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Direct Bounds on the Tau Neutrino Mass from LEP

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A review of direct bounds on the mass of the tau neutrino obtained at the LEP collider is presented. In addition to published results it includes preliminary results presented at recent conferences and new results presented at the 1996 Tau Workshop. The different techniques and decay modes employed by the ALEPH, DELPHI and OPAL collaborations are compared. The impact of the theoretical modelling of tau decays is also discussed.

The most stringent 95% CL limit on the tau neutrino mass is now obtained by a preliminary ALEPH analysis which combines the results from $\tau \rightarrow 5\pi^{\pm}(\pi^{0})\nu_{\tau}$ and $\tau \rightarrow 3\pi^{\pm}\nu_{\tau}$ decays. This bound constraints the mass of the tau neutrino below 18.2 MeV/c^2.

1. Introduction

The mechanism ruling the mass pattern of elementary particles is one of the outstanding unsettled subjects in modern particle physics. Neutrinos deserve special attention because they are much lighter than all other fermions, a property which is not explained by the basic principles of the Standard Model and thus could provide good hints for its extension. In addition neutrinos are good candidates for dark matter and a fundamental ingredient of the evolution of the universe.

Although the tau neutrino has never been observed directly, it is still considered an excellent candidate for the detection of a neutrino mass by current theoretical prejudices. Essentially the possibility to accommodate an unstable tau neutrino with mass greater than about 1 MeV/c^2 in the framework of the Big Bang Nucleosynthesis [1] and GUT *see-saw* [2] models is still open. On the contrary the mass of a stable tau neutrino is limited below few eV/c^2 and could be detected only indirectly *via* neutrino oscillations.

Assuming the mixing between leptonic families to be small, the ALEPH, DELPHI and OPAL experiments have bounded the value of tau neutrino mass by the kinematic reconstruction of τ decays. Their results will be reviewed here.

1.1. LEP data

From 1990 to 1995 LEP operated at $\sqrt{s} \simeq m_Z$, accurately monitoring the beam energy by mean of a resonant depolarisation technique [3]. The luminosities collected during these years by the experiments are reported in Table 1. They have to be compared with the 514 pb^{-1} recorded by ARGUS and the huge 6 fb^{-1} collected so far by CLEO. Of course a simple comparison is misleading: while the cross sections of tau pairs production are very similar, $\sigma_{\tau\tau}(\sqrt{s} = m_Z^2) \simeq 1.50\, nb^{-1}$ and $\sigma_{\tau\tau}(\sqrt{s} = \Upsilon_{4S}) \simeq 0.87\, nb^{-1}$, at LEP $\tau^+\tau^-$ events are disentangled from all other processes with a much higher efficiency than at low energies. Typically the efficiency in selecting τ pairs ranges around 75%, almost independently of the tau decay mode and free of contamination, while at low energies clean samples of $\tau^+\tau^-$ events can be selected only by requiring a leptonic decay of one of the two taus.

Table 1
Luminosities recorded by the experiments during the years of LEP phase I.

Year	Integrated Luminosities (pb^{-1}) at $\sqrt{s} = m_Z - x/\ m_Z/\ m_Z + x$		
	ALEPH	DELPHI	OPAL
1990	1.6-3.9-2.0	1.2-2.8-1.4	1.5-3.5-1.6
1991	2.4-8.3-2.3	2.0-7.4-1.3	2.4-9.0-2.6
1992	25.7	24.1	25.1
1993	9.4-17.1-9.5	9.9-15.8-10.7	8.8-18.2-8.3
1994	59.3	46.4	55.8
1995	8.6-18.1-9.2	8.5-13.7-9.8	8.8-16.7-9.5

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PII: S0920-5632(97)00241-7

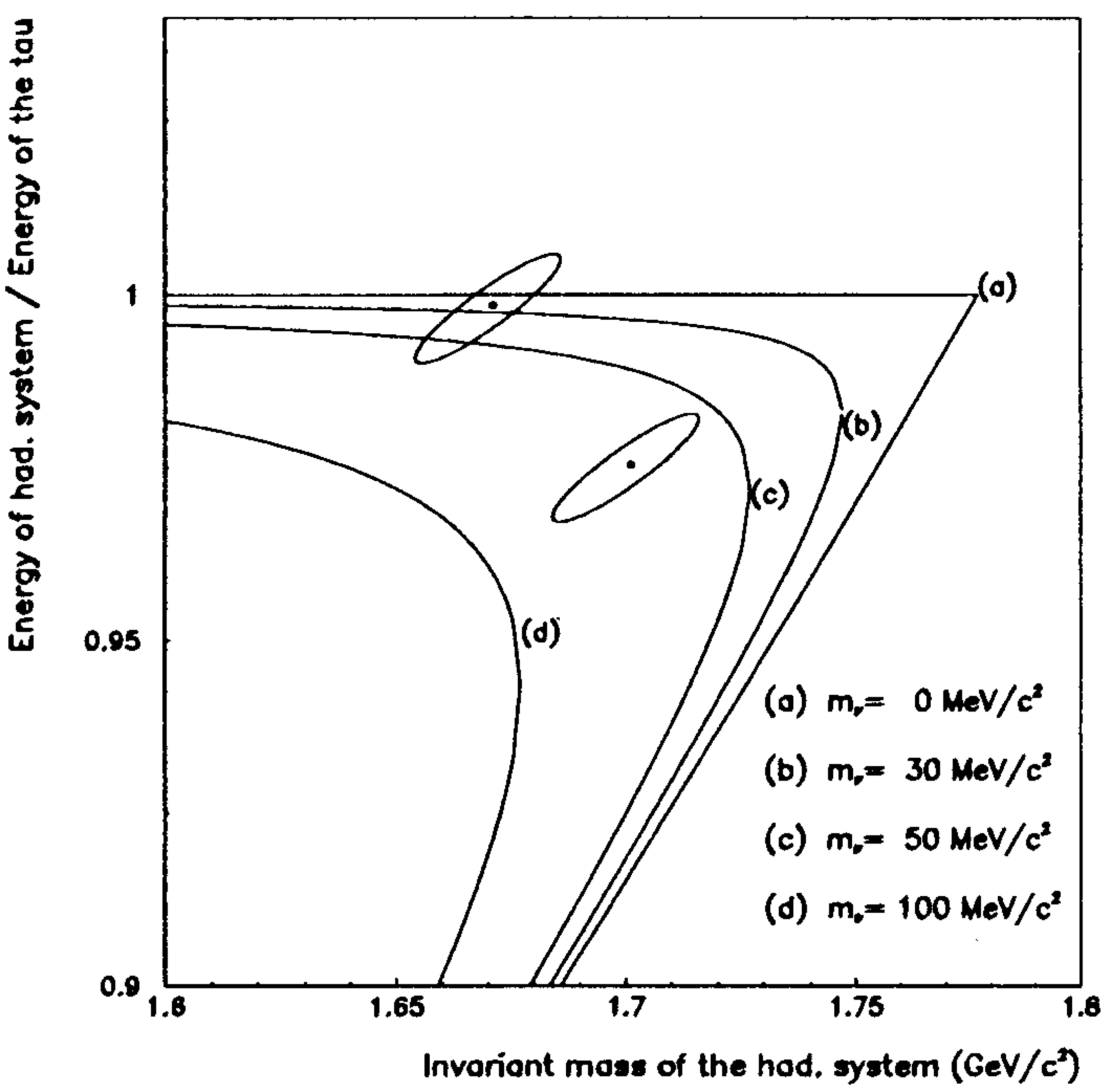

Figure 1. Allowed region on the $(m_{had}, E_{had}/E_\tau)$ plane for different neutrino masses. Two hypothetical events with equal error ellipses are drawn.

