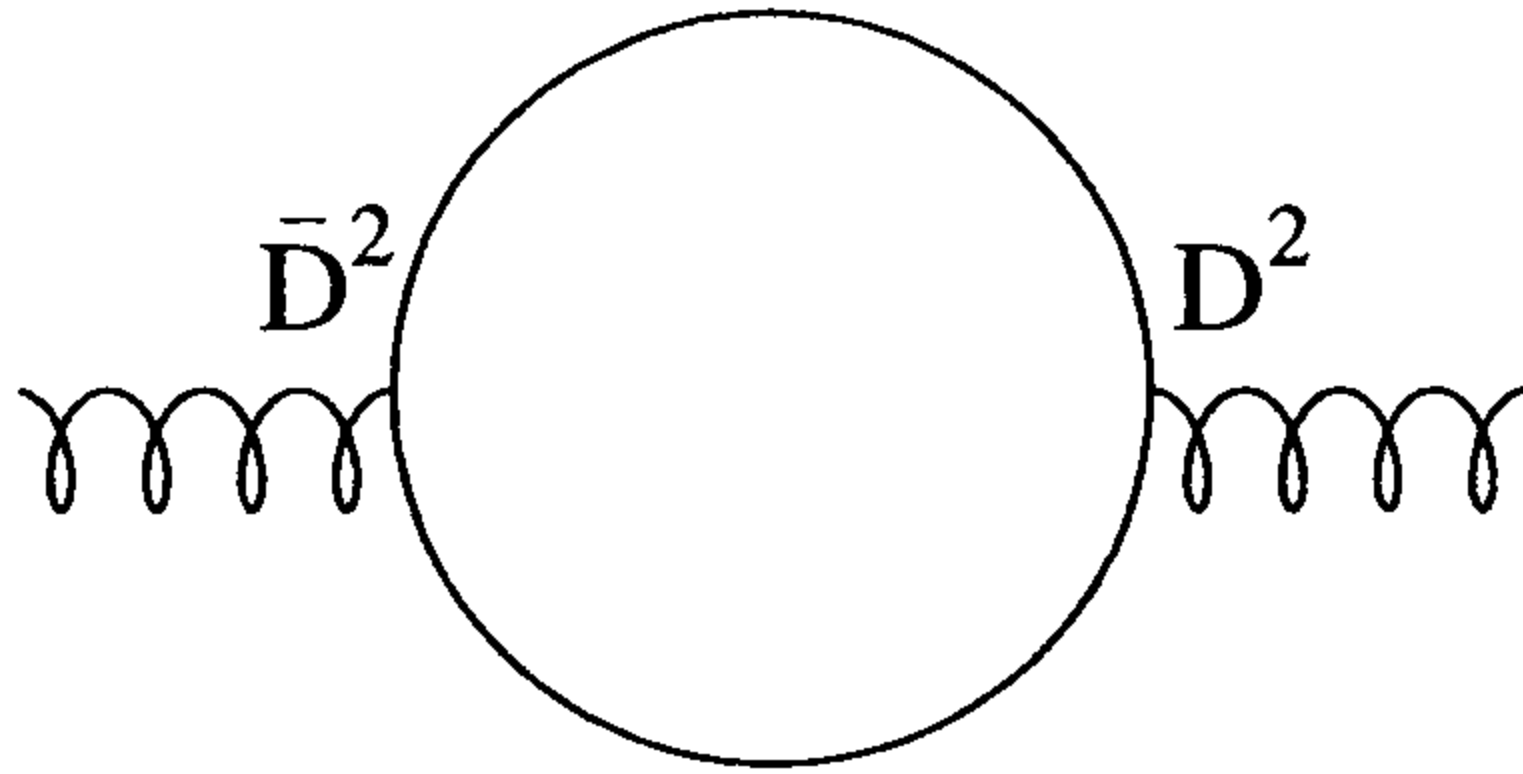
Figure 1: One-loop renormalization of gravitational actions from chiral matter (the $\hat{\eta}$ solid lines): the curly lines denote the compensator ϕ and its h.c. $\bar{\phi}$.

