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Transition Undulator Radiation (TUR) as a Beam Diagnostics and a High Brilliance Infrared Source

M. Castellano
INFN – Laboratori Nazionali di Frascati, P.O. Box 13, I–00044 Frascati (Rome), Italy

Abstract

The Transition Undulator Radiation (TUR), introduced by K. J. Kim, is derived as an interference between two bremsstrahlung amplitudes at the entrance and exit of the undulator.

The result is applied to the TTF FEL experiment, for which this radiation, in the visible bandwidth, may be used as an electron beam non intercepting diagnostics, while in the far infrared region, due to the coherent emission by each microbunch, is a high brilliance source synchronized with the higher energy FEL radiation.

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1 – Introduction

In a recent paper [1] Kwang–Je Kim has pointed out that a high energy electron beam going through an undulator emits also a rather strong radiation at wavelengths much longer that the fundamental spontaneous radiation.

This radiation has a wide bandwidth and an angular distribution peaked at an angle of the order of $1/\gamma$ with respect to the beam direction (γ being the reduced electron energy).

Kim has named this radiation Transition Undulator Radiation (TUR), suggesting that its origin derives from the sudden decrease of the beam average longitudinal velocity when entering the undulator, folloed by the symmetric velocity increase at the undulator exit.

In its derivation of the radiation intensity, Kim does not explicitly use this feature. In this paper I will show that TUR can indeed be obtained as the interference effect of two bremsstrahlung amplitudes at the undulator extremities.

This approach gives a better understanding of the dependence of the radiation from the various beam and undulator parameters, and opens also a question about the localization of the radiation source. In fact Kim's derivation seems to suggest an emission continuously distributed along the undulator, while the bremsstrahlung model points towards a localized emission at the entrance and exit of the undulator. I will show that, in particular situations, this can give rise to a sigificative difference for the radiation intensity and angular distribution.

Kim has evaluated the intensity of TUR from a standard undulator on a storage ring dedicated to synchrotron radiation production and compared it to that from a bending magnet of the same ring, obtaining that TUR has a lower intensity and its possible interest for users is only based on a significant better spatial coherence.

The situation is quite different for the linac based projects of UV or X–ray FEL using the SASE process. I will present numerical calculations using the parameters of the TTF FEL experiment [2]. In this case the high charge of the beam and the low wavelength of the first undulator harmonic allow a good TUR intensity in the optical bandwidth, sufficient for beam imaging for diagnostic purpose. The very short bunch length required to obtain the desired charge density makes possible a coherent emission from each microbunch at far infrared wavelengths, with many order of magnitude increase of TUR intensity, that becomes a high brilliance source.

2 – TUR as bremsstrahlung interference

When a high energy electron moving at a constant reduced velocity β enters an undulator, it follows an oscillatory trajectory emitting spontaneous radiation with a first harmonic wavelength λ_o. The electromagnetic field of the particle has a rather complex behavior, but for wavelengths $\lambda \gg \lambda_o$ can be well approximated to that of an electron moving on a straight line at a constant velocity equal to the average longitudinal velocity of the real electron β_1. Along the motion in the undulator no radiation can be emitted at this wavelength, but a sharp discontinuity is presented by the field at the entrance and the output of the undulator, identical to that of a bremsstrahlung in which the electron suddenly change its velocity from β to β_1 at the entrance and the reverse at the output as is shown in Figure 1.

The radiation amplitude due to a change of the electron velocity from β to β_1 can be usefully described as the sum of the amplitude of an electron with velocity β suddenly stopping and that of the same electron suddenly accelerating to the velocity β_1, as shown in Figure 2.

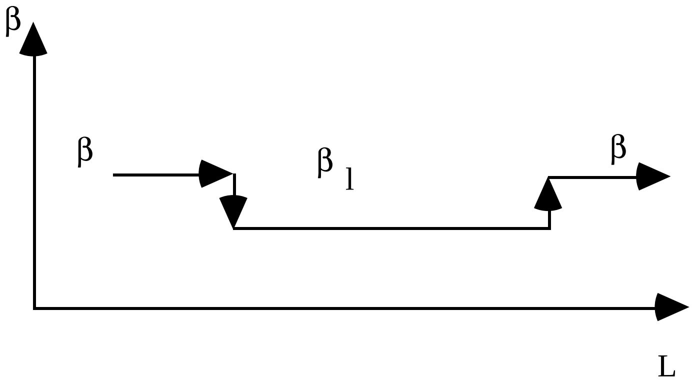

FIG. 1 – Schematic behavior of the electron average velocity before, along and after the undulator.

