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$\eta \rightarrow \pi^0\pi^0\gamma\gamma$ to 1-Loop in ChPT *

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(background suppression)

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1 Introduction

a) Motivation:

Phenomenological interest for the rare decay $\eta \to \pi^0\pi^0\gamma\gamma \Longleftarrow$ Large number of observed η's anticipated at various η–factories, e.g. CELSIUS [2 $\times 10^9$], ITEP [$\sim 10^9$], DAΦNE [3×10^8 ($\phi \to \eta\gamma$)] and other facilities, such as GRAAL [10^8], MAMI, ELSA, CEBAF, ([n]=#η's per year).

Theoretical interest: testing chiral perturbation theory (ChPT) (effect of chiral loops). Of a similar interest [1-9]: [$\gamma\gamma \to \pi^0\pi^0$, $\eta \to \pi^0\gamma\gamma$]=0 to lowest order (LO) $\Longrightarrow$ chiral loops are important.

b) Status of $\eta \to \pi^0\pi^0\gamma\gamma$:

$$A(\eta \to \pi^0\pi^0\gamma\gamma) = A_R + A_{NR} \ ,$$

physically distinct.
A_R has a pole at $s_{\gamma\gamma} = m^2_{\pi^0}$ ($\sqrt{s_{\gamma\gamma}}$ = diphoton invariant mass).

$A_R \propto A^{on-shell}_{\eta\to 3\pi^0} A^{on-shell}_{\pi^0\to\gamma\gamma} \Longrightarrow$ Get A_R (up to a phase) from data
(no ChPT calculation needed)

$$A_R = -\frac{A(\eta \to 3\pi^0)A(\pi^0 \to \gamma\gamma)}{s_{\gamma\gamma} - m^2_{\pi^0}} \ .$$

A_R dominates over the full kinematical range to LO [10].

A_{NR} must be calculated in ChPT (not from data). A_{NR} computed at tree level $O(p^4)$ [10]: only η-exchange diagram $\eta \to \pi^0\pi^0\eta^* \to \pi^0\pi^0\gamma\gamma$.
In [10] also η'-exchange, formally of higher order (HO).
For both diagrams A_{NR} is negligible with respect to A_R, because the LO $\eta\eta\pi^0\pi^0$ and $\eta\eta'\pi^0\pi^0$ vertices vanish in the limit $m_u = m_d = 0$.

Note: analogous suppression factor in the π^0–exchange contribution $\propto (m_u - m_d)$, but thanks to the enhancement due to the pole term A_R dominates over A_{NR}.

To one loop?
Presumably not: in the (related) $\gamma\gamma \to \pi^0\pi^0\pi^0$ amplitude $A_{1-loop} \approx 10 A_{tree}$ [11], because $A_{tree} \propto m_\pi^2$ and A_{1-loop} is not suppressed.

c) Purpose:
to calculate A_{NR} to one loop, neglecting $m_u - m_d$ and the (suppressed) η-exchange. Only 1PI diagrams contribute and A_{NR} is finite.
$O(p^6)$ counterterms (CT) [12] do not contribute (as in $\gamma\gamma \to 3\pi^0$).

d) Results:
A_{NR}^{1-loop} dominates A_{NR}^{tree} (at $m_u = m_d = 0$: $A_{NR}^{1-loop} \neq 0$, $A_{NR}^{tree} = 0$).

At large $s_{\gamma\gamma}$, A_R (background for A_{NR}) is suppressed $\Longrightarrow$ detect a pure $O(p^6)$ effect by mesuring $\Gamma(\eta \to \pi^0\pi^0\gamma\gamma)$.

