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The role of chiral loops in $\eta \to \pi^0\pi^0\gamma\gamma$ ⋆

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Abstract

We consider the rare decay $\eta \to \pi^0\pi^0\gamma\gamma$ and calculate the non-resonant contribution to the amplitude to one loop in Chiral Perturbation Theory. We display our result as both a diphoton energy spectrum and a partial decay rate as a function of the photon energy cut. It turns out that the one-loop correction can be numerically very important and could be detected, at sufficiently large center-of-mass photon energies, from a measurement of the partial decay width. © 1997 Elsevier Science B.V.

1. Introduction

The $\eta \to \pi\pi\gamma\gamma$ decays been been recently analyzed [1] in the framework of Chiral Perturbation Theory (CHPT) [2,3] (see [4] for a recent review). The experimental interest for such rare decays stems from the large number of observed η's anticipated at various η-factories, e.g. CELSIUS, ITEP and DAΦNE [5], as well as at other facilities, such as GRAAL, MAMI, ELSA and CEBAF.

In the neutral decay $\eta \to \pi^0\pi^0\gamma\gamma$ we can identify two physically distinct contributions to the amplitude:

$$A(\eta \to \pi^0\pi^0\gamma\gamma) = A_R + A_{NR}\,. \qquad (1.1)$$

The first contribution (that we call the 'resonant amplitude') is characterized by the π^0 pole in the diphoton invariant mass squared ($s_{\gamma\gamma}$) and is proportional to the on-shell amplitudes $\eta \to 3\pi^0$ and $\pi^0 \to \gamma\gamma$: [3]

$$A_R = -\frac{A(\eta \to 3\pi^0)A(\pi^0 \to \gamma\gamma)}{s_{\gamma\gamma} - m_{\pi^0}^2}\,. \qquad (1.2)$$

By construction, A_R can be predicted up to a phase from the experimental data on $\eta \to 3\pi^0$ and $\pi^0 \to \gamma\gamma$ and needs not to be calculated in CHPT. The main result of [1], in the $\eta \to \pi^0\pi^0\gamma\gamma$ channel, can be summarized as follows: the resonant contribution dominates over the tree-level non-resonant one in the whole phase space.

The non-resonant amplitude cannot be predicted using experimental data and must be calculated in CHPT. In Ref. [1] A_{NR} has been calculated at the tree level, i.e. to $O(p^4)$; at this order only the η-exchange diagram ($\eta \to \pi^0\pi^0\eta^* \to \pi^0\pi^0\gamma\gamma$) contributes. Knöchlein et al. have considered also the η'-exchange diagram (formally of higher order in CHPT), but for

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[3] The minus sign in Eq. (1.2) is due to our convention for the amplitudes.

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both η- and η'-exchange they found that $A_{\rm NR}$ is negligible with respect to $A_{\rm R}$. This is because the lowest-order $\eta\eta\pi^0\pi^0$ and $\eta\eta'\pi^0\pi^0$ vertices vanish in the limit $m_u = m_d = 0$. In spite of the analogous suppression factor in the π^0-exchange contribution, which is proportional to $m_u - m_d$, the enhancement due to the pole makes $A_{\rm R}$ dominant with respect to $A_{\rm NR}$ over the full range of kinematical parameters. However, there are good reasons to presume that this suppression does not occur to one loop. Indeed, recently Talavera et al. [6] have shown that the one-loop contribution to the $\gamma\gamma \to \pi^0\pi^0\pi^0$ amplitude – which is obviously connected to the $\eta \to \pi^0\pi^0\gamma\gamma$ one – dominates over the tree-level result (the former is one order of magnitude larger than the latter). The reason for this enhancement can be traced to the fact that the tree-level amplitude is proportional to m_π^2, whereas the one-loop correction is not affected by such suppression.

