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Top Mesons

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Abstract

The possibility of formation for a bound state of a t quark and a lighter one is investigated using potential model predictions and heavy quark effective theory approach. Resulting estimates for the 1S-2S splitting of the energy levels are compared to the total top decay width Γ_t . As for the case of toponium, our conclusions show that the probability of formation for T-mesons is negligibly small due to the high top mass value.

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1 Introduction

It is conventional wisdom [1] that the top quark has no probability of meson formation of any kind because the t quark decays in a way that is too fast to allow for a single orbit of the bound state. First calculations based on a relatively light top quark [2] have shown that the observability of $t\bar{t}$ bound states would be possible for a narrow window on m_t values. Subsequent estimates [3, 4] have demonstrated that there is only a small probability for creation of a $t\bar{t}$ bound state for higher top masses. From these results it could be inferred that even the formation possibility for a meson made out of a single t quark and a lighter one is small. In this note we discuss quantitatively the last point in light of the present high values for top mass [5, 6]. For this purpose we shall make use of calculations from potential models in QCD in both nonrelativistic and relativistic theories. The choice of the potentials is driven by considerations on the proper QCD scale for this problem. We will also give some results taken from heavy quark effective theory, based on an estimate for the inertia parameter $\bar{\Lambda}$. The latter method is particularly suitable for mesons containing a very light quark.

These results will be compared to the decay probability of the top quark, whose value is determined by m_t , in order to give quantitative answers on the possibility of creation for such “superheavy” mesons.

The paper is organized as follows: we review the toponium case in Sect. 2. The potential model approach for the T-mesons, both nonrelativistic and relativistic, is presented in Sect. 3. The heavy quark effective theory discussion for these mesons is in Sect. 4. Sect. 5 is devoted to the discussion and conclusions.

2 Toponium bound states revisited

The recent discovery of the top quark by CDF [5] and DØ collaboration [6], and subsequent measurements [7] and [8] have given large values for the top mass:

$$m_t = 175.6 \pm 9.3 \text{ GeV (CDF)} ; m_t = 169 \pm 11 \text{ GeV (DØ)} \quad (1)$$

Previous top mass values of around 130 GeV already excluded the possibility of formation of toponium bound states [3, 4].

This fact can be understood by comparing the time period of the would-be bound state with the decay probability of the top quark. For t mass above the Wb threshold, the top decays into a real W and a b quark [9, 10]. The single quark decay width of the top quark, with one-loop QCD correction and neglecting terms of order $(m_b/m_t)^2$, is given by [11] :

$$\Gamma_t = \frac{m_t^3}{16\pi v^2} \left[1 - \left(\frac{M_W}{m_t} \right)^2 \right]^2 \left[1 + 2 \left(\frac{M_W}{m_t} \right)^2 \right] \left[1 - \frac{2}{3} \frac{\alpha_s}{\pi} \left(\frac{2\pi^2}{3} - \frac{5}{2} \right) \right] \quad (2)$$

The width values increase from 438 MeV (for $m_t = 120$ GeV) to 1.02 GeV ($m_t = 160$ GeV), and 2.22 GeV for $m_t = 200$ GeV. With such a large width, the lifetime of T -mesons and toponium are dominated by single quark decays.

For a bound state, the typical formation time of a hadron is characterized by a revolution time driven by strong interactions. Thus, no bound states exist, if the revolution time, $t_R = 2\pi r/v$, is larger than the lifetime of the rotating quarks, $\tau_{t\bar{t}} = 1/\Gamma_{t\bar{t}}$ [9].

The value for the width of the toponium system is two times the width of the single top quark, since each one could decay in an independent manner: $\Gamma_{t\bar{t}} = 2\Gamma_t$. To discuss the bound state, let us begin with the toponium case through a Coulombic two-body potential:

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r} \quad (3)$$

for which analytic solutions exist. Here should also be considered corrections from Higgs boson exchange Yukawa type forces [12, 13]. For a top quark with mass less than ≈ 200 GeV these corrections are small, with an attractive potential for the quark-antiquark singlet state, and they amount to no more than 10% of the Coulomb term.