We start by discussing the renormalization of the gravitational action induced by a matter chiral superfield, through the presence of the Feynman diagram in figure 1. This yields the following divergent contribution:

$$am^2\frac{1}{\epsilon}\int d^4x d^4\theta \phi\bar{\phi} \quad , \tag{4.1}$$

with $a > 0$ in any case. One can doubt about the fact of interpreting this diagram as a renormalization of the pure supergravity action

$$-\frac{3}{\kappa^2}\int d^4x d^4\theta \phi\bar{\phi} \quad , \tag{4.2}$$

or the cosmological superspace action

$$\frac{\alpha}{\kappa^2}\int d^4x d^2\theta \phi^3 + h.c. \quad . \tag{4.3}$$

There are clear reasons to support this point of view, which goes in the direction of considering the above contribution as a renormalization of the second term, i.e. the cosmological action. In order to pursue this idea, we can translate the result in components, to read as follows:

$$am^2\alpha_R^2\frac{1}{\epsilon}\int d^4x\sqrt{-g} \quad . \tag{4.4}$$

Here, and throughout this section, we denote with subscript indices R, B the renormalized and the bare parameters of the action, respectively..

On the other hand, we know that the final expression for the gravitational actions has to be

$$S = -\frac{1}{\kappa^2}\int d^4x\sqrt{-g}R(r) + \frac{6}{\kappa^2}\int d^4x\sqrt{-g}\alpha_R^2 \quad , \tag{4.5}$$
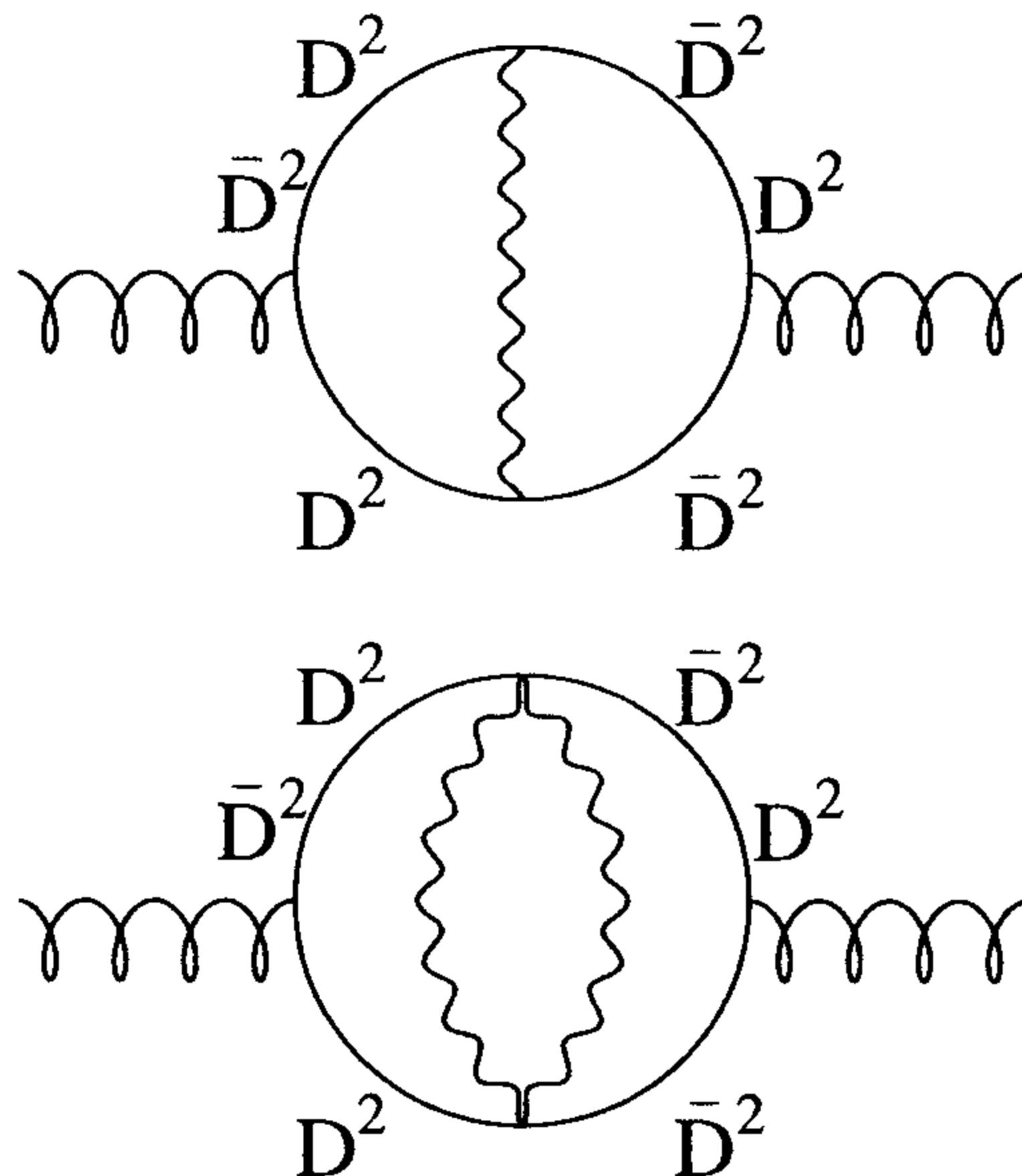
7

