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Corrections of Order Λ^2_{QCD}/m^2_c to Inclusive Rare B Decays

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Abstract

We calculate nonperturbative $\mathcal{O}(\Lambda^2_{QCD}/m^2_c)$ corrections to the dilepton invariant mass spectrum and the forward–backward charge asymmetry in $B \to X_s e^+e^-$ decay using a heavy quark expansion approach. The method has recently been used to estimate long–distance effects in $B \to X_s \gamma$. We generalize this analysis to the case of non–vanishing photon invariant mass, $q^2 \neq 0$, relevant for the rare decay mode $B \to X_s e^+e^-$. In the phenomenologically interesting q^2 region away from the $c\bar{c}$ resonances, the heavy quark expansion approach should provide a reasonable description of possible non–perturbative corrections. In particular this picture is preferable to the model–dependent approach relying on the tails of Breit–Wigner resonances, which has been employed so far in the literature to account for these effects. We find that the $\mathcal{O}(\Lambda^2_{QCD}/m^2_c)$ corrections to the dilepton invariant mass spectrum and to the forward–backward asymmetry in $B \to X_s e^+e^-$ amount to several percent at most for $q^2/m^2_b \lesssim 0.3$ and $q^2/m^2_b \gtrsim 0.6$ respectively. The $\mathcal{O}(\Lambda^2_{QCD}/m^2_c)$ correction to the $B \to X_s \gamma$ decay rate is also computed and found to be $+3\%$, which agrees in magnitude with previous calculations. Finally, we comment on long–distance effects in $B \to X_s \nu\bar{\nu}$, which in this case are extremely suppressed due to the absence of virtual photon contributions.

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1 Introduction

Inclusive rare decays of B mesons, such as $B \to X_s\gamma$, $B \to X_s e^+e^-$ or $B \to X_s\nu\bar{\nu}$, provide important opportunities to test the Standard Model of flavor physics. These processes are particularly suited for this purpose, since they occur only at the loop level and are dominated by contributions that are reliably calculable in perturbation theory. Heavy quark ($1/m_b$) expansion and renormalization group improved perturbative QCD form a solid theoretical framework to describe these decays. The rates of the B meson decays are essentially determined by those of free b quarks, which have been computed at next–to–leading order in QCD for $B \to X_s\gamma$ [1,2], $B \to X_s e^+e^-$ [3,4] and $B \to X_s\nu\bar{\nu}$ [5]. The first subleading $\mathcal{O}(1/m_b^2)$ power corrections in the heavy quark expansion have also been studied and are in general well under control [6–9]. However, to properly assess the prospects for precise tests of the Standard Model and its extensions (see for instance [10]), further possible sources of theoretical uncertainty, beyond those from $1/m_b$ expansion or QCD perturbation theory, have to be investigated.

It had been realized that there are, in principle, long–distance contributions to $B \to X_s\gamma$ and $B \to X_s e^+e^-$ related to the $c\bar{c}$ intermediate state, whose account requires going beyond QCD perturbation theory and which are also not described by non–perturbative higher order contributions in the $1/m_b$ expansion. They originate from the quark level process $b \to sc\bar{c} \to s\gamma$ and have previously been estimated using model calculations based on $c\bar{c}$ resonance exchange. For $B \to X_s\gamma$ one may have $b \to s\Psi \to s\gamma$, where the Ψ (or a higher resonance) converts into an on–shell photon ($q^2 = 0$). This requires extrapolating the Ψ couplings to far off–shell values (from $q^2 = M_\Psi^2$ to $q^2 = 0$). From estimates perfomed along these lines effects of typically $\sim 10\%$ at the amplitude level were found in [11,12]. Although the accuracy of this approach is difficult to quantify, a sizable uncertainty from this source could not strictly be excluded.

In the case of $B \to X_s e^+e^-$ *, the intermediate $c\bar{c}$ resonances can be on–shell for appropriate values q^2 of the dilepton invariant mass. In this region of q^2, the resonance contributions form a very large background to the flavor–changing neutral current signal. Experimental cuts are therefore necessary to remove this part of the q^2–spectrum in the analysis of $B \to X_s e^+e^-$, which is then restricted to the range of q^2 below (and in principle also above) the resonance region. Far from the resonances the situation with respect to long–distance effects is analogous to the case of $B \to X_s\gamma$. The tails of the resonances can extend into this region, however, the Breit–Wigner form usually employed in modeling the resonance peaks cannot be expected to give a correct description of the tail region

*For simplicity we write $B \to X_s e^+e^-$ throughout the paper with the understanding that our discussion is equally applicable to $B \to X_s\mu^+\mu^-$.

away from the peak. Model dependent treatments of the long–distance effects related to the tails of $c\bar{c}$ resonances have been discussed in the literature [7,13,14], but again cannot be considered as a fully conclusive account.
Recently, it has been proposed to treat in a model independent way, employing a heavy quark expansion in inverse powers of the charm–quark mass, the nonperturbative contributions in $B \to X_s\gamma$ related to the intermediate $c\bar{c}$ state [15–18]. This interesting approach, which leads to $\mathcal{O}(\Lambda_{QCD}^2/m_c^2)$ corrections, has the advantage of being well defined and systematic. It avoids problems of double–counting of certain contributions, which are present in model descriptions involving both charm–quark and hadronic charm degrees of freedom simultaneously. Furthermore, for $c\bar{c}$ states far off–shell, the quark picture appears to be more appropriate than a description in terms of hadronic resonances (Ψ, Ψ', ...). Irrespective of the ultimate numerical accuracy that can be expected from an expansion in Λ_{QCD}/m_c, we believe that this method can still yield a useful order of magnitude estimate of the effect, which is at least complementary, and probably superior, to alternative model dependent calculations.
The main purpose of the present article is to study nonperturbative long–distance effects in $B \to X_s e^+e^-$ in a model independent way, using the methods previously applied to $B \to X_s\gamma$. We will calculate the $\mathcal{O}(\Lambda_{QCD}^2/m_c^2)$ corrections to the dilepton invariant mass spectrum and to the forward–backward charge asymmetry in $B \to X_s e^+e^-$. We also obtain the $\mathcal{O}(\Lambda_{QCD}^2/m_c^2)$ correction to the branching ratio for $B \to X_s\gamma$ as a special case of this analysis. Contrary to model estimates (involving $c\bar{c}$ resonances) both the magnitude and the sign of the long–distance effect are calculable and we find $\Delta B(B \to X_s\gamma)/B(B \to X_s\gamma) = +0.02$. Our result agrees in magnitude with previous calculations [15,17,18].
We remark that in comparison to the case of $B \to X_s\gamma$ and $B \to X_s e^+e^-$, long–distance effects in $B \to X_s\nu\bar{\nu}$ are strongly suppressed by factors of $\mathcal{O}(m_c^2/M_Z^2)$. Obviously the reason is that neutrinos do not couple to photons but to weak bosons only, whose contributions are likewise completely negligible for the long–distance effects in $B \to X_s\gamma$ and $B \to X_s e^+e^-$. From these observations it is clear that $B \to X_s\nu\bar{\nu}$ is a particularly clean channel from a theoretical point of view.