We shall use the two-loop expression for α_s [14],

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \log[Q^2/\Lambda_{\overline{MS}}]} \left\{ 1 - \frac{2\beta_1 \log[\log[Q^2/\Lambda_{\overline{MS}}]]}{\beta_0^2 \log[Q^2/\Lambda_{\overline{MS}}]} \right\} \quad (4)$$

with $\beta_0 = 11 - \frac{2}{3} n_f$, $\beta_1 = 51 - \frac{19}{3} n_f$ evaluated at a fixed scale $Q^2 = 1/r_B^2$ [4], where r_B is the Bohr radius

$$r_B = \frac{3}{4\mu\alpha_s} \quad (5)$$

In figure (1) we show the behavior of the Bohr radius as a function of the quark mass for a quarkonia bound state under the effect of a Coulombic interaction. The energy is given by

$$E_n = -\frac{8}{9} \frac{\mu\alpha_s^2}{n^2} \quad (6)$$

where μ is the reduced mass.

To estimate t_R , we shall make use of the virial theorem, which, for the Coulombic potential states that $\langle T \rangle = -\frac{1}{2}\langle V \rangle$ (T and V are respectively the kinetic and potential energies). From the energy expression (6), we have $\langle v^2 \rangle = 8/9 \alpha_s^2$, which leads to $t_R = \sqrt{2}\pi 9/(4\alpha_s^2 m_t)$. As $\Gamma_{t\bar{t}} \approx 2cm_t^3$ (c is a constant), this existence criterion allows for toponium formation if $\tau_{t\bar{t}} > t_R$, namely for $m_t < \sqrt{\sqrt{2}/(9\pi)} \alpha_s/\sqrt{c}$.

Another slightly different criterion [4] states that the formation of a hadron can occur only if the level splitting between the lowest lying levels of the bound state, which depends upon the strength of the strong force between the quarks and their relative distance [1], is larger than the natural width of the state. In this case, we should have a bound state if $\Delta E_{2S-1S} \geq \Gamma_{t\bar{t}}$. From (6) we read the splitting of the levels, $\Delta E_{2S-1S} = 4/3m_t\alpha_s^2$. In this case, the estimate for toponium existence is for values of top quark mass such that $m_t < 1/\sqrt{6} \alpha_s/\sqrt{c}$.

A comparison with the previous method shows that the mass bound is larger for a factor $\sqrt{3\pi}/(2\sqrt{2})$, that is approximately two times larger, and we conclude that the energy level splitting criterion is less stringent than the comparison of the revolution time. We decide therefore to employ the looser condition: if the formation is prohibited by the ΔE_{2S-1S} criterion, then it is certainly prohibited also from the other method.

In figure (2), we show a comparison of the energy splitting and the toponium width $\Gamma_{t\bar{t}}$, as a function of the top mass. We shall use the value of $\Lambda_{\overline{MS}}$ for which $\alpha_s(M_Z) = 0.118$ [15], and we use for α_s the scale (5). From this figure, it is possible to see that for most recent top masses and the average value for the top mass, $m_t = 172.9 \pm 7.0 \text{ GeV}$, toponium bound state formation is rather unlikely.

3 T–Mesons from potential models

For the T–mesons, the less stringent criterion of level splittings will be used. Notice that from [4] the comparison of the Coulombic potential results with some other models gave substantially similar results, thus confirming the validity of this approach.

The problem for the T–meson, unlike toponium, is difficult due to the identification of the proper QCD scale for this problem in the Coulombic potential. The interquark force cannot clearly be set simply at the top mass scale, but it has to include both the information of the top and the light quark mass scale. In order to achieve it, we use for the scale the inverse of the Bohr radius (5) calculated in terms of the reduced mass of the T–meson, which in turn, because of the heavy top, is approximately the light quark mass. In addition, we should also include calculations using other potentials, such as Martin’s [16]

$$V(R) = -8.064 + 6.8678 r^{0.1} \quad (7)$$

(the units are in GeV), and the model of Grant, Rosner and Rynes [17]:

$$V(r) = \frac{\lambda}{\alpha} (r^\alpha - 1) + c \quad (8)$$

These have the property of being independent of QCD coupling α_s , an important consideration since the energy scale considered is low, of the order of few fractions of GeV . For quark masses we use the constituent one, whose approximate values are $m_b = 5.0$, $m_c = 1.5$, $m_s = 0.5$, $m_u = m_d = 0.3$ (in GeV). In table (1), we present a table of energy splitting values, for the bound states from the potential models. For sake of comparison we also include the prediction of the Coulombic model and α_s values. As for toponium, we shall use $\Lambda_{\overline{MS}}$ such that $\alpha_s(M_Z) = 0.118 GeV$ [15], and the scale from (5).