2 Kinematics and couplings

a) Kinematics of $\eta(q) \to \pi^0(p_1)\pi^0(p_2)\gamma(k_1, \epsilon_1)\gamma(k_2, \epsilon_2)$:

Five independent scalar variables:

$$s_{\pi\pi} = (p_1 + p_2)^2 \ , \qquad z_{1,2} = k_{1,2} \cdot (p_1 + p_2) \ ,$$
$$s_{\gamma\gamma} = (k_1 + k_2)^2 \ , \qquad z_3 = (k_1 + k_2) \cdot (p_1 - p_2) \ .$$

Decay amplitude:

$$A(\eta \to \pi^0\pi^0\gamma\gamma) \doteq e^2 \epsilon_1^\mu \epsilon_2^\nu A_{\mu\nu} \ .$$

Decay width:

$$\Gamma(\eta \to \pi^0\pi^0\gamma\gamma) = \frac{\alpha_{em}^2}{2^{11}\pi^6 m_\eta} \int \frac{d^3p_1}{p_1^0} \frac{d^3p_2}{p_2^0} \frac{d^3k_1}{k_1^0} \frac{d^3k_2}{k_2^0} \delta^{(4)}(p_1 + p_2 + k_1 + k_2) A^{\mu\nu} A^*_{\mu\nu} \ .$$

$A_{\mu\nu}$ is $O(p^4)$
(contributions only from odd–intrinsic parity sector of ChPT $\Longleftarrow$ process involving the electromagnetic interaction of an odd number of pions).

b) Interaction terms (couplings):

$O(p^4)$ ChPT $\mathcal{L}$:

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} \ ,$$

$$\mathcal{L}^{(2)} = \frac{F^2}{4} \mathrm{tr} \left(D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \right)$$

$\mathcal{L}^{(4)}$ splits into the odd-intrinsic anomalous part (i.e. the Wess-Zumino term [13]) and the $O(p^4)$ Gasser–Leutwyler lagrangian [14]

$$\mathcal{L}^{(4)} = \mathcal{L}_{WZ} + \sum_{i=1}^{10} L_i \mathcal{L}_i^{(4)} \ .$$

Usual exponential parametrization: $U = \exp(i\sqrt{2}P_8/F)$
$P_8 = SU(3)$ octet matrix of pseudoscalar mesons
$F|_{LO} \equiv \pi^+$ decay constant $F_\pi = 92.4$ MeV [14,15].

Covariant derivative:
$D_\mu U = \partial_\mu U + ieA_\mu[Q, U]$
Q =diag$(2/3, -1/3, -1/3)$.

In the external scalar sources: $\chi = \chi^\dagger = 2B\mathcal{M}$
$\mathcal{M} =$ diag(m_u, m_d, m_s) quark mass matrix
$B|_{LO} \equiv$ mass ratio $B_0 = m_\pi^2/(m_u + m_d)$.

Neglect in A_{NR} the $\eta - \eta'$ mixing, i.e. mixing of P_8 with the singlet–field η_0
$\Longrightarrow$ mass–eigenstate $\eta \equiv \eta_8$ octet–field.
Also $m_\pi = m_{\pi^0}$ (we neglect isospin–breaking in A_{NR}).

Couplings for tree–level calculation [10]:

$$\begin{aligned}
A^{(2)}(\eta_8 \to \pi^0\pi^0\pi^0) &= 3A^{(2)}(\eta_8 \to \pi^0\pi^+\pi^-) = \frac{B_0(m_u - m_d)}{\sqrt{3}F_\pi^2} \ , \\
A^{(2)}(\eta_8 \to \eta_8\pi^0\pi^0) &= A^{(2)}(\eta_8 \to \eta_8\pi^+\pi^-) = \frac{B_0(m_u + m_d)}{3F_\pi^2} \ , \\
A^{(4)}(\pi^0 \to \gamma\gamma) &= \sqrt{3}A^{(4)}(\eta_8 \to \gamma\gamma) = \frac{e^2}{4\pi^2 F_\pi}\epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu k_1^\nu \epsilon_2^\alpha k_2^\beta \ .
\end{aligned}$$