The paper is organized as follows. After this Introduction, in section 2, we briefly review non–perturbative effects in $B \to X_s\gamma$ and $B \to X_s e^+e^-$. We also discuss the $\bar{s}b$–photon–gluon vertex, which will be of central importance for our subsequent calculations. Using the results of section 2 in the limiting case of an on–shell photon, we derive the $\mathcal{O}(\Lambda_{QCD}^2/m_c^2)$ correction to the $B \to X_s\gamma$ rate in section 3. The main results of our paper, the $\mathcal{O}(\Lambda_{QCD}^2/m_c^2)$ effects on the dilepton invariant mass spectrum and the forward–backward asymmetry in $B \to X_s e^+e^-$, are calculated and discussed in section 4. Section 5 contains some comments on long–distance effects in $B \to X_s\nu\bar{\nu}$. A short summary is presented in section 6.

2 Non–perturbative Effects in $B \to X_s\gamma^*$ and the $\bar{s}b\gamma g$–Vertex

As mentioned before, $B \to X_s\gamma$ and $B \to X_s e^+e^-$ are dominated by perturbatively calculable short–distance contributions, including those from virtual top quarks. At NLO in QCD the intrinsic uncertainty of the perturbative calculation, indicated by residual renormalization scale dependence, is about $\pm 6\%$ for $B \to X_s\gamma$ [1,2] and of similar magnitude (or somewhat smaller) for the q^2–spectrum in $B \to X_s e^+e^-$ [4]. It is useful to keep these numbers in mind for comparison with uncertainties from other sources. Several effects that are beyond a perturbative treatment lead to corrections to the purely partonic decay picture and have to be considered for a more complete assessment of the total theoretical uncertainty:

- *Power corrections in $1/m_b$:* Sub–leading corrections in the heavy quark expansion start at $\mathcal{O}(\Lambda^2_{QCD}/m_b^2)$ and amount typically to several percent. The effects are calculable, generally well under control, and can be taken into account in a complete analysis [7,6]. An exception is the photon energy spectrum in $B \to X_s\gamma$ [19] and the endpoint region of the dilepton invariant mass (q^2) spectrum in $B \to X_s e^+e^-$ [7], where the $1/m_b$ expansion breaks down. This is not a problem for the integrated total $B \to X_s\gamma$ rate and for the $B \to X_s e^+e^-$ spectrum in the lower q^2 region. It is more troublesome for the shape of the photon energy spectrum in $B \to X_s\gamma$, which is needed to extract $B(B \to X_s\gamma)$ from the data. At present this fact still introduces some amount of model dependence in the measurement of this quantity [20].

- *On–shell $c\bar{c}$ resonances:* In the case of $B \to X_s\gamma$ there is a background from the process $B \to X_s\Psi$, with the (on–shell) Ψ subsequently decaying into a photon plus light hadrons. However, the energy spectrum of photons from this cascade process peaks at lower values than the one from the short–distance decay $B \to X_s\gamma$. In the window 2.2 GeV $< E_\gamma < 2.7$ GeV used to measure $B(B \to X_s\gamma)$ [21] this background is fairly small and can be corrected for. In the case of $B \to X_s e^+e^-$ the $c\bar{c}$ resonance background shows up as large peaks in the dilepton invariant mass spectrum. As discussed in the Introduction, it has to be removed by appropriate cuts.

- *Off–shell $c\bar{c}$ non–perturbative effects:* A particular non–perturbative contribution to the short–distance process $B \to X_s\gamma$ comes from the diagrams shown in Fig. 1. It arises when the gluon is soft and interacts with the spectator cloud of the b quark inside the B meson. This is the effect pointed out in [15,16] and further discussed in

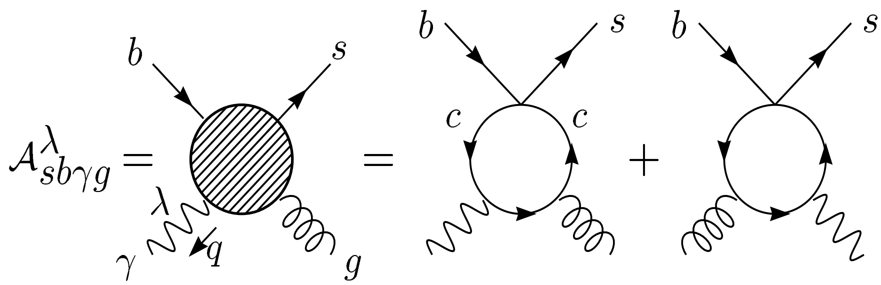

Figure 1: The $\bar{s}b\gamma g$ vertex at lowest order in QCD.

[17,18]. The same mechanism affects also $B \to X_s e^+ e^-$, in which case the photon invariant mass is $q^2 \neq 0$.

It is this last point we would like to focus on in the present paper. Since the gluon is very soft and interacting with the light spectator cloud, the process in Fig. 1 corresponds to a $c\bar{c}$ pair from b decay converting into a (on– or off–shell) photon. Previous attempts to describe long–distance contributions based on the conversion of intermediate $(c\bar{c})$ resonances into a photon (for both $B \to X_s\gamma$ and $B \to X_s e^+e^-$) can be viewed as model calculations aimed at addressing essentially the same effect.
Staying with the quark picture, one may evaluate the diagrams in Fig. 1 directly. To first order in the gluon momentum k (with $k^2 = 0$), the vertex function can be written as

$$\begin{aligned} \mathcal{A}^{\lambda}_{sb\gamma g} &= -i\frac{G_F}{\sqrt{2}}\lambda_c C_2 egQ_c \bar{s}\gamma_\mu(1-\gamma_5)G_{\alpha\varrho} b \, \frac{F(r)}{24\pi^2 m_c^2} \\ &\qquad \times \left[\varepsilon^{\alpha\beta\mu\varrho} q_\beta q^\lambda - \varepsilon^{\alpha\lambda\mu\varrho} q^2 + \varepsilon^{\beta\lambda\mu\varrho} q_\beta q^\alpha\right] \, . \end{aligned} \tag{1}$$

Here $^\lambda$ is the Lorentz index of the photon, q the photon momentum and $G_{\alpha\varrho} = G^a_{\alpha\varrho} T^a$ the gluon field strength. C_2 is the Wilson coefficient of the operator $Q_2 = (\bar{s}c)_{V-A}(\bar{c}b)_{V-A}$ in the standard effective weak Hamiltonian. In the leading logarithmic approximation,

$$C_2 = \frac{1}{2}\left[\left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right)^{6/23} + \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right)^{-12/23}\right] \, . \tag{2}$$

Furthermore, $\lambda_c = V^*_{cs} V_{cb}$ and $Q_c = +2/3$ is the charm–quark electric charge. The form factor $F(r)$ as a function of

$$r = \frac{q^2}{4m_c^2} \tag{3}$$

is given by

$$F(r) = \frac{3}{2r} \begin{cases} \dfrac{1}{\sqrt{r(1-r)}} \arctan \sqrt{\dfrac{r}{1-r}} - 1 & 0 < r < 1\,, \\ \dfrac{1}{2\sqrt{r(r-1)}} \left(\ln \dfrac{1-\sqrt{1-1/r}}{1+\sqrt{1-1/r}} + i\pi \right) - 1 & r > 1\,. \end{cases} \tag{4}$$

Our sign conventions are such that $\varepsilon^{0123} = +1$ and the covariant derivative is

$$D_\mu = \partial_\mu - igT^a A^a_\mu + ieQ_f A_\mu\,. \tag{5}$$

The diagrams in Fig. 1 generate a finite amplitude. However, in addition to the gauge–invariant m_c–dependent structure in (1), also terms proportional to $\varepsilon^{\alpha\lambda\mu\varrho}(q-k)_\alpha$ appear. We have not included these terms in $\mathcal{A}^\lambda_{sb\gamma g}$ because they are cancelled by the top–quark contribution via the GIM mechanism. This can be easily seen by considering the hierarchy $m_c \ll m_t \ll M_W$, which is sufficient since we are not interested in the detailed form of the top–quark contribution to $\mathcal{A}^\lambda_{sb\gamma g}$ and neglect $\mathcal{O}(1/m_t^2)$ terms anyway. We note also that by virtue of the GIM cancellation between top– and charm–quark contributions, the integrals are all manifestly convergent and the full calculation, including the Dirac algebra, can be safely performed in four dimensions.

