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Can we extract short–distance information from $B(K_L \rightarrow \mu^+ \mu^-)$?

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Abstract

A new analysis of the long–distance two–photon dispersive amplitude of $K_L \rightarrow \mu^+ \mu^-$ is presented. We introduce a phenomenological parametrization of the $K \rightarrow \gamma^* \gamma^*$ form factor, constrained at low energies by $K_L \rightarrow \gamma l^+ l^-$ ($l = e, \mu$) data and at high energies by perturbative QCD. Using this form factor we provide a reliable estimate of magnitude and relative uncertainty of the two–photon dispersive contribution in $K_L \rightarrow \mu^+ \mu^-$. We finally discuss the implications of this analysis for the extraction of short–distance information from $B(K_L \rightarrow \mu^+ \mu^-)$.

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1 Introduction

Historically the $K_L \rightarrow \mu^+ \mu^-$ decay provided a very important tool for understanding the flavor structure of electroweak interactions [1,2] and nowadays it still represents an interesting window on short-distance dynamics. The amplitude of this process can be conveniently decomposed in to two distinct parts: a long-distance contribution generated by the two-photon intermediate state (fig. 1a) and a short-distance part that, within the Standard Model, is due to W and Z exchange (fig. 1b). The latter turns out to be dominated by the top quark and it is known to the next-to-leading order in QCD [3]. If we were able to disentangle this contribution from the measured $K_L \rightarrow \mu^+ \mu^-$ branching ratio we could extract interesting information on the Cabibbo–Kobayashi–Maskawa (CKM) matrix element V_{td} [4]. Moreover, a model-independent determination of the short-distance amplitude could be useful to put constraints on possible Standard Model extensions [5].

To fully exploit the potential of $K_L \rightarrow \mu^+ \mu^-$ in probing short-distance dynamics, it is necessary to have a reliable control on its long-distance amplitude. However, the dispersive contribution generated by the two-photon intermediate state cannot be calculated in a model-independent way and it is subject to various uncertainties [6–11]. The purpose of this paper is to re-analyze this contribution, using all available information on the $K \rightarrow \gamma^* \gamma^*$ transition and trying to evaluate the error due to the model dependent assumptions. We will introduce a new low-energy parametrization of the $K \rightarrow \gamma^* \gamma^*$ form factor in terms of two parameters α and β measurable from $K_L \rightarrow \gamma l^+ l^-$ ($l = e, \mu$) and $K_L \rightarrow e^+ e^- \mu^+ \mu^-$. Moreover, we will discuss the matching of this approach with the behavior of the form factor in perturbative QCD. Finally, using our estimate of the two-photon dispersive contribution, we will derive new bounds on the CKM parameter ρ [12] and on possible new-physics flavor-changing couplings.

The plan of the paper is as following. In section 2 we briefly discuss the general decomposition of the $K_L \rightarrow \mu^+ \mu^-$ branching ratio and the main formulae for the bounds on short-distance parameters. In section 3 we introduce our low-energy parametrization of the $K \rightarrow \gamma^* \gamma^*$ form factor, we discuss the determination of α and β and the matching with the QCD calculation. Finally, in section 4, we analyze the numerical results.

2 Decomposition of $B(K_L \rightarrow \mu^+ \mu^-)$

The $K_L \rightarrow \mu^+ \mu^-$ branching ratio can be generally decomposed in the following way

$$B(K_L \rightarrow \mu^+ \mu^-) = |\Re \mathcal{A}|^2 + |\Im \mathcal{A}|^2, \quad (1)$$