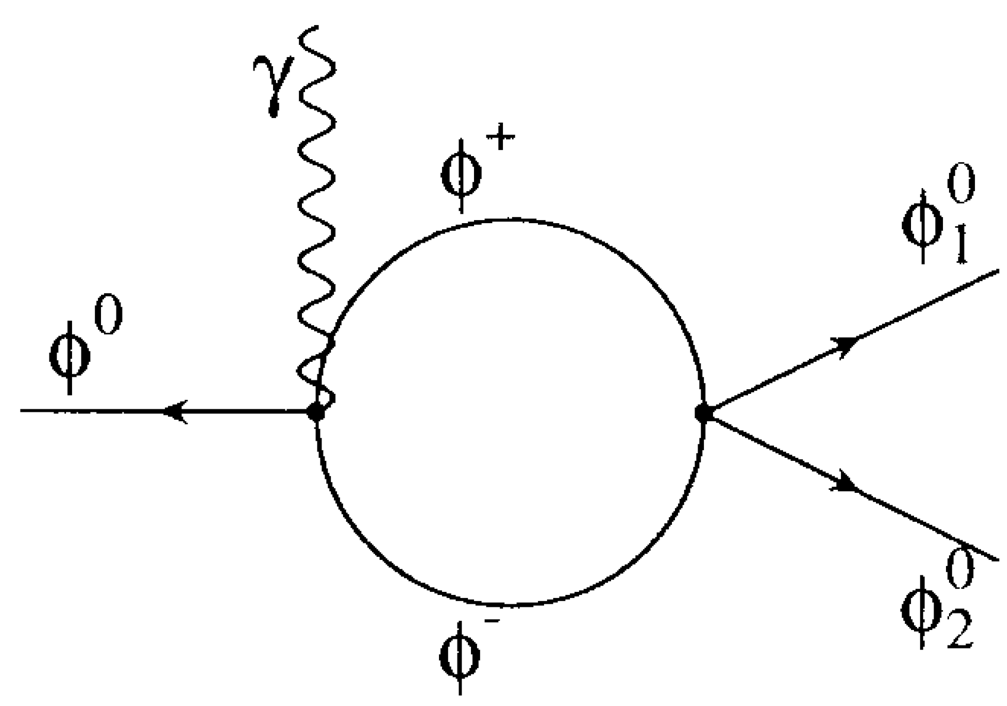

Figure 1: 1PI one–loop diagrams for the $\phi^0 \to \phi_1^0\phi_2^0\gamma\gamma$ transition. The second photon line has to be attached to the charged lines running in the loop and to the vertices.

Additional couplings for 1-loop diagrams in fig. 1:

$$\begin{aligned} A^{(2)}(\phi^+\phi^- \to \phi_1^0\phi_2^0) &= as_{\pi\pi} + bm_\pi^2 + c(p_+^2 - m_\pi^2) + d(p_-^2 - m_\pi^2) \, , \\ A^{(4)}(\phi^0 \to \phi^+\phi^-\gamma) &= f\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu k^\nu p_+^\alpha q^\beta \, , \end{aligned}$$

q, $p_\pm$, $p_{1,2}$, k: (outgoing) momenta of the pseudoscalars ϕ^0, $\phi^\pm$, $\phi_{1,2}^0$ and of A_μ.
a, b, c and d are constants with dim$= m^{-2}$; $[f] = m^{-3}$.

c, d are 'off–shell couplings' and irrelevant (they cancel in the amplitude, due to gauge invariance (GI)).
In $\pi^+\pi^- \to \pi^0\pi^0$ and $\eta_8 \to \pi^+\pi^-\gamma$ we find

$$a = -b = \frac{1}{F_\pi^2} \qquad \text{and} \qquad f = -\frac{e}{4\sqrt{3}\pi^2 F_\pi^3} \, .$$

(useful to estimate dominant π-loops).