1.2. LEP Results

The first LEP bounds on the tau neutrino mass were discussed at the 1994 Tau Workshop [4]. At that time the most stringent analysis were that of ARGUS and CLEO. The latter are based on a maximum likelihood fit of the hadronic invariant mass distribution of $\tau \to 5\pi\ \nu_\tau$, where the five pions were respectively all charged [5] or either in one of the in two charge configurations $5\pi^\pm/3\pi^\pm 2\pi^0$ [6]. ALEPH and OPAL analysed the modes $\tau \to 5\pi^\pm(\pi^0)\nu_\tau$ [7] and $\tau \to 5\pi^\pm\nu_\tau$ [8], both introducing in the fit the use of the energy of the hadronic system in addition to the invariant mass.

The advantage of fitting in the two variables is graphically explained in Fig. 1. The lines delimiting the region of the plane $(m_{had}, E_{had}/E_\tau)$ kinematically accessible to the hadronic system are drawn for different values of the neutrino mass, together with two hypothetical events. It is clearly visible that the event with a lower value of m_{had} constraints the neutrino mass more than the event at higher m_{had}, thanks to the higher value of E_{had}. In the rest frame of the tau, where the allowed region of the plane shrinks to the parabola $E^\star_{had} = (m_\tau^2 + m_{had}^2 - m_\nu^2)/2m_\tau$, the combined measurement of m_{had} and E_{had} for *any* event determines the value of the neutrino mass. In the laboratory frame this information is still present but it is diluted because the boost direction is unknown. In fact, by using the 2-D method ALEPH almost doubled the sensitivity to a massive neutrino. Table 2 reports the features of the published analysis of the four experiments. Scaling the 1-D limits by $\sigma/\sqrt{N}$ it is seen that the ALEPH result lies in between those of ARGUS and CLEO. Of course this comparison reflects the observed shape of the m_{had} distributions but, seen the low statistics, it should be taken with some caution. Also it is evident that the OPAL 2-D result is strongly penalised by the low efficiency in selecting the signal and the limited amount of data analysed.

In the recent past ALEPH, DELPHI and OPAL have performed a series of new analysis using the $\tau \to 3\pi^\pm\nu_\tau$ mode. OPAL presented for the first time its results at the EPS Conference in 1995 [9]; DELPHI at the APS Meeting in August 1996 [10] and ALEPH in September 1996 at Tau Workshop. In addition ALEPH had already presented the update of the five pion analysis at the 1995 EPS Conference [11].

The reason for which all experiments have used the three and the five prongs mode is that the ideal decay channel for the neutrino analysis should be reconstructed both with good resolution on the visible four-momenta and with low contamination from lower multiplicities modes. This kind of background is potentially dangerous because it biases the limit on the neutrino mass to lower values. The above conditions are difficult to achieve for decay modes with $\pi^0 s$. On one hand the energy resolution of the electromagnetic calorimeters is much worst than that of the tracking devices (for the relevant range of energy), on the other it is difficult to reduce the background from fake photons even with very restrictive cri-

Table 2
Main features of the published tau neutrino analysis.

	ARGUS	CLEO	ALEPH		OPAL
Produced $\tau^+\tau^-$	325 K	1.77 M	76 K		36 K
Selected Mode	$5\pi^\pm$	$5\pi^\pm/3\pi^\pm 2\pi^0$	$5\pi^\pm/5\pi^\pm\pi^0$		$5\pi^\pm$
Opposite hem. Mode	1 prong	μ, e	1,3 prong		1 prong
Selection Eff. (%)	4.2	2.6/0.3	24.7/7.0		6.4
Selected Events	20	60/53	23/2		5
Sel. Ev. with $m_{had} > 1.7\ MeV/c^2$	3	3/4	2/1		1
Mass Resolution (MeV/c^2)	20	10	15		20
Energy Resolution (MeV)	-	-	350		500
Method	1-D	1-D	1-D	2-D	2-D
95% CL limit (MeV/c^2)	31.0	32.6	40.2	23.8	74.0

teria. In fact, due to the high boost of the tau, the photon showers and the showers produced by charged hadrons are often too close to be fully resolved by the spatial granularity of the calorimeters. Therefore photon identification is difficult while fake π^0s are often constructed from the fluctuations of other showers. In this aspect ALEPH possesses some advantages: its electromagnetic calorimeter has a higher granularity and lies immediately after the tracking system, while DELPHI and OPAL have placed their calorimeters after the Cĕrenkov detectors or the magnet coil (for more details see *e.g.* [12]).

Moreover the momentum resolution of the detectors is not precise enough to use the $\tau \to h\ \nu_\tau$ mode. In this case the momentum of the hadron ranges from about 200 MeV/c to about 45 GeV/c. At the two extremes, where the spectrum becomes sensitive to a massive neutrino, the resolution is degraded either by the influence of multiple scattering or by the dependence on p^2 of the resolution. Differently, the average momentum of the hadrons in the three or five prongs modes is about 10 GeV/c, which is ideal from the point of view of detector performances.

2. The Methods

There are basically four methods to limit the tau neutrino mass:

a) By full kinematic reconstruction:
this method uses the maximum of information but it requires the knowledge of the tau direction $\hat{\tau}$. In principle $\hat{\tau}$ can be measured in $\tau^+\tau^-$ events where $\tau^- \to 3\pi^-\nu_\tau$ and $\tau^+ \to 3\pi^+\bar{\nu}_\tau$, by reconstructing the two vertexes of the three hadrons. Another possibility is to approximate $\hat{\tau}$ in some way, e.g. with the thrust direction. OPAL chose this method for its $3\pi^{+/-}$ analysis.

b) By a fit to the mass spectrum:
this method has the advantage that the endpoint of the m_{had} spectrum depends linearly on m_ν. Moreover it does not suffer from the energy degradation of the τs due to initial/final radiation. The $3\pi^\pm$ mode is disfavoured w.r.t. the $5\pi^\pm$ mode because the $m_{3\pi}$ distribution is shaped by the $a_1(1260)$ resonance, which depopulates the region near the kinematic boundary. This method is used in the DELPHI analysis.

c) By a fit to the energy spectrum:
this method, originally employed to bound the neutrino mass, was abandoned because of the quadratic dependence of the endpoint on the neutrino mass.

d) By a 2-D fit to the mass-energy spectrum:
this method is the natural combination of the previous two. It has been employed by ALEPH in all its analysis and by OPAL for the analysis of the $5\pi^\pm$ mode. In fact, given the actual distributions on the (m_{had}, E_{had}) plane of the two modes, the analysis of the $3\pi^\pm$ mode is very similar to an energy endpoint.

