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PROTON ELECTROMAGNETIC TIME-LIKE FORM FACTORS UPDATED AT THRESHOLD

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Abstract

A revised expression for the electromagnetic form factor of the proton in time-like region $(q^2 > 0)$ near threshold is given. Recent data on $p\bar{p}$ annihilation cross section as well as the well-known $p\bar{p}$-branching into e^+e^--pairs from atomic states are used to extract the value of form factor at threshold $(s = q^2 = 4m_p^2)$. At this point $|G_p(4m_p^2)| = 0.41^{+0.02}_{-0.08}$.

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1 Introduction

The proton electromagnetic form factor in the time-like region is usually obtained studying the cross section on the reactions $e^+e^- \to p\bar{p}$ and $p\bar{p} \to e^+e^-$[1]-[6]. At low energy (i.e. for antiproton momentum $\leq 300\ MeV/c$ in the labnoratory system) a precise measurement of the form factor has been performed at LEAR [2,3]. At these energies, the procedure used in [2,3] to extract the value of the form factor is not straightforward, as the antiproton flux was not directly measured; these values have been obtained using data on $p\bar{p}$ total cross section as well as partial cross sections into $\pi^+\pi^-$ and K^+K^-.

Recently, a new precise measurement of $p\bar{p}$ annihilation cross section has been performed by the OBELIX collaboration [7] at very low energies ($p_{lab} \sim 50 MeV/c$). These data can be used to get the value of the proton electromagnetic form factor at threshold. In a previous paper [8] we studied the form factor at threshold using preliminary data from OBELIX [9]. In this paper we take into account both the recent improved data on $p\bar{p}$ annihilation cross section [7] some information on $p\bar{p}$ atomic cascade in liquid hydrogen LH_2 [10].

2 The main equation

In this section we study the basic expression for the proton electromagnetic form factor at rest.

Let us start looking at the reaction $p\bar{p} \to e^+e^-$ in flight. The corresponding Feynman diagram is shown in Fig. 1. The cross section of this process is related to the proton form factor, however, to get its value at rest the authors of papers [2,3] were forced to use some addition information about $(p\bar{p})$ atom. In the following, we demonstrate that the relation between atomic data and the proton form factor is not as simple as one may think.

Let us define the branching ratio into e^+e^- channel in flight,

$$Br_{e^+e^-}(k) = \frac{\sigma^{e^+e^-}(k)}{\sigma^{ann}(k)}, \tag{1}$$

where $\sigma^{e^+e^-}(k)$ is partial annihilation cross section at c.m. antiproton momentum k and σ^{ann} is the total one. It is suitable to rewrite this expression in terms of singlet (σ_s) and triplet (σ_t) annihilation cross sections. As the reaction $p\bar{p} \to e^+e^-$ goes through the triplet $^3S_1$ state, we get:

$$Br_{e^+e^-}(k) = \frac{\frac{3}{4}\sigma_t^{e^+e^-}}{\frac{3}{4}\sigma_t^{ann} + \frac{1}{4}\sigma_s^{ann}} = \frac{\frac{3}{4}Br_{e^+e^-}(k, trip)}{\frac{3}{4} + \frac{1}{4}\frac{\sigma_s^{ann}}{\sigma_t^{ann}}}, \tag{2}$$

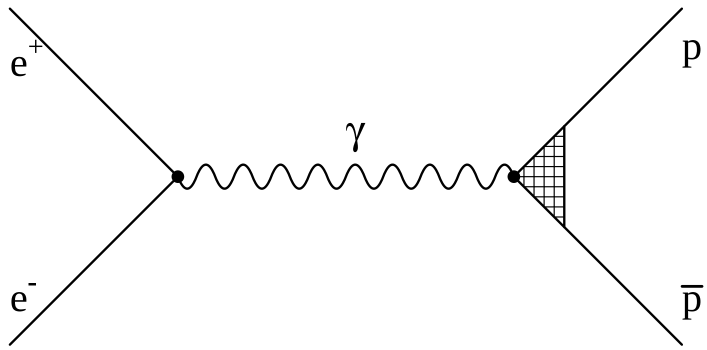

Figure 1: Feynman diagrams for the amplitude of the reaction $p\bar{p} \to e^+e^-$.