In this Letter we compute $A_{\rm NR}$ to one loop, neglecting isospin-breaking effects and the suppressed η-exchange diagrams. In this limit only one-particle-irreducible (1PI) diagrams contribute and the result is finite. Furthermore, $A_{\rm NR}$ does not receive any contribution from the $O(p^6)$ counterterms analyzed in [7], just as it happens in the $\gamma\gamma \to \pi^0\pi^0\pi^0$ case. We find that the one-loop contribution to $A_{\rm NR}$ is dominant, with respect to the corresponding tree-level one. Once again, the reason is that the tree-level amplitude goes to zero in the limit $m_u = m_d = 0$, whereas the one-loop result does not vanish in this limit. We find, in addition, that there exists a kinematical region in the phase space, i.e. a region of sufficiently large values of $s_{\gamma\gamma}$, where the resonant amplitude – which is a background that shadows $A_{\rm NR}$ – is suppressed, and a measurement of the decay width $\Gamma(\eta \to \pi^0\pi^0\gamma\gamma)$ would allow us to detect a purely $O(p^6)$ effect.

The outline of this Letter is as follows. We begin in Section 2 with the description of the kinematical variables for the decay $\eta \to \pi^0\pi^0\gamma\gamma$ and a list of interaction terms relevant to the calculation of the one-loop amplitude. In Section 3, after recalling the expression of the decay amplitude at the tree level, we describe the calculation of the one-loop corrections to the non-resonant amplitude and give the analytic expression of the result. We proceed in Section 4 to calculate the decay width, starting from the sum of $A_{\rm R} + A_{\rm NR}$. We display the result in the form of both a diphoton energy spectrum and a partial decay rate as a function of the energy cut around $s_{\gamma\gamma}^{1/2} = m_{\pi^0}$. Then we determine the phase-space region, in terms of a $s_{\gamma\gamma}$ range, where the suppression of the background due to $A_{\rm R}$ may allow to detect the one-loop effects in $A_{\rm NR}$. We end the Letter with some concluding remarks and a short discussion of further developments, including possible extensions of the one-loop calculation to the charged pions channel, as well as to the reactions $\gamma\gamma \to \eta\pi\pi$.

2. Kinematical variables and interaction terms

The kinematics of the decay

$$\eta(q) \to \pi^0(p_1)\pi^0(p_2)\gamma(k_1,\epsilon_1)\gamma(k_2,\epsilon_2)$$

can be described in terms of five independent scalar variables which we choose as

$$s_{\pi\pi} = (p_1+p_2)^2\,, \quad z_{1,2} = k_{1,2}\cdot(p_1+p_2)\,, \tag{2.1}$$

$$s_{\gamma\gamma} = (k_1+k_2)^2\,, \quad z_3 = (k_1+k_2)\cdot(p_1-p_2)\,. \tag{2.2}$$

If we write the decay amplitude in the following way:

$$A(\eta \to \pi^0\pi^0\gamma\gamma) = e^2\epsilon_1^\mu\epsilon_2^\nu A_{\mu\nu}\,, \tag{2.3}$$

the decay width is given by

$$\Gamma(\eta \to \pi^0\pi^0\gamma\gamma) = \frac{\alpha_{\rm em}^2}{2^{11}\pi^6 m_\eta}\int \frac{d^3p_1}{p_1^0}\frac{d^3p_2}{p_2^0}\frac{d^3k_1}{k_1^0}\frac{d^3k_2}{k_2^0} \times \delta^{(4)}(p_1+p_2+k_1+k_2)A^{\mu\nu}A^*_{\mu\nu}\,. \tag{2.4}$$

Since the process $\eta \to \pi^0\pi^0\gamma\gamma$ involves the electromagnetic interaction of an odd number of pions, the decay amplitude receives contributions only from the odd-intrinsic parity sector of CHPT and thus is at least $O(p^4)$.

The CHPT lagrangian, expanded up to $O(p^4)$, is given by

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)}\,, \tag{2.5}$$

where

$$\mathcal{L}^{(2)} = \frac{F^2}{4}{\rm tr}\left(D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\right) \tag{2.6}$$

and $\mathcal{L}^{(4)}$ can be split into the odd-intrinsic anomalous part (i.e. the Wess–Zumino term [8]) and the $O(p^4)$ Gasser–Leutwyler lagrangian [3]

$$\mathcal{L}^{(4)} = \mathcal{L}_{WZ} + \sum_{i=1}^{10} L_i \mathcal{L}_i^{(4)} . \qquad (2.7)$$

As usual, we assume the exponential parametrization $U = \exp(i\sqrt{2}P_8/F)$, where P_8 is the $SU(3)$ octet matrix of pseudoscalar mesons and F coincides to the lowest order with the charged pion decay constant $F_\pi = 92.4$ MeV [3,9]. The covariant derivative in Eq. (2.6) is given by $D_\mu U = \partial_\mu U + ieA_\mu[Q,U]$, where A_μ is the photon field and $Q = \mathrm{diag}(2/3, -1/3, -1/3)$. Finally, we employ the identification $\chi = \chi^\dagger = 2B\mathcal{M}$ in the external scalar sources, where $\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$ is the quark mass matrix and B can be identified to the lowest order with the mass ratio $B_0 = m_\pi^2/(m_u + m_d)$.