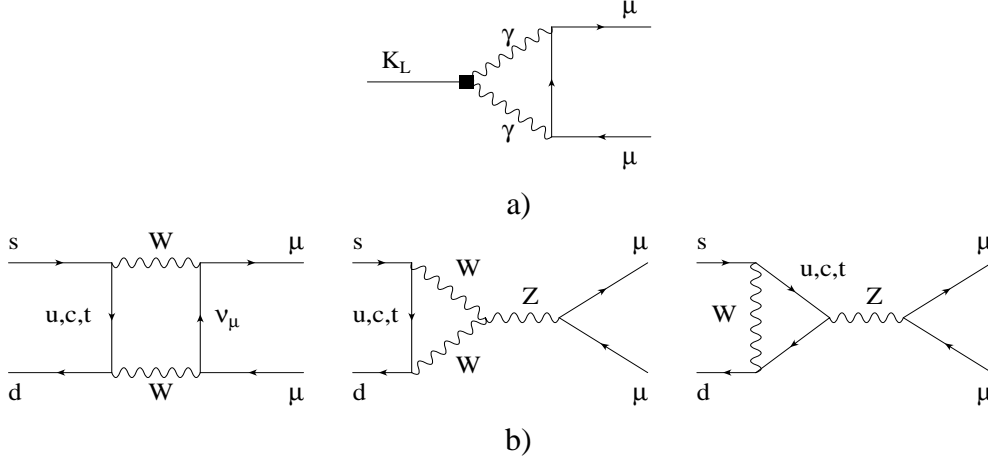


Figure 1: Long–distance (a) and lowest–order short–distance (b) contributions to $K_L \rightarrow \mu^+ \mu^-$

where $\Re\mathcal{A}$ denotes the dispersive contribution and $\Im\mathcal{A}$ the absorptive the one. The former can be rewritten as

$$|\Re\mathcal{A}| = |\Re\mathcal{A}_{long} + \Re\mathcal{A}_{short}|, \quad (2)$$

whereas the latter can be determined in a model independent way from the $K_L \rightarrow \gamma\gamma$ branching ratio¹

$$|\Im\mathcal{A}|^2 = \frac{\alpha_{em}^2 m_\mu^2}{2m_K^2 \beta_\mu} \left[\ln \frac{1 - \beta_\mu}{1 + \beta_\mu} \right]^2 B(K_L \rightarrow \gamma\gamma), \quad \beta = \sqrt{1 - \frac{4m_\mu^2}{m_K^2}}. \quad (3)$$

The recent measurement of $B(K_L \rightarrow \mu^+ \mu^-)$ [4] is almost saturated by the value of $|\Im\mathcal{A}|^2$, leaving a very small room for the dispersive contribution [4]

$$\begin{aligned} |\Re\mathcal{A}_{exp}|^2 &= B(K_L \rightarrow \mu^+ \mu^-) - |\Im\mathcal{A}|^2 = (-1.0 \pm 3.7) \times 10^{-10} \quad \text{or} \\ |\Re\mathcal{A}_{exp}|^2 &< 5.6 \times 10^{-10} \quad (90\% \text{ C.L.}). \end{aligned} \quad (4)$$

Within the Standard Model the NLO short–distance amplitude can be written as [3, 14]

$$|\Re\mathcal{A}_w|^2 = 0.9 \times 10^{-9} (1.2 - \bar{\rho})^2 \left[\frac{\overline{m}(m_t)}{170 \text{ GeV}} \right]^{3.1} \left[\frac{|V_{cb}|}{0.04} \right]^4, \quad (5)$$

¹ In principle the absorptive amplitude receives contributions also from intermediate states other than two–photons, like the two–pion one, but these are completely negligible [13].

where $\bar{\rho} = \rho(1 - \lambda/2)$ [15] and ρ, λ are the usual Wolfenstein parameters [12]. Using this result we can write

$$\bar{\rho} = 1.2 - \left| \frac{|\Re\mathcal{A}_{exp}| \pm |\Re\mathcal{A}_{long}|}{3 \times 10^{-5}} \right| \left[\frac{\overline{m}(m_t)}{170\text{GeV}} \right]^{-1.55} \left[\frac{|V_{cb}|}{0.04} \right]^{-2}, \quad (6)$$