where $Br_{e^+e^-}(k, trip)$ is the branching ratio into e^+e^- from the pure triplet state. The PS-170 collaboration [2,3] calculated this ratio using measurements with the formation of the $(p\bar{p})$ atoms in liquid hydrogen (LH_2 target). Let us get the expression for this branching ratio in terms of the observables for the $(p\bar{p})$ atom.

The number of e^+e^- events from atomic S-levels is related to the number of captured antiprotons $N_{\bar{p}}$ by the following expression:

$$N_{e^+e^-}(atom) = N_{\bar{p}}(1 - f(P))w_t Br_{e^+e^-}(trip), \tag{3}$$

where $f(P)$ is the fraction of captured antiproton which annihilate from P-states, and w_t is the relative population of $^3S_1$ state in $(p\bar{p})$ atom. If the population of S-levels is purely statistical, $w_t = 3/4$; however the exact value of w_t depends on the dynamic of absorption from P-states. In a recent paper [10] the relation between w_t and the target density was studied using the the potential approach. Using the same notation of the paper we can write

$$w_t = (3/4)E_t,$$

where E_t is the so called enchancement factor.

Regarding the value of $f(P)$, the situation is not absolutely clear. According to ref. [11]

$$f(P) = 0.085 \pm 0.015,$$

but, taking into account the $\pi^0\pi^0$ branching ratio, measured by the Cristal Barrel Collaboration [12], the value of $f(P)$ should be much larger, $f(P) = 0.27 \pm 0.02$. However, when the enchancement factor is taken into account, $f(P)$ decreases and, according to [10], its value is:

$$f(P) = 0.13 \pm 0.04. \tag{4}$$

We shall use this value to extract the value of the form factor at threshold.
Using eqs. (2), (3) and (4), we get

$$\sigma^{e^+e^-}(k) = \sigma^{ann}(k)\frac{N_{e^+e^-}(atom)}{N_{\bar{p}}}\frac{1}{E_t(1-f(P))}\frac{1}{\frac{3}{4}+\frac{1}{4}\frac{\sigma_s^{ann}}{\sigma_t^{ann}}}. \tag{5}$$

The value of the proton electromagnetic form factor at threshold $|G(0)| = |G_E(0)| = |G_M(0)|$ is related to the annihilation cross section into e^+e^- by the following equation [2,3]:

$$\frac{\pi\alpha^2|G(0)|^2}{2m_p} = \lim_{k\to 0}[\frac{k\sigma^{e^+e^-}(k)}{C^2(k)}], \tag{6}$$

where

$$C^2(k) = \frac{2\pi}{ka_B}/[1-\exp(-\frac{2\pi}{ka_B})]$$

is the Gamov factor and $a_B = 57.6\ fm$. We get:

$$\frac{\pi\alpha^2|G(0)|^2}{2m_p} = \lim_{k\to 0}[\frac{k\sigma^{ann}(k)}{C^2(k)}]\frac{N_{e^+e^-}S_F}{N_{\bar{p}}E_t(1-f(P))}, \tag{7}$$

where S_F is the spin factor

$$S_F = \lim_{k\to 0}[3/4 + 1/4\sigma_s/\sigma_t]^{-1}.$$

The right side of this equation is finite and may be used to extract $|G(0)|$. Writing the partial cross sections σ_s^{ann} and σ_t^{ann} in terms of the corresponding Coulomb nuclear scattering length $A_{s,t}^{CS}$,

$$\lim_{k\to 0}[\frac{k\sigma_{s,t}^{ann}}{C^2(k)}] = \frac{4\pi ImA_{s,t}^{CS}}{(1+4\pi Im_{s,t}^{CS}/a_B)}. \tag{8}$$

The Coulomb-nuclear scattering length A^{CS} is related to the shift ΔE and the width Γ of the $1S$-level for the $(p\bar{p})$ atom through the relation [13]:

$$\Delta E - i\Gamma/2 = -2(A^{CS}/a_B)E_C(1+3.1544A^{CS}/a_B), \tag{9}$$

where $E_C = 25\ keV$ is the Coulomb energy and $ImA^{CS} > 0$ by definition. Using eqs. (8) and (9), the value of the spin factor S_F can be estimated.