After the choice of the favourite distribution $\mathcal{F}(\vec{x}, m_\nu) = 1/\Gamma \cdot d^2\Gamma/dm_{had}\ dE_{had}, 1/\Gamma \cdot d\Gamma/dm_{had}$, etc. ($\vec{x}$ = any set of measured variables), the bound on m_ν is derived by a maximum likelihood fit. The likelihood is computed after convolving $\mathcal{F}(\vec{x}, m_\nu)$ with the experimental effects of the selection efficiency $\epsilon(\vec{x})$ and the resolution $\mathcal{R}(\vec{x}-\vec{\xi})$, where $\vec{\xi}$ is the measured value of $\vec{x}$. If the energy of the tau is used the initial/final radiation should be accounted for by a further convolution with a radiation kernel $\mathcal{G}(E_\tau, E_{beam})$. The total convolution gives the probability density $\mathcal{K}_i(\vec{\xi}, m_\nu)$ of observing the i-th event, from which the total (normalised) likelihood is build:

$$\mathcal{L}(m_\nu) = \frac{\prod_{i=1}^{N} \mathcal{K}_i(\vec{\xi}, m_\nu)}{\int_0^{+\infty} [\ \prod_{i=1}^{N} \mathcal{K}_i(\vec{\xi}, m_\nu)\ ]\ dm_\nu}$$

$$\mathcal{K}_i(\vec{\xi}, m_\nu) = \mathcal{F}(\vec{x}, m_\nu) \otimes \mathcal{R}(\vec{x} - \vec{\xi}) \otimes \epsilon(\vec{x})$$

This approach is used in all analysis but the ALEPH $3\pi^\pm$. In that analysis the fitted region is binned and for each bin the probability of finding the observed events is computed using the Poissonian statistics. The number of expected events is a function of the neutrino mass in which the signal contribution is given by the integral of $\mathcal{K}(\vec{\xi}, m_\nu)$ over the bin and the background is taken from the Monte Carlo (MC) prediction.

After the dependence of the likelihood on m_ν has been determined, the 95% CL is derived. DELPHI and OPAL have applied the Bayesian definition and computed the 95% CL interval $(0, m_{95})$ by integrating $\mathcal{L}(m_\nu)$:

$$\int_0^{m_{95}} \mathcal{L}(m_\nu)\ dm_\nu = 0.95 \int_0^{+\infty} \mathcal{L}(m_\nu)\ dm_\nu$$

Note that in the Bayesian approach the probability density to observe the i-th event is modified by the *a priori* probability density function $f_0(m_\nu)$ of the tau neutrino having a given mass m_ν:

$$\mathcal{P}_i(\vec{\xi}, m_\nu) \equiv \frac{\mathcal{K}_i(\vec{\xi}\ |m_\nu)\ f_0(m_\nu)}{\int \mathcal{K}_i(\vec{\xi}\ |m_\nu)\ f_0(m_\nu)\ dm_\nu}$$

and that the best estimate of m_ν is given by its expected value: $m_\nu^{best} = \int_0^{+\infty} m_\nu\ \mathcal{L}(\vec{x}, m_\nu)\ dm_\nu$, not by the value m_ν^{max} which maximises $\mathcal{L}$. In fact both experiments have assumed a uniform distribution for $f_0(m_\nu)$ so that $\mathcal{P}(\vec{\xi}, m_\nu)$ is equal to $\mathcal{K}(\vec{\xi}, m_\nu)$. ALEPH has used the frequentist definition, according to which $\mathcal{L}$ is an estimator of m_ν and is *wrong* to integrate it. Also it is forbidden to define a probability density function of m_ν (other than a Dirac δ) so that there is no *subjective* choice to do. The best estimate m_ν is m_ν^{max} and the 95% confidence level is defined through the relation:

$$\mathcal{L}(m_{95})/\mathcal{L}(m_\nu^{max}) = 1/1.92$$

which is valid for any reasonable functional dependence of $\mathcal{L}$ (see e.g. [13]) on m_ν up to corrections of order $\mathcal{O}(1/N)$. The two definitions are numerically equivalent for a Gaussian $\mathcal{L}$. This is not a coincidence since the likelihood becomes Gaussian in the limit of a large numbers of events thanks to the central limit theorem. In fact ALEPH and OPAL likelihoods are extremely well described by Gaussians, so that the different definitions imply a negligible numerical difference.

Another important point in the derivation of the limit is the question of defining $\mathcal{L}$ for negative values of m_ν^2. Numerically this is trivial when doing a mass endpoint. In the case of the energy endpoint it becomes possible only by defining an -arbitrary- analytical continuation of $\mathcal{F}(\vec{x}, m_\nu)$. This is necessary since the extremes of integration in $\mathcal{K}(\vec{x}, m_\nu)$ become imaginary in a region of (m_{had}, E_{had}) when $m_\nu^2 < 0$ is allowed, which is nothing but the consequence of the two body decay kinematics. This kind of approach has been made very popular by the tritium β decay experiments. However a negative m_ν^2 violates the Relativity principles and therefore it is illogical to introduce it in this way. The correct approach would be to modify the metric of the space, like *e.g.* in [14].

3. Modelling of Tau decays

The matrix element $\mathcal{M}$ for the decay of the tau can be written following Tsai notation [15]:

$$\mathcal{M} = \mathcal{L}^{\mu\nu}\ H_{\mu\nu} =$$

$$[p_\alpha k_\beta [g^{\alpha\mu} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\nu} - g^{\alpha\beta} g^{\mu\nu}] + i k_\alpha p_\beta \epsilon^{\alpha\mu\beta\nu}]$$

$$\times\ [(g_{\mu\nu} - q_\mu q_\nu/q^2) \cdot v(q^2) + q_\mu q_\nu \cdot a(q^2)]$$

where p(k) is the four-momentum of the $\tau(\nu_\tau)$, $q^2 = (p-k)^2 = m^2_{had}$, and $v(q^2), a(q^2)$ are -by definition- the spectral functions (s.f.) for $J = 1,0$ final states. This gives:

$$\overline{|\mathcal{M}|^2} = \frac{1}{2}[\omega_1(q^2, m^2_\nu) \cdot v(q^2) + \omega_0(q^2, m^2_\nu) \cdot a(q^2)]$$

$$\omega_1 = (m^2_\tau + 2q^2)(m^2_\tau - q^2) - m^2_\nu(2m^2_\tau - q^2) + m^4_\nu$$

$$\omega_0 = m^2_\tau\ (m^2_\tau - q^2) - m^2_\nu(2m^2_\tau + q^2) + m^4_\nu$$

The relevant portion of phase space of the decay is given by:

$$\lambda(q^2, m^2_\nu) = \sqrt{(m^2_\tau - q^2)^2 - 2m^2_\nu(m^2_\tau + q^2) + m^4_\nu}$$

$$\simeq (m^2_\tau - q^2) - m^2_\nu \cdot \frac{m^2_\tau + q^2}{m^2_\tau - q^2} + \mathcal{O}(m^4_\nu)$$

From these formulas it is clear that the effects of m_ν in the matrix element are almost negligible and that the sensitivity to a massive neutrino increase rapidly near the kinematic boundary. It is also useful to remember that for a fixed value of q^2 the distribution of E_{had} is just the typical two-body box distribution, distorted by initial/final state radiation, τ polarisation and detector effects.

For what concerns the functional form of the s.f., they are not predicted by the theory nor it is possible to infer them from the e^+e^- data *via* CVC because both modes are G odd. Nevertheless the uncertainty in the form of the s.f. plays a minor role in the determination of the bound on m_ν because the s.f. are assumed to be mildly dependent on q^2 in the small region at the edge of phase space. If this is not the case their influence could be much bigger. This point will be discussed later.