where the sign inside the modulo is positive if $\Re\mathcal{A}_w$ and $\Re\mathcal{A}_{long}$ interfere destructively and $|\Re\mathcal{A}_w| > |\Re\mathcal{A}_{long}|$. In principle the above equation could be used to put both a lower and an upper bound on $\bar{\rho}$. However $|\Re\mathcal{A}_{exp}|$ is compatible with zero and, as we will show in the following, the same is true for $|\Re\mathcal{A}_{long}|$, thus the upper bound on $\bar{\rho}$ is useless being above unity. On the other hand, independently of the interference sign between $\Re\mathcal{A}_w$ and $\Re\mathcal{A}_{long}$, we can derive a possibly meaningful lower bound on $\bar{\rho}$

$$\bar{\rho} > 1.2 - \max \left\{ \frac{|\Re\mathcal{A}_{exp}| + |\Re\mathcal{A}_{long}|}{3 \times 10^{-5}} \left[\frac{\overline{m}(m_t)}{170\text{GeV}} \right]^{-1.55} \left[\frac{|V_{cb}|}{0.04} \right]^{-2} \right\}. \quad (7)$$

Beyond the Standard Model we can parametrize new–physics contributions as in [5], introducing a flavor–changing Zds coupling at the tree level. Using the Lagrangian

$$\mathcal{L}_{NP}^Z = \frac{g}{2 \cos \theta_w} \sum_{i \neq j} U_{ij} \bar{d}_L^i \gamma^\mu d_L^j Z_\mu, \quad (8)$$

we obtain $|\Re\mathcal{A}_{NP}| = 3.7|\Re U_{ds}|$. Then, assuming $\Re\mathcal{A}_{short} = \Re\mathcal{A}_{NP} + \Re\mathcal{A}_w$, the most conservative bound on $|\Re U_{ds}|$ is given by

$$|\Re U_{ds}| < 0.27 \max \{ |\Re\mathcal{A}_{exp}| + |\Re\mathcal{A}_{long}| + |\Re\mathcal{A}_w| \}. \quad (9)$$

3 The $K_L \rightarrow \gamma^* \gamma^*$ form factor and $\Re\mathcal{A}_{long}$

The necessary ingredient for the evaluation of $\Re\mathcal{A}_{long}$ is the construction of a suitable $K \rightarrow \gamma^* \gamma^*$ amplitude. Assuming CP conservation, gauge and Lorentz invariance implies the following general decomposition [16]

$$A(K_L \rightarrow \gamma^*(q_1, \epsilon_1) \gamma^*(q_2, \epsilon_2)) = i \epsilon_{\mu\nu\rho\sigma} \epsilon_1^\mu \epsilon_2^\nu q_1^\rho q_2^\sigma F(q_1^2, q_2^2), \quad (10)$$

where F is a symmetric function of q_1^2, q_2^2 and $|F(0, 0)|$ can be determined by the $K_L \rightarrow \gamma\gamma$ width [17]

$$|F(0, 0)| = \left[\frac{64\pi\Gamma(K_L \rightarrow \gamma\gamma)}{m_K^3} \right]^{1/2} = (3.51 \pm 0.05) \times 10^{-9} \text{GeV}^{-1}. \quad (11)$$

Using (10) we obtain

$$|\Re\mathcal{A}_{long}|^2 = \frac{2\alpha_{em}^2 m_\mu^2 \beta_\mu}{\pi^2 m_K^2} B(K_L \rightarrow \gamma\gamma) |\Re R(m_K^2)|^2, \quad (12)$$

where [18]

$$R(q^2) = \frac{2i}{\pi^2 q^2} \int d^4 l \frac{q^2 l^2 - (q \cdot l)^2}{l^2 (l - q)^2 [(l - p)^2 - m_\mu^2]} \frac{F(l^2, (l - p)^2)}{F(0, 0)} \quad (13)$$

and $p^2 = m_\mu^2$.