Eq. (1) will be sufficient for our actual calculations of $\mathcal{O}(\Lambda^2_{QCD}/m_c^2)$ effects in $B \to X_s e^+ e^-$ and $B \to X_s \gamma$. It is, however, useful to also consider the more general result for the $sb\gamma g$–vertex obtained without expanding in $k \cdot q$. This will allow us to get some idea about the reliability of the lowest order approximation and its stability against higher order effects. Of course, there are further contributions, beyond those from higher powers in $k \cdot q$. For instance, one could have terms with more than one gluon field. But these are suppressed by additional powers of Λ^2_{QCD}/m_c^2 and we shall neglect them. The vertex function (valid to all orders in q^2 and $k \cdot q$, but with $k^2 = 0$) is then given by

$$\begin{aligned} \overline{\mathcal{A}}_{sb\gamma g} &= -i\frac{G_F}{\sqrt{2}} \lambda_c C_2 e g Q_c \bar{s}\gamma_\mu (1-\gamma_5) T^a b \, \frac{1}{48\pi^2 m_c^2} \varepsilon^{\alpha\lambda\mu\varrho} \\ &\times \left[\overline{F}(r+t,t)(G^a_{\lambda\sigma}\partial^\sigma F_{\varrho\alpha} + F_{\lambda\sigma}\partial^\sigma G^a_{\varrho\alpha} + G^a_{\alpha\varrho}\partial^\sigma F_{\sigma\lambda}) + \overline{F}(r,-t) G^a_{\alpha\varrho}\partial^\sigma F_{\sigma\lambda} \right]\,, \end{aligned} \tag{6}$$

where

$$t = \frac{k \cdot q}{2m_c^2} \tag{7}$$

and $F^{\alpha\beta}$ is the electromagnetic field strength. The form factor $\overline{F}$ is given by[†]

[†] Note that the form factor $\overline{F}$ can be written as $\overline{F}(r,t) = -384\pi^2 m_c^2 \widetilde{C}_{20}(q^2, k \cdot q)$ in terms of the subtracted three–point function $\widetilde{C}_{20}$ defined in [22]; the explicit expression of $\widetilde{C}_{20}$ can be found in the second ref. in [22].

$$\overline{F}(r+t,t) = 6\int_0^1 dz \left\{ -\frac{1-z}{t} - \frac{1-4z(1-z)r}{4zt^2} \ln \frac{1-4z(1-z)(r+t)}{1-4z(1-z)r} \right\} . \quad (8)$$

The following relations hold

$$\overline{F}(r,0) = F(r) , \qquad\qquad F(0) = 1 . \quad (9)$$

Expanding $\overline{F}$ in powers of t, (6) can be understood as a sum over an infinite number of operators involving all powers of the gluon momentum [17,18]. Unfortunately, for the physical processes we want to study, only the lowest order contribution can be calculated. The higher derivative operators lead to matrix elements that are unknown and one has to resort to a merely qualitative discussion in this case.

Since q is of order m_b and we are interested in the limit where $k \sim O(\Lambda_{QCD})$, we expect the matrix elements of these operators to be suppressed by corresponding powers of $m_b\Lambda_{QCD}/m_c^2$ (suppression of t^{n+1} with respect to t^n) and Λ_{QCD}/m_b (suppression of $F\partial G$ versus $G\partial F$). If both of the following inequalities were satisfied

$$m_b \gg \Lambda_{QCD} , \quad (10)$$

$$m_c^2/m_b \gg \Lambda_{QCD} , \quad (11)$$

we could neglect all the operators involving higher powers of the gluon momentum and use (1) instead of (6). However, whereas (10) is well justified, (11) is not a priori a good approximation in the real world. Thus we can safely neglect the $F\partial G$ term in (6) but we must have a closer look at the $O(t^n)$ corrections to (1).
In the $B \to X_s\gamma$ case ($q^2 = 0$) these corrections are estimated to be small [17,18]. Indeed, even though $k\cdot q/m_c^2 \sim O(1)$, the Taylor expansion of $\overline{F}(t,t)$ in terms of $k\cdot q/m_c^2$ involves small coefficients

$$\overline{F}(t,t) = 1 + \frac{4}{15}\frac{k\cdot q}{m_c^2} + \frac{3}{35}\left(\frac{k\cdot q}{m_c^2}\right)^2 + \frac{16}{525}\left(\frac{k\cdot q}{m_c^2}\right)^3 + \frac{8}{693}\left(\frac{k\cdot q}{m_c^2}\right)^4 + \ldots \quad (12)$$

A typical value for t would be $t \approx 0.3$ [17]. Then we have $\overline{F}(0.3, 0.3) \approx 1.2$, compared with $\overline{F}(0,0) = 1$. This indicates that the summed contribution of the t^n–terms ($n \geq 1$) is probably not too important, owing to the small numerical coefficients in the series [17, 18]. Of course, this is a rather crude argument, since the matrix elements of the higher derivative operators may be weighted differently for different n. Furthermore, coherent addition of all the terms may not be correct either. Nevertheless we take this at least as a conservative assumption. (Indeed, the matrix element of the $n = 1$ operator turns out to be vanishing in the case of $B \to X_s\gamma$ [18].) This estimate, despite being rough, provides a certain measure of the possible importance of neglected effects, exploiting the information contained in the form factor.

In the same spirit one may investigate the general case $q^2 \neq 0$. The ratio $\overline{F}(r+t,t)/\overline{F}(r,0)$, measuring the sensitivity to a nonvanishing t, reads ≈ 1.3, 1.4, 1.5, 1.75 for $r = 0.3$, 0.4, 0.5 and 0.6, respectively, using $t = 0.3$ (these values of r correspond to $q^2/m_b^2 \simeq 0.1$, 0.14, 0.17 and 0.2). This suggests that the expansion in t makes sense provided that q^2 is far enough from the $4m_c^2$ threshold. On the other hand, due to the physical cuts at $q^2 = 4m_c^2$ and $(k+q)^2 = 4m_c^2$ of the diagrams in Fig. 1, the functions $\overline{F}$ diverge near the $4m_c^2$ threshold. This breakdown of the gluon–momentum expansion can be viewed as an indication of the appearance of large genuine long–distance effects due to the nearby $c\bar{c}$ resonances. Hence, we conclude that if

$$4m_c^2 - q^2 \gtrsim m_c^2 \tag{13}$$

the lowest order ($t = 0$) result is still reasonable, as a sensible approximation for smaller q^2 and as an order–of–magnitude estimate closer to this bound.

