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Gravity Quantized
(the High Tension String)

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Abstract
A candidate theory of gravity quantized is reviewed.

1 The perturbative approach

1.1 Minimal gravity

This short review following and extending on a recent proposal [Shiekh, 1994, 1995, 1996, 1997; Akhundov, Bellucci and Shiekh, 1996; Bellucci and Shiekh; 1997] is about Einstein gravity quantized. Why the strange wording? It is well known that Einstein gravity is *not* quantizable; this however does not preclude the existence of a quantum form, and this talk is all about this subtle but important difference.

It is well known that Einstein gravity fails to quantize for the simple reason that the infinities cannot be accommodated within the starting Lagrangian. For the purpose of illustration we will be discussing minimal coupled gravity with massive scalar particles, as governed by the

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Lagrangian:

$$\mathcal{L} = \sqrt{-g}\left(-2\Lambda + R + \tfrac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \tfrac{1}{2}m^2\phi^2\right) \tag{1}$$

The counter terms that carry the infinities cannot be accommodated back within this starting Lagrangian, and so the theory retains its divergent nature.

One often speaks of the starting Lagrangian as the classical Lagrangian arguing that this is the starting point for quantization, while the final Lagrangian, which is of the same form, is referred to as the quantized Lagrangian. This is a misleading notation, as the original Lagrangian is divergent, having taken up the counter terms, and the classical limit actually arises in the $\hbar \to 0$ limit from the final complete Lagrangian. This distinction will become especially poignant in what follows.

It is worth repeating for clarity that Einstein gravity is not quantizable, and we will be making no attempt to get around this fact.

1.2 Maximal gravity

Having noted that the failure to quantize minimal gravity stemmed from the fact that the counter terms did not fall back into the starting Lagrangian, one can resort to extending the Lagrangian so as to ensure that the theory is 'formally' renormalizable. In this way we arrive at maximal gravity, which is constrained by symmetry to be:

$$\mathcal{L}_0 = \sqrt{-g_0}\begin{pmatrix} -2\Lambda_0 + R_0 + \frac{1}{2}p_0^2 + \frac{1}{2}m_0^2\phi_0^2 + \frac{1}{4!}\phi_0^4\lambda_0(\phi_0^2) + p_0^2\phi_0^2\kappa_0(\phi_0^2) + R_0\phi_0^2\gamma_0(\phi_0^2) \\ +p_0^4 a_0(p_0^2,\phi_0^2) + R_0 p_0^2 b_0(p_0^2,\phi_0^2) + R_0^2 c_0(p_0^2,\phi_0^2) + R_{0\mu\nu}R_0^{\mu\nu} d_0(p_0^2,\phi_0^2) + \ldots \end{pmatrix} \tag{2}$$

(using units where $16\pi G = 1$, $c = 1$)

where p_0^2 is shorthand for $g_0^{\mu\nu}(\partial_\mu\phi_0)(\partial_\nu\phi_0)$ and not the independent variable of Hamiltonian mechanics. λ_0, κ_0, γ_0, a_0, b_0, c_0, d_0 ... are arbitrary analytic functions, and the second line carries all the higher derivative terms.

Strictly this is formal in having neglected gauge fixing and the resulting presence of ghost particles. Quantum anomalies arise from a conflict between symmetries, where only one can be maintained [Mann, 1988]. For this reason no such trouble is present here. Had we had massless particles present, we would accept the conformal anomaly as disrupting the conformal symmetry.

The price for having achieved 'formal' renormalization, is that the theory (with its infinite number of arbitrary renormalized parameters) is now devoid of predictive content, even if it is finally finite. The failure to quantize has been rephrased from a problem of non-renormalizability to one of non-predictability.

Despite this, after renormalization we are lead to:

$$\mathcal{L} = \sqrt{-g}\begin{pmatrix} -2\Lambda + R + \frac{1}{2}p^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\phi^4\lambda(\phi^2) + p^2\phi^2\kappa(\phi^2) + R\phi^2\gamma(\phi^2) \\ +p^4a(p^2,\phi^2) + Rp^2b(p^2,\phi^2) + R^2c(p^2,\phi^2) + R_{\mu\nu}R^{\mu\nu}d(p^2,\phi^2) + \ldots \end{pmatrix} \tag{3}$$

1.3 Physical criteria

Up to this point we have just rephrased the problem of the non-renormalizability of gravity. Again we take the liberty to emphasize that we are no longer quantizing Einstein gravity, but rather some hideously large theory under the name maximal gravity.

However, there remain physical criteria to pin down some of these arbitrary factors. Since in general the higher derivative terms lead to acausal classical behavior, their renormalized coefficient can be put down to zero on physical grounds. This still leaves the three arbitrary functions: $\lambda(\phi^2)$, $\kappa(\phi^2)$ and $\gamma(\phi^2)$, associated with the terms ϕ^4, $p^2\phi^2$, and $R\phi^2$ respectively. The last may be abandoned on the grounds of defying the equivalence principle. To see this, begin by considering the first term of the Taylor expansion, namely $R\phi^2$; this has the form of a mass term and so one would be able to make local measurements of mass to determine the curvature, and so contradict the equivalence principle (charged particles, with their non-local fields have this term present with a fixed coefficient). The same line of reasoning applies to the remaining terms, $R\phi^4$, $R\phi^6$, ... etc.