The basic description of $\tau \to 3\pi^\pm\nu_\tau$ is done by neglecting the scalar term, which is suppressed by PCAC and by helicity considerations, and invoking the dominance of the $a_1(1260)$ for the vector term. Successively the a_1 decay is described through the chain $a_1^- \to \rho^0(\rho'^0)\pi^- \to \pi^+\pi^-\pi^-$.

This approach has been refined by several authors considering the distortions to the a_1 propagator due to the effect of the $K^*(892)\overline{K}$ threshold [16], by introducing a scalar $\pi'(1300)$ resonance [17] or a non-resonant term [18] in the decay amplitude.

All the experiments use a model by Kühn and Santamaria [19], inspired by the asymptotic limit $q^2 \to 0$ of chiral theory. This model is relatively popular because it has been implemented in the KORALZ [20] program. The hadronic current J^μ_{had} is written:

$$J^\mu_{had} = < \pi(q_1)\ \pi(q_2)\ \pi(q_3)|J^\mu_{had}(0)|0 > =$$

$$= -i\frac{2\sqrt{2}}{3f_\pi}BW_{a_1}(q^2)\ [B_\rho(s_1) \cdot V^\mu_1 + B_\rho(s_2) \cdot V^\mu_2]$$

where f_π the pion decay constant, q_j ($j = 1,2$) are the four-momenta of the two like-sign pions, $s_j = (q_j + q_3)^2$, V^μ_j and B_ρ are given by:

$$V^\mu_j = q^\mu_j - q^\mu_3 - q^\mu\frac{q \cdot (q_j - q_3)}{q^2}$$

$$B_\rho = \frac{BW_{\rho(770)} + \beta\ BW_{\rho(1450)}}{1+\beta}$$

and $BW_x(Q^2)$ are Breit-Wigner functions with energy dependent widths. The values of m_{a_1}, Γ_{a_1} and β are fitted to the ARGUS data.

The inclusion of a $\pi(1300)$ scalar term considered in [17] would introduce an additional contribution to the total width $\Gamma_{3\pi}$ of about 5%. The scalar part of the spectral function peaks at $q^2 \simeq m^2_{\pi(1300)}$, becoming almost constant after $\sqrt{q^2} \simeq 1.5\ GeV/c^2$. Therefore its effect are not likely to be relevant for the determination of the tau neutrino mass limit.

The situation for the $\tau \to 5\pi(\pi^0)\nu_\tau$ mode is very different from the 3π mode. There are very few studies of the spectral functions, mainly because the statistics available is extremely low. Experimentally it is seen that the invariant mass of the hadronic system peaks at high values of q^2, and seems unlikely to be dominated by a single resonance. In the published works by ARGUS, CLEO and OPAL a crude model with pure phase space was used. Such model could be improved by the addition of a spin one wave between the tau decay products. Some studies in this direction have been performed by assuming a $\tau^- \to \rho\rho\pi^-\nu_\tau$ decay as in [21], or either a $\tau^- \to a_1^-\pi^+\pi^-\nu_\tau$ or a $\tau^- \to \rho^0\pi^+\pi^-\pi^-\nu_\tau$ decay as in the work of ALEPH. In all cases the inclusion of the intermediate resonances has the effect of distorting to higher q^2 the shape of the spectral functions. The numerical effect on the ALEPH bound however is negligible.

4. The Modelling of Detector Effects

Distortions of $\mathcal{F}(\vec{x}, m_\nu)$ are produced by the effects of the selection criteria and detector resolution. For what concerns event selection the efficiencies are relatively high, ranging around 43% for $\tau \to 3\pi\nu_\tau$, and 25% for ALEPH $\tau \to 5\pi\nu_\tau$. The inefficiencies derive mainly from the limited acceptance of the detectors at low polar angles and the poor separation of very close trajectories of like sign charged particles. The non-τ background surviving the selection criteria is negligible. Moreover it is measurable on the data themselves thanks to extreme separation of the two decay hemispheres. Using one-hemisphere dedicated tags, both ALEPH and OPAL build control samples from the data where they measured the amount of $q\bar{q}$ background. The background from other τ modes is very small in the case of $\tau \to 5\pi^\pm\nu_\tau$, while it is relatively high for $\tau \to 3\pi^\pm\nu_\tau$, from the ALEPH 10% up to DELPHI 25%, but since it originates almost exclusively from higher multiplicities modes like $\tau \to 3\pi^\pm\ n\pi^0\ \nu_\tau$, or higher q^2 modes like $\tau \to K\pi\pi\ \nu_\tau$, it is not dangerous for the limit. In all cases the selection efficiency is mildly dependent on $\vec{x}$. In the $3\pi^\pm$ mode the dependence is bigger because the average angle between the pions is smaller and the trajectories overlap more than in the case of $5\pi^\pm$.

Differently from the selection effects the modelling of the detector resolution is a delicate matter. In general the form of $\mathcal{R}$ be derived in three ways:

a) The direct technique:
in this case the error matrix of the tracking algorithms is used. The advantage is that $\mathcal{R}$ is computed event by event, so that the dependence on the kinematical topology of the decay is automatically taken into account. However such algorithms have a strong tendency to underestimate tails in the resolution produced by non-Gaussian effects. Moreover it is difficult to tag independent samples where these tails can be measured. This method has been adopted by OPAL in the $3\pi^{+/-}$ analysis.

b) The duplication technique:
in this case each data event defines a kinematic topology; for each topology huge samples of identical events are generated and processed through the detector simulation so that high statistics samples of MC *duplicated* events are available. For each sample the resolution is then computed by fitting the distribution of $(\vec{x} - \vec{\xi})$. For example ALEPH and OPAL employ this approach for the $5(6)\pi$ analysis using for $\mathcal{R}$ the sum of a Bigaussian and a flat tail.
This method is the most accurate because it takes properly into account both the non-Gaussian effects and the dependence on the kinematic topology of the observed events.

c) The standard technique:
The drawback of the previous technique is the necessity to generate huge samples of MC events. When the statistics of the data is high this becomes difficult. The resolution used is then obtained by averaging over on all topologies; the high statistics ensuring that all topologies are encountered. In practice this means that the parameters entering $\mathcal{R}$ are a function of m_{had} and E_{had} alone and not of the four-momenta of the single pions. This approach is used both by ALEPH and DELPHI for their $3\pi^\pm$ analysis. ALEPH uses the sum of two Bigaussians and a flat tail, while DELPHI uses three Gaussians.

It should be remembered that the choice between these methods is also conditioned by the choice of $\mathcal{F}(\vec{x}, m_\nu)$. For example OPAL $3\pi^{+/-}$ are less disturbed by the tails in the measurement of $m_{3\pi}$ and $E_{3\pi}$ thanks to the correlation with the opposite hemisphere. On the other hand an additional systematics on the determination of the thrust direction had to be considered.

5. Results of the analysis

Apart from the update of the ALEPH $5\pi(\pi^0)$ analysis, all new LEP results come from the 3π mode. Their comparison is very interesting because, the statistics of the fitted samples is very high. In addition the use of three different techniques permits a comparison of methods and a cross-check of the results.