So far we have been considering the region of q^2 below the resonance domain. Above the resonances r is parametrically large, $r \sim q^2/m_c^2 \approx m_b^2/m_c^2 \gg 1$, and one may expand $\overline{F}(r+t,t)$ not only in t but subsequently also in inverse powers of r. Keeping only the leading term in $1/r$ in each of the coefficients of t^n one has

$$\overline{F}(r+t,t) \approx \sum_{n=0}^{\infty}(-1)^{n+1}\frac{3}{(n+2)r^{n+1}}t^n = -\frac{3}{2r} + \frac{t}{r^2} + \ldots \tag{14}$$

This shows that the leading corrections to the $t = 0$ result behave as a series in powers of $t/r \sim k\cdot q/q^2 \sim \Lambda_{QCD}/m_b$. Numerically, using the full expression for $\overline{F}(r+t,t)$ and taking $r = 2$ as a relevant example, we find that $\overline{F}(2-0.3,-0.3) = -1.40+1.10i$. To be more conservative we have here chosen $t < 0$ so that all contributions add coherently. The result is reasonably close to the $t = 0$ value $\overline{F}(2,0) = -1.22+0.83i$. It appears justified to neglect the higher dimensional operators in this case, hence, we expect the approximation $t = 0$ to work in the large q^2 region of the spectrum.

In the previous discussion we have restricted our attention to the function $\overline{F}(r+t,t)$, but similar conclusions hold also for $\overline{F}(r,-t)$. The latter contributes only for $q^2 \neq 0$ and its expansion in t shows a better behavior than the one of $\overline{F}(r+t,t)$.

At this point it is natural to address the question of the effect generated by the diagrams in Fig. 1 in case the charm quark inside the loop is replaced by an up quark. At first sight, the explicit m_c^2 dependence of (6) would suggest that the effect diverges as $1/m_q^2$ when $m_q \to 0$. This is still not a problem for $B \to X_s\gamma^*$, where the up–quark contribution is suppressed by the CKM hierarchy, but could be a dramatic effect in $B \to X_d\gamma^*$.

However, it should be noted that the $1/m_q^2$ behavior of (6) is correct only if the expansion of $\overline{F}$ in terms of $k \cdot q/m_q^2$ is allowed. This is simply not true for the up–quark contribution. In this case, and considering $q^2 = 0$ for the moment, it is more appropriate to expand $\overline{F}(t,t)$ in inverse powers of t: the leading term in this expansion goes like $t^{-1} \sim m_q^2/k \cdot q$ and cancels the artificial $1/m_q^2$ divergence of (6). The operators generated by this expansion are nonlocal and their matrix elements cannot be estimated reliably. However, from naive dimensional counting, we expect the leading contribution to be of order Λ_{QCD}/m_b. For the up–quark contribution in $B \to X_{s,d} e^+ e^-$ the situation is, in a sense, even more favorable. If $q^2 \gtrsim (2\text{GeV})^2$, one stays above the region of $u\bar{u}$ resonances and q^2 itself provides the relevant short–distance scale. The leading long–distance contribution is then of the order Λ_{QCD}^2/q^2. Corrections to this behavior arise as powers of $t/r \sim k \cdot q/q^2 \sim \Lambda_{QCD}/\sqrt{q^2}$, corresponding to a series of local, higher–dimensional operators. These results follow from (14). The situation is analogous to the one for the high–q^2 end of the spectrum in the case of intermediate $c\bar{c}$, except that for the $u\bar{u}$ case the operator product expansion approach remains valid down to smaller q^2 due to the absence of heavy resonances.

We finally remark that the diagrams in Fig. 1 actually contribute in two distinct ways to the decays $B \to X_s \gamma^*$. First, the gluon can be soft and couple to the light cloud in the B meson. This is the effect we are mainly interested in here and which we shall calculate, far from the resonance region, using (1). Second, the gluon, now not necessarily soft, may be radiated from the charm–quark loop to end up in the final state X_s. These processes are calculable in perturbation theory, it is infrared finite and contributes as part of the matrix element calculation at NLO in $B \to X_s \gamma$ [23,1]. For $B \to X_s e^+ e^-$ the situation is analogous, except that the diagrams of Fig. 1 would enter only beyond the next–to–leading order (at 'next–to–next–to–leading order'). The first of these processes is not contained within the second one, which is entirely perturbative. Therefore in calculating the nonperturbative correction there is no double counting of contributions already taken into account in the higher order perturbative result.

3 $B \to X_s \gamma$

In this section we re–derive the $\mathcal{O}(\Lambda_{QCD}^2/m_b^2)$ correction to $\Gamma(B \to X_s \gamma)$ via the results of the previous chapter in the limit of an on–shell photon.

By means of the optical theorem the total rate of a B meson decay can be expressed as the expectation value

$$\Gamma = \frac{1}{2M_B} \langle B | \mathcal{T} | B \rangle \tag{15}$$

of a forward transition operator $\mathcal{T}$ defined by

$$\mathcal{T} = \text{Im}\, i \int d^4x\, T\mathcal{H}_{eff}(x)\mathcal{H}_{eff}(0)\ . \tag{16}$$

Here $\mathcal{H}_{eff}$ is the low energy effective Hamiltonian governing the decay under consideration. The states $|B\rangle$ in (15) are to be taken in conventional relativistic normalization ($\langle B|B\rangle = 2EV$).

Since we are analyzing a small correction to the full decay width, it is sufficient to work to leading logarithmic accuracy and to neglect consistently all relative $\mathcal{O}(\alpha_s)$ effects. In this approximation we may write

$$\mathcal{H}_{eff} = \mathcal{H}^{(0)}_{eff} + \mathcal{H}^{(1)}_{eff}\ , \tag{17}$$

where

$$\mathcal{H}^{(0)}_{eff} = -\frac{G_F}{\sqrt{2}} V^*_{ts} V_{tb}\, C_7 \mathcal{O}_7 + \text{h.c.}\ , \qquad \mathcal{O}_7 = \frac{e}{8\pi^2} m_b \bar{s}\sigma^{\mu\nu}(1+\gamma_5) b\, F_{\mu\nu}\ , \tag{18}$$

represents the well known leading–log effective Hamiltonian for $b \to s\gamma$ decay [24] (for a review see [25]). $C_7 \simeq -0.30$ is the scheme independent ('effective') Wilson coefficient, whose analytic form can be found for instance in [25].

The correction $\mathcal{H}^{(1)}_{eff}$ can be derived from the amplitude (1) in the limit where the photon is put on–shell. In this case the vertex (1) is equivalent to a local operator. Performing the usual matching procedure one obtains

$$\mathcal{H}^{(1)}_{eff} = \frac{G_F}{\sqrt{2}} V^*_{cs} V_{cb}\, C_2\, \mathcal{O}_{11} + \text{h.c.}\ , \tag{19}$$

$$\mathcal{O}_{11} = \frac{eQ_c}{48\pi^2 m_c^2} \bar{s}\gamma_\mu(1-\gamma_5) g G_{\nu\lambda} b\, \varepsilon^{\mu\nu\varrho\sigma} \partial^\lambda F_{\varrho\sigma}\ . \tag{20}$$

To leading order only $\mathcal{H}^{(0)}_{eff}$ contributes to (15), (16). One finds the well–known result ($V^*_{cs}V_{cb} \simeq -V^*_{ts}V_{tb}$)

$$\Gamma(B \to X_s\gamma) = \frac{G_F^2 m_b^5}{192\pi^3} (V^*_{cs}V_{cb})^2\, \frac{6\alpha}{\pi} C_7^2\ . \tag{21}$$

