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Clustering-Related Aspects in the Compound nucleus Formation

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Clustering-related aspects in the compound nucleus formation

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Summary. — A new approach to describe the compound nucleus formation process, where clustering aspects of the nucleus are taken into account, is worked out. It is shown that quantitative differences between the compound nucleus cross-section of ^{232}Th and ^{238}U, as observed in recent photofission experiments, could be due to the different cluster preformation probabilities of these nuclei.

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1. – Introduction

The nuclear photofissility W_f is defined as the ratio between the photofission cross-section σ_f and the total inelastic cross-section σ_T. It has been shown recently [1] that at intermediate photon energies ω, where fission takes place through a thermodynamically equilibrated system, the photofissility of actinide nuclei is directly related to the compound nucleus cross-section $\langle \sigma_{CN} \rangle$. More specifically,

$$W_f \approx \frac{\langle \sigma_{CN} \rangle}{\sigma_T(\omega)}, \tag{1}$$

where $\langle \sigma_{CN} \rangle$ is the «mean compound nucleus cross-section» (more details are given in ref. [1]). In this case, therefore, σ_f is approximately equal to the entire compound nucleus cross-section.

It is a well-established experimental fact that the photofissility of actinides saturates ($W_f \approx 1$) for $\omega \gtrsim 50$ MeV [2,3], except for ^{232}Th where $W_f \approx 0.7$ even at

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energies as high as 1200 MeV [4]. This somewhat surprinsing finding was attributed to a possible smaller CN cross-section of ^{232}Th comparatively to other actinides [1].

An explanation of the fact that $\sigma_{CN}(\mathrm{Th}) < \sigma_{CN}(\mathrm{U})$ was worked out by us elsewhere [1], which points to the possibility that ^{232}Th would be more transparent than ^{238}U; that is, the mean free path of nucleons inside ^{232}Th is greater than in ^{238}U. There is to date no explanation, in terms of nuclear structure, for such difference between ^{232}Th and the heavier actinides.

In this work we present a new approach to describe the compound nucleus formation process, where clustering aspects of the nucleus are taken into account. We show that quantitative differences between $\sigma_{CN}(\mathrm{Th})$ and $\sigma_{CN}(\mathrm{U})$ may well be due to the different cluster preformation probabilities in ^{232}Th and ^{238}U. In other words, we show that preformed clusters could interfere in the intranuclear cascade process.

The paper is organized in the following way: in sect. 2 we develop the formalism; in sect. 3 we adapt the formalism for photon-nucleus interaction; sect. 4 shows some general results and the application of the formalism to the analysis of recent ^{232}Th and ^{238}U photofissility data; the conclusions are reported on sect. 5.

2. – Cascade formalism

Our starting point is a formalism developed by Kikuchi and Kawai (KK) [5]. In this formalism, the cascade process is analysed step by step, each step being characterized by a mean energy $\langle E_s \rangle$ of the cascade nucleons (*i.e.* nucleons that have received any amount of energy from the intranuclear cascade process). The index s refers to the step of the cascade.

In fig. 1*a*) we show a schematic view of the cascade process. In the KK formalism [5], only two nucleons collisions are considered: one nucleon of the cascade and another in the nuclear Fermi sea. E_0 is the energy of the nucleon that has triggered the cascade. Each vertex in the figure represents a collision,