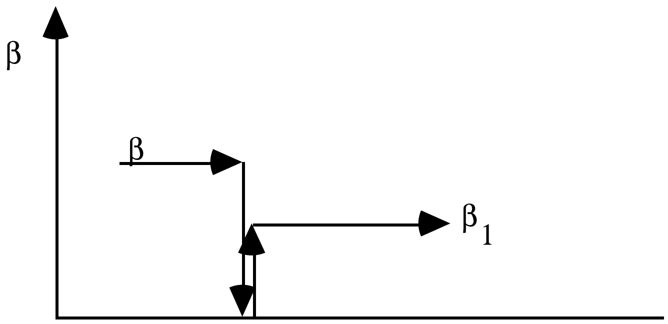

FIG. 2 – Change in the electron velocity described as a sudden stop and start.

In this case the number of photons emitted at wavelength λ for unit solid angle is given by

$$\frac{dN_f}{d\Omega} = \frac{d\lambda}{\lambda}\frac{\alpha}{4\pi^2}|A|^2 \qquad (1)$$

with α the fine structure constant and

$$A = \left(\frac{\beta\sin\theta}{1-\beta\cos\theta} - \frac{\beta_1\sin\theta}{1-\beta_1\cos\theta}\right) \qquad (2)$$

the bremsstrahlung amplitude.

In the case of an undulator we have two equal amplitudes, with opposite sign, separated by a phase difference Φ, so that

$$A = \left(\frac{\beta\sin\theta}{1-\beta\cos\theta} - \frac{\beta_1\sin\theta}{1-\beta_1\cos\theta}\right)\left(1-e^{i\Phi}\right)$$

with

$$\Phi = \frac{2\pi}{\beta_1}\frac{L}{\lambda}\left(1-\beta_1\cos\theta\right) = 2\pi N\frac{\lambda_o(\theta)}{\lambda}$$

in which L is the undulator length.

We therefore have

$$\frac{dN_f}{d\Omega} = \frac{d\lambda}{\lambda}\alpha\frac{\sin^2\theta}{(1-\beta\cos\theta)^2}\frac{(\beta-\beta_l)^2}{\beta_l^2}\left(\frac{L}{\lambda}\right)^2\frac{\sin^2 \Phi/2}{(\Phi/2)^2} \tag{3}$$

It is easy to demonstrate that, being

$$\beta_l = \beta\left(1-\frac{k^2}{4\gamma^2\beta^2}\right)$$

with **k** the standard undulator parameter, and

$$\langle\beta_\perp^2\rangle = \frac{k^2}{2\gamma^2}$$

for **γ»1** and **θ«1** we obtain exactly Kim's formula

$$\frac{dN_f}{d\Omega} = \frac{d\lambda}{\lambda}\alpha\frac{\sin^2 x}{x^2}\langle\beta_\perp^2\rangle^2\left(\frac{L}{\lambda}\right)^2\frac{\gamma^4\theta^2}{(1+\gamma^2\theta^2)^2} \tag{4}$$

with

$$x = \frac{\Phi}{2} = \pi N\frac{\lambda_o(\theta)}{\lambda}$$

Written as function of the undulator parameters, it becomes

$$\frac{dN_f}{d\Omega} = \frac{d\lambda}{\lambda}\frac{\alpha}{4}\frac{\sin^2 x}{x^2}k^4\left(\frac{L}{\lambda}\right)^2\frac{\theta^2}{(1+\gamma^2\theta^2)^2} \tag{5}$$

$$x = \frac{\pi L}{2\gamma^2\lambda}\left(1+\frac{k^2}{2}+\gamma^2\theta^2\right) \tag{6}$$

The dependence of (5) from the various parameters is rather complex, but in the framework of this model the physics is clear: we have two bremsstrahlung amplitudes, equal in magnitude but of opposite sign, separated by a phase difference depending on the ratio L/λ and on θ. For a zero–length undulator the two amplitudes exatly subtrac from each other and no radiation is present, while for a short undulator and sufficiently long wavelength, for which we can cosider Φ«1, and for a fixed angle θ, the sum of the amplitudes will linearly depend on Φ, and thus on L/λ. The intensity will thus be proportional to the square of this parameter. Increasing L/λ, the intensity will increase up to a maximum for Φ = π, and then decrease to a new zero when Φ = 2 π, but for a given undulator length this will be true only for a well defined wavelength and emission angle θ.