Energy splitting values ΔE for potential models (GeV)

<i>quark</i>	<i>Martin</i>	<i>Rosner</i>	<i>Coulomb</i>	α_s
<i>b</i>	0.56	0.65	0.29	0.30
<i>c</i>	0.60	0.61	0.20	0.45
<i>s</i>	0.31	0.41	0.20	0.78
<i>u, d</i>	0.08	0.02	0.24	1.09

Table 1: Values of ΔE from different potential models (PM). We show ΔE as a function of the light quark mass for the potentials given in formulæ(7), (8) and (3).

We see that ΔE increases with the mass of the light quark, except for the Coulombic case, whose high α_s values for light quarks make perturbative calculations unreliable.

In the following, we investigate the bound state system by means of a model that includes relativistic corrections. The t quark inside the $t\bar{q}$ system moves nonrelativistically ($v^2/c^2 \sim 0.01$ for $m_t \approx 173$ GeV). In order to show whether the lighter quarks need a relativistic treatment we employ the Salpeter equation:

$$\left[\sqrt{-\nabla^2 + m_t^2} + \sqrt{-\nabla^2 + m_q^2} + V(r) \right] \psi = E\psi \quad (9)$$

with potentials from the nonrelativistic models. Instead of dealing with the mathematical difficulties of finding the eigenvalues of (9) because of the square root operator, we shall apply the Rayleigh–Ritz variational method with suitable trial functions. This procedure has already been successfully applied for the B and D meson decay constants [18]. By means of a Fourier transform on (9), we have

$$\left[\sqrt{p^2 + m_t^2} + \sqrt{p^2 + m_q^2} + V(r) \right] \psi = E\psi \quad (10)$$

Writing explicitly the dependence of ψ upon an extremum parameter ξ , we find that the energy of the state is given by minimizing the expectation value of H in (9)

$$\langle H \rangle = \langle \psi(\xi) | H | \psi(\xi) \rangle = E(\xi) ; \frac{dE(\xi)}{d\xi} = 0 \text{ for } \xi = \bar{\xi} \quad (11)$$

The energy levels depend on the number of nodes of the trial function: no nodes for the ground state, and one node for the first excited level. We have performed the calculation for two kinds of trial wavefunctions: hydrogen-type wavefunction, coming from a Coulombic potential, and Gaussian wavefunction, from the harmonic oscillator, thus mimicking the short and the long range behavior of the interquark force respectively.

For the 1S hydrogen-like function, we choose

$$\psi(r) = \frac{1}{\sqrt{4\pi}} \frac{2}{a^{3/2}} e^{-r/a} \quad (12)$$

and its Fourier transform, which reads:

$$\hat{\psi}(p) = \frac{2\sqrt{2}}{\pi} \frac{a^{3/2}}{(1+a^2p^2)^2} \quad (13)$$

a being the variational parameter. The 2S function is:

$$\psi(r) = \frac{1}{\sqrt{8\pi}} a^{-3/2} \left(1 - \frac{r}{2a}\right) e^{-r/2a} \quad (14)$$

and

$$\hat{\psi}(p) = \frac{16}{\pi} a^{3/2} \frac{(4a^2p^2 - 1)}{(1+a^2p^2)^2} \quad (15)$$

respectively. For the Gaussian wavefunction, the 1S is:

$$\psi(r) = \left(\frac{\mu}{\sqrt{\pi}}\right)^{3/2} e^{-\mu^2 r^2/2} \quad (16)$$

and

$$\hat{\psi}(p) = \frac{1}{(\sqrt{\pi}\mu)^{3/2}} e^{-p^2/2\mu^2} \quad (17)$$

The 2S function is given by:

$$\psi(r) = \sqrt{\frac{1}{(5\sqrt{\pi} - 8)\pi}} \mu^{3/2} (1 - r\mu) e^{-\mu^2 r^2/2} \quad (18)$$

and

$$\hat{\psi}(p) = \left[\frac{2}{\sqrt{2\pi}} - L_{1/2}^{1/2} \left(\frac{p^2}{2\mu^2} \right) \right] \sqrt{\frac{1}{(5\sqrt{\pi} - 8)\pi}} \mu^{-3/2} e^{-p^2/2\mu^2} \quad (19)$$

where μ is a different minimization parameter from the previous one. $L_n^a(x)$ is the generalized Laguerre polynomial, obeying the differential equation $xy'' + (a+1-x)y' + ny = 0$ and the orthogonality relation $\int_0^\infty L_m^a(x)L_n^a(x)x^a e^{-x} dx = 0$ for $m \neq n$ (for $a = 0$, we retrieve the usual Laguerre polynomials).