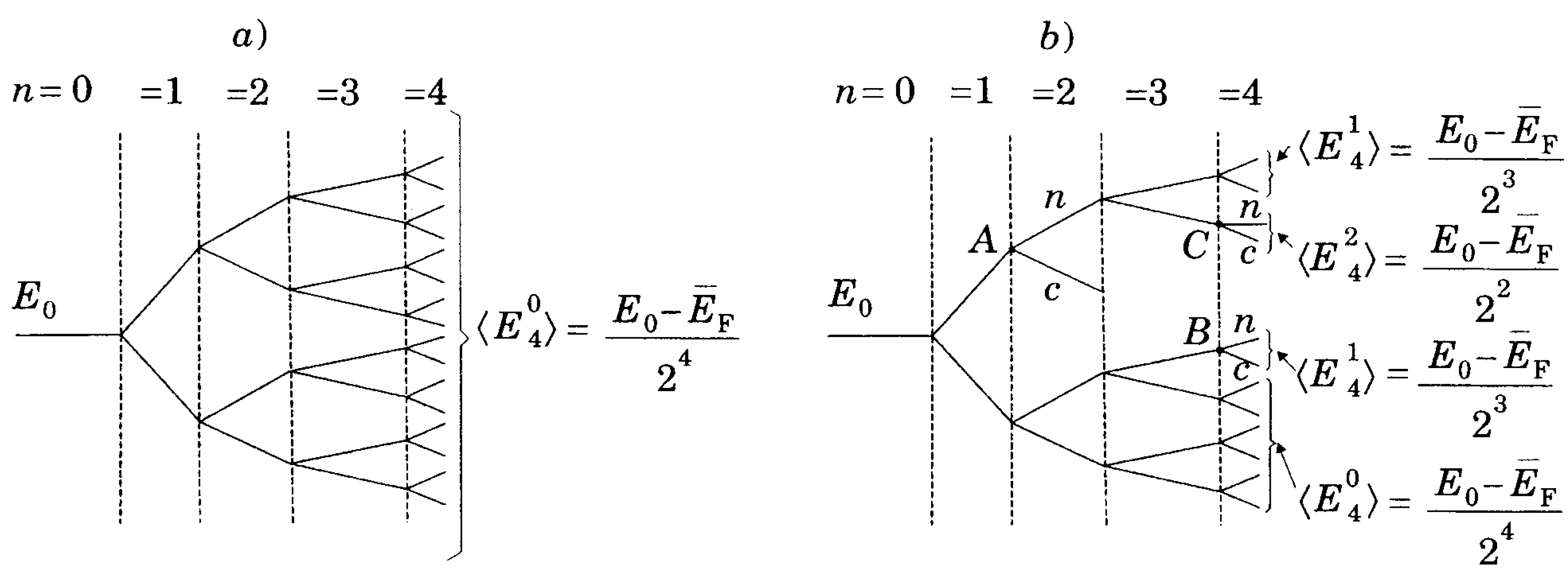

Fig. 1. – Schematic view of some cascade processes («cascade tree»). The mean energy received by the cascade nucleons at the end of the processes is indicated (see eq. (9)). We show cascades: *a*) with 4 steps and without clusters collisions; *b*) with 4 steps and collisions with clusters indicated by A, B and C. In the branch c we represent the clusters nucleons, which do not participate in the cascade process. The branch n represents the cascade nucleon carring out all its initial energy.

and the two outgoing lines from the vertex represent the nucleons emerging after the collision.

As indicated in fig. 1*a*), the process is divided in many steps, each one characterized by an energy $\langle E_s \rangle$ of the 2^s nucleons involved in the cascade at that step.

The mean energy of the nucleons, after a collision in the s-th step of the cascade process, is given by

$$\langle E_s \rangle = \frac{1}{2}\left(\langle E_{s-1} \rangle + \overline{E}_{\mathrm{F}}\right), \tag{2}$$

where $\overline{E}_{\mathrm{F}}$ is the mean energy of the nucleons due to Fermi motion. In ref. [5] this equation was written in the form

$$\langle E_s \rangle = \frac{1}{2}\left(\langle E_{s-1} \rangle - \overline{E}_{\mathrm{F}}\right) + \overline{E}_{\mathrm{F}} = \frac{E_0 - \overline{E}_{\mathrm{F}}}{2^s} + \overline{E}_{\mathrm{F}}\,, \tag{3}$$

where E_0 is the incident projectile energy, in order to emphasize that the energy excess of the cascade nucleon, with respect to the energy of the nucleon in the Fermi sea, $\langle E_{s-1} \rangle - \overline{E}_{\mathrm{F}}$, is equally divided between the two colliding nucleons.