The structure of the $K \rightarrow \gamma^* \gamma^*$ form factor has been already discussed and parametrized in different ways in the literature [6–11]. However, all the existing analyses are inspired by model dependent assumptions and suffer from various theoretical uncertainties. In order to be as model independent as possible and to evaluate the size of the theoretical errors, we propose the following low-energy parametrization

$$f(q_1^2, q_2^2) = \frac{F(q_1^2, q_2^2)}{F(0, 0)} = 1 + \alpha \left(\frac{q_1^2}{q_1^2 - m_V^2} + \frac{q_2^2}{q_2^2 - m_V^2} \right) + \beta \frac{q_1^2 q_2^2}{(q_1^2 - m_V^2)(q_2^2 - m_V^2)}, \quad (14)$$

where α and β are arbitrary real parameters and m_V is conventionally chosen to be the ρ mass. The above expression has at least three interesting features:

1. It is the most general parametrization compatible with the chiral expansion of the $K_L \rightarrow \gamma^* \gamma^*$ amplitude up to $O(p^6)$ [16,19].
2. It includes the poles of the lowest vector meson resonances with arbitrary residues.
3. The parameters α and β , expected to be $\mathcal{O}(1)$ by naive dimensional chiral power counting, are in principle directly accessible by experiments in $K_L \rightarrow \gamma l^+ l^-$ ($l = e, \mu$) and $K_L \rightarrow e^+ e^- \mu^+ \mu^-$.

Clearly the expression (14) cannot be considered correct for arbitrary values of q_1^2 and q_2^2 . To be more general we should consider α and β as q^2 -dependent couplings. However, we believe a reasonable assumption to treat α and β as constants up to $q_1^2 \sim q_2^2 \sim 1 \text{ GeV}^2$. Moreover, being just a phenomenological description, we do not expect the form factor (14) to produce a finite result in the $K_L \rightarrow \mu^+ \mu^-$ amplitude. Indeed, using (14) and (13) we obtain

$$\begin{aligned} \Re R(m_K^2) &= -3[\ln(\Lambda/m_0) + 2\alpha \ln(\Lambda/m_\alpha) + \beta \ln(\Lambda/m_\beta)] \\ &= -3[\ln(m_\beta/m_0) + 2\alpha \ln(m_\beta/m_\alpha)] - 3(1 + 2\alpha + \beta) \ln(\Lambda/m_\beta), \end{aligned} \quad (15)$$

where

$$m_0 = 140 \text{ MeV}, \quad m_\alpha = 452 \text{ MeV}, \quad m_\beta = 806 \text{ MeV}, \quad (16)$$

and Λ is an ultraviolet cutoff. As one could expect from (13), the cutoff sensitivity of (15) is determined by the value of the combination $(1 + 2\alpha + \beta)$. Indeed, for large values of

the loop–momentum, the integrand in (13) is proportional to

$$f(l^2, l^2) \xrightarrow{l^2 \gg m_V^2} 1 + 2\alpha + \beta. \quad (17)$$

The following subsections are devoted to the determination of α and β . At first we shall analyze the experimental information coming from $K_L \rightarrow \ell^+ \ell^- \gamma$ and $K_L \rightarrow \mu^+ \mu^- e^+ e^-$. Then we will constraint the value of $(1 + 2\alpha + \beta)$ analyzing the behavior of $f(q^2, q^2)$ at large q^2 in the framework of perturbative QCD. Finally we will discuss the consistency of the previous findings with a model–dependent determination of α and β within the approach proposed in [19].

3.1 Experimental determination of α and β

As anticipated, α and β are in principle accesible by experiments in the decay $K_L \rightarrow \ell^+ \ell^- \gamma$ and $K_L \rightarrow \mu^+ \mu^- e^+ e^-$, dominated by $K_L \rightarrow \gamma \gamma^*$ and $K_L \rightarrow \gamma^* \gamma^*$ form factors respectively. The differential decay rates of $K_L \rightarrow \ell^+ \ell^- \gamma$ and $K_L \rightarrow \mu^+ \mu^- e^+ e^-$, normalized to $\Gamma_L^{\gamma\gamma} \equiv \Gamma(K_L \rightarrow \gamma\gamma)$, are given by