3 Decay amplitude (analytic)

a) Tree-level:

$$A_R^{(4)} = -\frac{e^2}{4\sqrt{3}\pi^2 F_\pi^3} \frac{B_0(m_u - m_d)}{(s_{\gamma\gamma} - m_{\pi^0}^2)} \epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu k_1^\nu \epsilon_2^\alpha k_2^\beta \,,$$

$$A_{NR}^{(4)} = -\frac{e^2}{12\sqrt{3}\pi^2 F_\pi^3} \frac{B_0(m_u + m_d)}{(s_{\gamma\gamma} - m_\eta^2)} \epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu k_1^\nu \epsilon_2^\alpha k_2^\beta \,.$$

$A_R^{(4)}$ enhanced and dominant over $A_{NR}^{(4)}$ in the entire kinematical space.

b) One loop:

$O(p^6)$ loop and CT divided in three GI subgroups: reducible π^0–exchange diagrams, reducible η_8–exchange diagrams and 1PI diagrams.

i. π^0–exchange diagrams (include both loops and CT) contribute mainly to A_R.

   In principle they contribute also to A_{NR}. Decompose the $\eta \to \pi^0\pi^0(\pi^0)^*$ amplitude:

   $$A(\eta \to \pi^0\pi^0(\pi^0)^*) = A_{on-shell}(\eta \to 3\pi^0) + (s_{\gamma\gamma} - m_\pi^2) \times A_{off-shell} \,.$$

   Non–resonant contribution $\propto \mathcal{A}_{off-shell}$ vanishes in the limit $m_u = m_d$ $\Longrightarrow$ neglected.

   Extract $|\mathcal{A}_{on-shell}|$ from experiments, no need to evaluate it in ChPT.

ii. η_8–exchange diagrams (both loops and CT) contribute only to A_{NR} and can be neglected.

We explicitly checked that they are same order as tree–level $A_{NR}^{(4)}$ (small).

Reason of suppression: π-π loops (dominant contribution) are suppressed by $(m_u + m_d)$ (as the tree level).

K-K loops and $\mathcal{L}^{(4)}$ are not suppressed by $(m_u + m_d)$. Nonetheless negligible

(we are far below the kaon threshold and the CT combinations involved, i.e. $(L_1 + L_3/6)$, $(L_2 + L_3/3)$ and L_4, are small [16]).

iii. The 1PI diagrams: fig. 1 (at least four distinct diagrams).

Their sum is finite and is the dominant contribution to A_{NR}.

Calculation of loop diagrams in fig. 1 similar to [17]: the radiative four–meson amplitudes, with one pseudoscalar replaced by one photon (difference).
Results simply dictated by QED

$$
\begin{aligned}
A_{NR}^{1PI} &= 4ef(as_{\pi\pi} + bm_\pi^2) \times \\
&\times \left\{ \widetilde{C_{20}}(s_{\pi\pi}, -z_2) \epsilon_{\mu\nu\alpha\beta} \epsilon_1^\mu k_1^\nu \left[(\epsilon_2 \cdot p_{12}) k_2^\alpha - z_2 \epsilon_2^\alpha \right] q^\beta \ + \ (\epsilon_1, k_1 \leftrightarrow \epsilon_2, k_2) \right\} ,
\end{aligned}
$$

$p_{12} = p_1 + p_2$

function $\widetilde{C_{20}}(x,y)$ defined in terms of the three–denominator one–loop scalar functions [17].
In π-π case and for $x,\ x - 2y > 4m_\pi^2$ the explicit expression is:

$$
\begin{aligned}
(4\pi)^2 \Re e \widetilde{C_{20}}(x,y) &= \frac{x}{8y^2} \left\{ \left(1 - 2\frac{y}{x}\right) \left[\beta \log\left(\frac{1+\beta}{1-\beta}\right) - \beta_0 \log\left(\frac{1+\beta_0}{1-\beta_0}\right) \right] \right. \\
&\left. + \frac{m_\pi^2}{x} \left[\log^2\left(\frac{1+\beta_0}{1-\beta_0}\right) - \log^2\left(\frac{1+\beta}{1-\beta}\right) \right] + 2\frac{y}{x} \right\} , \\
(16\pi) \Im m \widetilde{C_{20}}(x,y) &= -\frac{x}{8y^2} \left\{ \left(1 - 2\frac{y}{x}\right) [\beta - \beta_0] \right. \\
&\left. + \frac{2m_\pi^2}{x} \left[\log\left(\frac{1+\beta_0}{1-\beta_0}\right) - \log\left(\frac{1+\beta}{1-\beta}\right) \right] + 2\frac{y}{x} \right\} ,
\end{aligned}
$$

$$
\text{where} \qquad \beta_0 = \sqrt{1 - \frac{4m_\pi^2}{x}} \qquad \text{and} \qquad \beta = \sqrt{1 - \frac{4m_\pi^2}{(x-2y)}} .
$$