If we assume that one nucleon of the cascade can interact simultaneously with a cluster of N nucleons, then eqs. (2) and (3) are not valid anymore. In fact, in this case the total energy involved in the collision,

$$\langle E_{s-1} \rangle + N\overline{E}_{\mathrm{F}} = \left(\langle E_{s-1} \rangle - \overline{E}_{\mathrm{F}}\right) + (N+1)\overline{E}_{\mathrm{F}}\,, \tag{4}$$

is not equally distributed, but, for kinematical reasons, the cluster energy after the collision will be

$$E_{\text{cluster}} \leqslant \frac{4\,N}{(N+1)^2}\left(\langle E_{s-1} \rangle - \overline{E}_{\mathrm{F}}\right) + N\overline{E}_{\mathrm{F}}\,, \tag{5}$$

and that for the cascade nucleon,

$$\langle E_s \rangle \geqslant \frac{N-3}{N+1}\left(\langle E_{s-1} \rangle - \overline{E}_{\mathrm{F}}\right) + \overline{E}_{\mathrm{F}}\,. \tag{6}$$

Assuming now that when the correlation between the nucleons that formed the cluster disappears the energy E_{cluster} received by the cluster is equally divided among its N nucleons, the energy of the cluster's nucleons is

$$\langle E_s^c \rangle \leqslant \frac{4}{(N+1)^2}\left(\langle E_{s-1} \rangle - \overline{E}_{\mathrm{F}}\right) + \overline{E}_{\mathrm{F}}\,. \tag{7}$$

For sufficiently small energies $\langle E_{s-1} \rangle$, the nucleon-cluster collision is elastic because of the Pauli blocking effects to the final states of the cluster's nucleons. Thus, the incident nucleon carries out all the energy $\langle E_{s-1} \rangle$, and the cluster's nucleons keep their energy $\overline{E}_{\mathrm{F}}$.

In fig. 1*b*) are illustrated the modifications which occur in the cascade process if collisions with clusters are taken into account. For example, if the collision in the point A of the cascade tree is with a cluster (elastic collision), then, the energy of the nucleon from branch n is higher than the average energy in the cascade. On the other hand, the cluster's nucleons (branch c) will not get any energy from the cascade and, therefore, they will not participate in this cascade. As a consequence, at the end of the cascade process we would have a smaller number of participating nucleons, with some of them (those performing one or more collisions with clusters all along the «cascade history») having energies higher than the average energy of the remaining nucleons. This is shown in fig. 1*b*).

If $\gamma \ll 1$ is the probability of a collision between a cascade nucleon and a nuclear cluster, it is straightforward to conclude that after s steps of the cascade process, the average number of cascade nucleons present in a branch with k collisions with clusters is approximately given by

$$N_s^{(k)} = \gamma^k 2^{s-k}, \tag{8}$$

with γ^k being the relevant probability, since the k collisions with a cluster do not contribute to the increase of the number of nucleons in the cascade. The average energy of these nucleons is

$$\langle E_s^k \rangle = \langle E_{s-k} \rangle = \frac{E_0 - \overline{E}_\mathrm{F}}{2^{s-k}} + \overline{E}_\mathrm{F}. \tag{9}$$

In the KK formalism the excitation energy of the residual nucleus, E^*, is calculated by summing up the excitation energies E_s^* resulting from all possible cascades (each characterized by the relevant number s of steps) weighted by its probability of occurring, $\sigma^{(s)}/\sigma_\mathrm{a}$, that is

$$E^* = \sum_{s=s_\mathrm{c}}^{\infty} E_s^* \frac{\sigma^{(s)}}{\sigma_\mathrm{a}}, \tag{10}$$

where σ_a is the total absorption cross-section, and $\sigma^{(s)}$ is the cross-section to get a cascade with s steps; s_c, the «critical number» of steps, is the average number of collisions which produce nucleons with sufficient energy to escape from the nucleus.