Equation (7) is similar to the one used in [2,3] to extract the value of form factor at rest, but for the enchancement factor E_t, which is absent in the corresponding formula of the papers [2,3]. Furthermore, spin factor S_F differs from that is given in the refs. [2,3]. Namely, eqs. (8) and (9) reproduce the relationship between Γ, ΔE and A^{CS} with second order corrections in powers of the ratio A^{CS}/a_B.

Notice that in the experiment [2,3] the total number of captured antiprotons $N_{\bar{p}}$ was not measured directly, but was determined according to:

$$N_{\bar{p}} = \frac{N_{\pi^+\pi^-}}{Br_{\pi^+\pi^-}(atom)}, \tag{10}$$

where $N_{\pi^+\pi^-}$ is the total number of $\pi^+\pi^-$ pairs following $p\bar{p}$ annihilation at rest and $Br_{\pi^+\pi^-}(atom)$ is the atomic branching ratio into $\pi^+\pi^-$ final state. Using eq. (10), we get:

$$\frac{\pi\alpha^2|G(0)|^2}{2m_p} = \lim_{k\to 0}[\frac{k\sigma^{ann}(k)}{C^2(k)}]\frac{N_{e^+e^-}}{N_{\pi^+\pi^-}}\frac{Br_{\pi^+\pi^-}(atom)(1-\delta)S_F}{E_t\ (1-f(P))}\frac{F_h < \epsilon_{\pi^+\pi^-}(0) >}{< \epsilon_{e^+e^-}(0) >}. \tag{11}$$

In this equation the factor $(1-\delta)$ is introduced. It takes into account the radiative corrections to the process $p\bar{p} \to e^+e^-$ and it is the same introduced in [2,3]. We also introduced in (11) the experimental effectivities for registration of e^+e^- and $\pi^+\pi^-$ pairs $< \epsilon_{e^+e^-} >$ and $< \epsilon_{\pi^+\pi^-} >$ and the reduction factor F_h in agreement with the notations of the papers [2,3].

3 Numerical results

Equation (11) has been used to get the value of form factor $|G(0)|$ at threshold. We used the experimental numbers for $N_{e^+e^-}$ and $N_{\pi^+\pi^-}$ as well as the quantities F_h, $< \epsilon_{e^+e^-} >$ and $< \epsilon_{\pi^+\pi^-} >$ from the papers [2,3].

The leading correction to the value of $|G(0)|$ comes from the factor

$$\lim_{k\to 0}[\frac{\beta\sigma^{ann}}{C^2(k)}]. \tag{12}$$

Nowdays we can use the direct data on $p\bar{p}$ annihilation cross section at extremely low energies, measured by OBELIX collaboration [7], instead of the extrapolation from high energy data, as in [2,3]. The experimental data for $\beta\sigma^{ann}/C^2(k)$ are shown in the Fig. 2. Here $\beta = p_{lab}/m_p = 2k/m_p$. Lines in Fig. 2 are theoretical χ^2 fits to these data. A reasonable interval for the quantity (12) is in the limits 32-34 mb, corresponding to