Figure 2: $O(g^2)$ two-loop corrections to the cosmological renormalization factor Z_α in (4.7); the wavy lines denote the gauge superfields.

where, let us say, R is restricted to metrics of the anti-de Sitter type, with arbitrary radius r. Then it is clear to us that the contribution (4.4) can be consistently interpreted as a renormalization of only the cosmological term, i.e. the second term on the r.h.s. of Eq. (4.5). This interpretation is also compatible with the fact that the renormalization of the higher order gravitational tensors (i.e. the contributions of order α_R^4) does not take place. In our opinion, this is not a coincidence, rather it means that any vacuum diagram can be seen as a renormalization of only one parameter in the gravitational action, namely α_R.

So, let us write then

$$am^2\frac{1}{\epsilon}\int d^4x d^4\theta \phi\bar{\phi} = am^2\frac{1}{\epsilon}\alpha_R\int d^4x d^2\theta \phi^3 \quad , \qquad (4.6)$$

where we make use of the equation of motion (2.11) for the chiral compensator superfield ϕ. Considering this contribution alone, we would have

$$\alpha_B = \alpha_R - am^2\alpha_R\frac{1}{\epsilon}\kappa^2 \equiv Z_\alpha\alpha_R \quad , \qquad (4.7)$$

but then it is obvious that this renormalization factor Z_α cannot lead to a running α_R, since the above m and κ parameters do not depend on the renormalization scale μ (the m parameter that appears here cannot be anything, other than the bare mass m_B).