The dependence of the phase from the emission angle θ makes the angular distribution of the TUR different from that of a single bremsstrahlung, resulting in a wider angular spread.

For Φ » 2 π, the oscillations of the radiation intensity as function of θ and λ are so rapid that a measurement over a finite solid angle and bandwidth will give only an average value. In this case we can replace in (5) the square of the sine by its average value of 1/2 and we obtain that the intensity is now the sum of the intensities of the two single bremsstrahlungs, without interference between them, and is independent from the undulator length and from wavelegth. In a different language we can say that the two amplitudes are now separated by more than the radiation formation length.

3 – TUR in the TTF FEL experiment

The TTF FEL [2] is a project to experimentally demonstrate the feasibility of a short wavelength Free Electron Laser using the Self Amplified Spontaneous Emission (SASE) process [3] and exploiting the high beam quality produced by the TTF superconducting linac [4]. This project is a first step towards the production of a high intensity, coherent x–ray radiation in the framework of the TESLA linear collider complex.

The main parameters of the beam and of the undulator are shown in Table I.

Table I – Beam and Undulator Parameters.

Beam energy	1 GeV
N. of electrons/microbunch	$6\ 10^9$
N. of microbunches	7200
Bunch length (rms)	50 μm
Bunch radius (rms)	50 μm
Undulator length	6 x 5 m
Undulator gap width	12.5 mm
Fundamental harmonics wavelength	6.4 nm
Undulator k parameter	1.32

The undulator consists of 6 modules of 5 meters each separated by short straight sections containing focusing elements. In this case the total TUR amplitude is the sum of 12 bremsstrahlung amplitudes, but the phase difference between the exit from one module and the entrance in the next one is so small that it can be neglected. In this way amplitudes from neighbouring discontinuities subtract from each other and we are left with only the first and the last one, that is, the result is the same of that of a single undulator of length equal to the sum of the lengths of all modules.

The number of emitted photons in a bandwidth of .1% is plotted in Figure 3 as function of wavelength.

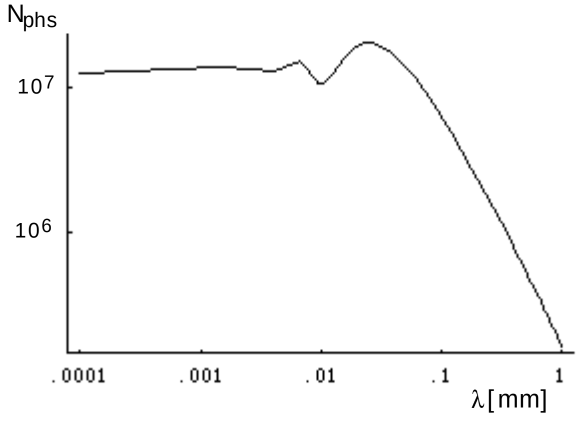

FIG. 3 – LogLog plot of the number of emitted photons in a .1% bandwidth,

from eq. (5), applied to the TTF FEL project.

In the optical region, where Φ»2π and the approximation of the sum of two bremsstrahlung intensities holds, the number of photons is almost constant. In the near and mid infrared region it oscillates. A detail from 1 to 50 μm is given in Figure 4, showing that it reaches a maximum around 20 μm and then decreases very rapidly in the far infrared, when Φ«π and the two bremsstrahlung amplitudes almost cancel.

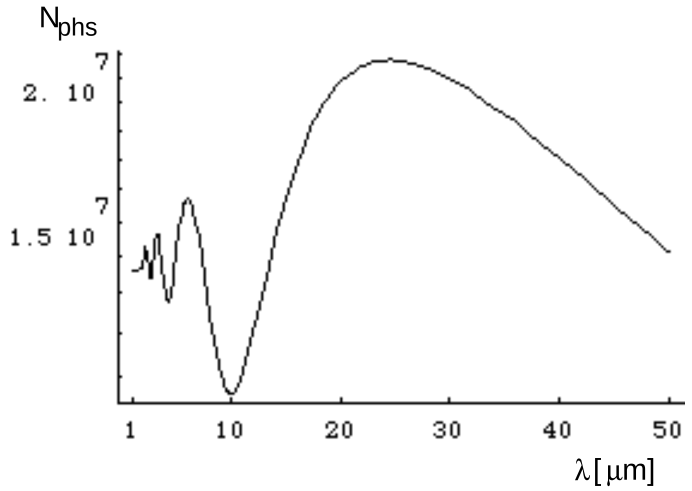

FIG. 4 – A particular of Figure 3: from 1 to 50 μm wavelength.