The interference between $\mathcal{H}^{(0)}_{eff}$ and $\mathcal{H}^{(1)}_{eff}$ leads to a correction of the transition operator

$$\Delta\mathcal{T} = -\frac{G_F^2 m_b^5}{192\pi^3} (V^*_{cs}V_{cb})^2\, \frac{\alpha}{9\pi}\, \frac{C_2\, C_7}{m_c^2}\, \bar{b} g\sigma\cdot G b\ , \qquad \sigma_{\mu\nu} \equiv \frac{i}{2}[\gamma_\mu, \gamma_\nu]\ . \tag{22}$$

Using [8]

$$\frac{\langle B|\bar{b}g\sigma\cdot Gb|B\rangle}{2M_B} = \frac{3}{2}(M^2_{B^*} - M^2_B), \tag{23}$$

we finally obtain

$$\frac{\Delta\Gamma(B \to X_s\gamma)}{\Gamma(B \to X_s\gamma)} = -\frac{C_2}{36C_7}\frac{M_{B^*}^2 - M_B^2}{m_c^2} \approx +0.03\ . \tag{24}$$

This result suggests that the potentially problematic non–perturbative effects in $B \to X_s\gamma$ should indeed be negligible, re–enforcing the role of the $B \to X_s\gamma$ process as a significant test of the Standard Model.

We stress that the correction (24) has a definite sign, contrary to previous estimates of nonperturbative $c\bar{c}$ effects based on off–shell resonance exchange [11,12]. In this context we note that for a positive matrix element (23) and negative C_7, the consistent covariant derivative, specifying the sign of strong and electromagnetic couplings and thus of (24), is the one given in (5).

The non–perturbative correction in (24) may be compared with model calculations of the long–distance effect. Assuming that the dominant contribution to (24) is generated by the Ψ exchange, the above result can be used to fix sign and magnitude of the non–factorizable $bs\Psi$ coupling at $q^2 = 0$ (g_2 in the notation of [12]). The result thus obtained is consistent with the lower values given in [12] ($g_2 \sim 10^{-2}$).

4 $B \to X_s e^+ e^-$

In the present section we proceed to compute the non–perturbative correction to the rare decay $B \to X_s e^+e^-$. The effective Hamiltonian for $b \to se^+e^-$ at next–to–leading order has been reviewed in [25]. In the following we will make use of the results collected in this article and adopt its notation. At next–to–leading order, the amplitude for $B \to X_s e^+e^-$ can be written as

$$\mathcal{A} = -i\frac{G_F}{\sqrt{2}}\lambda_c\frac{\alpha}{2\pi}\left[2m_bC_7J_7^{\kappa\nu}\frac{iq_\kappa}{q^2}\ \bar{u}\gamma_\nu v + \tilde{C}_9^{eff}J_9^\nu\ \bar{u}\gamma_\nu v + \tilde{C}_{10}J_9^\nu\ \bar{u}\gamma_\nu\gamma_5 v\right]\ , \tag{25}$$

where

$$J_7^{\kappa\nu} = \langle X_s|\bar{s}\sigma^{\kappa\nu}(1+\gamma_5)b|B\rangle \tag{26}$$

$$J_9^\nu = \langle X_s|\bar{s}\gamma^\nu(1-\gamma_5)b|B\rangle \tag{27}$$

and u, v denote the electron and positron spinor, respectively. C_7 and $\tilde{C}_{10}$ are Wilson coefficients; $\tilde{C}_9^{eff}$ is not strictly speaking a Wilson coefficient, since it includes also contributions from the $b \to se^+e^-$ matrix elements of the operators Q_1, ... Q_6 in the Hamiltonian [25]. However, the amplitude expressed in terms of $\tilde{C}_9^{eff}$ provides a convenient notation. In particular, $\tilde{C}_9^{eff}$ already contains contributions from intermediate charm–quark states entering the one–loop matrix elements (these are similar to the graphs in Fig. 1 but without the gluon and a lepton line connected to the photon propagator).

In writing (25) we have used $V_{ts}^* V_{tb} \approx -V_{cs}^* V_{cb} = -\lambda_c$. Note also that, to calculate $B \to X_s e^+ e^-$ at NLO, only the leading logarithmic approximation is necessary for C_7 ($\equiv C_{7\gamma}^{(0)eff}$ in the notation of [25]).

The non–perturbative correction we want to evaluate stems from the vertex function in (1), which yields the following correction to the leading amplitude in (25)

$$\mathcal{A}^c = -i\frac{G_F}{\sqrt{2}}\lambda_c \frac{\alpha}{2\pi}\frac{C_2 Q_c}{3m_c^2}\frac{F(r)}{q^2}\; J_{G,\mu\alpha\varrho}\left[\varepsilon^{\beta\lambda\mu\varrho}q_\beta q^\alpha - \varepsilon^{\alpha\lambda\mu\varrho}q^2\right]\; \bar{u}\gamma_\lambda v \tag{28}$$

with

$$J_{G,\mu\alpha\varrho} = \langle X_s|\bar{s}\gamma_\mu(1-\gamma_5)gG_{\alpha\varrho}b|B\rangle\,. \tag{29}$$

Note that the term proportional to q^λ in (1) vanishes when multiplied by $\bar{u}\gamma_\lambda v$ due to current conservation.

Adding $\mathcal{A} + \mathcal{A}^c$, squaring the amplitude, summing over inclusive states X_s and performing the necessary phase space integration, one may calculate the differential decay rate for $B \to X_s e^+ e^-$. Defining $s = q^2/m_b^2$ and

$$R(s) = \frac{\frac{d}{ds}\Gamma(B \to X_s e^+ e^-)}{\Gamma(B \to X_c e\nu)}\,, \tag{30}$$

one has at NLO in QCD perturbation theory ($z = m_c/m_b$)

$$R(s) = \frac{\alpha^2}{4\pi^2}\frac{(1-s)^2}{f(z)\kappa(z)}\Big[\quad (1+2s)\left(|\tilde{C}_9^{eff}|^2 + |\tilde{C}_{10}|^2\right) + 4\left(1+\frac{2}{s}\right)|C_7|^2 + 12 C_7 \mathrm{Re}\tilde{C}_9^{eff}\Big]\,. \tag{31}$$

Here $f(z)$ is the phase space factor and $\kappa(z)$ the QCD correction factor entering $\Gamma(B \to X_c e\nu)$; they can be found in [25].

For the $\mathcal{O}(\Lambda_{QCD}^2/m_c^2)$ correction term, arising from the interference of $\mathcal{A}^c$ with $\mathcal{A}$, we obtain

$$\Delta R(s) = -\frac{\alpha^2}{4\pi^2}C_2\frac{2(M_{B^*}^2 - M_B^2)}{9m_c^2}\frac{(1-s)^2}{f(z)\kappa(z)} \times \mathrm{Re}\left\{F(r)\left[C_7^*\frac{1+6s-s^2}{s} + \tilde{C}_9^{eff*}(2+s)\right]\right\}\,. \tag{32}$$

For the evaluation of ΔR we have used the identity [26]

$$\langle B|\bar{b}\Gamma G_{\alpha\beta}b|B\rangle = \frac{1}{48}\langle B|\bar{b}\sigma\cdot Gb|B\rangle\;\; \mathrm{tr}\left\{\Gamma(1+\not{v})\sigma_{\alpha\beta}(1+\not{v})\right\}, \tag{33}$$

which is valid to leading order in the $1/m_b$ expansion. Here v denotes the B meson four–velocity and Γ an arbitrary string of γ–matrices. For handling the Dirac algebra throughout our calculations we have used the program Tracer [27].

