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A Status Report Concerning Theoretical Predictions for Various Kaon Decays

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Abstract

A short overview of theoretical predictions for various kaon decays is presented. Particular attention is devoted to pure and radiative nonleptonic decays in the framework of Chiral Perturbation Theory. The relevance of KLOE's future results [1, 2] to improve our knowledge of kaon physics and more generally of the Standard Model at low energy is also emphasized.

1 Introduction

Kaon decays offer a unique possibility to test strong, weak and electromagnetic interactions (i.e. the Standard Model) at low energies. At the same time, through flavour–changing neutral current and CP-violating processes, kaon decays are sensitive to new physics up to the TeV scale. The interest and the variety of such decays (see e.g. Tab. 1) is definitely too large to be covered in this talk and we refer to some recent reviews [2, 3, 4, 5, 6] for a comprehensive analysis.

Table 1: Incomplete list of what we can learn from kaon decays. $\mathcal{L}_S$ and $\mathcal{L}_W^{|\Delta S|=1}$ denote strong and weak nonleptonic chiral Lagrangians, respectively.

channel	$\mathcal{L}_S$	$\mathcal{L}_W^{|\Delta S|=1}$	other (CP, CPT, U_{CKM})
$l\nu\gamma$ $l\nu e^+e^-$	determination of $L_9 + L_{10}$		bounds on tensor couplings
$\pi l\nu$	determination of λ_+ and λ_0		bounds on t. c., $\mathcal{T}$. CPT tests
$\pi l\nu\gamma$	test of the WZW sector		
$\pi\pi l\nu\gamma$	$< 0|\bar{q}q|0 >$ ($\pi\pi$ phase shifts)		
2π		$O(p^4)$ operators	ϵ, ϵ'/ϵ, CPT tests
3π	3π phase shifts ($K_{L,S} \to \pi^+\pi^-\pi^0$)	$O(p^4)$ operators	$\delta g/g$, ϵ'_{000}, ϵ'_{+-0}
$\gamma\gamma$		$O(p^6)$ operators	$\epsilon'_\perp$, $\epsilon'_{\parallel}$
$\pi\gamma\gamma$		$O(p^4) + O(p^6)$ operators, unitarity corrections	intermediate role in $K \to \pi e^+e^-$
$2\pi\gamma$ $3\pi\gamma$		$O(p^4)$ operators, weak anomalous sector	$\epsilon'_{+-\gamma}$, $\delta\Gamma/\Gamma$
$\mu^+\mu^-$ $e^+e^-\gamma(\gamma^*)$		$O(p^6)$ operators	$|V_{td}V_{ts}^*|$ from $K_L \to \mu^+\mu^-|_{SD}$
πe^+e^-		$O(p^4)$ operators	CP-dir. in $K_L \to \pi^0 e^+e^-$
$\pi\nu\bar{\nu}$			$|V_{td}V_{ts}^*|$ ($K^+ \to \pi^+\nu\bar{\nu}$) CP-dir. ($K_L \to \pi^0\nu\bar{\nu}$)

The natural tool to analyze the Standard Model at low energies is Chiral Perturbation Theory [7] (CHPT), in its $SU(3)$ version if we are interested in processes involving the strange quark. Within this framework kaon decays play a twofold role. On one side semileptonic transitions let us to explore the strong sector of the chiral Lagrangian, answering to fundamental questions like the chiral behaviour of the quark condensate. On the other side nonleptonic and radiative decays help us to understand the chiral realization of the weak four–quark effective hamiltonian, unravelling possibly short–distance effects.

In this talk we concentrate on nonleptonic processes. Few interesting topics in semilepto-

nic decays are just mentioned whereas the problem of CP violation in $K \to 2\pi$ is completely omitted. Both these subjects are discussed elsewhere in these proceedings [7, 8, 9].

2 Semileptonic decays

The $SU(3)$ version of CHPT is based on the assumption that the $SU(3)_L \times SU(3)_R$ symmetry of the QCD Lagrangian in the chiral limit ($m_u = m_d = m_s$) is spontaneously broken and that the corresponding Goldstone modes can be identified with the octet of light pseudoscalar mesons (π, K, η_8). The approach is then to build the most general Lagrangian consistent with the $SU(3)_L \times SU(3)_R$ symmetry in terms of the Goldstone boson fields, and add to it the soft–breaking terms induced by quark masses. Such a Lagrangian is not renormalizable and in principle contains an infinite number of arbitrary constants. Nevertheless, if we are interested in low energy processes we can expand the transition amplitudes up to a given order in powers of pseudoscalar masses and momenta and consider only a finite number of such constants.

Table 2: Occurrence of the low–energy coupling constants $L_1, \ldots, L_{10}$ and of the anomaly in kaon semileptonic decays [11]. In $K_{\mu 4}$ decays, the same constants as in the electron mode (displayed here) occur. In addition, L_6 and L_8 enter in the channels $K^+ \to \pi^+\pi^-\mu^+\nu_\mu$ and $K^+ \to \pi^0\pi^0\mu^+\nu_\mu$.

	$K_{l2\gamma}$	K_{l2ll}	K_{l3}	$K_{l3\gamma}$	$K^+ \to$ $\pi^+\pi^-e^+\nu_e$	$K^+ \to$ $\pi^0\pi^0e^+\nu_e$	$K^0 \to$ $\pi^0\pi^-e^+\nu_e$
L_1					×	×	
L_2					×	×	
L_3					×	×	×
L_4					×	×	
L_5					×	×	×
L_9		×	×	×	×	×	×
$L_9 + L_{10}$	×	×		×			
Anomaly	×	×		×	×		×

Gauging the global symmetry $SU(3)_L \times SU(3)_R$ leads to describe, in terms of the 'strong chiral Lagrangian, also semileptonic transitions at $O(G_F)$. The relevance of these processes for the determination of the constants L_i appearing in the $O(p^4)$ (next–to–leading order) chiral Lagrangian [10] is shown in Tab. 2 [11]. Since to date all the L_i have been determined phenomenologically, the picture of semileptonic decays is complete and offers the possibility of precise and interesting tests of the Standard Model.