Technically, one splits $H = T + V$ and then computes the average

$$E(\xi) = \langle \hat{\psi}(p) | T(p) | \hat{\psi}(p) \rangle + \langle \psi(r) | V(r) | \psi(r) \rangle =$$

$$4\pi \int_0^\infty dp p^2 \hat{\psi}(p) \left[\sqrt{p^2 + m_t^2} + \sqrt{p^2 + m_q^2} \right] \hat{\psi}(p) + 4\pi \int_0^\infty dr r^2 \psi(r) V(r) \psi(r) \quad (20)$$

Below, we present a comparative table with results from both the potential models and two kinds of wavefunctions.

ΔE for potential and relativistic models (GeV)

quark	PM		RPM, Coulomb		RPM, Gauss	
	Martin	Rosner	Martin	Rosner	Martin	Rosner
<i>b</i>	0.56	0.65	0.47	0.57	0.53	0.61
<i>c</i>	0.60	0.61	0.45	0.39	0.55	0.35
<i>s</i>	0.31	0.41	0.41	0.09	0.44	0.00
<i>u, d</i>	0.08	0.02	0.38	0.01	0.34	0.00

Table 2: Comparison of ΔE results from potential models (PM) and relativistic potential models (RPM) for potentials (7) and (8). “Coulomb” and “Gauss” labels indicate the two different trial wavefunctions.

As we should expect, major differences between the nonrelativistic and relativistic potential models arise for the lighter (and hence faster) quarks. Also, the Martin potential is less sensitive to mass changes. We also notice that there are no significant differences between different kinds of trial wavefunctions; confirming thus the reliability of this approach.

4 Heavy Quark Theory approach

For performing these calculations, we shall now use a different method borrowed from heavy quark effective theory (HQET). This has the advantage of giving results at a scale (u and d quarks) for which some potential models predictions may not be reliable enough. The Coulombic model is one of these, because α_s is of order 1 in this case.

In this model, a meson (hadron) containing a single heavy quark ($m_Q \gg \Lambda_{\overline{MS}}$) is considered. The heavy quark's momentum can be written as

$$p_Q = m_Q \cdot v + k \quad (21)$$

k is the “residual” momentum, which measures the degree to which the quark is off-shell [19]; v is the velocity satisfying $v^2 = 1$. The quark Q exchanges only small momenta with the rest of the hadron, so it is essentially on shell, $p_Q^2 = m_Q^2$. Q behaves like a static electric and chromomagnetic field source. The properties of the light degrees of freedom do not depend upon the flavor and mass of the heavy field. Since the top quark is very heavy, it plays the role of the heavy quark and we could apply HQET to give some information on the energy of the bound state.

The mass M of the meson is here expressed as [20]

$$M = m_Q + \bar{\Lambda} + O\left(\frac{1}{m_Q}\right) \quad (22)$$

where $\bar{\Lambda}$ is a positive contribution to M . The “inertia” parameter $\bar{\Lambda}$ has no dependence on the heavy degrees of freedom:

$$\bar{\Lambda} = \bar{\Lambda}_q \equiv \lim_{m_Q \rightarrow \infty} (M - m_Q) \quad (23)$$

On the other hand, from the potential models (PM) it is possible to write, for the meson mass,

$$M = m_Q + m_q - E_b \quad (24)$$

where m_q is the mass of the light quark, $E_b (> 0)$ is the binding energy [4]. Comparing (22) and (24), we arrive at the identification

$$\bar{\Lambda}_q = m_q - E_b \quad (25)$$

neglecting terms of order $O(1/m_Q^2)$, an operation valid for $m_t \approx 173 \text{ GeV}$. We find therefore that the quantity $\bar{\Lambda}_q$ gives substantially the energy of the light degrees of freedom². In order to compute ΔE , we need some estimates for $\bar{\Lambda}_q$.