$$\lim_{k\to 0}[\frac{k\sigma^{ann}}{C^2(k)}] = 15 - 16(mb\ GeV/c).$$

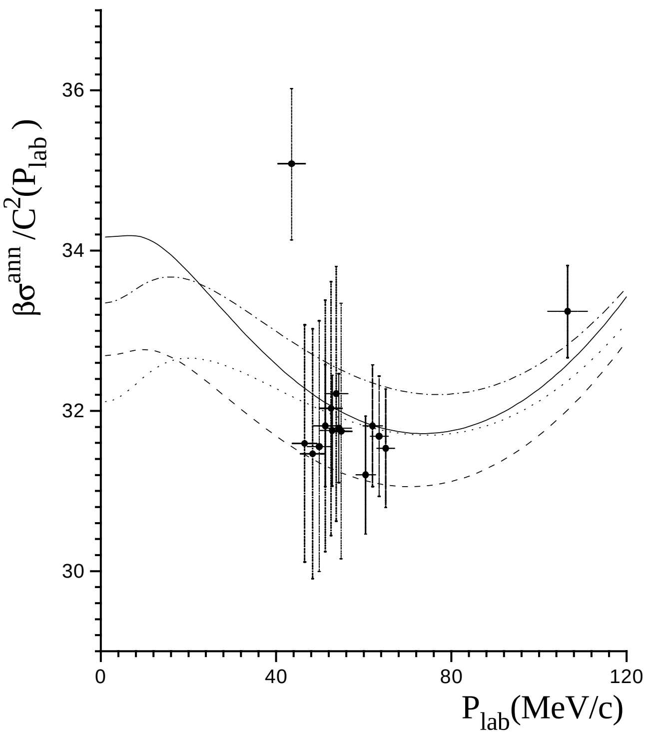

Figure 2: Experimental results for the $p\bar{p}$ annihilation cross section. The lines correspond to theoretical χ^2 fits to this cross section under different reasonable hypotheses on $ReA_{p\bar{p}}$.

This number must be compared with 26.9 mb GeV/c, used in refs. [2,3] to estimate $|G(0)|$.

The second essential contribution to the value of the form factor at rest comes from the spin factor S_F. In Fig. 3 our fits to the existing experimental data on annihilation cross section under different hypothesis for the ratio ImA_s^{CS}/ImA_t^{CS} are shown. The curves 1-4 in this figure reproduce reasonably the total annihilation cross section. According to these solutions the quantity

$$R_F \equiv S_F \lim_{k\to 0}[\frac{k\sigma^{ann}}{C^2(k)}]$$

has the following boundaries:

$$11.2 < R_F < 17.2(mb\, GeV/c).$$

These variations of spin factor correspond to variations of the ratio Γ_s/Γ_t in the range $0.5 \leq \Gamma_s/\Gamma_t \leq 3$.

Another way to estimate the uncertainties coming from the spin factor is to use direct information on width of $\Gamma(^3S_1)$ state of the $(p\bar{p})$ atom. According to the results of

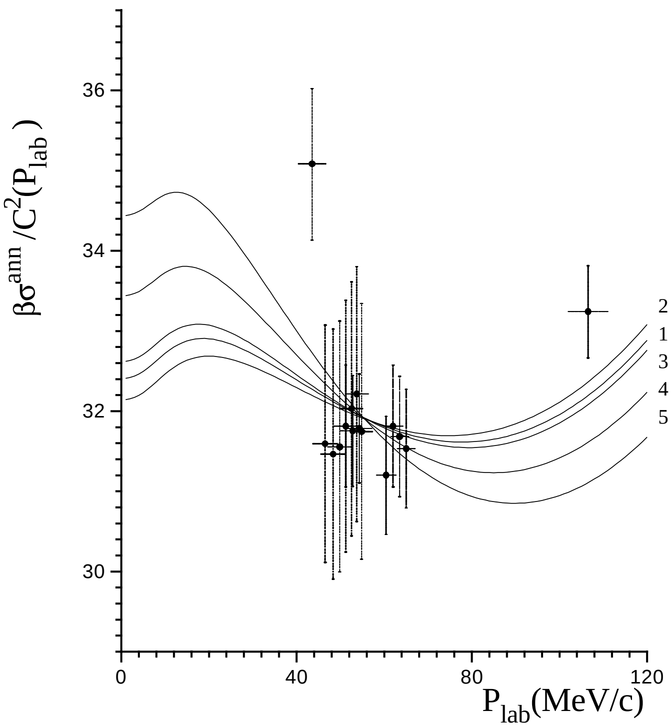

Figure 3: Experimental $p\bar{p}$ annihilation cross section. The curves 1-5 correspond to the χ^2 fit to this cross section under different hypotheses for ratio ImA_s^{CS}/ImA_t^{CS}.