Among the various tests it is worthwhile to mention at least two examples of particular interest for KLOE: the determination of the scalar form–factor λ_0 in $K_{\mu 3}$ and the determination of $\pi\pi$ phase shifts near threshold in K_{e4} (Figs. 1-2). In both cases there are firm CHPT predictions whereas the present experimental situation is not clear [12]. In the case of $\pi\pi$ phase shifts, recently calculated up to two loops, i.e. at $O(p^6)$, both in CHPT [14] and
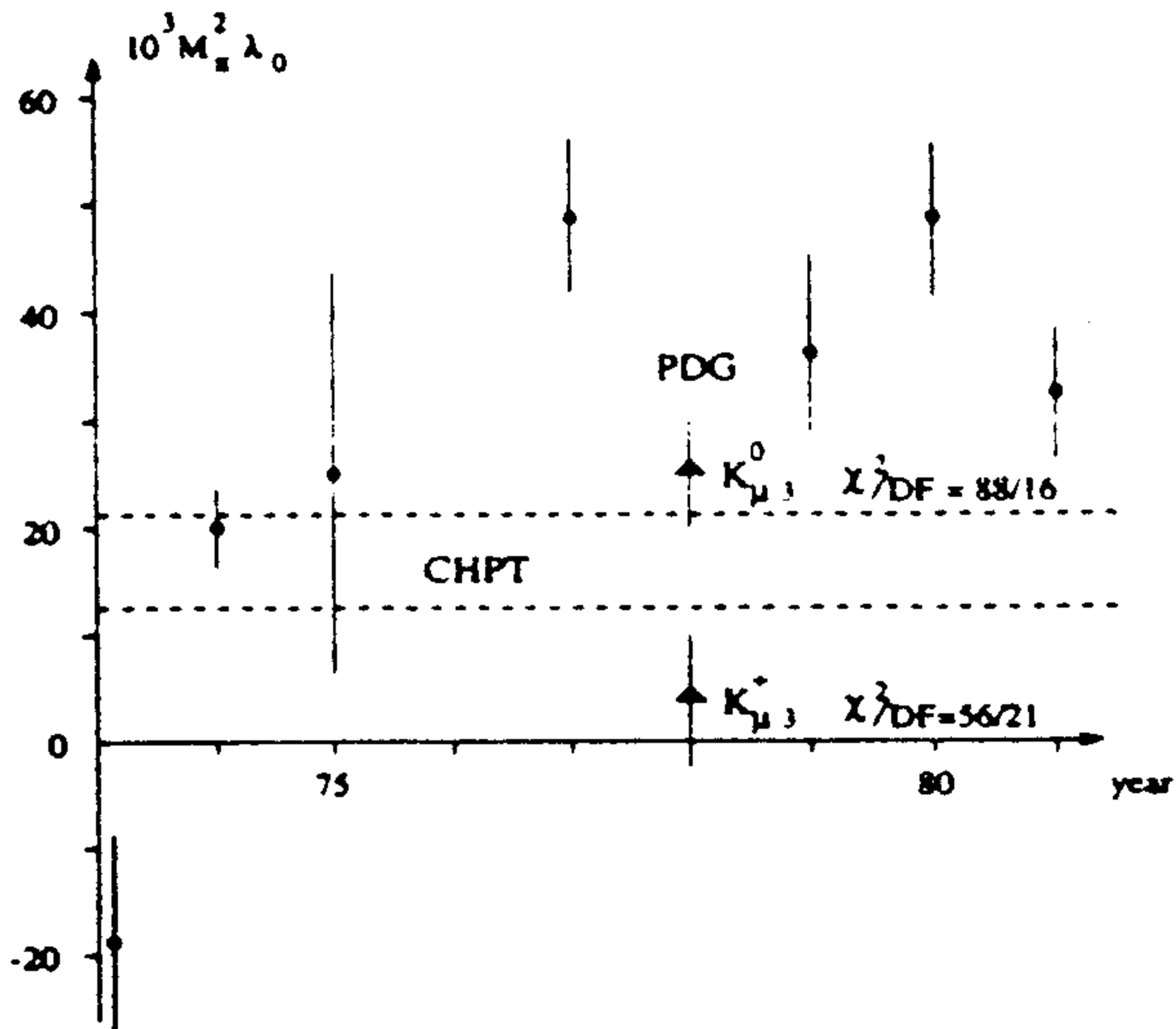
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Figure 1: Comparison between the experimental data and the CHPT prediction for the scalar form factor λ_0 [12, 13]. Dots denote $K^0_{\mu3}$ measurements; the two triangles indicate the PDG averages for $K^0_{\mu3}$ and $K^+_{\mu3}$ [16]; the two dashed lines show the CHPT prediction $\lambda_0 = 0.17 \pm 0.04$.
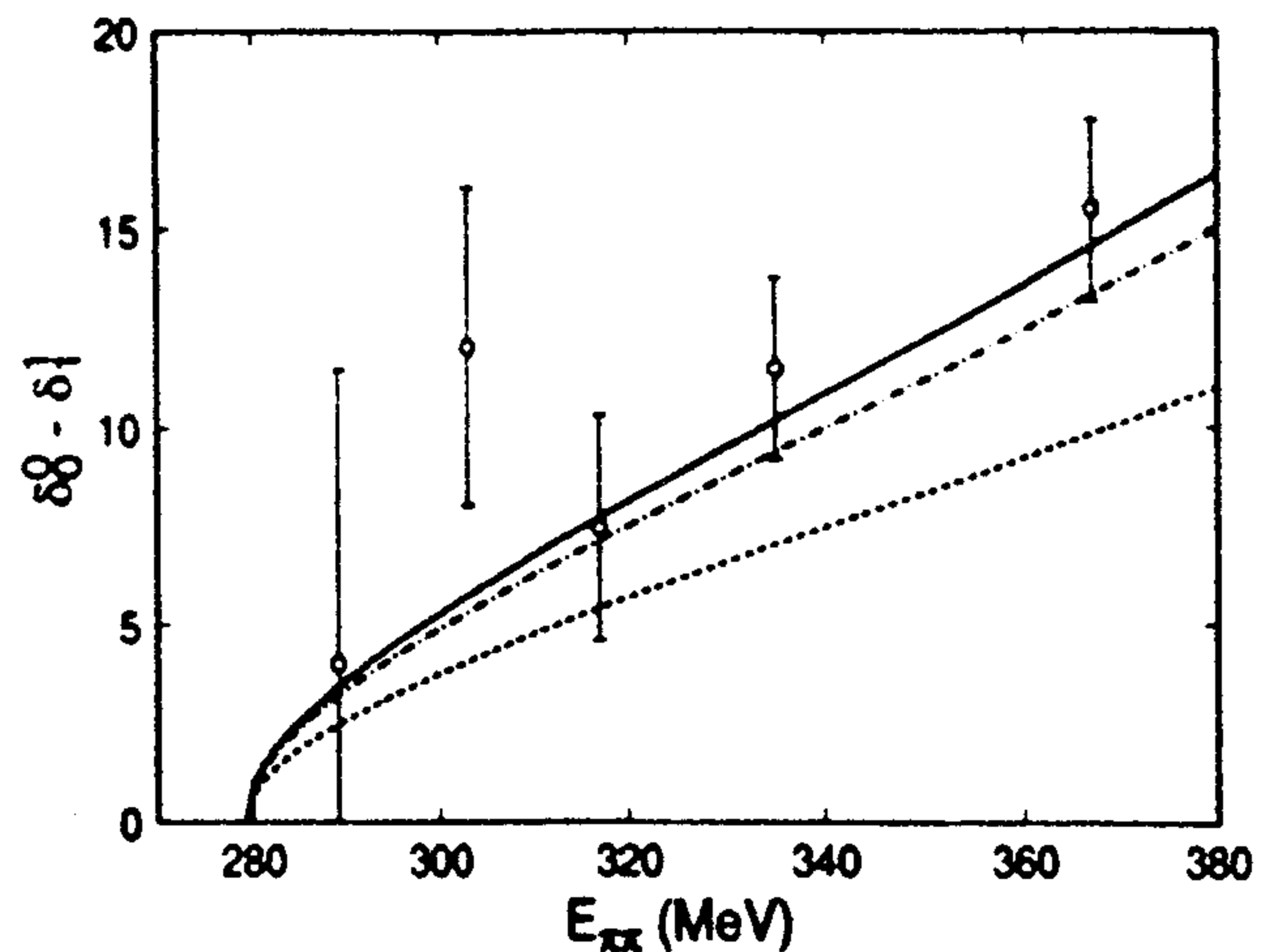
Figure 2: The phase shift difference $\delta^0_0 - \delta^1_1$ (in degrees) as a function of the $\pi\pi$ center-of-mass energy. Dotted, dash–dotted and full lines denote respectively tree, one–loop and two–loop CHPT results [14]. The bars indicate the results of the K_{e4} experiment of Rosselet et al. [17].