A rigorous lower bound on the value of $\bar{\Lambda}_q$ has been derived [22], and leads to the values (in GeV)

$$\bar{\Lambda}_q(Q\bar{u}) \geq 0.057, \quad \bar{\Lambda}_q(Q\bar{d}) \geq 0.076, \quad \bar{\Lambda}_q(Q\bar{s}) \geq 0.343 \quad (26)$$

for a meson formed by a heavy quark Q and an u, d and s quark respectively. This lower bound has been obtained using the Euclidean path integral formulation of QCD. The Cauchy-Schwarz inequality has been used to derive inequalities among Euclidean correlation functions, obtaining $m(\bar{Q}\Gamma q) - m_q \geq 1/2 m'(\bar{q}i\gamma_5 q)$. The first term of the LHS is the heavy meson mass, while the RHS is the mass of the meson created by the light degrees of freedom, neglecting annihilation diagrams.

Equation (22) gives us the binding energy of the lowest state, while we need the difference in energy. From the above

$$m_q - E_b = \bar{\Lambda}_q \geq \gamma_q \quad (27)$$

where γ_q is the lower bound on $\bar{\Lambda}_q$. Moreover, we have $E_b \geq \Delta E_{2S-1S}$, and we obtain from the lower bound on $\bar{\Lambda}_q$, an upper bound for the quantity ΔE_{2S-1S} :

$$\Delta E_{2S-1S} \leq m_q - \gamma_q \quad (28)$$

From equation (24) and constituent mass values for the light quarks, we obtain the following results:

$$\Delta E_{2S-1S}(t\bar{u}) \leq 0.243, \quad \Delta E_{2S-1S}(t\bar{d}) \leq 0.224, \quad \Delta E_{2S-1S}(t\bar{s}) \leq 0.257 \quad (29)$$

²A similar equivalence has been obtained in [21].

with $m_u = m_d = 0.3 \text{ GeV}$; $m_s = 0.5 \text{ GeV}$.

The values in equation (29) have been derived using chiral perturbation theory, and such a method is not applicable for heavier quarks. There is still the bound $\bar{\Lambda}_q \geq \frac{1}{2} m'(\bar{q}i\gamma_5q)$ [22] , and therefore $\bar{\Lambda}_q \geq 0$. With this less stringent bound and using the aforementioned values, we have (in GeV):

$$\Delta E_{2S-1S}(t\bar{c}) \leq 1.5 , \Delta E_{2S-1S}(t\bar{b}) \leq 5.0 \quad (30)$$

assuming $m_c = 1.5 \text{ GeV}$, $m_b = 5.0 \text{ GeV}$. We notice that the estimate of the HQET for the ΔE_{2S-1S} for the heavier quarks is rather large due to a poor estimate on $\bar{\Lambda}_q$, yet it is consistent with the potential models.

5 Discussion and conclusions

In table (3), we present a summary of the results which have to be compared to the width of the T-meson. If $\Delta E_{2S-1S} \geq \Gamma_{t\bar{q}}$, then there is an opportunity of formation for the bound state. Since the quarks could decay in an independent manner from each other, one has $\Gamma_{t\bar{q}} = \Gamma_t + \Gamma_q$. We know however that $\Gamma_q \approx m_q^5$ for a light quark, while $\Gamma_t \approx m_t^3$, leading to $\Gamma_t \gg \Gamma_q$, thus $\Gamma_{t\bar{q}}$ is essentially given by the top width.

Summary of ΔE results (GeV)

quark	PM	RPM	HQET
b	0.29 – 0.65	0.47 – 0.61	0.00 – 5.00
c	0.20 – 0.61	0.35 – 0.45	0.00 – 1.50
s	0.20 – 0.41	0.00 – 0.44	0.00 – 0.26
u, d	0.02 – 0.24	0.00 – 0.38	0.00 – 0.24

Table 3: Summary results of ΔE ranges from potential models (PM), relativistic models (RPM) and heavy quark effective theory estimates (HQET).

The tables (1), (2) and (3) have to be compared with the width values

of the top. Considering the average value $m_t = 172.9 \pm 7.0 \text{ GeV}$ for the top mass, we have from (2) (in GeV):

$$\Gamma_t = 1.17 (m_t = 165.9), \Gamma_t = 1.35 (m_t = 172.9), \Gamma_t = 1.55 (m_t = 179.9) \quad (31)$$

Figure (3) shows the window in energy ranges with respect to the decay width of the T -meson, which is drawn in full line. The dashed horizontal lines represent the indetermination due to the error on the top mass.