As said before, the inclusion of collisions with clusters in the cascade changes this formula because some cascade nucleons, those colliding with clusters, have a mean energy greater than that predicted by the KK formalism [5]. Then, one has to subtract the energy carried by all cascade nucleons coming from a branch with one or more collisions with clusters, and thus eq. (10) becomes

$$\mathcal{E}^* = \sum_{s=s_\mathrm{c}}^{\infty} \left(E_s^* - \sum_{k=s-s_\mathrm{c}+1}^{\infty} N_s^{(k)} \left(\langle E_s^k \rangle - \overline{E}_\mathrm{F} \right) \right), \tag{11}$$

where the second term in brackets gives approximately the mean energy of all cascade nucleons which have undergone collisions with clusters.

In the case of $s = s_\mathrm{c}$, k varies from 1 to ∞. When $s = s_\mathrm{c} + 1$, the nucleons coming from a branch with only one collision with a cluster have mean energy smaller than that needed to escape from the nucleus. Therefore, in this case k varies from 2 to ∞. In an analog way, when $s = s_\mathrm{c} + l$, k varies from $l + 1$ to ∞.

By using eqs. (8) and (9), and $E_s^* = E = E_0 - \overline{E}_\mathrm{F}$, as calculated in the KK formalism, eq. (11) writes

$$\mathcal{E}^* = \sum_{s=s_c}^{\infty}\left(1 - \sum_{k=s-s_c+1}^{\infty} \gamma^s \frac{\sigma^{(s)}}{\sigma_a}\right)(E_0 - \overline{E}_F), \tag{12}$$

which can be rewritten in the form

$$\frac{\sigma_a \mathcal{E}^*}{E} = \sum_{s=s_c}^{\infty}(1 - \gamma S_\infty)\sigma^{(s)} + \sum_{s=s_c}^{\infty}\sum_{k=1}^{s-s_c} \gamma^k \sigma^{(s)}, \tag{13}$$

where $S_\infty = 1/(1-\gamma)$.

As a crosscheck of our deduction we note that, in the absence of nucleon-cluster collisions ($\gamma = 0$), eq. (13) is simply

$$\frac{\sigma_c}{\sigma_a} = \frac{E^*}{E} \tag{14}$$

which is, exactly, the relation deduced by KK in ref. [5].

Therefore, our result (eq. (13)) shows that the quantity $\sigma_a \mathcal{E}^*/E$ is no longer equal to the compound nucleus cross-section σ_c; instead, it is equal to a more complex quantity which contains σ_c and γ.

We define C so that

$$\frac{\sigma_c}{C} = \sum_{s=s_c}^{\infty}(1 - \gamma S_\infty)\sigma^{(s)} + \sum_{s=s_c}^{\infty}\sum_{k=1}^{s-s_c} \gamma^k \sigma^{(s)}, \tag{15}$$

thus eq. (13) is now given by

$$\sigma_c = C\frac{\mathcal{E}^*}{E}\sigma_a . \tag{16}$$

In the next section we show that one has to introduce slight modifications into the formalism before applying it to the photonuclear reactions. However, since photons below pion threshold are primarily absorbed by a quasi-deuteron pair, we assumed that the photon energy is equally divided between the two absorbing particles. Then, since both proton and neutron from the pair have an energy $\overline{E}_F$ inside the nucleus, they will initiate two independent cascade processes with initial energy given by

$$E_0 = \frac{\omega}{2} + \overline{E}_F , \tag{17}$$

where ω is the incident photon energy.

From eq. (16) we get, for photonuclear reactions,

$$\sigma_c = C\frac{\mathcal{E}^*}{\omega}\sigma_a . \tag{18}$$

It is interesting to note that this expression was empirically proposed in ref. [1], for photonuclear reactions (see eq. (29) of ref. [1]), where it was found experimentally that $C \approx$ 2-3 for actinides [1].