in 'generalized CHPT' [15], accurate data could tell us which is the behaviour of the quark condensate in the chiral limit [7, 8].

3 Nonleptonic decays

The operator product expansion (OPE) let us to compute nonleptonic $|\Delta S| = 1$ transitions at low energies ($\mu \ll M_W$) by means of an effective four–fermion Lagrangian [18, 19]:

$$\mathcal{L}^{|\Delta S|=1}_{eff} = \frac{4G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) O_i + \text{h.c..} \tag{1}$$

The four–fermion operators O_i contain only light fermion fields ψ_l ($m_f < \mu$), whereas the Wilson coefficients $C_i(\mu)$ depend on the heavy degrees of freedom integrated out. The renormalization scale μ introduces an artificial distinction between short- and long-distance effects: the μ-dependence of the $C_i(\mu)$ that parametrizes short-distance effects is cancelled by the μ-dependence of the four-fermion operator matrix elements between initial and final hadronic states.

The Wilson coefficients can be calculated reliably using renormalization group techniques down to $\mu \gtrsim m_c$, where QCD is still in a perturbative regime. Recently the $C_i(\mu)$ have been calculated at the next–to–leading order [18, 19] drastically reducing the theoretical uncertainties on the short–distance part of Eq. (1). The main source of uncertainties in nonleptonic kaon decays is therefore the evaluation of the four-fermion operator matrix elements.

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There are two ways to approach this problem. On one side we can try to use non-perturbative techniques (lattice QCD, $1/N_c$ expansion, resonance saturation, etc...) to estimate the matrix elements of the operators O_i. On the other side, following CHPT assumptions, we can use the symmetry properties of the four-fermion lagrangian under $SU(3)_L \times SU(3)_R$ to construct its realization in terms of the pseudo Goldstone–boson fields. The second solution is certainly less predictive, since a number of low–energy constants must be introduced, but is the most systematic and in many cases the most reliable. Nonetheless the two approaches are complementary. Up to date non-perturbative techniques are not accurate enough to fix all the unknown couplings of the CHPT Lagrangian, but in future we may expect that the two approaches will merge yielding a systematic and completely predictive picture of nonleptonic decays.

The lowest–order chiral realization of the four-fermion Lagrangian (1) is very simple

$$\mathcal{L}_W^{(2)} = F^4 \left[G_8 W_8^{(2)} + G_{27} W_{27}^{(2)} \right] + \text{h.c.} \tag{2}$$

where $W_8^{(2)}$ and $W_{27}^{(2)}$ are $O(p^2)$ operators transforming under $SU(3)_L \times SU(3)_R$ as $(8_L, 1_R)$ and $(27_L, 1_R)$, respectively[1]. The two unknown couplings, naively expected to be of the order of $\sin\theta_C \times G_F$, can be fixed from the $K \to 2\pi$ widths. In this case one finds

$$|G_8| \simeq 9.1 \times 10^{-6} \text{ GeV}^{-2}, \qquad G_{27}/G_8 \simeq 1/18, \tag{3}$$

where the suppression of the ratio G_{27}/G_8 is a consequence of the phenomenological enhancement of $\Delta I = 1/2$ transitions.

The Lagrangian (2) let us to predict the decay amplitudes of $K \to 3\pi$, $K \to 2\pi\gamma$ and $K \to 3\pi\gamma$ in terms of G_8 and G_{27}. However, in many cases, the lowest order predictions are not accurate enough to describe present data and a complete $O(p^4)$ analysis is needed. This is even more evident for processes like $K \to \pi l^+ l^-$, $K \to \gamma\gamma$ and $K \to \pi\gamma\gamma$ where the lowest order predictions vanish.