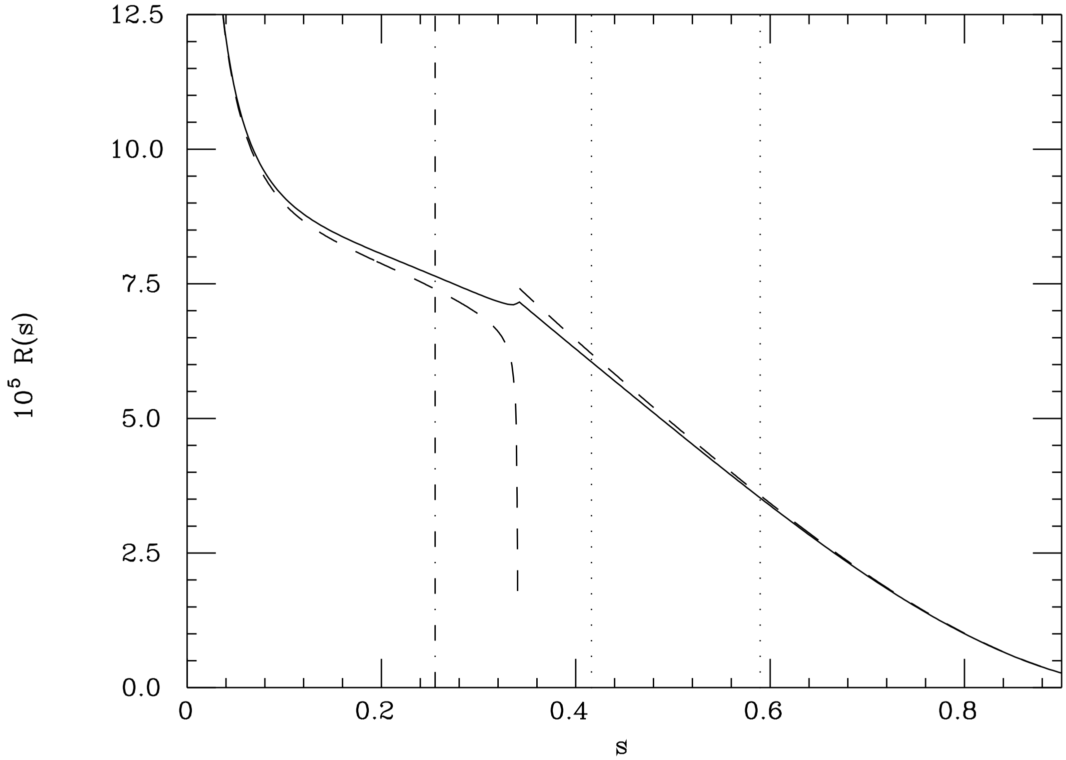

Figure 2: The dilepton invariant mass spectrum of $B \to X_s e^+ e^-$ normalized to the semileptonic width (30). The solid curve is the short–distance result (31) whereas the dashed one includes the $\mathcal{O}(\Lambda_{QCD}^2/m_c^2)$ corrections. The dotted curves indicate the position of the Ψ and Ψ' resonances. A dash–dotted curve is shown at $q^2 = 3m_c^2$. The corrections diverge at $q^2 = 4m_c^2$. In this plot we have used $m_c = 1.4$ GeV and $m_b = 4.8$ GeV.

The final result is displayed in Fig. 2, showing the corrected and uncorrected $R(s)$. The conditions for the validity of the corrections have already been discussed in section 2. They are best satisfied for the high–q^2 end of the spectrum, above the resonances, and for very low q^2 respectively. The reliability of the approximation used deteriorates somewhat as q^2 is increased from ≈ 0 towards the resonance region. As an order of magnitude estimate the calculation should still make sense around $q^2 \approx 3m_c^2$, indicated in Fig. 2 by the dash–dotted curve. For larger q^2 one gets too close to the lowest resonance and the heavy quark expansion breaks down. Further up, at $q^2 = 4m_c^2$, the correction develops a (unphysical) square root divergence (in our choice $m_c = 1.4$ GeV). The difference between M_Ψ and $2m_c$ is of $\mathcal{O}(\Lambda_{QCD})$ and corresponds to non–perturbative effects that are beyond the control within our approximation.

As can be noticed, the $\mathcal{O}(\Lambda_{QCD}^2/m_c^2)$ corrections are very small in the region where this calculation should be trusted, viz. below $3m_c^2$ and above the resonance peaks. Actually, in the high q^2 region the effect is further suppressed since $F(r)$ decreases as $1/r$ for large r, as discussed in section 2. For $q^2 \sim O(m_b^2)$ the corrections behave in fact more as

$O(\Lambda^2_{QCD}/m_b^2)$ instead of $O(\Lambda^2_{QCD}/m_c^2)$. We also observe that for s below 0.1 the corrections become vanishingly small due to a cancellation among the terms in (32) (C_7 is negative, Re$\tilde{C}_9^{eff}$ positive). This is related to the fact that the correction $\Delta R(s)$ is negative for s above 0.1, where $\tilde{C}_9^{eff}$ dominates, but positive for s close to zero, which essentially corresponds to the case of $B \to X_s \gamma$. In fact, one may obtain (24) as a special case of $\Delta R/R$ from (31) and (32) in the limit $q^2 \to 0$.
The sign of the effect for s around 0.2 is different from the one obtained by the approaches based on resonance exchange [13,14,7]. This is not a problem since the sign of the correction determined in model calculations is not reliable far from the resonances. Moreover, as discussed in the previous section, the corrections we are considering are essentially related to the non–factorizable contributions of charmed resonances. The latter are usually considered in $B \to X_s \gamma$, where the factorizable terms vanish, but are always neglected in $B \to X_s e^+ e^-$.
The factorizable resonance contributions can be identified, to the lowest order in α_s, with diagrams as those in Fig. 1 but without gluons. The effect of such diagrams is positive but is already included in the NLO calculation as the matrix element contribution to $\tilde{C}_9^{eff}$. As pointed out in [28] within the context of exclusive decays, adding the factorizable resonance effects to the NLO calculation leads to a double counting problem related to the simultaneous use of quark and hadronic ($c\bar{c}$ resonances) degrees of freedom. On the other hand, this is not the case for the Λ^2_{QCD}/m_c^2 effect we have evaluated, where we have consistently used a pure quark level description. Of course, some model–dependent treatment may still be useful close to the Ψ–resonance (an interesting approach has been presented in [29]). If one should attempt this, the $\mathcal{O}(\Lambda^2_{QCD}/m_c^2)$ effect could be used to fix the small q^2 limit.