3. – Adaptation of the formalism for photonuclear interaction

To apply the KK formalism to the photon-nucleus interaction, we had to modify the cross-section

$$\sigma^{(s)} = 2\pi \int_0^R b\,\mathrm{d}b\, \frac{1}{s!} \left(\frac{2\sqrt{R^2 - b^2}}{\lambda} \right)^s \exp\left[- \frac{2\sqrt{R^2 - b^2}}{\lambda} \right], \tag{19}$$

given by KK to account for cascade processes initiated at the nuclear surface by projectiles inciding with impact parameter b. Here, R is the nuclear radius and λ is the mean free path of nucleons inside the nucleus. In fact, when the projectile is a photon, the cascade process does not necessarily start at the nuclear surface, but it can start at any place inside the nucleus along the photon trajectory.

We modified eq. (19), by introducing a factor f wich can vary from 0 to 1, resulting as

$$\sigma^{(s)} = 2\pi \int_0^R b\,\mathrm{d}b\, \frac{1}{s!} \int_0^1 \mathrm{d}f \left(\frac{2f\sqrt{R^2 - b^2}}{\lambda} \right)^s \exp\left[- \frac{2f\sqrt{R^2 - b^2}}{\lambda} \right]. \tag{20}$$

We want to notice that the formalism developed is valid if the energy transfer to the cluster is not sufficient to excite its nucleons to higher energy states. From the Fermi energy distribution it results that the nucleon mean energy inside the nucleus is $\varepsilon \cong 15\,\mathrm{MeV}$. Then, for a Fermi energy $\overline{E}_\mathrm{F} \cong 30\,\mathrm{MeV}$, to excite the cluster's nucleons it is necessary to transfer an energy $E_s^\mathrm{c} > 15\,\mathrm{MeV}$ to each of them.

The maximum energy transfer to a cluster of N nucleons is

$$E_\mathrm{c} = \frac{4\,N\,(\omega/2)}{(N+1)^2}\,, \tag{21}$$

where we have assumed that the photon is absorbed by a quasi-deuteron and its energy is equally shared by the proton and neutron (see eq. (5) and discussion at the end of sect. 2).

In this case, the energy transfer to each nucleon of the cluster is

$$E_s^\mathrm{c} = \frac{4(\omega/2)}{(N+1)^2}\,, \tag{22}$$

and then, in order to have $E_s^\mathrm{c} < 15\,\mathrm{MeV}$, the following inequality must be fulfilled:

$$15 \geqslant \frac{2\omega}{(N+1)^2}\,; \tag{23}$$

thus

$$\omega \leqslant \frac{15}{2}(N+1)^2\,\mathrm{MeV}\,. \tag{24}$$

For alpha clusters ($N = 4$) we get $\omega < 185\,\mathrm{MeV}$. It should be remembered that this maximum energy limit of 185 MeV is valid only for the *first step* of the cascade process, since in each step the energy involved in the collision is generally divided by a factor of 2. In addition, the inclusion of heavier clusters ($N > 4$) should increase the limit above for ω. In conclusion, it is expected that our approach can be safely used at energies somewhat below 400 MeV.

4. – Results

At this point we have achieved our main objective, namely, to show that the inclusion of clusters interactions in the cascade process can qualitatively explain the observed differences in the photofissility of Th and U. We are now able to compare our calculations with experimental data.

In fig. 2 the behaviour of C *vs.* γ is shown; as it is seen, C is always greater than unit, and it is an increasing function of γ. Therefore, according to eq. (18), the existence of clusters inside the nucleus causes an enhancement of the compound nucleus cross-section. In ref. [1] it was concluded that C must be approximately constant with ω. From its definition (eq. (15)), we see that C may depend on ω through σ_c.

In fig. 3 we plotted C *vs.* ω, showing that C is weakly dependent on ω, in complete agreement with the conclusions of ref. [1].