The next–to–leading order structure of the nonleptonic Lagrangian is quite complicated since the number of independent operators substantially increase. The analysis of such operators, started by Ecker, Pich and de Rafael in the radiative sector [21, 22], has been completed by Kambor, Missimer and Wyler both for the octet and the 27-plet components [23]. In the case of the octet sector a useful basis has been introduced in Ref. [24], where the number of independent operators has been reduced to 37. As can be noticed from Fig. 3, in the basis of Ecker, Kambor and Wyler [24] the octet operators are organized in such a way that only few of them occur in a definite nonleptonic process. In principle there are enough observables (particularly in the radiative sector) both to disentangle several N_i and to perform consistency checks of the CHPT approach [25, 26]. Unfortunately present data do not allow to fulfill this program but the situation will certainly change after the completion of the KLOE experiment. We stress that a better knowledge of the N_i is not only useful by itself but also to discriminate among the various non–perturbative hadronization models.
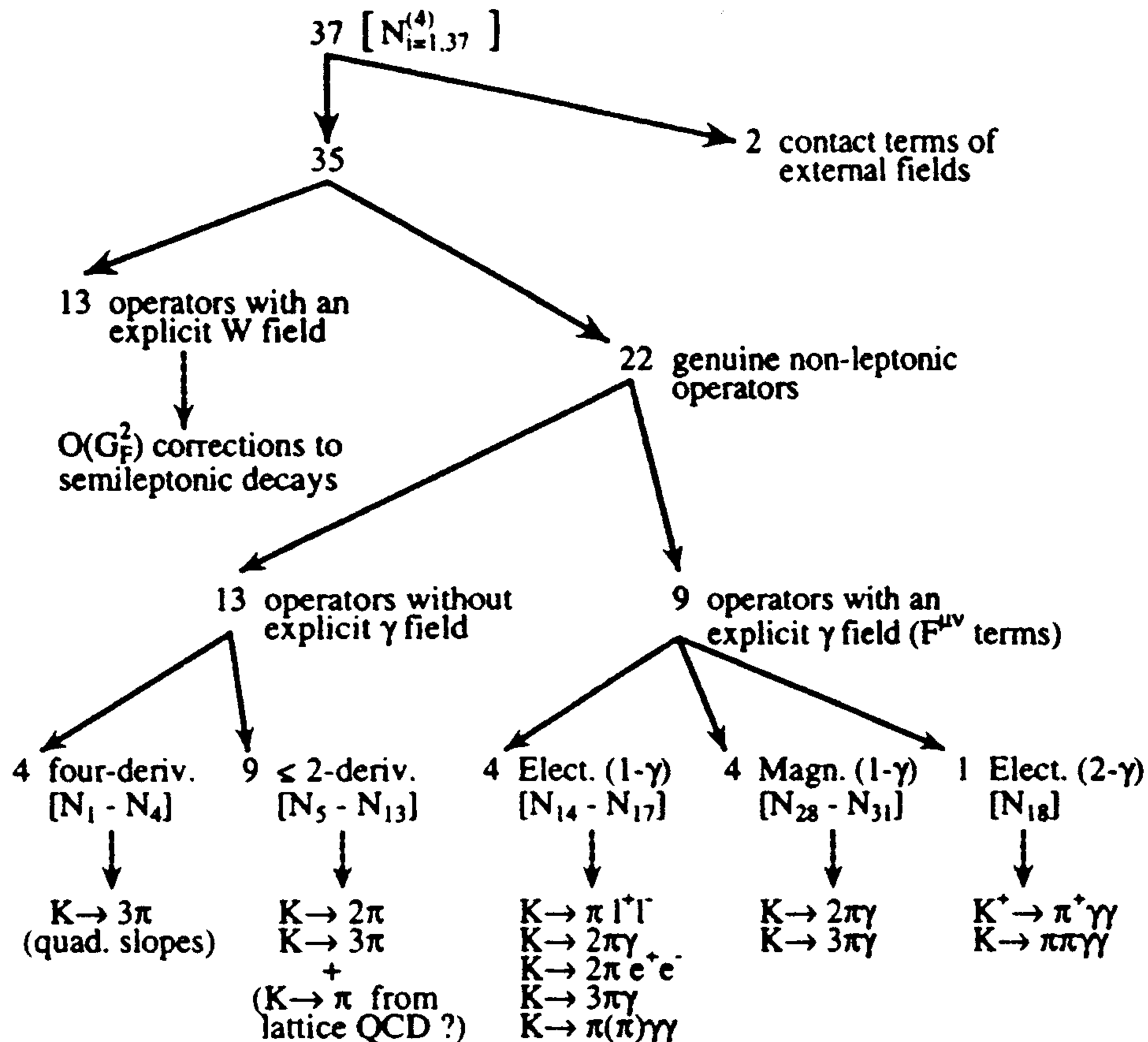
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$$37\ [N^{(4)}_{i=1,37}]$$

35 → 2 contact terms of external fields

13 operators with an explicit W field

$O(G_F^2)$ corrections to semileptonic decays

22 genuine non-leptonic operators

13 operators without explicit γ field

9 operators with an explicit γ field ($F^{\mu\nu}$ terms)

4 four-deriv. $[N_1 - N_4]$

9 ≤ 2-deriv. $[N_5 - N_{13}]$

4 Elect. (1-γ) $[N_{14} - N_{17}]$

4 Magn. (1-γ) $[N_{28} - N_{31}]$

1 Elect. (2-γ) $[N_{18}]$

$K \to 3\pi$ (quad. slopes)

$K \to 2\pi$
$K \to 3\pi$
+
($K \to \pi$ from lattice QCD ?)

$K \to \pi\, l^+l^-$
$K \to 2\pi\gamma$
$K \to 2\pi\, e^+e^-$
$K \to 3\pi\gamma$
$K \to \pi(\pi)\gamma\gamma$

$K \to 2\pi\gamma$
$K \to 3\pi\gamma$

$K^+ \to \pi^+\gamma\gamma$
$K \to \pi\pi\gamma\gamma$

Figure 3: Role of the low–energy constants N_i appearing in the $O(p^4)$ nonleptonic octet Lagrangian of Ref. [24].

3.1 $K \to 3\pi$

$K \to 3\pi$ amplitudes are usually expanded in terms of the Dalitz Plot variables X and Y [25]:

$$A(K \to 3\pi) = a + bY + b'X + O(X^2, XY, Y^2),\qquad (4)$$

where

$$X = \frac{s_1 - s_2}{M_\pi^2},\qquad Y = \frac{s_3 - s_0}{M_\pi^2},\qquad s_i = (p_k - p_{\pi_i})^2,\qquad s_0 = (s_1 + s_2 + s_3)/3\ .\qquad (5)$$

Present data are well described by an expansion up to quadratic terms (higher powers of X and Y belong to high angular momentum states). The $O(p^2)$ CHPT predictions are non-vanishing only for constant and linear terms [27]. Quadratic slopes and re–scattering phases

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[1] We neglect the suppressed $(8_L . 8_R)$ electromagnetic-penguin operators (see Ref. [6, 20] for a recent discussion about this point).

Table 3: Number of independent isospin amplitudes and $O(p^4)$ free parameters in $K \to 3\pi$.

	$\Delta I = 1/2$	$\Delta I = 3/2$	phases
const.	1	1	1
	(α_1)	(α_3)	
linear	1	2	3
	(β_1)	(β_3, γ_3)	
quad.	2	3	-
	(ζ_1, ξ_1)	(ζ_3, ξ_3, ξ'_3)	
$O(p^4)$ free par.	2	3	-

Table 4: Experimental results and CHPT predictions for $K \to 3\pi$ amplitudes.