In principle, in this decay one could take into account also the η-η' mixing, i.e. the mixing of P_8 with the singlet-field η_0. However, as shown in [1], this effect can be safely neglected in the non-resonant amplitude. Hence in the loop calculation we identify the mass-eigenstate η with the octet field η_8. In the same spirit, since we neglect isospin-breaking effects in $A_{\rm NR}$, in the following we assume $m_\pi = m_{\pi^0}$.

For the interaction terms necessary to calculate the tree-level amplitudes one can refer to [1]. However, not just for the sake of completeness, but also given that we use a small subset of the couplings in [1], we collect them in the following:

$$A^{(2)}(\eta_8 \to \pi^0\pi^0\pi^0) = 3A^{(2)}(\eta_8 \to \pi^0\pi^+\pi^-) = \frac{B_0(m_u - m_d)}{\sqrt{3}F_\pi^2} , \qquad (2.8)$$

$$A^{(2)}(\eta_8 \to \eta_8\pi^0\pi^0) = A^{(2)}(\eta_8 \to \eta_8\pi^+\pi^-) = \frac{B_0(m_u + m_d)}{3F_\pi^2} , \qquad (2.9)$$

$$A^{(4)}(\pi^0 \to \gamma\gamma) = \sqrt{3}A^{(4)}(\eta_8 \to \gamma\gamma) = \frac{e^2}{4\pi^2 F_\pi}\epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu k_1^\nu \epsilon_2^\alpha k_2^\beta . \qquad (2.10)$$

In addition, in order to compute the one-loop diagrams in Fig. 1, we introduce the generic couplings

$$A^{(2)}(\phi^+\phi^- \to \phi_1^0\phi_2^0) = a s_{\pi\pi} + b m_\pi^2 + c(p_+^2 - m_\pi^2) + d(p_-^2 - m_\pi^2) , \qquad (2.11)$$

$$A^{(4)}(\phi^0 \to \phi^+\phi^-\gamma) = f\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu k^\nu p_+^\alpha q^\beta , \qquad (2.12)$$

where q, $p_\pm$, $p_{1,2}$ and k are the (outgoing) momenta of the pseudoscalars ϕ^0, $\phi^\pm$, $\phi_{1,2}^0$ and of the photon, respectively. The constants a, b, c and d have the dimensions of inverse mass squared, whereas f has those of an inverse mass cubed. As we will show in the next section, the 'off-shell couplings' c and d are irrelevant for our process, since their contribution to the amplitude cancels out as a consequence of the gauge invariance. In the $\pi^+\pi^- \to \pi^0\pi^0$ and $\eta_8 \to \pi^+\pi^-\gamma$ cases, useful in order to estimate the dominant pion loops, we find

$$a = -b = \frac{1}{F_\pi^2} \quad \text{and} \quad f = -\frac{e}{4\sqrt{3}\pi^2 F_\pi^3} . \qquad (2.13)$$

3. The decay amplitude to one loop

In this Section we would like first to briefly recall the expression of the tree-level amplitudes $A_{\rm R}^{(4)}$ and $A_{\rm NR}^{(4)}$, obtained by considering the π^0 and η_8 exchange diagrams [1]

$$A_{\rm R}^{(4)} = -\frac{e^2}{4\sqrt{3}\pi^2 F_\pi^3}\frac{B_0(m_u - m_d)}{(s_{\gamma\gamma} - m_{\pi^0}^2)}\epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu k_1^\nu \epsilon_2^\alpha k_2^\beta , \qquad (3.1)$$

$$A_{\rm NR}^{(4)} = -\frac{e^2}{12\sqrt{3}\pi^2 F_\pi^3}\frac{B_0(m_u + m_d)}{(s_{\gamma\gamma} - m_\eta^2)}\epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu k_1^\nu \epsilon_2^\alpha k_2^\beta . \qquad (3.2)$$

As already stated in the introduction, the enhancement factor due to the pion pole makes $A_{\rm R}^{(4)}$ dominant – in spite of its suppression factor $(m_u - m_d)$ – with respect to $A_{\rm NR}^{(4)}$ in the entire kinematical space.