Update of ALEPH $5\pi^{\pm}(\pi^0)$ analysis

The addition of 1994 data to the published analysis of the 1991-93 data brought a minor improvement to the limit. The fitted sample is now composed of a total 38 $\tau^- \rightarrow 5\pi^{\pm}\nu_\tau$ and 3 $\tau^- \rightarrow 5\pi^{\pm}\pi^0\nu_\tau$ candidates. On the basis of a MC/data comparison it was expected to find the new events distant from the kinematic boundary. In fact while the 95% CL limit on m_ν improved from 23.8 MeV/c^2 to 23.1 MeV/c^2, the probability of observing the events, given the expected $\tau^- \rightarrow a_1^- \pi^+\pi^-\nu_\tau$ distribution, increased from 5% to 10%. The candidate events in the high q^2 region are shown in Fig. 2.

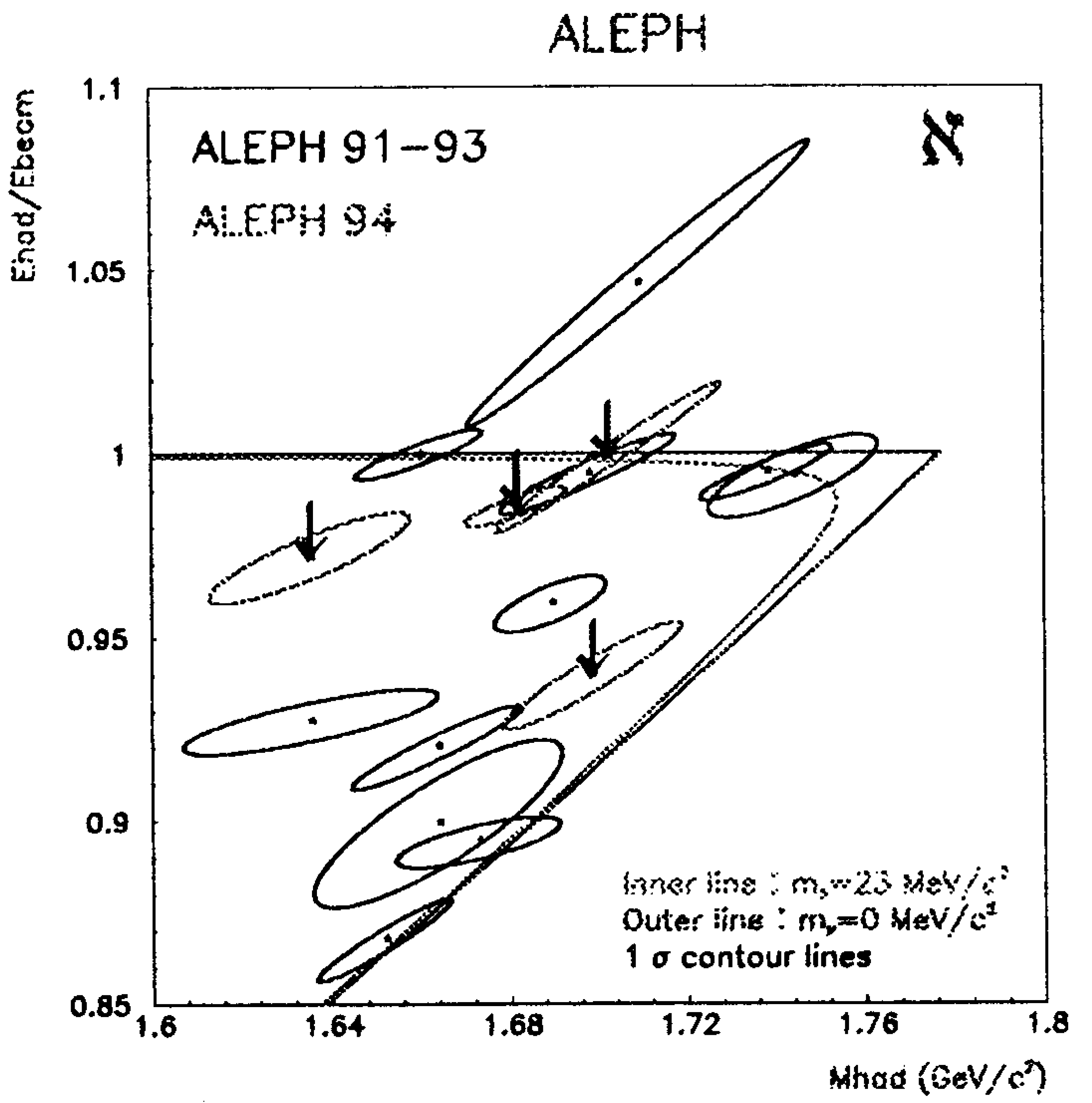

Figure 2. Distribution of ALEPH $\tau \rightarrow 5\pi^{\pm}(\pi^0)\nu_\tau$ events onto the $(m_{had}, E_{had}/E_\tau)$ plane shown as points with 1σ error ellipses. The continuous lines delimit the allowed region for a neutrino with $m_\nu = 0, 23$ MeV/c^2. The events added in the update of the analysis are indicated with an arrow.

The ALEPH $3\pi^{\pm}$ analysis

This analysis is based on the full 1991-95 statistics. The method employed is the same as in the previous mode, except that only a region of the $(m_{had}, E_{had}/E_{beam})$ is used for the fit. The region is defined by $m_{had} \in [0.757, 1.817]$ GeV/c^2 and $E_{had}/E_{beam} \in [0.871, 1.043]$ and contains approximately 19% of the $3\pi^{\pm}$ signal events. The distribution of the fitted events is shown in Fig. 3.