	$O(p^2)$	$O(p^4)$ [30, 28]	exp. fit [28]
α_1	74.0	input	91.71 ± 0.32
β_1	-16.5	input	-25.68 ± 0.27
ζ_1	$-$	-0.47 ± 0.18	-0.47 ± 0.15
ξ_1	$-$	-1.58 ± 0.19	-1.51 ± 0.30
α_3	-4.1	input	-7.36 ± 0.47
β_3	-1.0	input	-2.43 ± 0.41
γ_3	1.8	input	2.26 ± 0.23
ζ_3	$-$	-0.011 ± 0.006	-0.21 ± 0.08
ξ_3	$-$	0.092 ± 0.030	-0.12 ± 0.17
ξ'_3	$-$	-0.033 ± 0.077	-0.21 ± 0.51

are generated at $O(p^4)$ [28]. The assumption of isospin symmetry and the inclusion of re-scattering phases up to linear terms provide us with a parametrization of the decay amplitudes of the five CP-conserving processes,

$$K^\pm \to \pi^\pm \pi^\pm \pi^\mp, \quad K^\pm \to \pi^0 \pi^0 \pi^\pm, \quad K_L \to \pi^+ \pi^- \pi^0, \quad K_L \to \pi^0 \pi^0 \pi^0, \quad K_S \to \pi^+ \pi^- \pi^0, \quad (6)$$

in terms of 13 real parameters [28, 29] (see Tab. 3).

As can be noticed from Tabs. 3-4, at $O(p^4)$ it is possible to predict unambiguously $K \to 3\pi$ quadratic slopes and re-scattering phases. The first ones have been measured with reasonable accuracy only in the $\Delta I = 1/2$ sector and there the agreement with CHPT is good. An improvement in the experimental determination of $K \to 3\pi$ slopes, together with a theoretical analysis of isospin-breaking effects, is definitely needed to analyze the $\Delta I = 3/2$ sector and thus to understand better the so-called '$\Delta I = 1/2$ rule'.

Re-scattering phases cannot be extracted by a simple analysis of $K \to 3\pi$ widths. The best way to experimentally access to the 3π phases is through time-interference measurements in the neutral channel $K_{L,S} \to \pi^+ \pi^- \pi^0$ [31, 29]. This method has recently let to observe the rare $K_S \to \pi^+ \pi^- \pi^0$ decay [32, 33] (a pure $\Delta I = 3/2$ transition). Unfortunately present data are affected by large errors and do not to allow to test $O(p^4)$ effects both in the real and in the imaginary parts of the amplitudes. A significant improvement on this kind of measurements is expected at KLOE.

Another interesting aspect of $K \to 3\pi$ decays are the direct CP-violating asymmetries. In this case the CHPT approach does not allow to make firm predictions since we have not a good control on the imaginary parts of the low-energy constants. Nevertheless, using the available information from $K \to 2\pi$ (experimental and lattice QCD results) it is still possible to put interesting upper bounds on these asymmetries [34, 35, 6]. In particular, within the Standard Model we can exclude the possibility to observe such effects at KLOE.

3.2 $K \to 2\pi\gamma$ and $K \to 3\pi\gamma$

In processes with one photon in the final state it is useful to expand the decay widths in terms of the photon energy E_γ:

$$\frac{d\Gamma(K \to \pi\pi(\pi)\gamma)}{dE_\gamma} = \frac{a}{E_\gamma} + b + cE_\gamma + O(E_\gamma^2) \,. \tag{7}$$

QED implies that the first two terms in the expansion (i.e. the bremsstrahlung contribution) can be unambiguously predicted in terms of the corresponding non-radiative amplitude [36] and vanish if the transition involves only neutral particles. On the contrary, $O(E_\gamma)$ terms are 'structure dependent' and receive contributions from the so–called direct–emission amplitudes.

In the framework of CHPT bremsstrahlung amplitudes are non-vanishing already at $O(p^2)$ whereas direct emission ones, both of electric and magnetic type, are generated only at $O(p^4)$ [26]. Interestingly in four-meson processes CHPT allow to extend the concept of bremsstrahlung relating also the dominant $O(E_\gamma)$ effect in (7) to the corresponding non-radiative process [37][2].

A complete $O(p^4)$ analysis has been performed both for $K \to 2\pi\gamma$ [38, 39] and $K \to 3\pi\gamma$ [40] decays. The first ones turn out to be very promising to extract information on several N_i combinations, the others are interesting to test the concept of 'generalized bremsstrahlung'. Unfortunately present data on direct emission amplitudes are not very accurate, especially in the $K_S \to \pi^+\pi^-\gamma$ case and in all $K \to 3\pi\gamma$ channels. KLOE will certainly improve the present situation and with some effort might even succeed in detecting the very rare $K_S \to \pi^+\pi^-\pi^0\gamma$ transition: $BR^{th.}(K_S \to \pi^+\pi^-\pi^0\gamma)|_{E_\gamma>10 \text{ MeV}} \simeq 2 \times 10^{-10}$ [40].

3.3 Two photon decays

Decays with two photons in the final state are very interesting in the framework of CHPT [26]. The first non-vanishing contribution to the decay amplitudes of $K_S \to \gamma\gamma$ [41, 42] and $K_L \to \pi^0\gamma\gamma$ [44, 45] is of $O(p^4)$ and is generated only by (finite) loop diagrams. The above amplitudes are thus unambiguously predicted in terms of G_8 and G_{27}. In the K_S case the theoretical branching ratio is in good agreement with the experimental result [43], on the contrary $BR^{exp}(K_L \to \pi^0\gamma\gamma)/ BR^{O(p^4)}(K_L \to \pi^0\gamma\gamma) \gtrsim 2$. The reason of this discrepancy can be traced back to the problem of $O(p^6)$ unitarity corrections and resonance contributions which affect $K_L \to \pi^0\gamma\gamma$ but not $K_S \to \gamma\gamma$ (see e.g. Refs. [46, 47, 48]). Indeed, in $K_S \to \gamma\gamma$ unitarity corrections to the $K \to \pi\pi$ vertex are already included in the constant G_8 and vector-meson contributions are forbidden by kinematics. The estimate of the $O(p^6)$ local terms in $K_L \to \pi^0\gamma\gamma$ is model dependent and a good resolution on the diphoton energy spectrum is needed to distinguish among various models (Figs. 4–5).