This leaves us the two remaining infinite families of ambiguities with the terms $\phi^4\lambda(\phi^2)$ and $p^2\phi^2\kappa(\phi^2)$. In the limit of flat space in 3+1 dimensions this will reduce to a renormalized theory in the traditional sense if $\lambda(\phi^2) = constant$, and $\kappa(\phi^2) = 0$. So one is lead to proposing that the physical parameters should be:

$$\begin{gathered}\Lambda = \kappa(\phi^2) = \gamma(\phi^2) = 0 \\ a(p^2,\phi^2) = b(p^2,\phi^2) = c(p^2,\phi^2) = d(p^2,\phi^2) = \ldots = 0 \\ \lambda(\phi^2) = \lambda = scalar\ particle\ self\ coupling\ constant \\ m = mass\ of\ the\ scalar\ particle\end{gathered} \tag{4}$$

and so the renormalized theory of quantum gravity for a scalar field should have the form:

$$\mathcal{L} = \sqrt{-g}\left(R + \tfrac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \tfrac{1}{2}m^2\phi^2 + \tfrac{1}{4!}\lambda\phi^4\right) \tag{5}$$

This is a candidate for the long sought after Einstein gravity quantized; and not quantized Einstein gravity, and the classical theory arises in the $\hbar \to 0$ limit of this.

2 Self consistency

One might now worry about the renormalization group pulling the coupling constants around.

Since we are interested only that the zeroed couplings remain so, we shall name them as external couplings, in so much as they belong to terms outside the final renormalised Lagrangian (eq. 5). The finite number remaining will naturally take up the designation of internal couplings.

Now the beta functions which characterize the way in which the coupling 'run' with energy, stem from the divergent parts. They are in general polynomial functions of the various couplings, with no constant part [Ramond, 1990]. These observations should be sufficient to argue that the external coupling don't run, and so remain at zero at all energy scales.

Now, one of the physical criterion imposed on the physics was that the Lagrangian in flat space should be renormalizable in the traditional sense, that is to say the infinities should fall back upon that same Lagrangian. Now one can see that we need a slight generalization of this condition, namely that the infinities stemming from the Lagrangian corresponding to the flat case should not fall outside. This is most easily seen on dimensional grounds. For simplicity let us work in the mass independent scheme, since the zeros of the beta function are scheme independent. The fact that the external couplings are all dimensionful limits the possible dependence of the associated beta function. For example:

$$\beta_a \equiv \mu \frac{\partial a}{\partial \mu} = \textit{polynomial of the couplings with dimension of } a$$

and in this way we see that it is enough that the external couplings are zeroed in order that their beta functions also be zero. This given, one can immediately deduce that the beta functions of the external couplings have no dependence on the internal couplings, and so once zeroed will remain zero. The internal couplings continue to run as one might anticipate for good physics, good being defined as in accord with the observations made of nature.

This line of reasoning not only proves the self consistency of orthodox gravity, so making it a candidate, but also makes the condition of traditional renormalization seem very natural.

3 Non-perturbative perspective

The above argument was done completely within a perturbative context, and one might wonder if a non-perturbative perspective would lead to the same proposal, and then perhaps without the infinities of the perturbative approach.

3.1 The high tension string

String theory might be thought of as another attempt to quantize gravity by generalizing away from point particle theory (super-gravity having failed).

When viewing string theory as a higher derivative, infinitely large Lagrangian, one sees many similarities with orthodox gravity, excepting that string theory has only one, and not an infinity, of extra parameters in the form of the string tension.

It then becomes very natural to wonder about the point particle limit of the super-string, when one anticipates the appearance of super-gravity. There immediately arises a question of how to resolve the fact that super-gravity is not renormalizable, but that the string in the high tension limit exists. The above investigation makes the resolution rather transparent in so much as the starting Lagrangian is not that of super-gravity even in the limit, for one has no reason to suppose the higher derivative bare terms disappear in the high tension limit. Again, only the theory quantized reduces to super-gravity.

In this way one might view orthodox gravity as the point particle (high tension) limit of the string, and as such it is a second confirmation of the existence of orthodox gravity as a candidate for gravity quantized.

3.2 Occam's gravity

We generalised to super-gravity and string theory because our former candidates failed to give us quantum gravity. But why go to the complexities of higher dimensions, or a new set of particles if we can locate a simpler candidate? Naturally, at the end of the day, it is not a choice for us to make, but rather a question to be put to nature.

It is rather paradoxical that we have arrived at a minimalist proposal by having first resorted to a maximal theory. But it is satisfying in having added every ingredient to the broth and seeing it slim itself down on its own accord.

Being such a simple candidate, one can immediately go about calculating with this proposal.

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