In this way, we reach the conclusion that the relevant term in Z_α is the two-loop contribution to the vacuum, what forces us to introduce gauge interactions in the game. In fact, when introducing gauge interactions, of all the plethora of diagrams that one can imagine to the order g^2 in the gauge coupling constant, we believe there are only two that survive, after performing the D-algebra of the covariant derivatives, i.e. the diagrams in figure 2. We give, in the following, the computation of the first graph (at the top of figure 2), as the other one (at the bottom of figure 2) looks really frightening to compute, and anyhow it cannot
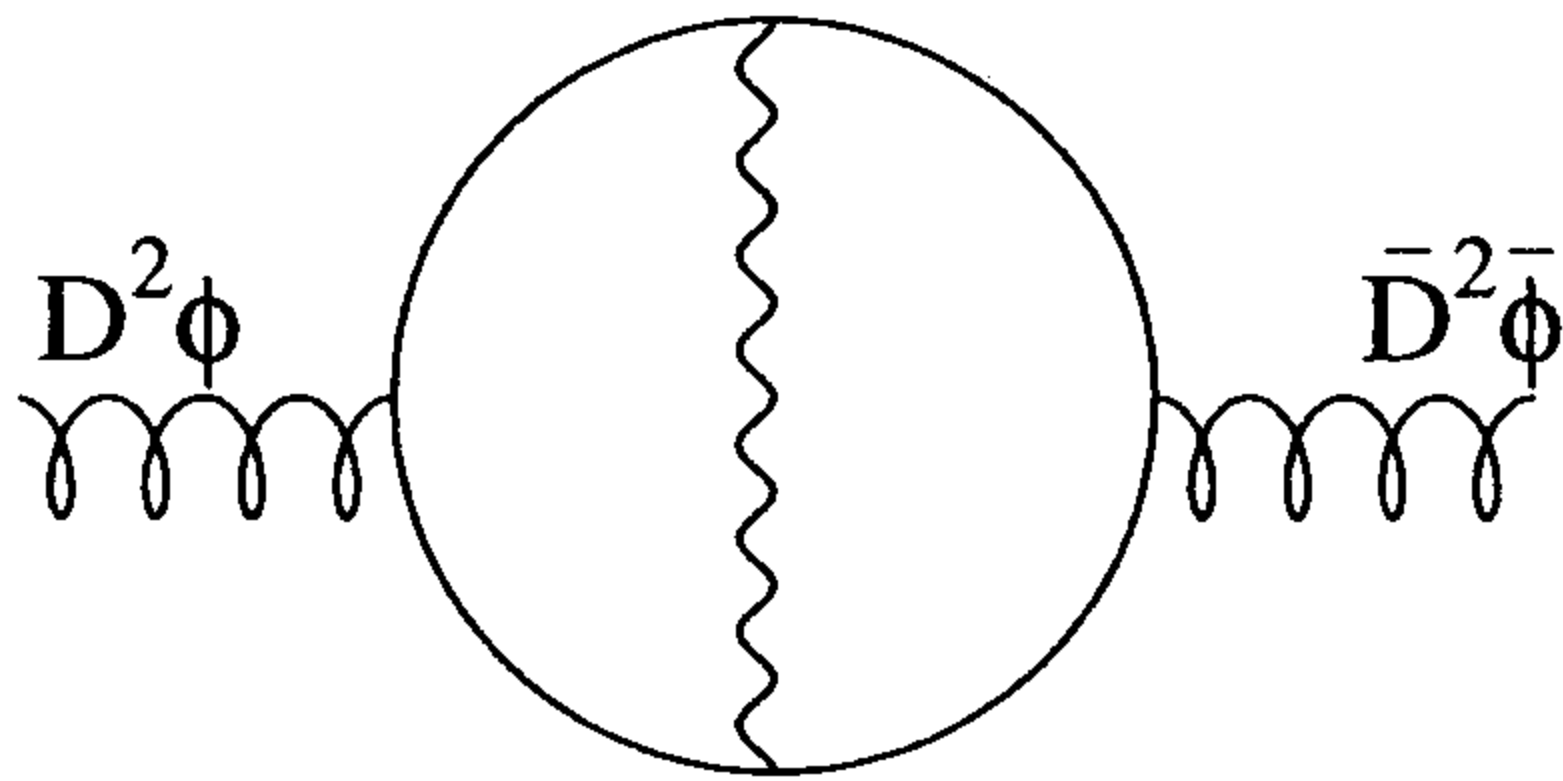
Figure 3: A finite contribution to the gravitational action produced, when integrating by parts the superspace covariant derivatives in the Feynman diagram at the top of figure 2, in order to obtain the graph of figure 4.

change the conclusion of this story. Using integration by parts for the superspace covariant derivatives and discarding a finite remainder given in figure 3, one can identify the divergent parts of the first graph in figure 2 and the diagram in figure 4.