Also in $K^+ \to \pi^+\gamma\gamma$ the decay amplitude is at least of $O(p^4)$ and the sum of the $O(p^4)$ loop diagrams is finite, however in this case there is an additional $O(p^4)$ contribution from

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[2]Actually the $O(E_\gamma)$ effect of 'generalized bremsstrahlung' is dominant only if the corresponding non-radiative amplitude is not suppressed [40].
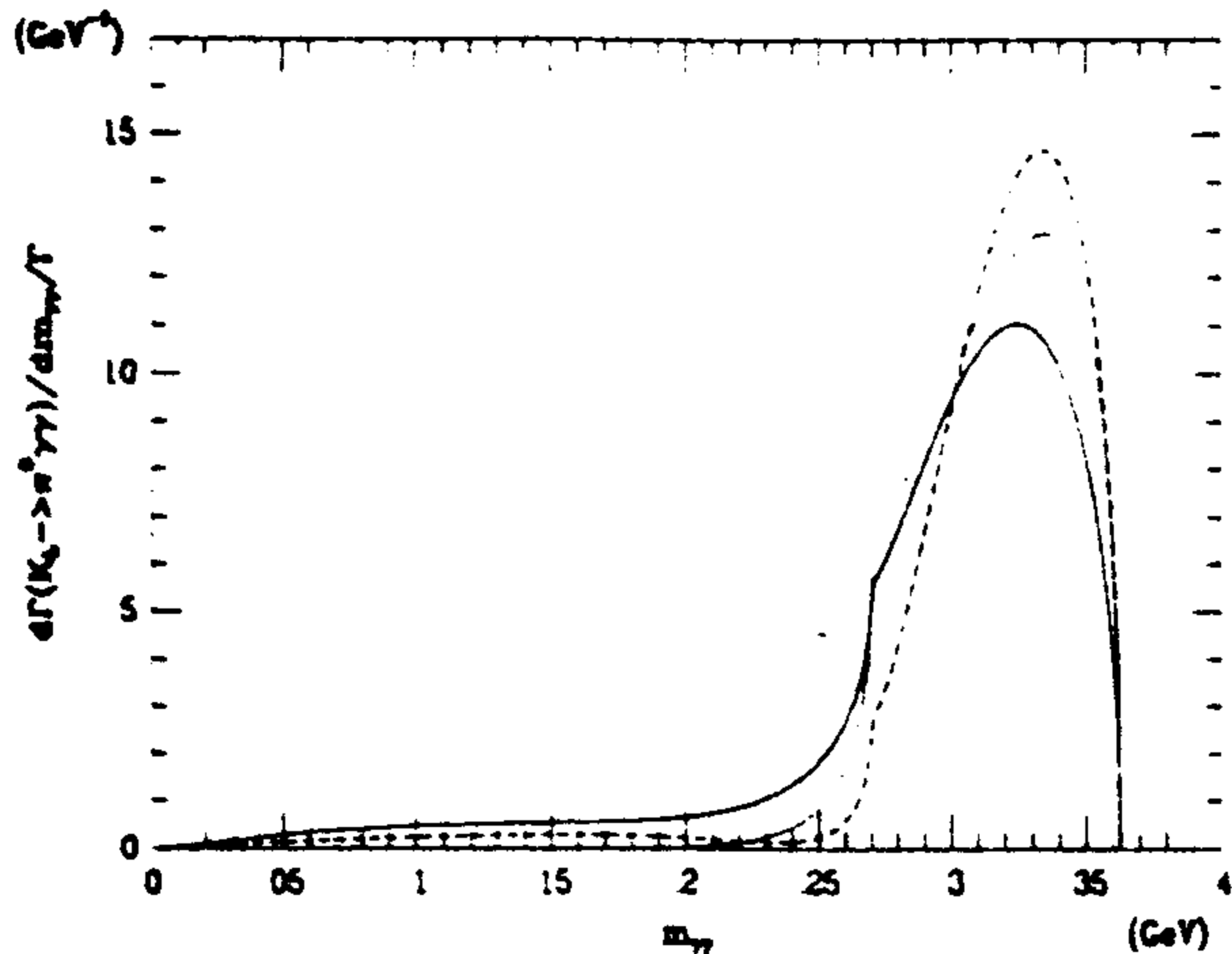
Figure 4: Theoretical predictions for the width of $K_L \rightarrow \pi^0\gamma\gamma$ as a function of the two-photon invariant mass. The dotted curve is the $O(p^4)$ contribution, dashed and full lines correspond to the $O(p^6)$ estimates [46, 49] for $a_V = 0$ and $a_V = -0.8$, respectively. The three distributions are normalized to the 50 unambiguous events of NA31.
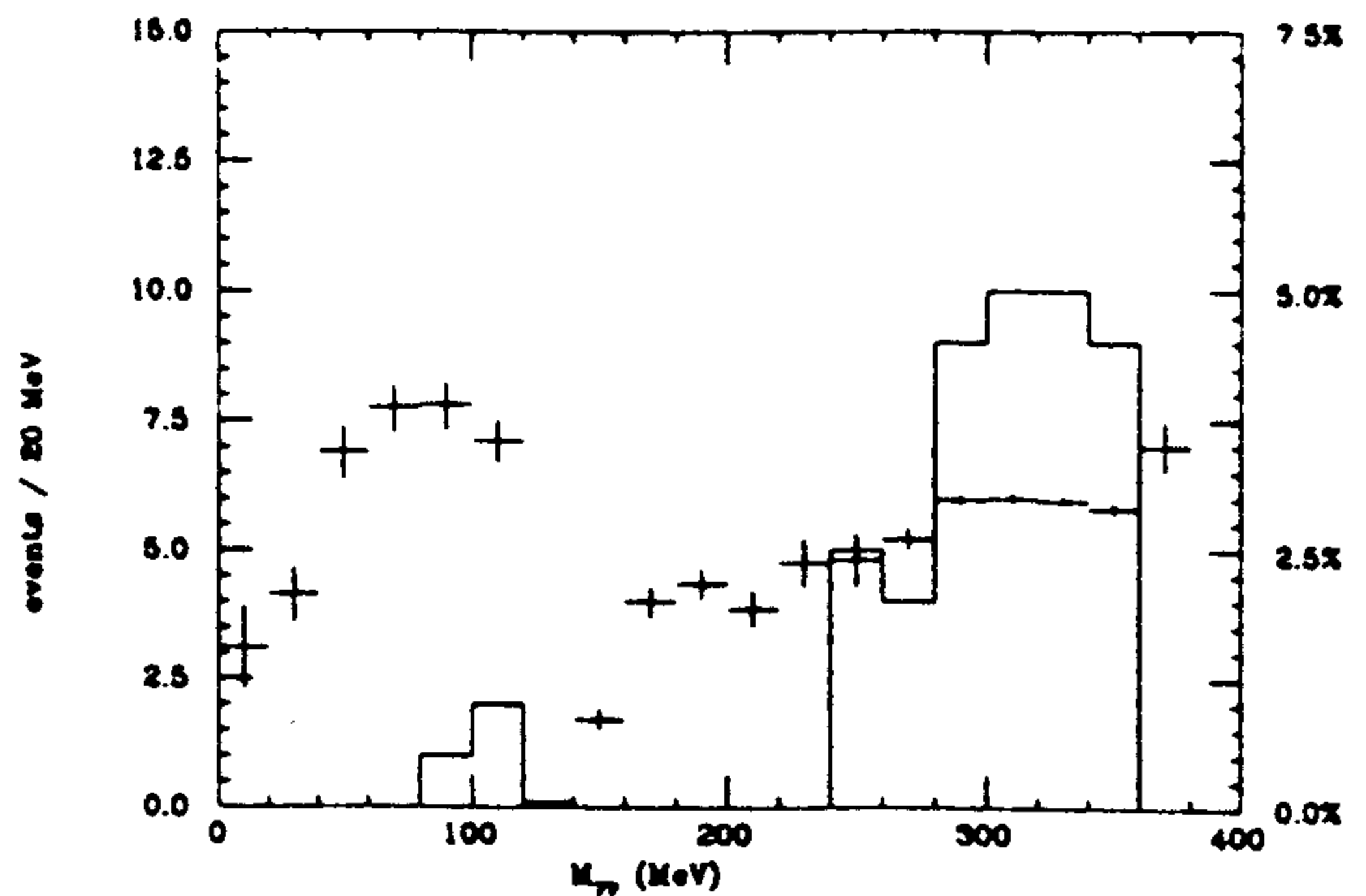
Figure 5: Distributions of the 50 unambiguously $K_L \rightarrow \pi^0\gamma\gamma$ events reconstructed by NA31[50] (histograms). Crosses indicate the experimental acceptance (scale on the right).