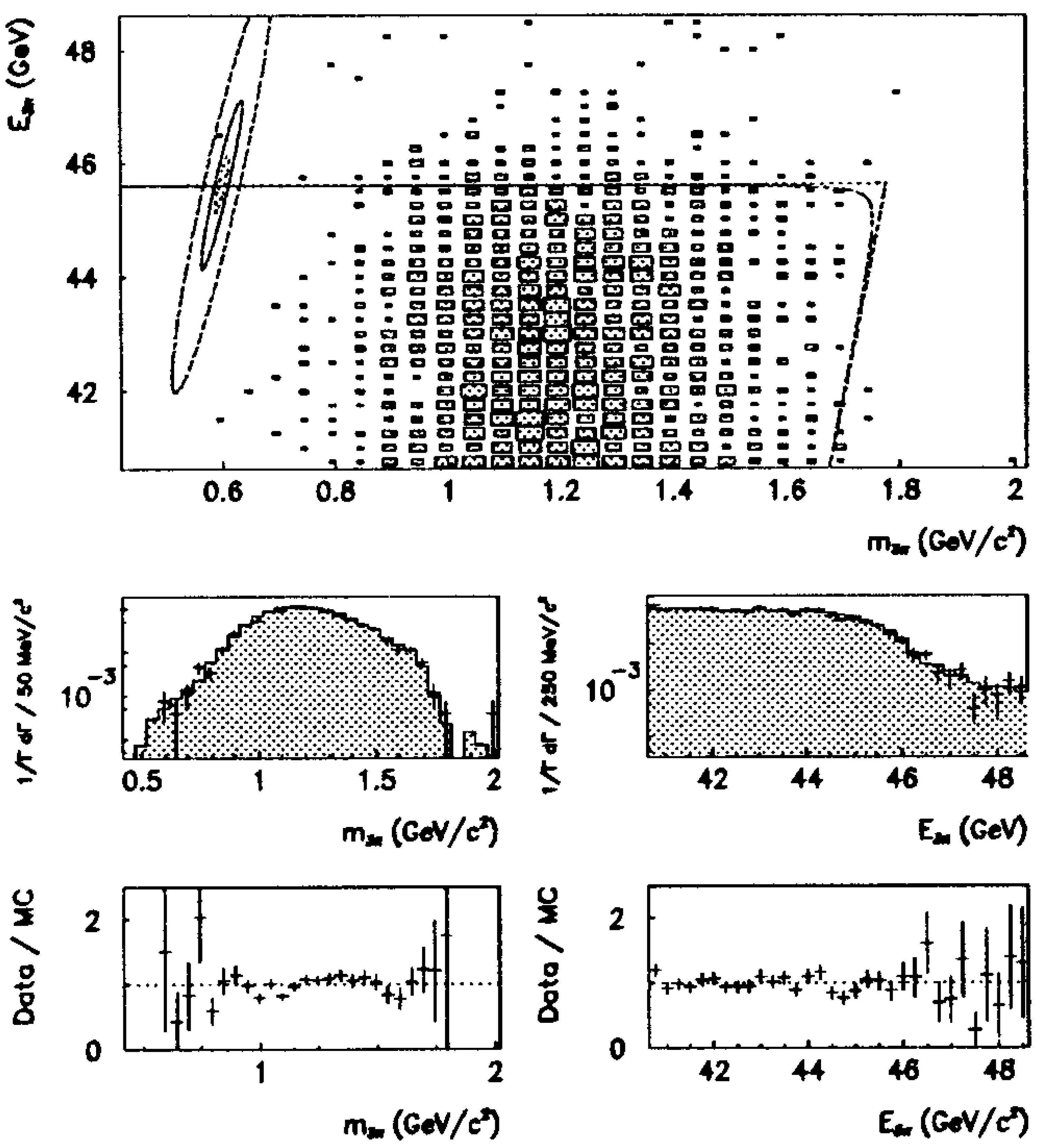

Figure 3. Distribution of ALEPH $\tau \rightarrow 3\pi\nu_\tau$ events onto the (m_{had}, E_{had}) plane. The continuous lines are defined as in Fig. 2. The three components of the resolution are also shown, centred at $(0.6\ GeV/c^2, E_{beam})$. The lower plots show the projection onto the two axes compared with the MC prediction (filled histogram), and the ratio of the data to MC prediction.

The DELPHI $3\pi^{\pm}$ analysis

This analysis is based on the 1992-94 statistics and uses only the hadronic mass as variable. The q^2 distribution is shown in Fig. 4 for the high q^2 region only. A clear excess of events in the proximity of the endpoint is seen.

According to the DELPHI simulation this region should be populated only by signal events. The excess is such that the result of the fit is $m_\nu = (-33 \pm 23)\ MeV/c^2$, which results in a 27 MeV/c^2 95% CL limit. The result however is still *very preliminary*.

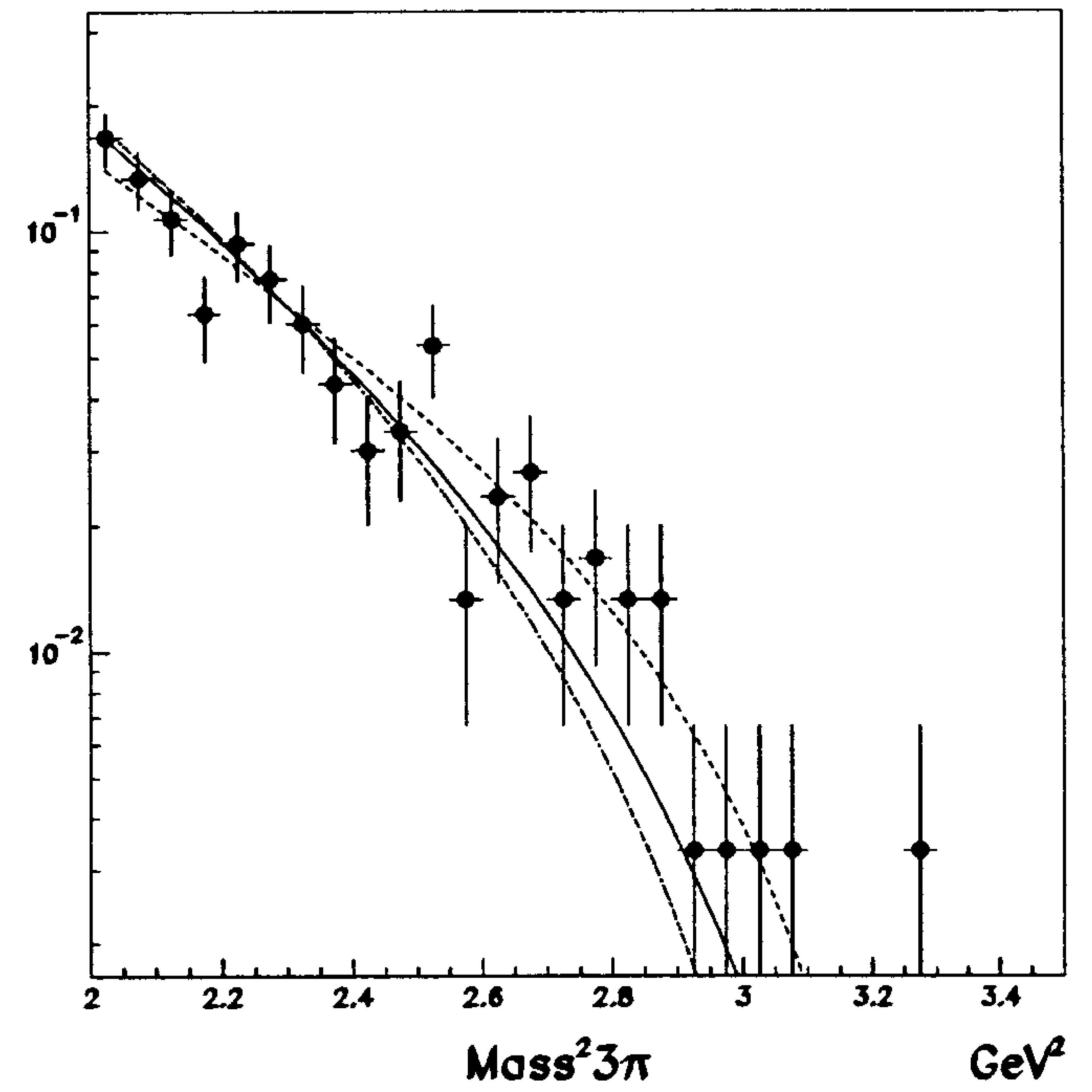

Figure 4. Distribution of m^2_{had} for DELPHI selected events in proximity of m_{had} endpoint is shown as solid points with error bars. Full and dot dashed lines are fit result for input $m_\nu = 0, 27\ MeV/c^2$ neutrino masses. The dashed line is for the addition of a higher $a_1'(1800)$ resonance.