Due to GI, amplitude depends only on 'on–shell couplings' a, b and f.
Result is $O(k_1, k_2)$ (analogy to $O(k)$ direct–emission amplitudes of [17]).

Vertices in a general form $\Longrightarrow$ not only dominant pion loops, but also kaon loops are represented in result.
We recover, as a particular case, part of the result of [11] (i.e. the 1PI diagrams).
Correspondence of $\widetilde{C_{20}}(x,y)$ with their function $R(x,y)$:

$$R(x,y) = 32\pi^2 y \widetilde{C_{20}}(x,y) \ .$$

Result depends only on $\widetilde{C_{20}}$ and thus is finite $\Longleftarrow$

1. GI of the amplitude and
2. on–shell $\pi^+\pi^- \to \pi^0\pi^0$ amplitude independent of loop variables (it depends only on $s_{\pi\pi}$).

Sum of 1PI diagrams is no more finite if the two external π^0's are replaced by a π^+-π^- pair $\Longleftarrow$

1. on–shell $\pi^+\pi^- \to \pi^+\pi^-$ amplitude depends on loop momenta, also
2. sum is not GI (to get GI result, add reducible diagrams with a photon emission from external legs).

4 Decay width (numerical)

Results of numerical analysis: figs. 2 and 3 from [18].
Plots obtained integrating numerically the (modulus-square of) decay amplitude:

$$A(\eta \to \pi^0\pi^0\gamma\gamma) = A_R^{phys} + \left[A_{NR}^{(4)} + A_{NR}^{1PI}\right] \ .$$

$A_{NR}^{(4)}$, A_{NR}^{1PI} are the ChPT results.

A_R^{phys} is a phenomenological expression for the resonant amplitude:

$$A_R^{phys} = A_R^{(4)} \rho e^{i\alpha_0} \ .$$

Factor $\rho e^{i\alpha_0}$ = corrections to tree-level $\eta \to 3\pi^0$ amplitude (known to be large [19]).

ρ can be obtained from data:
assume a flat Dalitz Plot for $\eta \to 3\pi^0$ decay
(no experimental evidence of a D-wave contribution)
and use [19]

$$B_0(m_u - m_d) = m_{K^0}^2 - m_{K^+}^2 - m_{\pi^0}^2 + m_{\pi^+}^2 \ ,$$

$\Longrightarrow$ we find $\rho \simeq 2$.

Phase α_0 cannot be extracted from $\eta \to 3\pi^0$ data.

Evaluate α_0, similarly to $K \to 3\pi$ analysis of [20]:
expand the one-loop $\eta \to 3\pi^0$ amplitude of [19] around the center of the Dalitz Plot

$$\Longrightarrow \alpha_0 = \frac{1}{32\pi F_\pi^2}\left(1 - \frac{4m_\pi^2}{s_0}\right)^{1/2}(2s_0 + m_\pi^2) \simeq 0.18 \ ,$$

where $s_0 = (m_\eta^2 + 3m_\pi^2)/3$.

Figs. 2,3 show that:

1. A_{NR}^{1PI} dominates over $A_{NR}^{(4)}$ in the whole phase space.
2. For $s_{\gamma\gamma} \gtrsim 0.15 m_\eta^2$: A_{NR} becomes non-negligible with respect to A_R.
3. For $s_{\gamma\gamma} \gtrsim 0.20 m_\eta^2$: A_{NR} gives the dominant contribution.

Used $\rho = 2$ in A_R^{phys} and dominant π-π loops only in A_{NR}^{1PI}.
Kaon loops give a very small contribution (checked).