the experiment [14],

$$\Gamma(^3S_1) = (708 \pm 160 \pm 10)eV.$$

Using equation (9), we get $ImA_t^{CS} = 0.45\ fm$ at $ReA_t^{CS} = -0.83\ fm$. Using the experimental value [7] for σ^{ann} at $p_{lab} = 50\ MeV/c$, we get the value of ImA_s^{CS}:

$$ImA_s^{CS} \approx 2\ fm.$$

This solution corresponds to the curve 5 in the Fig. 3 and we get from this solution the lowest value for $|G(0)| = 0.33$.

To obtain the solution for $|G(0)|$, we use $Br_{\pi^+\pi^-}(LH_2) = (3.07 \pm 0.13)10^{-3}$ according to [15]. The value $f(P) = 0.13$ and $E_t = 0.98$ have been used according to [10].

Taking into account all these factors, we get:

$$|G(0)| = 0.41^{+0.02}_{-0.08}$$

instead of the result of [2]:

$$|G(0)| = 0.53 \pm 0.02^{+0.04}_{-0.06}.$$

4 Conclusion

The resulting dependence of $|G(s)|$ on s for several lowest points is shown in Fig. 4. Black points are taken from the refs. [2,3], the circle point at threshold is the result of this analysis. Solid curve in Fig. 4 is drawn by hand to link the experimental points of refs. [2,3]; the dashed one is the result of χ^2 fit to the four lowest points including our point at threshold. The dotted line is the result of a vector dominance model (VDM) calculation[16].

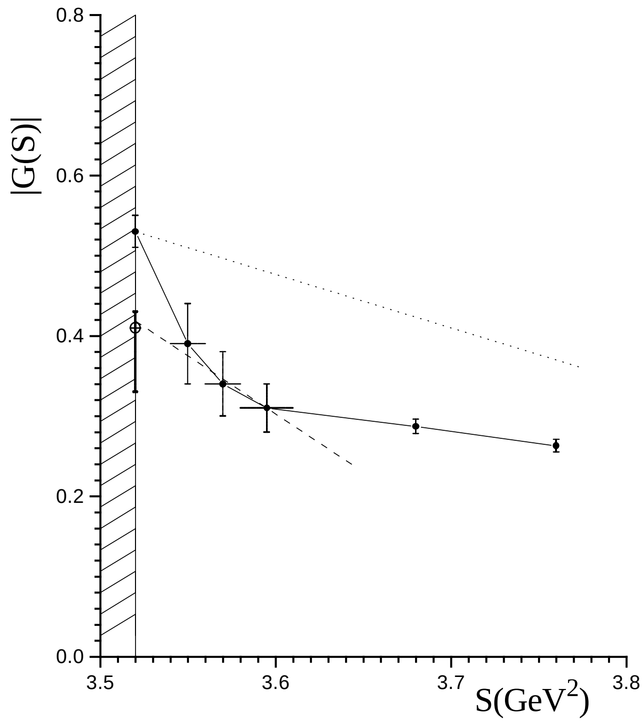

Figure 4: Proton form factor in time-like region. The solid line goes through the experimental points of ref.[2,3]. The dashed line shows the slope of the form factor with the updated point at threshold beeing included. The dotted line demonstrates the VDM prediction for the form factor slope.

Taking into account our solution for the electromagnetic form factor at rest, we conclude that the slope of form factor is getting not so sharp as it was stressed in the refs. [2,3]. As shown in Fig. 4, the slope of the form factor at threshold is $-1.41\ GeV^{-1}$, to be compared with $-4.8\ GeV^{-1}$ from old analysis in papers [2,3].

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