We see that the values for Γ_t are larger than the one predicted from both the PM and the RPM. It must be noticed that some of the values from the HQET, the one for the heavier quarks (see table (3)), seem to allow for T -meson formations, but it has to be stressed that the estimates for the c and b quarks are rather poor, since at the present time a more precise estimate on $\bar{\Lambda}_{c,b}$ is lacking.

From a combination of these calculational techniques we may conclude that there is little evidence of possibility of formation for bound states of t quark and lighter quarks.

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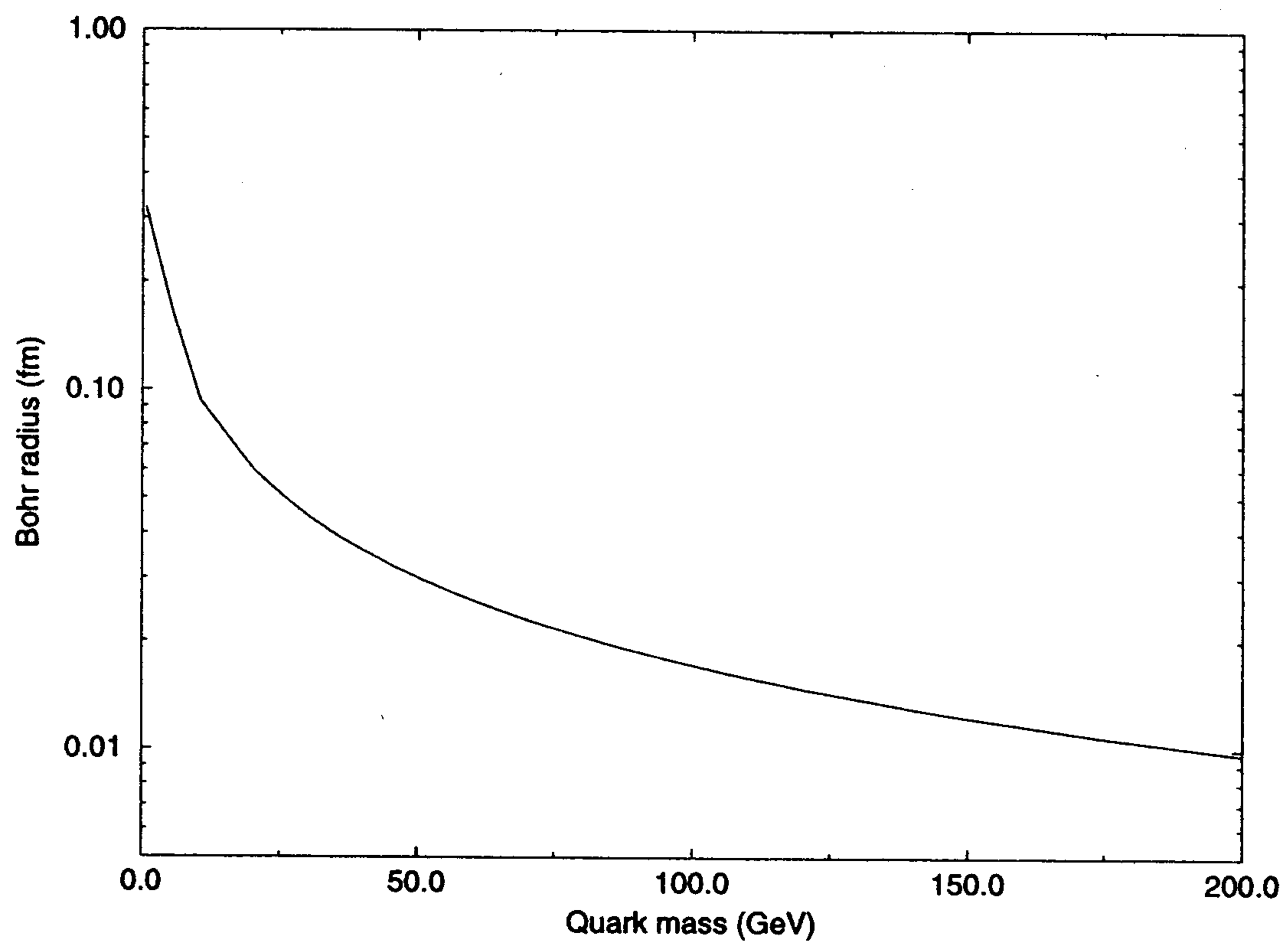