The $O(p^6)$ loop and counterterm (CT) contributions can be divided in three gauge-invariant subgroups: reducible π^0-exchange diagrams, reducible η_8-exchange diagrams and 1PI diagrams.

(i) The π^0-exchange diagrams, which include both loops and CT, contribute mainly to $A_{\rm R}$. In principle such diagrams generate also a contribution to $A_{\rm NR}$. Indeed, if we decompose the $\eta \to \pi^0\pi^0(\pi^0)^*$ amplitude as follows:

$$A(\eta \to \pi^0\pi^0(\pi^0)^*) = A_{\text{on-shell}}(\eta \to 3\pi^0) + (s_{\gamma\gamma} - m_\pi^2) \times A_{\text{off-shell}} , \qquad (3.3)$$

the $\mathcal{A}_{\text{off-shell}}$ term drops out of $A_{\rm R}$. However this non-resonant contribution vanishes in the limit

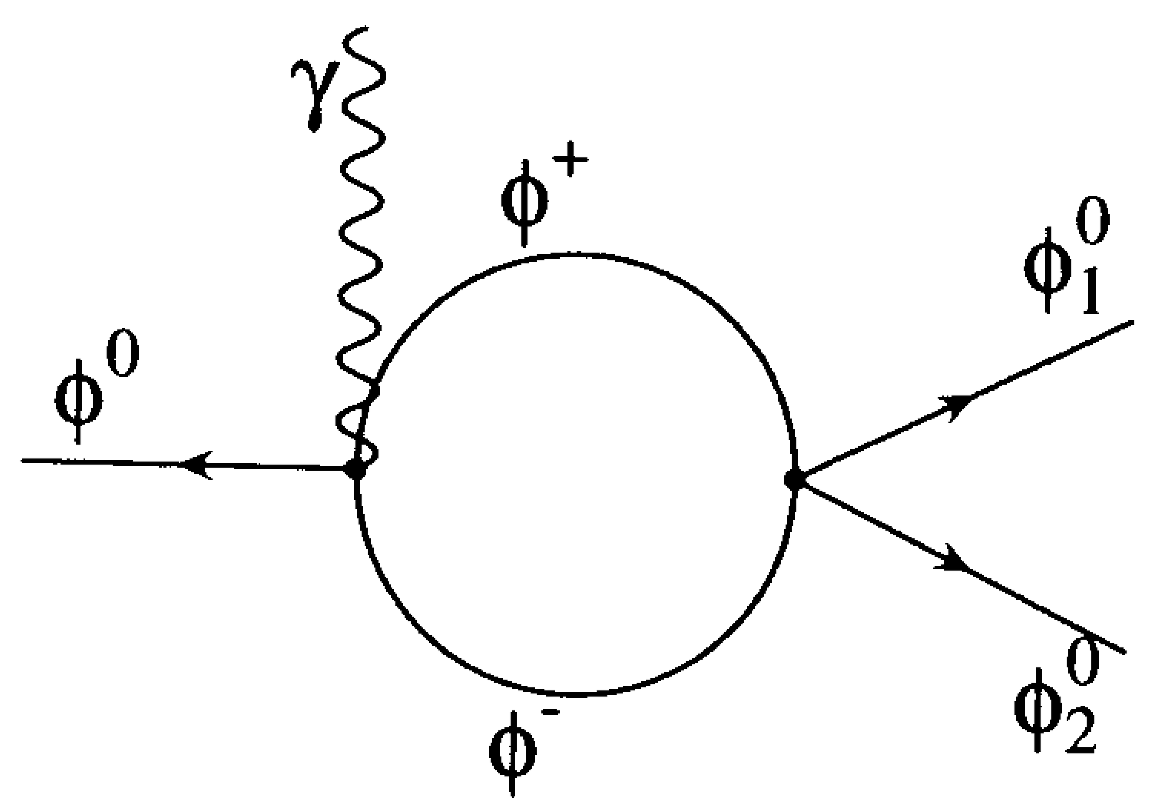

Fig. 1. 1PI one-loop diagrams for the $\phi^0 \to \phi_1^0\phi_2^0\gamma\gamma$ transition. The second photon line has to be attached to the charged lines running in the loop and to the vertices.