The OPAL $3\pi^+/3\pi^-$ analysis

This analysis is based on the statistics collected in the 1990-94 period. OPAL selected events in which both taus decayed to charged three prongs. The thrust of the event:

$$\hat{P}_{thrust} = \frac{\vec{P}_{3\pi^-} - \vec{P}_{3\pi^+}}{|\vec{P}_{3\pi^-} - \vec{P}_{3\pi^+}|} = \hat{P}_\tau$$

approximates the direction of the tau momenta, which was then used to build the normalised missing energy ω and the missing mass squared η:

$$\omega = \frac{E_{miss}}{E_\tau} = \frac{E_\tau - E_{3\pi}}{E_\tau}$$

$$\eta = m^2_{miss} = (E_\tau - E_{3\pi})^2 - (\vec{P}_\tau - \vec{P}_{3\pi})^2$$

It has to be noticed that the value of η does not coincide with m^2_ν because of the approximation used, but rather:

$$\eta = m^2_\nu + 2\ P_\tau\ P_{3\pi}\ (cos\theta_{thrust} - cos\theta_\tau)$$

the angles θ being defined w.r.t. $\vec{P}_{3\pi}$.

The distributions of OPAL events are shown in Fig. 5. The disagreement between the data and the simulation around $\eta = 0$ is treated as an additional source of systematic error.

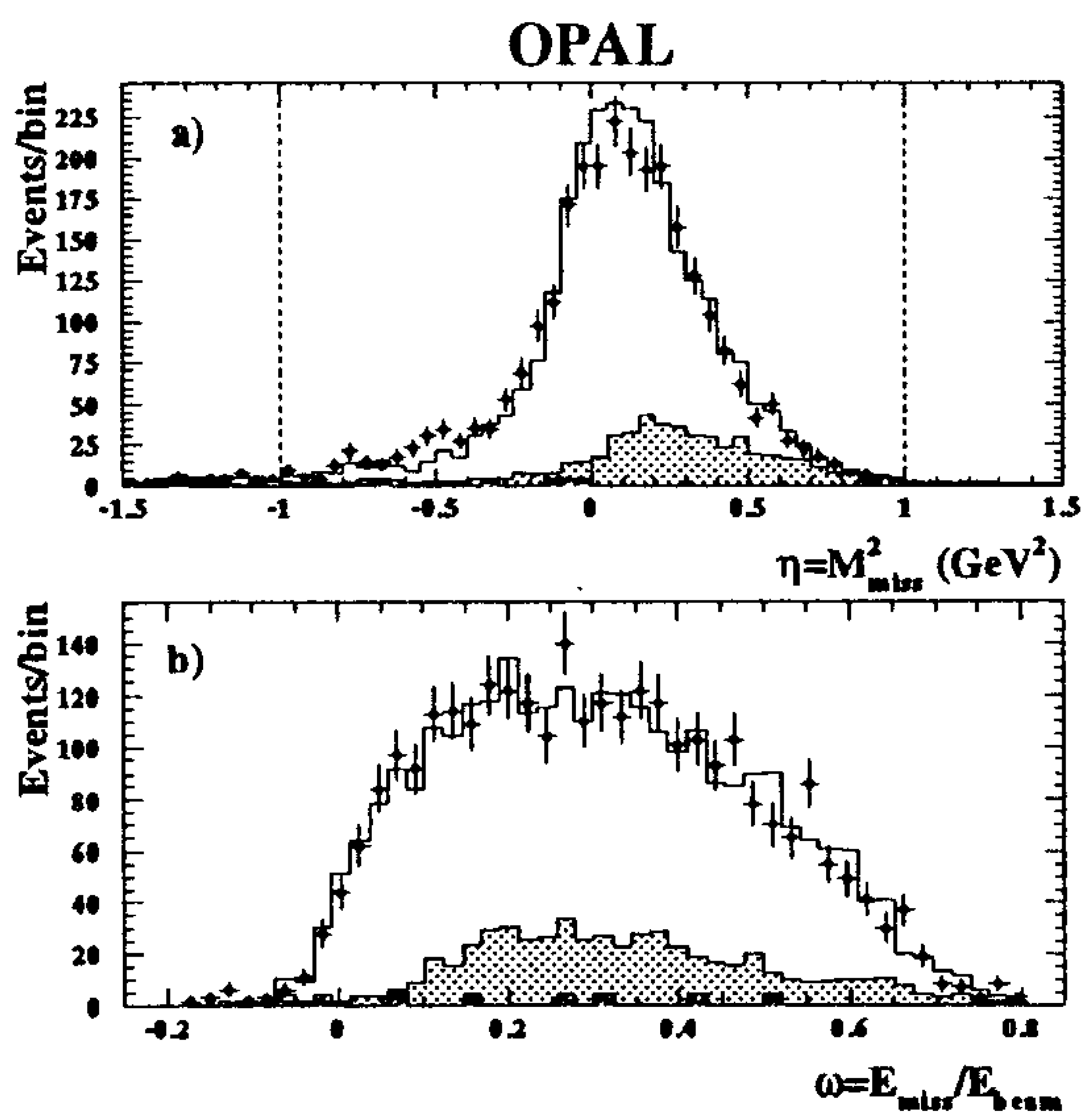

Figure 5. Distributions of η and ω for OPAL events are shown as solid points with error bars. The MC expectation for an input $m_\nu = 10\ MeV/c^2$ is shown as a solid histogram; the expected background is drawn shaded.

The results of the four unpublished analysis are reported in Table 3. It is clear that the limits obtained using the $3\pi^\pm$ mode are well competitive with those from $5\pi^\pm$ in spite of the unfavoured dynamics of the decay. Clearly the loss of sensitivity is recovered thanks to the higher branching fraction of the 3π mode (about 10% vs 0.1%). It is

Table 3
Main features of the unpublished tau neutrino analysis.

	ALEPH	ALEPH	DELPHI	OPAL
Produced $\tau^+\tau^-$	140 K	180 K	100 K	150 K
Selected Mode	$5\pi^\pm/5\pi^\pm\pi^0$	$3\pi^\pm$	$3\pi^\pm$	$3\pi^{+/-}$
Opposite hem. Mode	1,3 prongs	1,3 prongs	1 prong	$3\pi^{-/+}$
Selection Eff. (%)	25/7	49	38	42
Purity (%)	100	90	74	77
Fitted Events	38/3	2939[a]	6534	2514
Mass Resolution	10 MeV/c^2	10 MeV/c^2	20 MeV/c^2	0.1 GeV^2/c^4 [b]
Energy Resolution (MeV)	350	400	-	500
95% CL limit (MeV/c^2)[c]	22.3	21.5	27.0	32.1

(a) ALEPH fits only the 19% fraction of high E_{had} events
(b) refers to η (see text)
(c) statistics only (see text)

also interesting to see that the ALEPH and OPAL 3π results compare well if scaled by the number of fitted events and the resolutions. DELPHI result scales faster, a consequence of the excess of data near the endpoint.

6. Systematic errors

After the 95% CL limit m_{95} has been derived from the likelihood $\mathcal{L}$, the effect of each source $\sigma\epsilon$ of systematic error is computed. All experiments adopt the same method. For each $\sigma\epsilon$ they build a new likelihood $\tilde{\mathcal{L}}_{\sigma\epsilon}$ and derive a new 95% CL limit $\tilde{m}_{95}$. The total systematic error is given by:

$$\Delta m_{95} = \sqrt{\sum_{\sigma\epsilon}(\tilde{m}_{95} - m_{95})^2}$$

which is added linearly to m_{95} to get the final 95% CL limit. The sources of systematic errors can be grouped in four classes:

1) Tau properties

The mass and polarisation of the τ have been be varied according the best available measurements. These errors are perfectly under control. The τ energy is equal to the beam energy which is intensively studied by the LEP Calibration Group. Also this source of error is well under control.