The cluster collision probability γ can be related to the cluster preformation probability P_c by

$$\gamma = \frac{P_c \sigma_{\mathcal{NC}}}{(1 - P_c)\sigma_{\mathcal{NN}} + P_c \sigma_{\mathcal{NC}}}, \tag{25}$$

where $\sigma_{\mathcal{NN}}$ and $\sigma_{\mathcal{NC}}$ are the cross-sections for nucleon-nucleon and nucleon-cluster collision, respectively.

In fig. 3 we used as input the values $\gamma = 0.41$ for thorium, and $\gamma = 0.46$ for uranium, in order to obtain the corresponding values for C reported in ref. [1]. Using eq. (25), and assuming $\sigma_{\mathcal{NC}} \approx 4\,\sigma_{\mathcal{NN}}$, it results that the alpha preformation probability for thorium is $P_c(\mathrm{Th}) \approx 15\%$, and for uranium, $P_c(\mathrm{U}) \approx 17\%$. In table I we compare our results with some data from the literature. We see that our results are well consistent

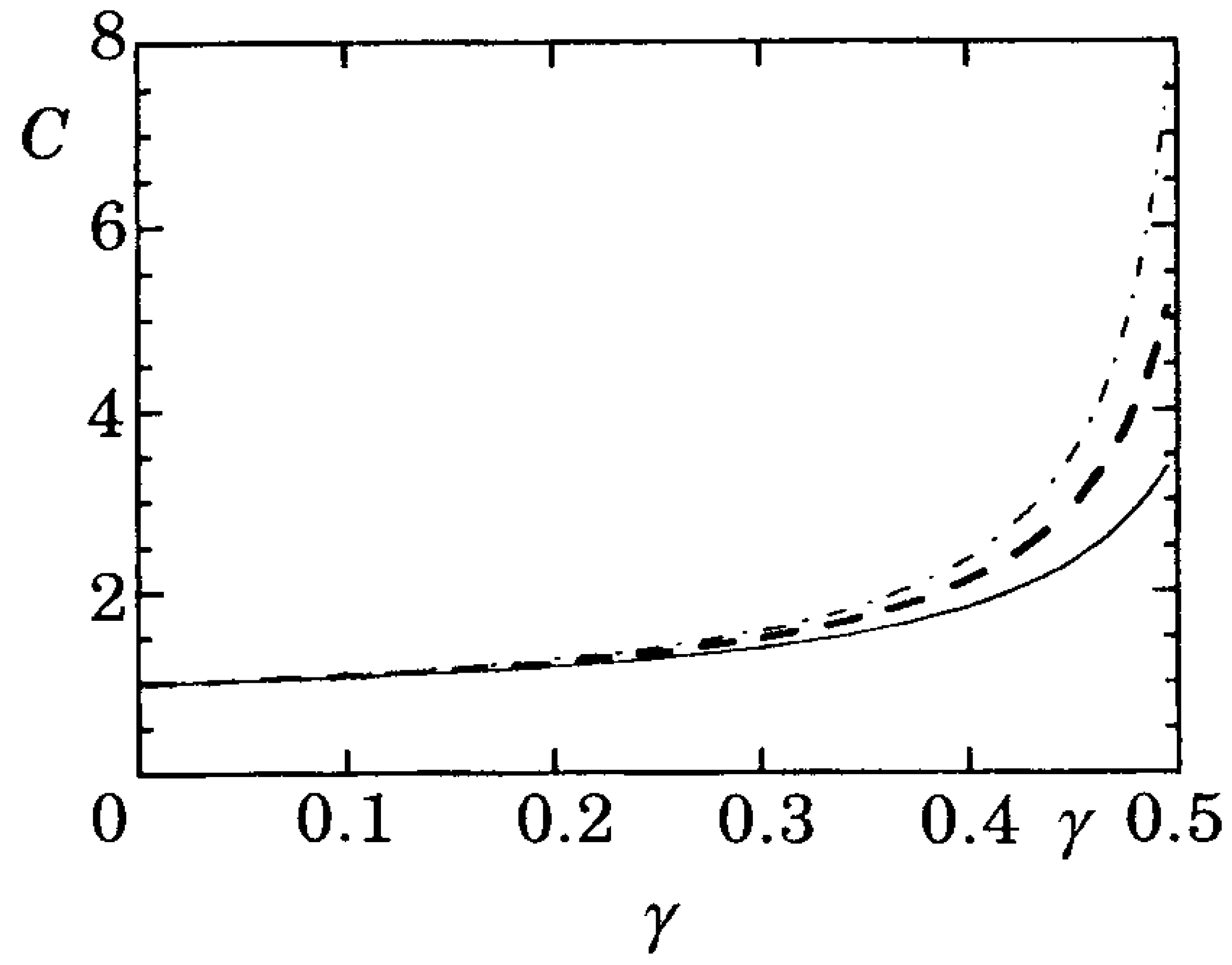

Fig. 2. – The factor C, defined in the text, as a function of the nucleon-cluster collision probability γ for three different values of nucleon mean free path in nuclear matter (—— $\lambda = 3.0$, --- $\lambda = 5.0$, —·— $\lambda = 8.0$). The used parameters are: binding energy $E_b = 6.0$ MeV, nuclear mass number $A = 238$, and photon energy $\omega = 300$ MeV.