The incoherent intensity is at all frequencies definitely too small to be considered as a source for experimental work, but two wavelength regions deserve a more detailed analysis: the optical bandwidth, because the radiation emitted in this frequency range can be used to mage the electron beam and derive its position and intensity profile, and the far infrared region, in which the coherent emission by all electrons in a microbunch can greatly enhance the intensity.

4 – TUR in the optical bandwidth

The short fundamental harmonics wavelength of the TTF FEL makes the condition λ » λ_0 well satisfied even in the optical bandwidth. From (6) it is clear that in this case Φ » 2π, and the total TUR intensity is the sum of the intensities of two single bremsstrahlungs.

From a practical point of view, there is a strong difference depending on whether the radiation is emitted by two distinct sources or continuously along the undulator: in the first case the radiation from the undulator entrance cannot reach a detector placed downstream, due to the small vacuum pipe diameter, other than for very small angles or through multiple reflections on the inner surface, and an image of the undulator exit will show a well focused beam image produced by the radiation emitted by bremsstrahlung there, with possibly a very tenuous diffuse background. In the second case a fraction of the radiation, produced in the last part of the undulator, may be detected, and, due to the finite field depth of the optical system, the image of the beam will not be well defined being, in any case, the average on a rather long trajectory.

This ambiguity will be solved experimentally but, in the meantime, I have computed the measurable intensity for both situations, even though, in my opinion, the two–source model will prove to be correct in the short wavelength region.

For the reason discussed above, in the case of a localized emission we can consider only the radiation from the bremsstrahlung at the undulator exit, that, from (1) and (2), can be written, in terms of the undulator parameters, as

$$\frac{dN_f}{d\Omega} = \frac{d\lambda}{\lambda}\frac{\alpha}{4\pi^2}\frac{k^4\gamma^4\theta^2}{\left(1+\frac{k^2}{2}+\gamma^2\theta^2\right)^2\left(1+\gamma^2\theta^2\right)^2} \tag{7}$$

The angular distribution of this radiation is shown in Figure 5.

The figure evidences that a simple way to extract the radiation from the vacuum pipe, without interfering with the high energy FEL radiation, exists: the FEL radiation being concentrated around the beam direction in a cone of the order of few tens of μrad aperture, a mirror tilted at 45° with a hole along the beam axis may extract most of the TUR and let the FEL radiation through the aperture.

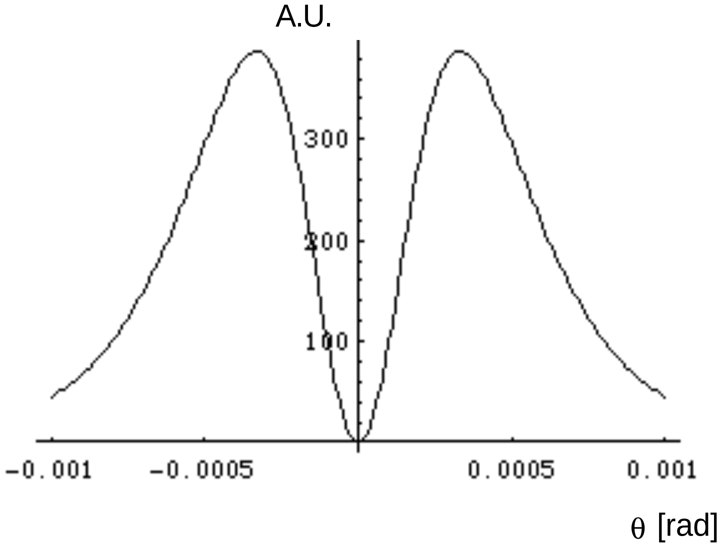

FIG. 5 – Angular distribution of the single bremsstrahlung radiation at the undulator exit.

With a hole subtending ± 200 μrad from the undulator exit, the 99% of the TUR will be extracted and the total intensity integrated over the optical bandwidth, from 400 to 800 nm, is

$$N_{opt} = 1\cdot 10^{-4}\,\frac{\text{Photons}}{\text{electron}}$$

This means that we can have 6 10^5 photons per microbunch, a number more than sufficient to give a well detailed image of the beam charge distribution using an intensified camera. It will then be possible to study the evolution of beam position and profile along the macrobunch.