Fig 3: result is quite independent of α_0.
Normalization factor ρ is very important.

More precise data on $\eta \to 3\pi^0 \Longrightarrow$ improve the accuracy on $A_R^{phys} \Longrightarrow$ include the (small) D-wave contribution we neglected.

Discrepancy with [10] in overall normalization factor. Analytic agreement.
Problem in the program used to produce [10] plots.

δm[MeV]	0	25	50	75
Br	0.3	10^{-7}	3×10^{-8}	10^{-8}
N/year	9×10^7	30	9	3
Γ_{NR}/Γ_R	$-$	0.4	1	1.5

At DAΦNE, assuming luminosity $\mathcal{L} = 5 \times 10^{32}$ cm^{-2}s^{-1}, 1 year = 10^7 s
$\Longrightarrow$ # of $\phi \to \eta\gamma$ decays per year = 2.8×10^8.
Used: $\Gamma_{tot}(\eta) = 1.18 \times 10^{-3}$ MeV, Br($\eta \to 3\pi^0$) = 32.1%
[recalling: Br($\pi^0 \to \gamma\gamma$) = 99%]

$\Longrightarrow$ total # of ($\eta \to 3\pi^0$) events (no cut, i.e. $\delta m = 0$) = 9×10^7 per year.

$$\Gamma_R = \int d\Gamma |A_R|^2$$

$$\Gamma_{NR} = \int d\Gamma \Big(|A_{NR}|^2 + 2Re(A_R^* A_{NR}) \Big)$$

5 Discussion: detectability of chiral loop effects (background suppression)

Dominant 1-loop corrections in ChPT to $\eta \to \pi^0\pi^0\gamma\gamma$, to go beyond the simple current algebra calculation of [10].

Phenomenological interest: experimental facilities acting effectively as η-factories.

Results on $\gamma\gamma \to \pi^0\pi^0\pi^0$ [11] inspiring:

1. lowest-order amplitude is suppressed and
2. the corrections due to chiral loops dominate the cross-section.

Similar result for the non-resonant contribution to $\eta \to \pi^0\pi^0\gamma\gamma$.

Despite this enhancement (due to 1-loop corrections), A_{NR} is shadowed from A_R (i.e. the π^0-exchange) over a large portion of the diphoton spectrum.

However, at large $s_{\gamma\gamma}$, A_{NR}^{1PI} dominates over A_R.

Measurement of $\eta \to \pi^0\pi^0\gamma\gamma$ partial width in this region $\Longrightarrow$ new test of ChPT at $O(p^6)$.

Future developments.

$\eta \to \pi^+\pi^-\gamma\gamma$ (statistically favored): dominated by the bremsstrahlung of $\eta \to \pi^+\pi^-\gamma$ [10]

(not suppressed already at the tree level) $\Longrightarrow$ 1-loop corrections not related to the $\eta \to \pi^+\pi^-\gamma$ amplitude will be hardly detectable.

$\gamma\gamma \to \pi^+\pi^-\eta$ and $\gamma\gamma \to \pi^0\pi^0\eta$ more interesting for studying chiral–loop effects.