$$\begin{aligned} \frac{1}{\Gamma_L^{\gamma\gamma}} \frac{d\Gamma_L^{\ell^+ \ell^- \gamma}}{dq^2} &= \frac{2}{q^2} \left(\frac{\alpha_{em}}{3\pi} \right) |f(q^2, 0)|^2 \lambda^{3/2} \left(1, \frac{q^2}{m_K^2}, 0 \right) G_\ell(q^2), \quad (18) \\ \frac{1}{\Gamma_L^{\gamma\gamma}} \frac{d\Gamma_L^{\mu^+ \mu^- e^+ e^-}}{dq_e^2 dq_\mu^2} &= \frac{2}{q_e^2 q_\mu^2} \left(\frac{\alpha_{em}}{3\pi} \right)^2 |f(q_e^2, q_\mu^2)|^2 \lambda^{3/2} \left(1, \frac{q_e^2}{m_K^2}, \frac{q_\mu^2}{m_K^2} \right) G_e(q_e^2) G_\mu(q_\mu^2) \end{aligned} \quad (19)$$

where

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ac) \quad (20)$$

and

$$G_\ell(q^2) = \left(1 - \frac{4m_\ell^2}{q^2} \right)^{1/2} \left(1 + \frac{2m_\ell^2}{q^2} \right). \quad (21)$$

Present data on both $K_L \rightarrow e^+ e^- \gamma$ [20,21] and $K_L \rightarrow \mu^+ \mu^- \gamma$ [22] let us to extract useful information about the q^2 dependence of $f(q^2, 0)$. The experimental results have been analyzed up to now assuming only the form factor proposed by Bergström, Massó and Singer (BMS model) [7]. The latter depends on one unknown parameter α_K^* and, expanding in powers of q^2/m_ρ^2 , can be written as

$$f(q^2, 0)_{BMS} \simeq 1 + (1 - 3.1\alpha_K^*) \frac{q^2}{m_\rho^2} + \mathcal{O} \left((q^2)^2/m_\rho^4 \right). \quad (22)$$

The fitted values of α_K^* are given by

$$\begin{aligned} \alpha_K^* &= -0.280 \pm 0.083 \begin{matrix} +0.054 \\ -0.034 \end{matrix} [20], \\ \alpha_K^* &= -0.28 \pm 0.13 [21], \\ \alpha_K^* &= -0.028 \begin{matrix} +0.115 \\ -0.111 \end{matrix} [22], \end{aligned} \quad (23)$$

and the corresponding weighted average is

$$\alpha_K^* = -0.204 \pm 0.062 . \quad (24)$$

Comparing the BMS form factor (22) with the one proposed in (14), we obtain the following relation

$$\alpha = -1 + (3.1 \pm 0.5)\alpha_K^* , \quad (25)$$

where the error is due to the different quadratic dependence on q^2/m_ρ^2 . Then, using (24) we find

$$\alpha = -1.63 \pm 0.22 . \quad (26)$$

As already pointed out in [19], it must be stressed that an improved determination of α would be possible if present data were not analyzed assuming only the BMS model.

Contrary to α , the experimental determination of β is much uncertain. In principle the $K_L \rightarrow e^+e^-\mu^+\mu^-$ rate should be sensitive, in the region where both dilepton pairs have a large invariant mass, to the higher structure in momenta carried by the β component of the form factor. However, the real sensitivity of this process to β is rather small. Thus, even if the first evidence for $K_L \rightarrow e^+e^-\mu^+\mu^-$ has been recently reported [23], it is unlikely that β will be measured with a reasonable accuracy in the short term.