The diagram in figure 4 yields the amplitude $\mathcal{V}$

$$\mathcal{V} = m^2 g^2 \int \frac{d^D k}{(2\pi)^D} \int \frac{d^D q}{(2\pi)^D} \frac{1}{k^2} \frac{1}{q^2} \frac{1}{(k-p)^2} \frac{1}{(k+q)^2} \quad , \tag{4.8}$$

where we work in dimension $D = 4 - \epsilon$. The usual Feynman parameter trick provides us with the first integral

$$\int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2} \frac{1}{(k+q)^2} = \frac{1}{(2\pi)^{4-\epsilon}} \pi^{2-\epsilon/2} \Gamma(\frac{\epsilon}{2}) \int_0^1 dx \frac{1}{[k^2 x(1-x)]^{\epsilon/2}}$$

$$= \frac{1}{(2\pi)^{4-\epsilon}} \pi^{2-\epsilon/2} \Gamma(\frac{\epsilon}{2}) B(1 - \frac{\epsilon}{2}, 1 - \frac{\epsilon}{2}) \frac{1}{(k^2)^{\epsilon/2}} \quad . \tag{4.9}$$

Plugging it into the amplitude $\mathcal{V}$ yields

$$\mathcal{V} = m^2 g^2 \frac{1}{(2\pi)^{4-\epsilon}} \pi^{2-\epsilon/2} \Gamma(\frac{\epsilon}{2}) B(1 - \frac{\epsilon}{2}, 1 - \frac{\epsilon}{2}) \mathcal{I} \quad , \tag{4.10}$$

where we define the momentum integral

$$\mathcal{I} \equiv \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2} \frac{1}{(k-p)^2} \frac{1}{(k^2)^{\epsilon/2}} \quad . \tag{4.11}$$
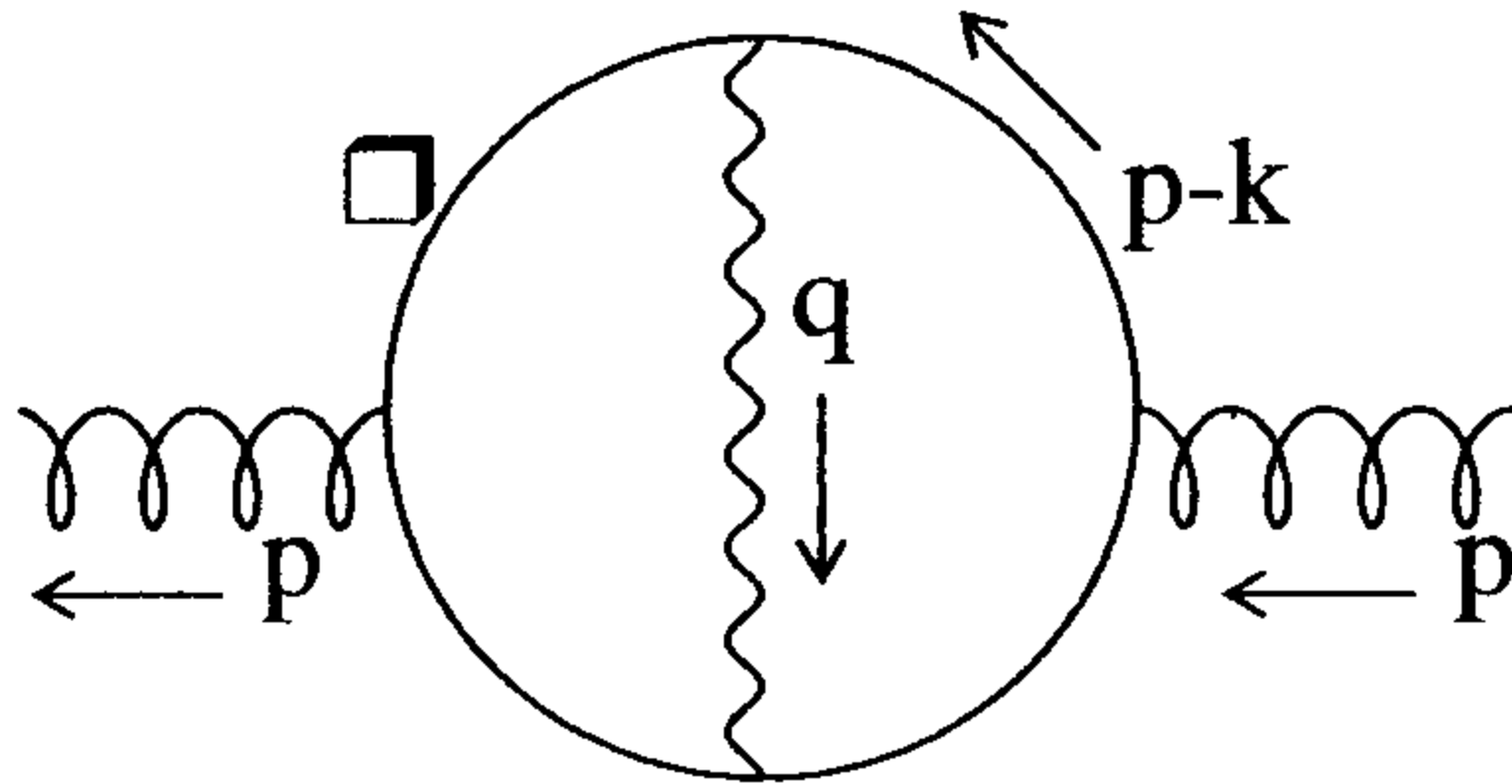
9