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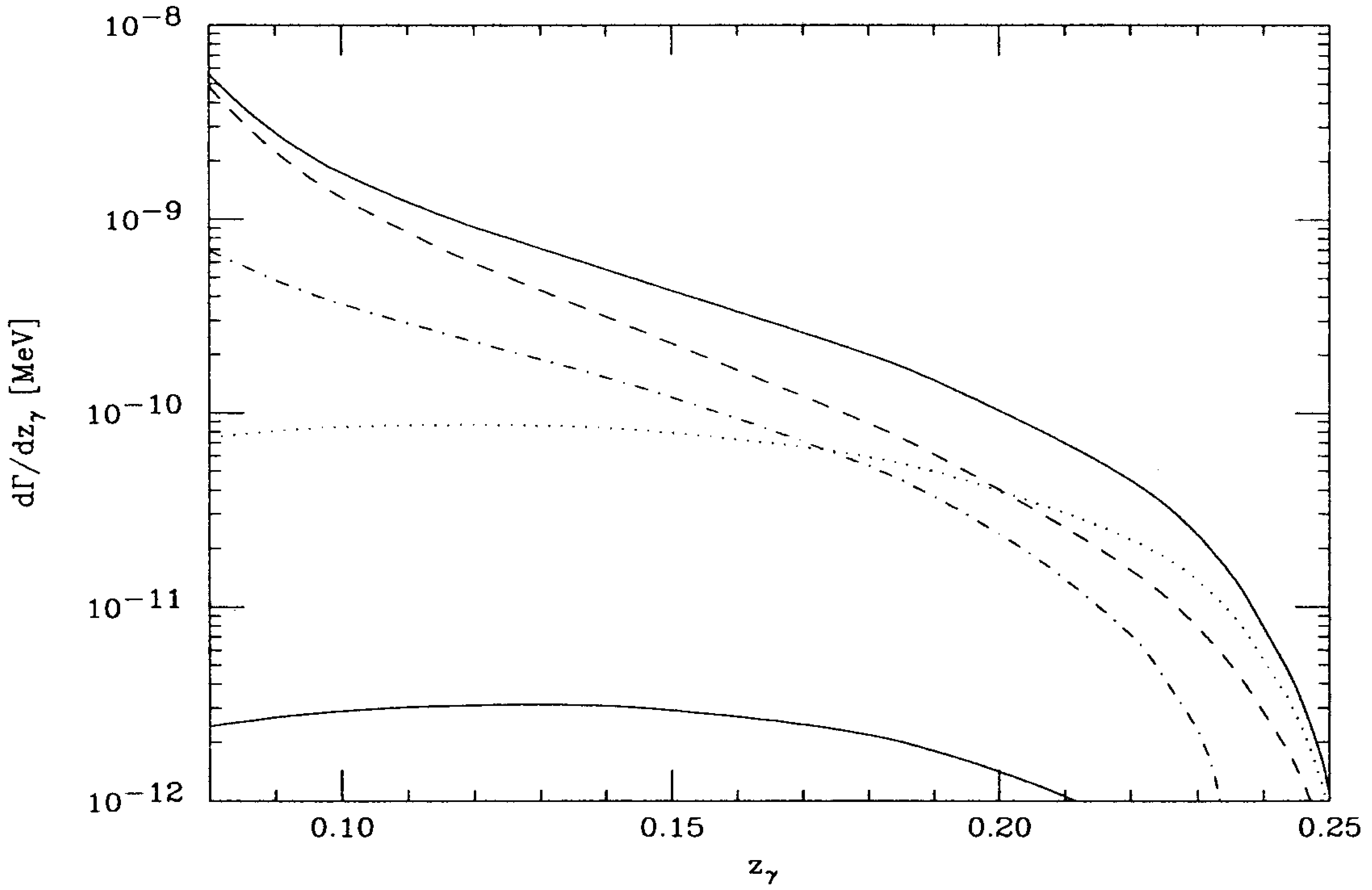

Figure 2: Diphoton spectrum ($z_\gamma = s_{\gamma\gamma}/m_\eta^2$) for the decay $\eta \to \pi^0\pi^0\gamma\gamma$. The upper full line is the total contribution. The dashed line is the resonant contribution ($|A_R^{phys}|^2$), the dotted line is the one–loop non-resonant contribution ($|A_{NR}^{1PI}|^2$) and the dash–dotted line is their interference ($\rho = 2$, $\alpha_0 = 0.18$). The lower full line is the tree-level non–resonant contribution ($|A_{NR}^{(4)}|^2$).