3.2 Perturbative evaluation of $f(q^2, q^2)$

In the limit $q_1^2 = q_2^2 = q^2 \gg m_K^2$ we can simply evaluate the form factor within perturbative QCD. At the lowest order in α_s , the only diagrams that contribute to $f(q^2, q^2)$ are those shown in fig. 2 [6]. Neglecting masses and momenta of the external quarks, as well as the contribution of the top quark inside the loop (suppressed by CKM factors [11]), the result can be written as

$$f^{QCD}(q^2, q^2) = N_F \left[g_u \left(\frac{q^2}{4m_u^2} \right) - g_c \left(\frac{q^2}{4m_c^2} \right) \right] , \quad (27)$$

where

$$g_q(r) = -r \frac{d}{dr} J(r) + \left[\frac{1+2r}{6r} J(r) + \frac{1}{3} \ln \frac{M_W^2}{m_q^2} \right] \quad (28)$$

and

$$J(r) = \begin{cases} -2\sqrt{1/r-1} \arctan \sqrt{r/(1-r)} + 2 & 0 < r < 1 , \\ \sqrt{1-1/r} \left(\ln \frac{1-\sqrt{1-1/r}}{1+\sqrt{1-1/r}} + i\pi \right) + 2 & r > 1 . \end{cases} \quad (29)$$

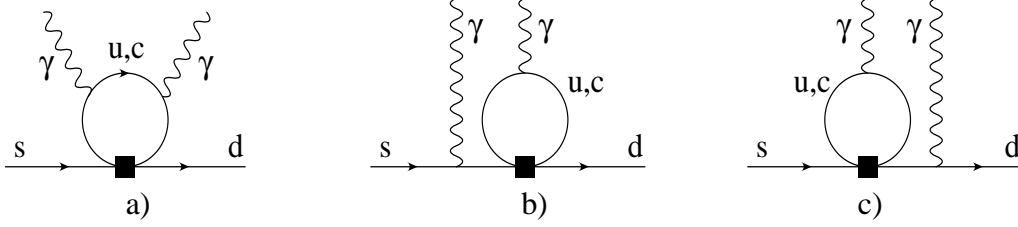


Figure 2: Lowest-order quark diagrams that contribute to the $K_L \rightarrow \gamma^* \gamma^*$ transition (any diagram is understood with the corresponding crossing-photon term).

The normalization factor of (27) is given by

$$|N_F| = \frac{16}{9} \frac{\lambda G_F F_\pi \alpha_{em}}{|F(0,0)| \pi \sqrt{2}} \simeq 0.20, \quad (30)$$

where λ denotes the sine of the Cabibbo angle [12] and $F_\pi \simeq 93$ MeV the pion decay constant. The first term in (28) is the contribution of the diagram in fig. 2a, whereas the second one is originated by the irreducible graphs in fig. 2b–c. We have neglected all the contributions independent from quark masses that cancels via the GIM mechanisms and, whenever possible, we have consider the limit $M_W \rightarrow \infty$ (this is always possible except for the $\ln(M_W^2/m_q^2)$ term originated by the reducible diagrams).

From the above equations it follows

$$|\Re e f^{QCD}(q^2, q^2)| = |N_F| \begin{cases} \mathcal{O}(m_c^2/q^2) & q^2 \gg 4m_c^2, \\ \frac{14}{9} + \frac{1}{3} \ln \frac{m_c^2}{q^2} & 4m_u^2 \ll q^2 \ll 4m_c^2. \end{cases} \quad (31)$$

Using this approximate expression in (13) and keeping in the final result only the dominant $\ln(m_c^2/m_u^2)$ terms, leads to the approximate formula of Voloshin and Shabalin for $\Re e \mathcal{A}_{long}$ [6]. This result indicates that the long-distance dispersive amplitude of $K_L \rightarrow \mu^+ \mu^-$ is very small, however it cannot be trusted in detail since the low q^2 limit of $f^{QCD}(q^2, q^2)$ is completely out of control in perturbative QCD. A more detailed analysis of $\Re e \mathcal{A}_{long}$ at the quark level has been recently presented in [11], where the leading QCD correction have been estimated. Nonetheless, also the final result of [11] cannot be considered fully conclusive since an arbitrary infrared cutoff is introduced in order to avoid the dangerous low q^2 region.