2) Event selection

This is a minor concern at LEP. The efficiency is mildly dependent on the fit variables and its absolute value is irrelevant for the analysis. The τ background has the effect of deteriorating the limit to higher values, and the non-τ background is negligible. The distribution of the variables used in the selection are investigated by MC/data comparison. In fact all the experiments found very small errors from these sources.

3) Detector simulation

The resolution function entering the determination can be wrongly centred because of calibration effects, too narrow because of an optimistic estimate of detector resolution, or too regular because of forgotten non-Gaussian effects. The first two effects have been tested directly by ALEPH and OPAL on the data. The error on $|\vec{p}|$ of a given particle is modelled as:

$$\delta p = k_1 \mid \vec{p} \mid + q\ k_2 \mid \vec{p} \mid^2$$

where $\vec{p}$ is the measured momentum, q the electric charge, and $k_{1/2}$ two constants.

The first term takes into account magnetic field distortions and the second sagitta errors. The constant k_1 is measured exploiting two favourable effects. The first is that the second term cancels out -at first order- when making the invariant mass of two particles and the second is the fact that in $D^0 \to K^-\pi^+$ decays the daughter tracks are emitted at large angles so that only the error on $|\vec{p}|$ contributes to the error on the measured value of $m^2_{D^0}$. Attention to a possible dependence of k_1 on q^2 has been paid by measuring $m^2_{D^0}$ as a function of the D^0 energy. The sagitta error has

Table 4
Systematic errors in the unpublished analysis.

Selected Mode	ALEPH $5\pi^\pm/5\pi^\pm\pi^0$	ALEPH $3\pi^\pm$	DELPHI $3\pi^\pm$	OPAL $3\pi^{+/-}$
Systematic Source	Variation of the 95% CL limit (MeV/c^2)			
Selection efficiency	< 0.1	0.1	t.b.d[a]	< 0.1
Background	0.3	0.1	1.0	0.6
Tau mass	0.2	0.3	t.b.d.	< 0.1
Beam Energy	0.3	0.1	-	0.1
Spectral functions	< 0.1	0.2	1.8	1.4
Calibration	0.3	2.6	1.0	2.2
Resolution	0.2	3.2	t.b.d.	1.2
Tails	0.6	0.6	t.b.d.	-
Fit Region	-	0.7	-	1.2
TOTAL	0.8	4.2	2.3	3.2
95% CL limit[b] (MeV/c^2)	23.1	25.7	29.3	35.3

(a) to be done
(b) systematics included

been measured by a more conventional technique, namely by comparing the difference in momenta of the two muons in $Z \to \mu^+\mu^-$ events. The error on the resolution $\sigma_{|\vec{p}|}$ is derived by comparing the measured width of the D^0 with the one predicted by the MC. However the small tails of the resolution (typically few % of the total events) are masked by the combinatorial background of the D^0 candidates and cannot be measured in this comparison. The amount of the tails is therefore determined with the MC and checked on low statistics test samples.

The above determinations are sufficient to estimate the systematic error on E_{had} but not on m_{had}. This is done by measuring the mass and the width of $D^- \to K^+\pi^-\pi^-$ which is the decay more similar to $\tau \to 3(5)\pi^\pm\nu_\tau$. Again the tails in the resolution are masked by the combinatorial background. In fact D^- decays have been used only by ALEPH. OPAL has used the $(D^\star)^+ \to D^0\pi^+ \to K^-\pi^+\pi^+$ invoking similar considerations. DELPHI has not measured any of the above systematics yet.

4) Spectral functions

In the published 5π analysis the variation of the spectral functions had little effects. In the 3π analysis they are also a minor source of error. However, if some new resonances with $m_{res} \simeq m_\tau$ are introduced, one expects bigger effects because the phase space is strongly perturbed near the kinematic boundary.

For the $5\pi^\pm$ mode an additional test was performed by ALEPH [22] by modelling the decay as $\tau \to \rho'(1800)\nu_\tau \to 5\pi^\pm\nu_\tau$. The limit computed in this way was 2 (6.2) MeV/c^2 higher than the published one for the 2(1)-D method.

For the $3\pi^\pm$ analysis DELPHI introduced a $a_1'(1800)$ resonance and fitted simultaneously its content and the tau neutrino mass. In this way the limit increased to 71 MeV/c^2.

It is useful to remind that a clear excess of events at high q^2 is observed only by DELPHI. On the contrary all experiments agree on the fact that the a_1 decay is poorly described using [19].

These tests are interesting because they give the magnitude of the most dramatic deformation of the s.f. one can think of. However they should be considered asymptotical errors. Further studies of the s.f. are certainly welcomed and it is very likely that they will be achieved quite soon, especially in the $3\pi^\pm$ mode.

The effects on the limit are reported in Table 4 for each source of systematic error. It can be seen that, in all analysis, the largest variations are produced by calibration and resolution errors.

7. Combining the analysis

ALEPH and OPAL have combined the analysis of the $3\pi^\pm$ and $5\pi^\pm$ modes. The combined statistical limit is easily derived from the combined likelihood, which is just the product of the two single likelihoods. In this way ALEPH obtained a 95% CL limit of 16.6 MeV/c^2 and OPAL of 26.7 MeV/c^2.

The systematic error on the combined limit has been derived differently by the two experiments.

ALEPH uses the likelihoods $\tilde{\mathcal{L}}_{\sigma\epsilon}$ computed for each decay mode and each source of systematic error. For common sources, e.g. the tau mass, they are combined in the form:

$$(\tilde{\mathcal{L}}_{comb})_{\sigma\epsilon} = \tilde{\mathcal{L}}^i_{\sigma\epsilon} \cdot \tilde{\mathcal{L}}^j_{\sigma\epsilon} \qquad i,j = \tau \to 3/5\pi$$

while for uncommon sources, e.g. the $3/5\pi$ spectral functions, they are combined in the form:

$$(\tilde{\mathcal{L}}_{comb})_{\sigma\epsilon} = \tilde{\mathcal{L}}^i_{\sigma\epsilon} \cdot \mathcal{L}^j$$

In this way ALEPH obtains a total systematic error of 1.6 MeV/c^2 and a final 95% CL limit of 18.2 MeV/c^2. OPAL adopted a simpler method. The total systematic error of 3.2 MeV/c^2 was added to the combined limit. This is justified by the fact that the combined limit is dominated by the $3\pi^{+/-}$ result. The OPAL final result is then 29.9 MeV/c^2.

8. Conclusions

LEP experiments have strongly contributed to bound the τ neutrino mass. New methods have been developed and precise measurements obtained. This has been made possible thanks to people ingenuity and excellent detector performances. However LEP detectors are penalised by the low statistics, while CLEO has all the means to improve the accuracy of the ALEPH 18.2 MeV/c^2 limit. In the long run, after each experiment had fully analysed its own data, there is hope that the limit can be further improved by combining the results of several experiments.

Nevertheless, at the present moment, it seems very difficult that the final result could exclude neutrino masses down to 1 MeV/c^2. This has probably to await the advent of B and/or τC factories.

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