Figure 4: $O(g^2)$ two-loop Feynman diagram corresponding to the amplitude $\mathcal{V}$ in (4.8); the two independent momenta (loop variables) of the internal lines are explicitly indicated, together with the momenta of the external compensators.

The integral $\mathcal{I}$ can be evaluated as follows:

$$
\begin{aligned}
\mathcal{I} &= \left(1 + \frac{\epsilon}{2}\right) \int_0^1 dy \int \frac{d^D k}{(2\pi)^D} y^{\epsilon/2} \frac{1}{[k^2 + p^2 y(1-y)]^{2+\epsilon/2}} \\
&= \frac{1}{(2\pi)^{4-\epsilon}} \pi^{2-\epsilon/2} \frac{\Gamma(\epsilon)}{\Gamma(1+\epsilon/2)} \frac{1}{(p^2)^\epsilon} \int_0^1 dy \frac{1}{y^{\epsilon/2}(1-y)^\epsilon} \quad . \tag{4.12}
\end{aligned}
$$

Plugging this expression into the amplitude $\mathcal{V}$ we get

$$
\mathcal{V} = m^2 g^2 \cdot \frac{1}{(2\pi)^{8-2\epsilon}} \pi^{4-\epsilon} \Gamma(\tfrac{\epsilon}{2}) B\left(1 - \frac{\epsilon}{2}, 1 - \frac{\epsilon}{2}\right) \frac{\Gamma(\epsilon)}{\Gamma(1+\epsilon/2)} \frac{1}{(p^2)^\epsilon} B\left(1 - \frac{\epsilon}{2}, 1 - \epsilon\right) \quad . \tag{4.13}
$$

Recalling the following property of the function $B(1 - z, 1 - w)$:

$$
\begin{aligned}
B(1 - z, 1 - w) &\equiv \int_0^1 dy \frac{1}{y^z (1-y)^w} \\
&= \frac{\Gamma(1-z)\Gamma(1-w)}{\Gamma(2-z-w)} \quad , \tag{4.14}
\end{aligned}
$$

we finally get the result

$$
\mathcal{V} = m^2 g_R^2 \frac{1}{(2\pi)^{8-2\epsilon}} \pi^{4-\epsilon} \frac{2}{\epsilon} \Gamma(\epsilon) \frac{1}{1-\epsilon} [\Gamma(1 - \frac{\epsilon}{2})]^3 \frac{1}{\Gamma(2-3\epsilon/2)} \left(\frac{\mu^2}{p^2}\right)^\epsilon \quad . \tag{4.15}
$$