As anticipated, our strategy is to use $f^{QCD}(q^2, q^2)$ to fix the high q^2 behavior of the low-energy parametrization (14). The simplest requirement that we can derive from (27) is that $f(q^2, q^2)$ must vanish for $q^2 \gtrsim 4m_c^2$. This condition can be implemented in the

phenomenological expression (15) in two ways: in a weak sense, assuming

$$\Lambda^2 \lesssim 4m_c^2, \quad (32)$$

or in a strong one, imposing the ‘‘sum–rule’’

$$1 + 2\alpha + \beta = 0. \quad (33)$$

To be conservative we will use only the weak bound in (32), the strong one would have been correct only if the low energy parametrization (14) was valid also above the charm threshold. A more realistic constraint on $|1 + 2\alpha + \beta|$ can be obtained imposing the matching between (14) and (27) for $\Lambda_{QCD} \ll q^2 \ll 4m_c^2$. In this case from the second line of (31) we obtain

$$|1 + 2\alpha + \beta| \simeq \frac{14}{9}|N_F| \simeq 0.3. \quad (34)$$

Interestingly, this result suggest that the sum–rule (33) is violated only in a mild way below the charm threshold. We recall, for comparison, that naive dimensional arguments could not exclude values of $|1 + 2\alpha + \beta|$ one order of magnitude larger than in (34). The smallness of $|1 + 2\alpha + \beta|$ is further supported by the leading QCD correction to f^{QCD} . Indeed, as discussed in [6,11], the main effect of these correction is an overall multiplicative factor smaller than one.

Combining (32) and (34), we believe that a realistic bound for the last term in (15) is given by

$$|1 + 2\alpha + \beta| \ln(\Lambda/m_\beta) < 0.4. \quad (35)$$

We finally note that is not possible to fix the absolute sign of $(1 + 2\alpha + \beta)$ in the framework of perturbative QCD. Indeed, since we do not trust the low q^2 limit of the perturbative calculation, we are not able to fix the relative sign between the un–normalized form factor ($F^{QCD}(q^2, q^2)$) and the $K_L \rightarrow \gamma\gamma$ amplitude ($F(0, 0)$).

3.3 Determination of α and β in the FMV model

A more precise, but also more model–dependent, determination of α and β can be achieved within specific hadronization models. The Factorization Model in the Vector couplings (FMV) was proposed in [19] as a framework to compute the factorizable contributions to weak vertices involving vector mesons. This model was proven to be efficient in achieving a satisfactory joint description of the vector meson exchange contributions to $K \rightarrow \pi\gamma\gamma$ and $K_L \rightarrow \gamma\gamma^*$, giving a slope parameter $\alpha_{FMV} \simeq -1.22$ quite near to the phenomenological determination in (26). The application of this model to the construction of the $K_L \rightarrow \gamma^*\gamma^*$ vertex through vector meson dominance and, consequently, the

Pseudoscalar–Vector–Vector (PVV) weak vertex is straightforward and gives (assuming only octet contributions)

$$\beta_{FMV} = \frac{256\pi}{3\sqrt{2}} \frac{G_8 \alpha_{em} m_V^2}{F_\pi |F(0,0)|} f_V^3 h_V \eta \simeq 1.43, \quad (36)$$

where f_V is fixed from $\Gamma(\rho^0 \rightarrow e^+e^-)$ to be $|f_V| \simeq 0.20$, $\Gamma(\omega \rightarrow \pi^0\gamma)$ gives $|h_V| \simeq 0.037$ and $m_V = m_\rho$. Moreover, $G_8 \simeq 9.2 \times 10^{-6} GeV^{-2}$ is the effective coupling of the octet $\mathcal{O}(p^2)$ weak chiral lagrangian determined from $K \rightarrow \pi\pi$, and $\eta \simeq 0.21$ was fixed in [19] from the weak $VP\gamma$ vertex. Note that the experimental value of the $\pi^0 \rightarrow \gamma\gamma^*$ slope implies $f_V h_V > 0$, thus the sign of β (and α) is completely determined by the one of $A(K_L \rightarrow \gamma\gamma)$. The sign in (36) is chosen to be positive by consistency with the sign of α (fixed to be negative by the experimental data).