Another interesting quantity that can be measured in $B \to X_s e^+ e^-$ decays is the forward–backward charge asymmetry [13]. Normalizing $\Gamma(B \to X_s e^+ e^-)$ to the semileptonic width, as in (30), we define

$$A(s) = \frac{1}{\Gamma(B \to X_c e \nu)} \int_{-1}^{1} d\cos\theta \, \frac{d^2\Gamma(B \to X_s e^+ e^-)}{ds \, d\cos\theta} \text{sgn}(\cos\theta) \,, \tag{34}$$

where θ is the angle between the e^+ and B momenta in the dilepton center–of–mass frame. Proceeding similarly to the case of $R(s)$, the NLO perturbative result turns out to be

$$A(s) = -\frac{3\alpha^2}{4\pi^2} \frac{(1-s)^2}{f(z)\kappa(z)} \text{Re} \left\{ \tilde{C}_{10}^* \left[2\, C_7 + s\, \tilde{C}_9^{eff} \right] \right\} \,, \tag{35}$$

whereas the $\mathcal{O}(\Lambda^2_{QCD}/m_c^2)$ correction term, arising from the interference of $\mathcal{A}^c$ and $\mathcal{A}$, is

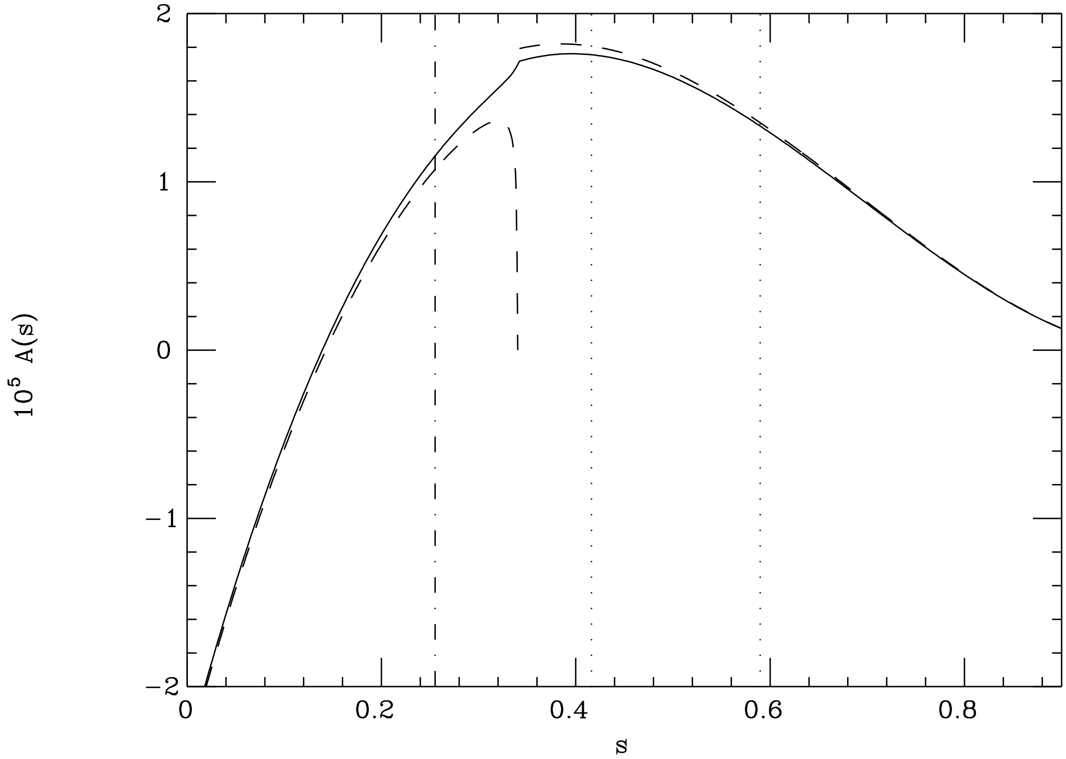

Figure 3: The differential forward–backward asymmetry defined in (34) as a function of the dilepton invariant mass. Notations are as in Fig. 2.

given by

$$\Delta A(s) = \frac{3\alpha^2}{4\pi^2} C_2 \frac{M_{B^*}^2 - M_B^2}{36 m_c^2} \frac{(1-s)^2}{f(z)\kappa(z)} \, \mathrm{Re} \left\{ \tilde{C}_{10}^* F(r) \right\} (1+3s) \ . \tag{36}$$

The corrected and uncorrected results for $A(s)$ are shown in Fig. 3. Concerning range of validity and size of the corrections similar comments apply to $A(s)$ as to the spectrum $R(s)$ discussed above. In addition we note that the small corrections to $A(s)$ and $R(s)$ have a tendency to further cancel in the normalized asymmetry $A(s)/R(s)$, a useful observable for direct comparison with experiment.

5 $B \to X_s \nu\bar{\nu}$

In $B \to X_s \nu\bar{\nu}$ there are no contributions from virtual photons. Consequently the pattern of GIM cancellation is not logarithmic ('soft'), as in $B \to X_s e^+ e^-$, but powerlike ('hard') due to the exchange of heavy gauge bosons. Therefore it is clear that long–distance effects in $B \to X_s \nu\bar{\nu}$ from charm quarks are additionally suppressed by a factor of order m_c^2/M_W^2. In principle, however, in this case also long–distance contributions related to the $c\bar{c}$ intermediate state do exist. The largest of these is associated with the cascade process

$B \to X_s\Psi \to X_s\nu\bar{\nu}$. In the following we shall try to estimate this effect to show more quantitatively to which extent the short–distance contribution dominates in $B \to X_s\nu\bar{\nu}$. Using the relation $\langle 0|\bar{c}\gamma^\mu c|\Psi\rangle = m_\Psi f_\Psi \epsilon^\mu$ and the coupling of the $\bar{c}\gamma^\mu c$ current with the electroweak fields

$$\mathcal{L} = -e\bar{c}\gamma^\mu c\left[\frac{2}{3}A_\mu + \frac{1}{4\sin\theta_w\cos\theta_w}\left(1-\frac{8}{3}\sin^2\theta_w\right)Z_\mu\right] , \qquad (37)$$

we obtain

$$R_\nu = \frac{|\mathcal{A}(\Psi\to\nu\bar{\nu})|}{|\mathcal{A}(\Psi\to e^+e^-)|} = \left(\frac{M_\Psi}{M_Z}\right)^2 \frac{3-8\sin^2\theta_w}{16\sqrt{2}\cos^2\theta_w\sin^2\theta_w} \simeq 3.3\times 10^{-4} . \qquad (38)$$

Then, using the theoretical result $B^{s.d.}(B \to X_s\nu\bar{\nu}) = (4.0\pm 1.0)\times 10^{-5}$ [5] and the experimental values of $B(B\to X_s\Psi)$ and $B(\Psi\to e^+e^-)$ [30], we find

$$\frac{B(B\to X_s\Psi(\nu\bar{\nu}))}{B^{s.d.}(B\to X_s\nu\bar{\nu})} = 3R_\nu^2\frac{B(B\to X_s\Psi)B(\Psi\to e^+e^-)}{B^{s.d.}(B\to X_s\nu\bar{\nu})} \simeq 5\times 10^{-6} . \qquad (39)$$

Unfortunately we cannot reliably estimate the interference terms between short– and long–distance contributions, since we have no useful information about the strong phases in $B\to X_s\Psi$. However, the result (39) indicates that the long–distance corrections to the amplitude are at the level of 10^{-3} at most. This is by far enough to conclude that $B\to X_s\nu\bar{\nu}$ is an extraordinarily clean channel [5] and, therefore, a very interesting probe of the Standard Model and its extensions [9].