Even with a standard CCD camera we have enough photons for a very good image of the beam integrated over the whole macrobunch.

In the case of a distributed source, to obtain the total flux we must integrate over a solid angle that depends on the position along the undulator. We neglect the possibility of internal reflection, a correct assumption if we want to obtain an image of the beam, because the reflected radiation givies only a diffuse background.

The angular distribution obtained in this case is compared to that of a single bremsstrahlung source in Figure 6.

The total flux in the optical bandwidth, through the same ± 200 μrad hole, is in this case 6.3 10^{-5} photons/electron, which is not very different from the previous one. The photon number is also in this case more than sufficient for a statistically good image, but the long source may give serious problems to obtain a well focused and detailed image.

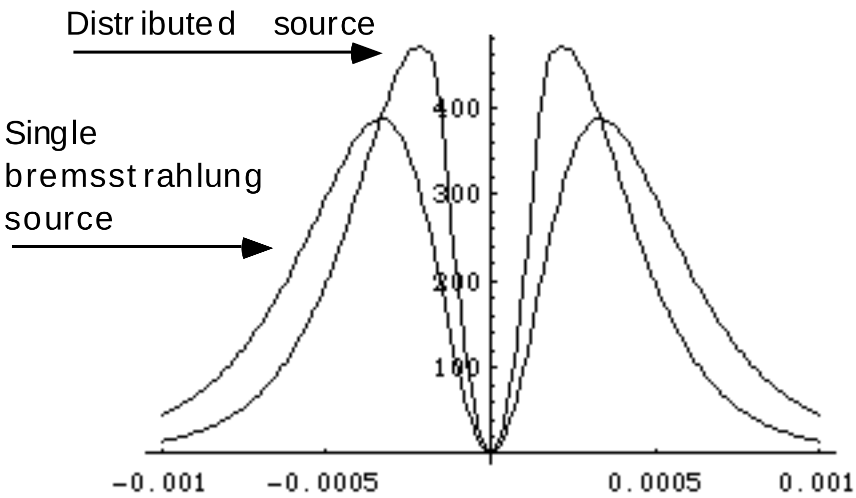

FIG. 6 – Comparison of angular distribution from a TUR distributed source and a single bremsstralung

The difference in angular distribution could be used to discriminate between the two possibilities, but the main difference remains in our opinion in the focusing properties.

Note that the diagnostics by means of TUR, being based on a radiation spontaneously emitted, does not in anyway perturb the electron beam and, more importantly, is really non intercepting, avoiding the danger of material damage by the high power density of the beam itself.

5 – TUR in the far infrared region

As we have seen in Paragraph 3, in the far infrared region the radiation intensity is strongly reduced by the almost complete cancellation of the two bremsstrahlung amplitudes due to the very small phase difference, however, for wavelengths longer than the microbunch length, all electrons emit more o less coherently, and the total intensity is no longer proportional to the number of electrons, but will increase with a power of that number that, for very long wavelengths, tends to the square.

More precisely, the total number of photons emitted by a microbunch with $\mathbf{N_e}$ electrons is

$$N_{fch} = N_e\left(1 + N_e f(\lambda)\right)N_f \qquad (8)$$

where $\mathbf{N_f}$ is the number of photons emitted by a single electron, while $\mathbf{f(\lambda)}$ is a form factor related to the charge distribution in the microbunch $\boldsymbol{\rho(z)}$:

$$f(\lambda) = \left| \int \rho(z) \exp[i2\pi z / \lambda] dz \right|^2 \tag{9}$$

in which I have neglected the effect of the transverse beam dimension.

In this wavelength region there are not efficient instruments for beam imaging, and, furthermore, the metallic vacuum pipe can be considered a perfect mirror. Only the total flux is therefore of interest.

Assuming the radiation will be extracted, as in the previous case, by a mirror with a hole covering an angle of ±200 μrad from the undulator exit, the flux given by formula 5, neglecting coherent emission, is shown in Figure 7, in which the total number of photons in a .1% bandwidth is plotted as a function of wavelength.