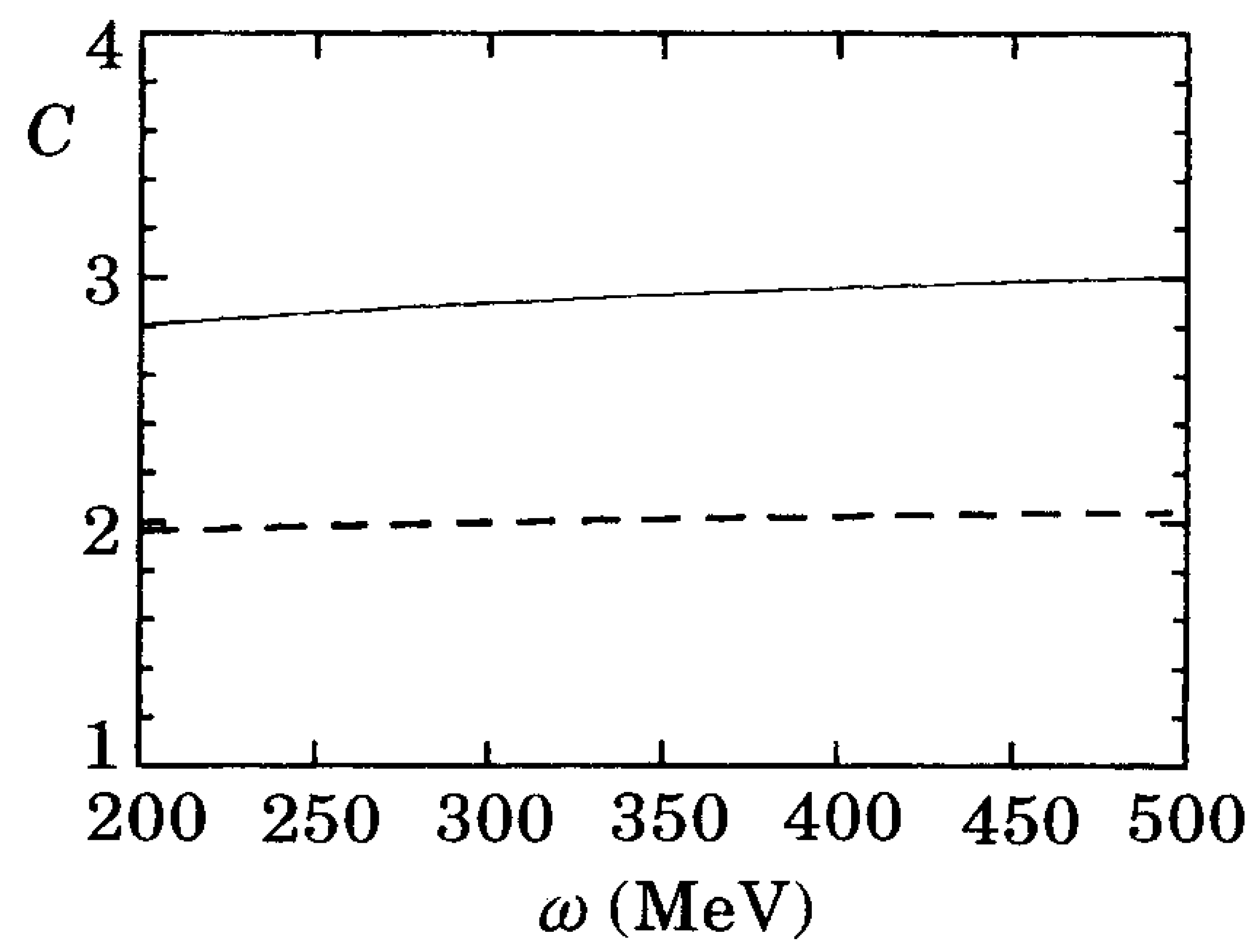

Fig. 3. – The factor C as a function of the photon energy ω. The used parameters for thorium (dashed line) and uranium (full ine) are: the nucleon mean free path inside nucleon $\lambda = 3.5$ fm, the binding energy $E_b = 6.0$ MeV, and the probabilities $\gamma(\mathrm{Th}) = 0.41$ and $\gamma(\mathrm{U}) = 0.46$.

with the 25% value estimated by Janecke *et al.* [6] from an analysis of (d, ^{6}Li) α-pick-up reactions on heavy nuclei, and in good agreement with the (19 ± 2)% value obtained by Genin *et al.* [7] from (e, e′ α) measurements on ^{6}Li.

For the sake of completeness, we have to say that the values for alpha cluster preformation probability (also called spectroscopic factor) obtained in alpha-decay analysis of mean and heavy nuclei [8-10], calculated from the natural alpha-decay constant, are much lower than our values. We note that the latter data suffer of uncertainties of orders of magnitude in the absolute preformation probability due to the difficult determination of some critical parameters [8,10]. However, these results confirm that one could expect $P_c(\mathrm{Th}) < P_c(\mathrm{U})$.