counterterms [22]. Preliminary results form the BNL-E787 experiment [51] indicate that the diphoton energy spectrum of this decay is consistent with the CHPT prediction for a reasonable value of the counterterm combination. In analogy to the $K_L \rightarrow \pi^0\gamma\gamma$ case, it is possible that $O(p^6)$ effects play an important role also in $K^+ \rightarrow \pi^+\gamma\gamma$: recently D'Ambrosio and Portolés have shown that unitarity corrections are large [52], at least as in $K_L \rightarrow \pi^0\gamma\gamma$, whereas local contributions are likely to be suppressed [49].

Substantially different from the previous decays are the $K_L \rightarrow \gamma\gamma$ and $K_L \rightarrow \gamma^*\gamma^*$ transitions. Here at $O(p^4)$ there are no loop contributions and the π^0- and η-pole diagrams cancel each other due to the Gell-Mann-Okubo relation. In general $O(p^6)$ effects are strongly model dependent, nevertheless a interesting and consistent picture of both $K_L \rightarrow \gamma\gamma^*$ and $K \rightarrow \pi\gamma\gamma$ decays has been recently proposed [49]. It is worthwhile to stress that a better understanding, both from the theoretical and the experimental point of view, of $K_L \rightarrow \gamma^*\gamma^*$ form factors could help to disentangle interesting short-distance effects in $K_L \rightarrow \mu^+\mu^-$ [53].

3.4 $K \rightarrow \pi l^+ l^-$ decays

The decay amplitudes of $K^+ \rightarrow \pi^+ l^+ l^-$ and $K_S \rightarrow \pi^0 l^+ l^-$ are at least of $O(p^4)$ and receive contributions from both loops and counterterms [21]. Up to date only the two K^+ decay channels have been observed. The measurement of the energy spectrum of the lepton pair in $K^+ \rightarrow \pi^+ e^+ e^-$ [54] let to determine the N_i combination which appears also in $K^+ \rightarrow \pi^+\mu^+\mu^-$

and thus to predict unambiguously $BR(K^+ \to \pi^+\mu^+\mu^-) = 6.2^{+0.8}_{-0.6} \times 10^{-8}$ [21, 57]. The preliminary measurement of BNL-E787 [51] is in good agreement with the above prediction. Unfortunately the N_i combination which appears in $K_S \to \pi^0 e^+ e^-$ is different and thus we cannot predict unambiguously this decay. Without model dependent assumptions we can state only [26, 6][3]

$$10^{-10} \lesssim BR(K_S \to \pi^0 e^+ e^-) \lesssim 10^{-8} . \tag{8}$$

The $K_L \to \pi^0 e^+ e^-$ decay is very interesting to study short–distance effects. Here we can distinguish three contributions: a short–distance dominated and direct CP–violating term [56], the indirect CP–violating process $K_L \to K_S \to \pi^0 e^+ e^-$ and the CP–conserving two–photon re–scattering $K_L \to \pi^0 \gamma\gamma \to \pi^0 e^+ e^-$. The short–distance contribution can be calculated unambiguously and yields [56]

$$BR_{CP-dir}(K_L \to \pi^0 e^+ e^-) = (4.5 \pm 2.6) \times 10^{-12}, \tag{9}$$

where the error is dominated by the poor knowledge of the CKM matrix elements involved. On the contrary the other two contributions are affected by large theoretical uncertainties. According to Eq. (8) the K_S contribution lies between 3×10^{-13} and 3×10^{-11}. The absorptive part of the photon re–scattering yields [46, 58, 49]

$$BR_{CP-cons}(K_L \to \pi^0 e^+ e^-)|_{abs} = (0.3 - 1.8) \times 10^{-12}, \tag{10}$$

where the error is related to the uncertainty on the diphoton spectrum of $K_L \to \pi^0 \gamma\gamma$ at small $m_{\gamma\gamma}$, the dispersive part –expected to be of the same order– is even more model dependent [58].

If the indirect CP–violating and the CP–conserving contributions were calculable with reasonable accuracy, or better if it was found that are negligible with respect to the direct CP–violating one, an observation of $K_L \to \pi^0 e^+ e^-$ (within the reach of future facilities [53]) would provide an important window on short–distance physics. To this purpose KLOE plays an important role. Indeed this experiment should be able both to measure $BR(K_S \to \pi^0 e^+ e^-)$ (or to put an upper bound on it at the level of 10^{-9}) and to improve our knowledge on the diphoton spectrum of $K_L \to \pi^0 \gamma\gamma$.

4 Conclusions

In this short overview we have discussed different aspects of kaon decays with particular attention to those cases where new measurements could provide important theoretical insights. The main points of our discussion are briefly summarized in Tab. 1 and we will not repeat them here. Our discussion was rather qualitative and far from being complete. Nonetheless we hope to have outlined some of the unique features of kaon decays in testing the Standard Model at different energy scales and, within this context, the important role foreseen for KLOE.

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[3]The counterterm combinations of $K_S \to \pi^0 e^+ e^-$ and $K^+ \to \pi^+\mu^+\mu^-$ are correlated within specific hadronization hypotheses, in a wide class of models [21, 55] one gets $BR(K_S \to \pi^0 e^+ e^-) \simeq 5 \times 10^{-10}$.

Acknowledgments

References

[1] A. Aloisio et al., 'The KLOE Detector: Technical Proposal', LNF-93-002-IR, Jan. 1993.