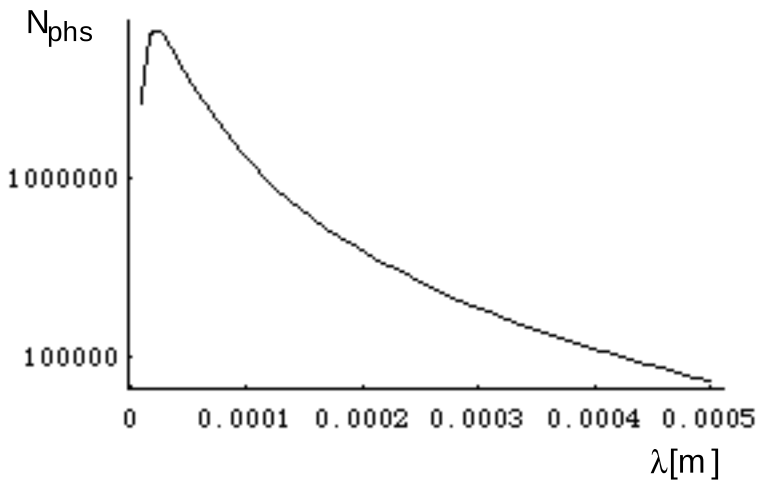

FIG. 7 – Photon flux at long wavelength without coherent emission.

The coherent emission changes the picture completely, but, to proceed with the calculation, a hypothesis on the electron distribution in the microbunch must be made. For a uniform distribution of length l = 100 μm, corresponding to a rms value of 50 μm, the form factor f(λ) from eq. 9 becomes

$$f(\lambda) = \frac{\sin^2\left(\frac{\pi l}{\lambda}\right)}{\left(\frac{\pi l}{\lambda}\right)^2} \tag{10}$$

and the total photon flux as function of wavelength, with the same bandwidth and extraction geometry of the previous figure, is shown in Figure 8.

For a gaussian electron distribution with σ = 50 μm, we have

$$f(\lambda) = e^{-\frac{l_b^2 \pi^2}{\lambda^2}} \tag{11}$$

and the relative total photons flux, in the same bandwidth and with the same mirror hole, is shown in Figure 9.

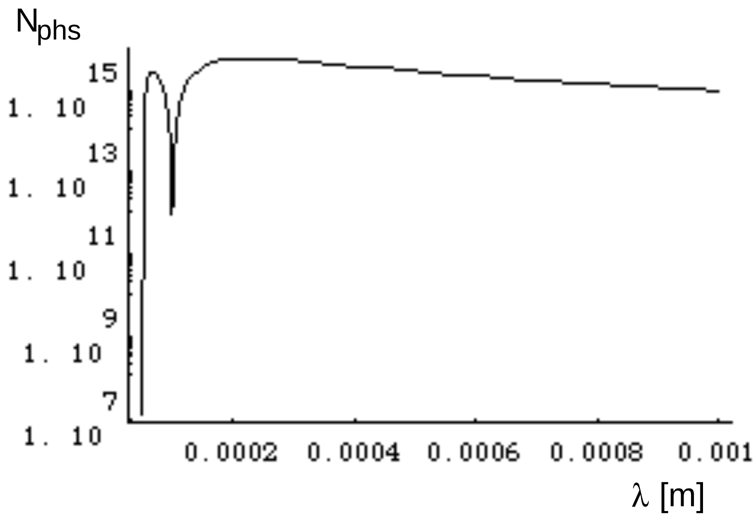

FIG. 8 – Photon flux from coherent emission for a uniform electron distribution.

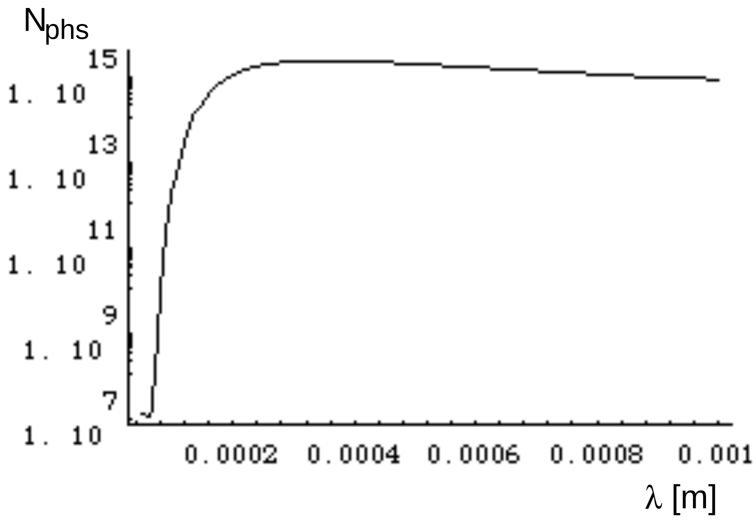

FIG. 9 – Photon flux from coherent emission from a gaussian electron distribution.