In conclusion, the formalism developed in this work helps to elucidate the clear difference between the photofissility of ^{232}Th and those of heavier actinides. In addition, it becomes now clear why the non-saturating photofissility of ^{232}Th is nearly energy independent in a wide range. In fact, by combining eqs. (1) and (18), one obtains

$$W_f(k) = C\left(\frac{\mathcal{E}^*}{\omega}\right), \tag{26}$$

where both C (see fig. 2) and $\mathcal{E}^*/\omega$ [1] are weakly energy dependent.

TABLE I. – *Preformation probability in the literature.*

Reference	Nucleus	Probability
Janecke *et al.* [6]	heavy nuclei	0.25 ± 0.10 (*)
Genin *et al.* [7]	^{6}Li	0.19 ± 0.02
our work	^{238}U	0.17
our work	^{232}Th	0.15

(*) The uncertainty was estimated by us.

5. – Conclusions

We developed a formalism to describe the intranuclear cascade in which the presence of preformed clusters inside the nucleus is taken into account and incorporated in the cascade process. It was shown that the inclusion of nucleon-cluster collisions significantly modifies the cross-section for compound nucleus formation.

This formalism was applied to the interpretation of recent experimental photofissility data for ^{232}Th and ^{238}U, which show that $\sigma_{CN}(U) > \sigma_{CN}(Th)$ [1]. According to our approach, this result could be explained by the fact that the probability of clusters preformation in ^{238}U is higher than in ^{232}Th.

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