6 Summary

In this paper we have calculated long–distance corrections of $\mathcal{O}(\Lambda^2_{QCD}/m_c^2)$ to the dilepton invariant mass (q^2) spectrum and to the forward–backward asymmetry in $B\to X_s e^+e^-$. For low $q^2/m_b^2 \lesssim 0.1$ these corrections can be expected to give a fairly accurate description of long–distance effects due to intermediate (far off–shell) $c\bar{c}$ pairs. Contributions from higher–dimensional operators, formally suppressed by powers of $m_b\Lambda_{QCD}/m_c^2$ relative to the Λ^2_{QCD}/m_c^2 effect, involve unknown hadronic matrix elements and have been neglected. Despite the fact that $m_b\Lambda_{QCD}/m_c^2 \approx 0.6$, this is justified for small q^2 due to small numerical coefficients accompanying these additional contributions. They could be more important for larger q^2. Close to the resonance region $q^2/m_b^2 \sim 4m_c^2/m_b^2 \sim 0.34$ the assumptions on which the calculation is based clearly break down. On the other hand, simple order–of–magnitude estimates suggest that even for $q^2/m_b^2 \sim 0.26$ the calculable leading $\mathcal{O}(\Lambda^2_{QCD}/m_c^2)$ effect should give a reasonable account of the approximate size of the corrections within, say, a factor of two. Above the resonance region $q^2/m_b^2 \gtrsim 0.7$ the calculated effect is again reliable. In spite of the various uncertainties we believe that the $1/m_c$ expansion approach is still preferable to model dependent estimates relying on off–shell $c\bar{c}$ resonances.

Numerically the corrections we find are small (at the one to two percent level) in the phenomenologically interesting region of q^2 away from the resonances. For comparison one may recall that uncertainties from QCD perturbation theory are at the level of $\pm 6\%$ at next–to–leading order. This indicates that long–distance corrections related to intermediate $c\bar{c}$ should not be a serious problem in the analysis of $B \to X_s e^+ e^-$ decay, unless the neglected effects of higher order in Λ_{QCD}^2/m_c^2 or $m_b \Lambda_{QCD}/m_c^2$ turned out to be substantially larger than anticipated.

We have pointed out that the $\mathcal{O}(\Lambda_{QCD}^2/m_c^2)$ effect is calculable in both magnitude and sign, which is to be contrasted with the situation in hadronic model calculations. The $\mathcal{O}(\Lambda_{QCD}^2/m_c^2)$ correction to $B(B \to X_s \gamma)$, considered in previous work, can be obtained as a special application of our analysis and is found to be $\approx +3\%$.

The decays $B \to X_s \gamma$ and $B \to X_s e^+ e^-$ may be contrasted with the mode $B \to X_s \nu \bar{\nu}$, where the corresponding nonperturbative effects due to intermediate $c\bar{c}$ are still much stronger suppressed, down to a level of $\sim 10^{-3}$ in the amplitude.

Long–distance effects are in general a notoriously difficult problem. However, they have to be controlled sufficiently well to address fundamental questions in flavor physics. The existence of a realistic limit in which such effects can in fact be computed from first principles is therefore of considerable theoretical interest in its own right, as well as of interest for phenomenology. It is particularly gratifying that for the important processes $B \to X_s \gamma$ and $B \to X_s e^+ e^-$ a calculation of this type is possible and a class of potentially large long–distance corrections is indeed found to be rather small and well under control.

Note added: After this paper was finished, we have received a preprint [31] in which the $\mathcal{O}(\Lambda_{QCD}^2/m_c^2)$ correction to the differential decay rate of $b \to s e^+ e^-$ has been calculated. We disagree with their result in sign.

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References

[1] C. Greub, T. Hurth and D. Wyler, Phys. Rev. **D54**, 3350 (1996).

[2] K.G. Chetyrkin, M. Misiak and M. Münz, ZU-TH-24-96, hep-ph/9612313.

[3] M. Misiak, Nucl. Phys. **B393**, 23 (1993); erratum ibid. **B439**, 461 (1995).

[4] A.J. Buras and M. Münz, Phys. Rev. **D52**, 186 (1995).

[5] G. Buchalla and A.J. Buras, Nucl. Phys. **B400**, 225 (1993).

[6] A.F. Falk, M. Luke and M.J. Savage, Phys. Rev. **D49**, 3367 (1994).

[7] A. Ali, G. Hiller, L.T. Handoko and T. Morozumi, Phys. Rev. **D55**, 4105 (1997).

[8] I.I. Bigi *et al.*, in *B-Decays* (2nd edition), ed. S.L. Stone, World Scientific, Singapore (1994), p. 132.

[9] Y. Grossman, Z. Ligeti and E. Nardi, Nucl. Phys. **B465**, 369 (1996); erratum ibid. **B480**, 753 (1996).

[10] J.L. Hewett and J.D. Wells, Phys. Rev. **D55**, 5549 (1997).

[11] N.G. Deshpande, X.-G. He and J. Trampetić, Phys. Lett. **B367**, 362 (1996); G. Eilam, A. Ioannissian, R.R. Mendel and P. Singer, Phys. Rev. **D53**, 3629 (1996).

[12] J.M. Soares, Phys. Rev. **D53**, 241 (1996).

[13] A. Ali, T. Mannel and T. Morozumi, Phys. Lett. **B273**, 505 (1991).

[14] M.R. Ahmady, Phys. Rev. **D53**, 2843 (1996); C.-D. Lü and D.-X. Zhang, Phys. Lett. **B397**, 279 (1997).

[15] M.B. Voloshin, Phys. Lett. **B397**, 275 (1997).

[16] A. Khodjamirian, R. Rückl, G. Stoll and D. Wyler, WUE-ITP-97-001, hep-ph/9702318.

[17] Z. Ligeti, L. Randall and M.B. Wise, CALT-68-2097, hep-ph/9702322.

[18] A.K. Grant, A.G. Morgan, S. Nussinov and R.D. Peccei, UCLA/97/TEP/5, hep-ph/9702380.

[19] M. Neubert, Phys. Rev. **D49**, 4623 (1994); J. Chay and S.-J. Rey, Z. Phys. **C68** (1995) 425.

[20] A. Ali and C. Greub, Phys. Lett. **B361**, 146 (1995).

[21] M.S. Alam *et al.* (CLEO Collaboration), Phys. Rev. Lett. **74**, 2885 (1995).

[22] G. D'Ambrosio, G. Ecker, G. Isidori and H. Neufeld, Phys. Lett. **B380**, 165 (1996); G. D'Ambrosio and G. Isidori, LNF-96/036(P), hep-ph/9611284.

[23] A. Ali and C. Greub, Z. Phys. **C49**, 431 (1991); *ibid.* **C60**, 433 (1993); N. Pott, Phys. Rev. **D54**, 938 (1996).

[24] M. Ciuchini, E. Franco, L. Reina and L. Silvestrini, Nucl. Phys. **B421**, 41 (1994).

[25] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

[26] A.F. Falk and M. Neubert, Phys. Rev. **D47** (1993) 2965.

[27] M. Jamin and M.E. Lautenbacher, Comput. Phys. Commun. **74**, 265 (1993).

[28] Z. Ligeti and M.B. Wise, Phys. Rev. **D53**, 4937 (1995).

[29] F. Krüger and L.M. Sehgal, Phys. Lett. **B380**, 199 (1996).

[30] Review of Particle Properties, Phys. Rev. **D54**, 1 (1996).

[31] J.-W. Chen, G. Rupak and M.J. Savage, DOE/ER/41014-10-N97, hep-ph/9705219.