[2] 'The Second DAΦNE Physics Handbook', eds. L. Maiani, G. Pancheri and N. Paver (Frascati, 1995).

[3] G. Ecker, Prog. Part. Nucl. Phys. **35** (1995) 1.

[4] A. Pich, Rep. Prog. Phys. **58** (1995) 563.

[5] E. de Rafael, in 'CP Violation and the limits of the Standard Model', TASI 1994 proceedings, ed. J.F. Donoghue (World Scientific, 1995).

[6] G. D'Ambrosio and G. Isidori, 'CP violation in kaon decays', INFNNA-IV-96-29, LNF-96/033(P), hep-ph/9611284, to appear in Int. J. Mod. Phys. A.

[7] H. Leutwyler, these proceedings.

[8] M. Pennington, these proceedings.

[9] L. Maiani, these proceedings.

[10] J. Gasser and H. Leutwyler, Nucl. Phys. **B250** (1985) 465.

[11] J. Bijnens, G. Ecker and J. Gasser, in [2].

[12] J. Bijnens, G. Colangelo, G. Ecker and J. Gasser, in [2].

[13] J. Gasser, lectures given at the '1996 LNF Spring School in Nuclear and Subnuclear Physics', LNF-96/030 (IR).

[14] J. Bijnens, G. Colangelo, G. Ecker, J. Gasser and M.E. Sainio, Phys. Lett. **B374** (1996) 210.

[15] M. Knecht, B. Moussallam, J. Stern and N.H. Fuchs, Nucl. Phys. **B457** (1995) 513.

[16] Particle Data Group, Phys. Rev. **D54** (1996) 1.

[17] L. Rosselet et al., Phys. Rev. **D15** (1977) 574.

[18] A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz, Nucl. Phys. **B400** (1993) 37; A.J. Buras, M. Jamin and M.E. Lautenbacher, ibid. **B400** (1993) 75.

[19] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, Nucl. Phys. **B415** (1994) 403.

[20] M. Fabbrichesi and E.I. Lashin, Phys. Lett. **B387** (1996) 609.

[21] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B291** (1987) 692.

[22] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B303** (1988) 665.

[23] J. Kambor, J. Missimer and D. Wyler, Nucl. Phys. **B346** (1990) 17.

[24] G. Ecker, J. Kambor and D. Wyler, Nucl. Phys. **B394** (1993) 101.

[25] L. Maiani and N. Paver, in [2].

[26] G. D'Ambrosio, G. Ecker, G. Isidori and H. Neufeld, in [2].

[27] J.A. Cronin, Phys. Rev. **161** (1967) 1483.

[28] J. Kambor, J. Missimer and D. Wyler, Phys. Lett. **B261** (1991) 496.

[29] G. D'Ambrosio, G. Isidori, N. Paver and A. Pugliese, Phys. Rev. **D50** (1994) 5767, erratum, ibid. **51** (1995) 3975.

[30] J. Kambor, J.F. Donoghue, B.R. Holstein, J. Missimer and D. Wyler, Phys. Rev. Lett. **68** (1992) 1818.

[31] G. D'Ambrosio and N. Paver, Phys. Rev. **D46** (1992) 352; ibid. **D49** (1994) 4560.

[32] G.B. Thomson et al., Phys. Lett. **B337** (1994) 411.

[33] R. Adler et al. (CPLEAR Collaboration), Phys. Lett. **B374** (1996) 313.

[34] G. D'Ambrosio, G. Isidori and N. Paver, Phys. Lett. **B273** (1991) 497.

[35] G. Isidori, L. Maiani and A. Pugliese, Nucl. Phys. **B381** (1992) 522.

[36] F.E. Low, Phys. Rev. **110** (1958) 974.

[37] G. D'Ambrosio, G. Ecker, G. Isidori and H. Neufeld, Phys. Lett. **B380** (1996) 165.

[38] G. Ecker, H. Neufeld and A. Pich, Phys. Lett. **B278** (1992) 337; Nucl. Phys. **B413** (1994) 321.

[39] G. D'Ambrosio and G. Isidori, Z. Phys. **C65** (1995) 649.

[40] G. D'Ambrosio, G. Ecker, G. Isidori and H. Neufeld, '$K \to \pi\pi\pi\gamma$ in Chiral Perturbation Theory', INFNNA-IV-96/47, UWthPh-1996-55, LNF-96/070(P), hep-ph/9612412.

[41] G. D'Ambrosio and D. Espriu, Phys. Lett. **B175** (1986) 237.

[42] J.L. Goity, Z. Phys. **C34** (1987) 341.

[43] G.D. Barr et al. (CERN-NA31), Phys. Lett. **B351** (1995) 579.

[44] G. Ecker, A. Pich and E. de Rafael, Phys. Lett. **B189** (1987) 363.

[45] L. Cappiello and G. D'Ambrosio, Nuovo Cimento **99A** (1988) 155.

[46] A.G. Cohen, G. Ecker and A. Pich, Phys. Lett. **B304** (1993) 347.

[47] L. Cappiello, G. D'Ambrosio and M. Miragliuolo, Phys. Lett. **B298** (1993) 423.

[48] J. Kambor and B. R. Holstein, Phys. Rev. **D49** (1994) 2346.

[49] G. D'Ambrosio and J. Portolés, 'Vector meson exchange contributions to $K \to \pi\gamma\gamma$ and $K_L \to \gamma l^+ l^-$', INFNNA-IV-96/21, hep-ph/9610244.

[50] G.D. Barr et al. (CERN-NA31), Phys. Lett. **B284** (1992) 440.

[51] T. Shinkawa (BNL-787), '$K^+ \to \pi^+\nu\bar{\nu}$ search', talk presented at the Workshop on K Physics (Orsay, May 30 – June 4, 1996).

[52] G. D'Ambrosio and J. Portolés, Phys. Lett. **B386** (1996) 403, erratum, ibid. **389** (1996) 770.

[53] L. Littenberg, 'Rare kaon decay experiments, summary and prospects', talk presented at the Workshop on K Physics (Orsay, May 30 – June 4, 1996).

[54] C. Alliegro et al., Phys. Rev. Lett. **68** (1992) 278.

[55] C. Bruno and J. Prades, Z. Phys. **C57** (1993) 585.

[56] A. Buras, M.E. Lautenbacher, M. Misiak and M. Munz, Nucl. Phys. **B423** (1994) 349.

[57] A. Pich, 'Rare kaon decays', talk presented at the Workshop on K Physics (Orsay, May 30 – June 4, 1996), hep-ph/9610243.

[58] J.F. Donoghue and F. Gabbiani, Phys. Rev. **D51** (1995) 2187.