Here we introduced the expression of the renormalized gauge coupling constant g_R in terms of the renormalization scale

$$
g_R = g\mu^{-\epsilon} \quad . \tag{4.16}
$$

10

Two remarks are in order about this result. First of all, the leading divergence is of order ϵ^{-2}, what makes it relevant for the purpose of running α_R. Secondly, it contains subleading nonlocal divergencies of the form $\log(p/\mu)$. We will have to prove that they cancel with similar terms coming from the other graph in figure 2, for this whole thing to make sense. What is important about this other graph, apart from the cancellation of the nonlocal divergencies, is that the leading divergence comes with a positive sign (as it is the case for the amplitude $\mathcal{V}$ computed above). So let us assume that the whole contribution from figure 2 is of the form

$$bm^2 g_R^2 \alpha_R (\frac{1}{\epsilon})^2 \int d^4x d^2\theta \phi^3 \quad , \tag{4.17}$$

with $b > 0$, plus perhaps $1/\epsilon$ subleading divergencies that are not relevant, when studying the renormalization group equation for α_R to the order g_R^2. Then, we have that

$$Z_\alpha = 1 - am^2\kappa^2 \frac{1}{\epsilon} - bm^2\kappa^2 g_R^2 (\frac{1}{\epsilon})^2 \tag{4.18}$$

$$\frac{d\alpha_B}{d\mu} = 0 = Z_\alpha \frac{d\alpha_R}{d\mu} + \frac{dZ_\alpha}{d\mu}\alpha_R \tag{4.19}$$

$$\frac{d\alpha_R}{d\mu} = -\frac{1}{Z_\alpha}\frac{dZ_\alpha}{d\mu}\alpha_R$$

$$\approx -\frac{1}{1 - am^2\kappa^2/\epsilon}[-2bm^2\kappa^2 g_R \frac{dg_R}{d\mu}(\frac{1}{\epsilon})^2]\alpha_R + O(g_R^3)$$

$$= -\frac{1}{1 - am^2\kappa^2/\epsilon}(bm^2\kappa^2 g_R^2 \frac{1}{\mu}\frac{1}{\epsilon})\alpha_R + O(g_R^3) \quad . \tag{4.20}$$

In the limit $\epsilon \to 0$, we have then

$$\frac{d\alpha_R}{d\mu} = \frac{b}{a}g_R^2 \frac{1}{\mu}\alpha_R \quad , \tag{4.21}$$

so that one can easily guess the kind of theories, in which the effective cosmological constant goes to zero in the infrared, as a power of μ. We can then write that, if $g_R^2 = \text{constant}$

$$\alpha_R = \alpha_0 \left(\frac{\mu}{\mu_0}\right)^{bg_R^2/a} \quad . \tag{4.22}$$

The important thing, which we checked repeatedly, is that the exponent is *positive*, though it is more obscure, under which circumstances it could be bigger than one.

5   Conclusion

We cannot add much more to the above considerations, apart from the fact that, if the interpretation $\mu^2 \approx R$ in a gravitational measurement is plausible, then this could explain why the effect of the cosmological constant is always shadowed, no matter what the value of α_0 might be, by the effects of newtonian gravity.

The calculation of the stress-tensor anomaly in AdS_4 supersymmetry showed [42] that the choice of the vacuum, around which the model is quantized, does not affect the renormalization of the purely geometrical tensors in the effective action, nor the trace anomaly

11

induced by matter multiplets invariant under the supersymmetry group $OSp(N,4)$ in curved space. These quantities are independent also from the boundary conditions for the free-field propagators, as proven in Ref. [42].

Notice, as a last remark, that the conformal anomaly, as an integrability condition for the supersymmetric sigma models corresponding to superstring theories, was obtained in Refs. [43-47].

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