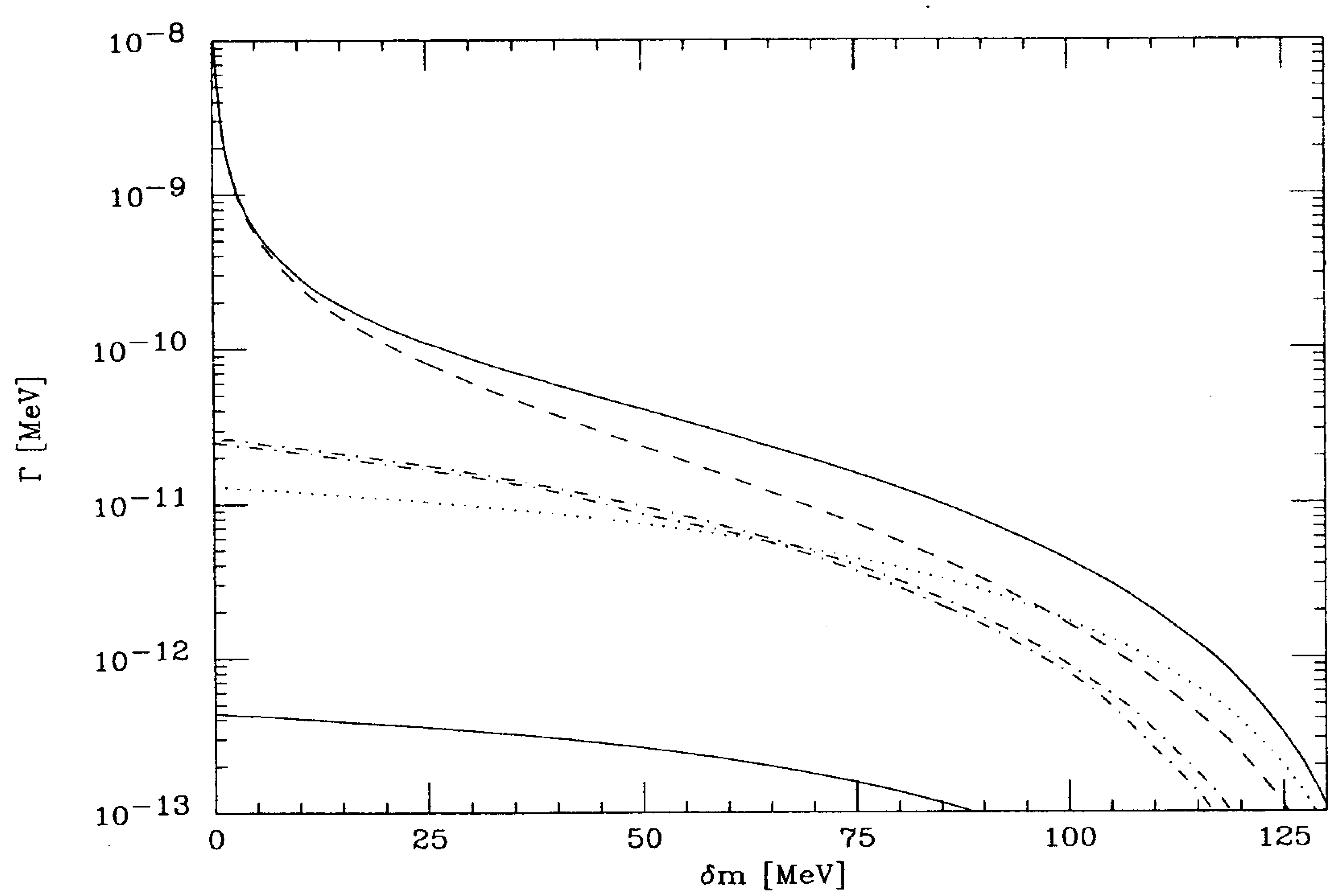

Figure 3: Partial decay rate of $\eta \to \pi^0\pi^0\gamma\gamma$ as a function of the energy cut $|s_{\gamma\gamma}^{1/2} - m_{\pi^0}| < \delta m$. Full, dashed and dotted curves as in fig. 2. The two dash-dotted lines, denoting the interference between A_R^{phys} and A_{NR}^{1PI}, have been obtained for $\alpha_0 = 0.16$ (upper line) and $\alpha_0 = 0.20$ (lower line).