The real electron distribution will probably be in between these two extreme cases.

The small transverse dimension of the source, less than the radiation vawelength, and the limited angular spread, give very high, diffraction limited, brilliance; averaged over a macropulse, it can reach 10^{22} Nphs/(s .1% mrad^2 mm^2).

A quantity of particular interest for the FEL project, as a possible source of perturbations, is the total power emitted by the electron beam through this new process.

It is evident that most of the power is carried by wavelengths for which the coherent emission occurs. The energy lost by the beam, in a 100% bandwidth and in the hypothesis of a gaussian electron longitudinal distribution, is shown in Figure 10 as a function of wavelength. The total energy, integrated on all wavelengths, is however only about 5 10^3 GeV, negligible compared to the beam energy. At a repetition rate of 10 Hz this corresponds to an average power of 6 μW, most of which due to wavelengths in a small interval around 200 μm.

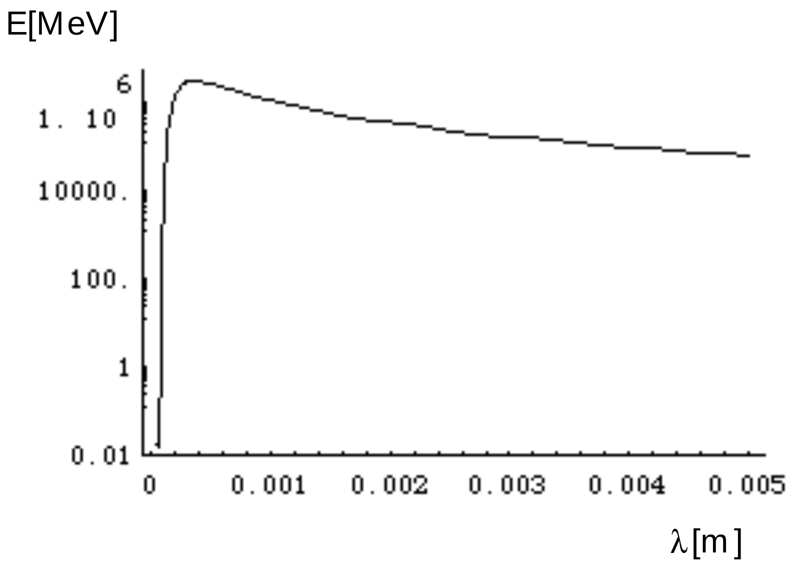

FIG. 10 – Energy loss per unit bandwidth from a macrobunch in the far infrared as function of wavelength.

6 – Conclusions

The Transition Undulator Radiation has been derived as an interference between two bremsstrahlung amplitudes produced at the entrance and exit of the undulator. This model gives a clear insight of the physics of the process and a simple explanation of the dependence of the radiation intensity from the various parameters.

The result is applied to the TTF FEL experiment, for which, due to the very short wavelength of the fundamental harmonics of spontaneous radiation, a good intensity is emitted even in the optical bandwidth. At these wavelengths the phase difference between the two bremsstrahlung is much larger then the radiation formation length, and the model predicts the existence of two distinct radiation sources, one at each end of the undulator. In these conditions the optical radiation may prove a very effective and non perturbing beam diagnostics tool.

In the far infrared region the short bunch length allows a coherent emission that strongly enhances the otherwise very low intensity. Even so the intensity is not very high but the brilliance is very good in a wavelength region in which intense sources are not easy to be found. This fact, and the natural synchronization with the FEL radiation, may result in a possible scientific use. Further analysis of this point is needed.

References

[1] K.–J. Kim – Phys. Rev. Letters 76, 1244, (1996)

[2] A VUV Free Electron Laser at the TESLA Test Facility at Desy
Conceptual Design Report
TESLA–FEL 95–03 Desy 1995

[3] R. Bonifacio, C. Pellegrini, L.M. Narducci – Opt. Commun. 53, 197, (1985)

[4] TESLA Test Facility Linac – Design Report
D.A. Edwards ed.– TESLA 95–01 Desy 1995