Figure 1: *Bohr radius versus quark mass*

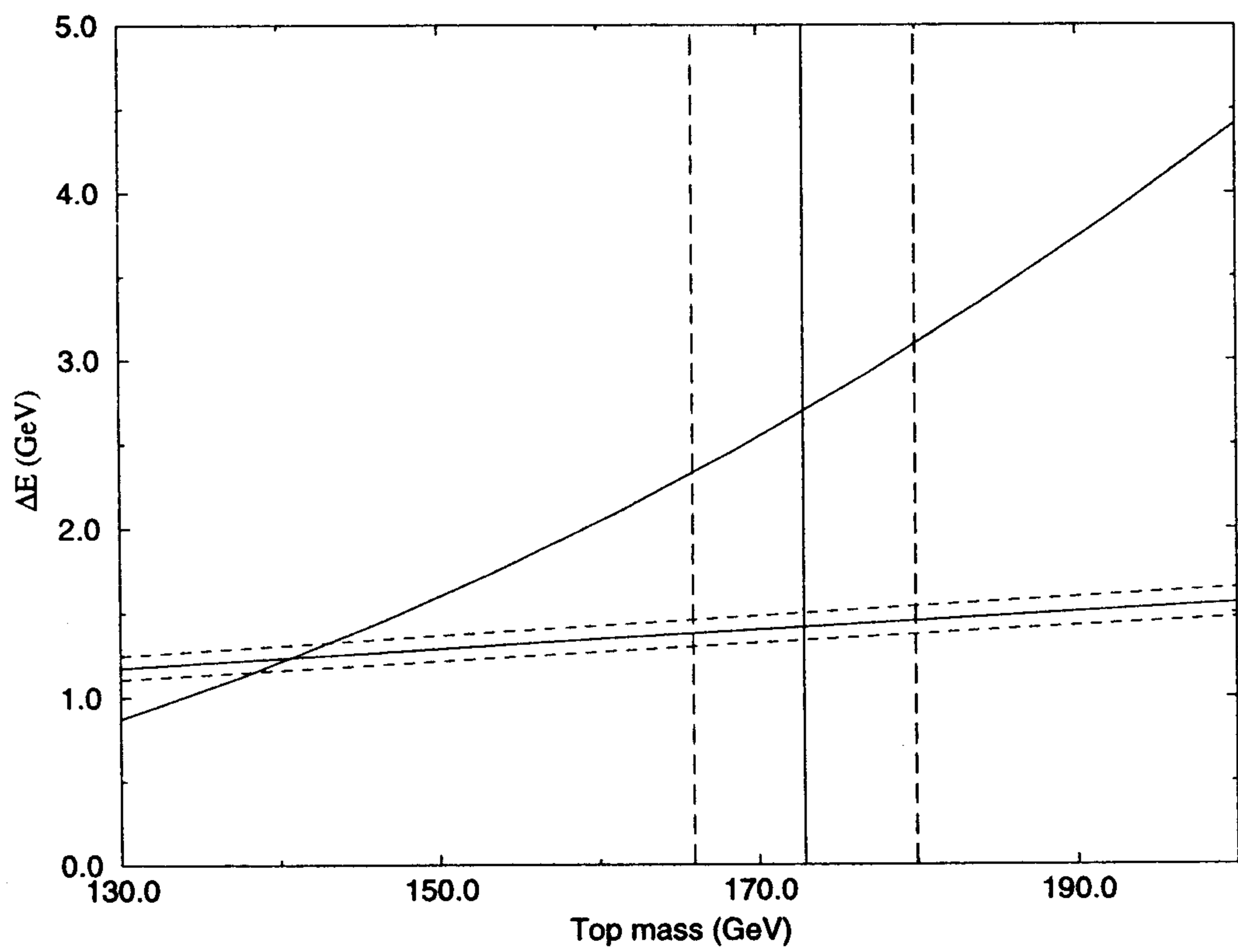


Figure 2: *Toponium width compared to energy separation of the Coulombic model. The stripe represents the indetermination of ΔE due to the value of α_s . The vertical bars show the indetermination of the top mass.*