Combining the predictions of α and β in the FMV model we get

$$1 + 2\alpha_{FMV} + \beta_{FMV} = -0.01. \quad (37)$$

This result is perfectly consistent with the QCD bound in (35).²

4 Numerical results

The theoretical bound on $(1 + 2\alpha + \beta) \ln(\Lambda/m_\beta)$ in (35), together with the experimental determination of α in (26), let us to estimate $|\Re e \mathcal{A}_{long}|$ by means of (12) and (15). In order to combine the two information we must assume a statistical distribution for $(1 + 2\alpha + \beta) \ln(\Lambda/m_\beta)$. Assuming for the latter a flat distribution between -0.4 and $+0.4$, and combining it with the gaussian distribution of α , we find

$$|\Re e \mathcal{A}_{long}| < 2.9 \times 10^{-5} \quad (90\% \text{ C.L.}). \quad (38)$$

The same result is obtained assuming for $(1 + 2\alpha + \beta) \ln(\Lambda/m_\beta)$ a gaussian distribution with central value 0 and $\sigma = 0.8/\sqrt{12}$ (the σ of the original flat distribution). However, in this case one can distinguish better the various contributions to the limit (38). Indeed we find

$$\begin{aligned} |\Re e \mathcal{A}_{long}| &= \left[\frac{2\alpha_{em}^2 m_\mu^2 \beta_\mu B(K_L \rightarrow \gamma\gamma)}{\pi^2 m_K^2} \right]^{1/2} \left| 5.25 + 3.47\alpha + 3(1 + 2\alpha + \beta) \ln \frac{\Lambda}{m_\beta} \right| \\ &= 1.61 \times 10^{-5} \times |0.41 \pm 0.76 \pm 0.69| = |0.66 \pm 1.65| \times 10^{-5}. \end{aligned} \quad (39)$$

² In [19] it was shown that a better estimate of the $K_L \rightarrow \gamma\ell^+\ell^-$ slope could be obtained adding to the FMV prediction a contribution generated by weak Vector–Vector transition (the main ingredient of the BMS model). However the two kind of contributions have a different momentum structure and cannot be consistently added at large q^2 , i.e. in the region where we are interested in the value of $(1 + 2\alpha + \beta)$.

Interestingly, the knowledge of the absolute sign of the central value of $\Re e \mathcal{A}_{long}$ (i.e. the relative sign between short and long distance) is not very important at this stage, given the large value of the error in (39). Moreover, at present the largest source of uncertainty is generated by the experimental error on α , thus a substantial improvement could be foreseen with the next generation of high-precision experiments in kaon decays.

Having derived a numerical estimate for $|\Re e \mathcal{A}_{long}|$ we are finally able to extract some short distance information from the measured value of $B(K_L \rightarrow \mu^+ \mu^-)$:

1. Bound on $\bar{\rho}$. Using the Bayesian prescription of the Particle Data Group [17], we construct a statistical distribution for $|\Re e \mathcal{A}_{exp}|$ that eliminates the unphysical values. Then, combining it with the gaussian distribution of $\Re e \mathcal{A}_{long}$ discussed above, we obtain a distribution function for $(|\Re e \mathcal{A}_{exp}| + |\Re e \mathcal{A}_{long}|)$. Finally, using this distribution in (7), together with $\bar{m}(m_t) = (167 \pm 6)$ GeV and $|V_{cb}| = (0.04 \pm 0.003)$ [14], we find

$$\bar{\rho} > -0.38 \quad \text{or} \quad \rho > -0.42 \quad (90\% \text{ C.L.}) . \quad (40)$$

2. Bound on $|\Re e U_{ds}|$. Similarly to the previous case we can derive a bound on $|\Re e U_{ds}|$ by means of Eq. (9). Treating also $|\Re e \mathcal{A}_w|$ as a statistical variable (assuming a flat distribution for ρ between -1 and $+1$) we find

$$|\Re e U_{ds}| < 2.1 \times 10^{-5} \quad (90\% \text{ C.L.}) . \quad (41)$$

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