$m_u = m_d$ and thus can be safely neglected. On the other hand, $|\mathcal{A}_{\text{on-shell}}|$ can be extracted by means of experimental data and we do not need to evaluate it in CHPT.

(ii) Also the η_8-exchange diagrams include both loops and CT. These diagrams contribute only to A_{NR} and can be safely neglected. Indeed, we have explicitly checked that the contribution of these diagrams is of the same order as the tree-level result (3.2) that is known to be small [1]. The reason of such suppression can be easily understood: the π–π loops, which are expected to provide the dominant contribution, are suppressed by the factor $(m_u + m_d)$ in (2.9) just as the tree level result. There are contributions from the K–K loops and $\mathcal{L}^{(4)}$ which are not suppressed by $(m_u + m_d)$. They are nonetheless negligible, since we are far below the kaon threshold and the CT combinations involved, i.e. $(L_1 + L_3/6)$, $(L_2 + L_3/3)$ and L_4, are small [4].

(iii) The 1PI diagrams are the loop diagrams in Fig. 1 (note that the figure indicates schematically at least four distinct diagrams). The sum of these contributions is finite and, as we will show in the following, turns out to be the dominant contribution to A_{NR}.

The calculation of the loop diagrams in Fig. 1 resembles a recent calculation by D'Ambrosio et al. [10] of the radiative four-meson amplitudes. The difference is that in our case one pseudoscalar field is replaced by one photon, but the main features of the result – simply dictated by QED – are the same. We find indeed

$$A_{\text{NR}}^{\text{1PI}} = 4ef(as_{\pi\pi} + bm_\pi^2)\Big\{\widetilde{C_{20}}(s_{\pi\pi}, -z_2) \times \epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu k_1^\nu\,[(\epsilon_2\cdot p_{12})k_2^\alpha - z_2\epsilon_2^\alpha]\,q^\beta + (\epsilon_1, k_1 \leftrightarrow \epsilon_2, k_2)\Big\}, \tag{3.4}$$

where $p_{12} = p_1 + p_2$ and the function $\widetilde{C_{20}}(x,y)$ is given in appendix. As in [10], gauge invariance forces the result to depend only from the 'on-shell couplings' a, b and f. Furthermore, the amplitude (3.4) is $O(k_1, k_2)$ in the limit of vanishing photon momenta, in analogy to the direct-emission amplitudes of [10] which are $O(k)$.

Since we have written the two vertices in a general form, not only the dominant pion loops, but also the kaon loops are represented in Eq. (3.4). From our general one-loop amplitude we recover, as a particular case, part of the result of Talavera et al. [6] (i.e. the contribution of 1PI diagrams). The precise correspondence between the function $\widetilde{C_{20}}(x,y)$ and the function $R(x,y)$ entering Eq. (10) of [6] is given by

$$R(x,y) = 32\pi^2 y\widetilde{C_{20}}(x,y). \tag{3.5}$$

The amplitude (3.4) depends only on the function $\widetilde{C_{20}}$ and thus is finite. This is a consequence of both the gauge invariance of the amplitude and the fact that the on-shell $\pi^+\pi^- \to \pi^0\pi^0$ amplitude depends only on $s_{\pi\pi}$ (i.e. it does not depend on the loop variables). We expect that the sum of 1PI diagrams is no more finite if the two external π^0's are replaced by a π^+–π^- pair. Indeed, in this case not only the on-shell $\pi^+\pi^- \to \pi^+\pi^-$ amplitude does depend on the loop momenta, but this sum is also not gauge invariant (in order to obtain a gauge invariant result, it is necessary to add the corresponding reducible diagrams with a photon emission from the external legs).

4. Numerical analysis

The results of our analysis are summarized in Figs. 2 and 3. The plots have been obtained integrating numerically Eq. (2.4) with the following decay amplitude:

$$A(\eta \to \pi^0\pi^0\gamma\gamma) = A_{\text{R}}^{\text{phys}} + \left[A_{\text{NR}}^{(4)} + A_{\text{NR}}^{\text{1PI}}\right]. \tag{4.1}$$

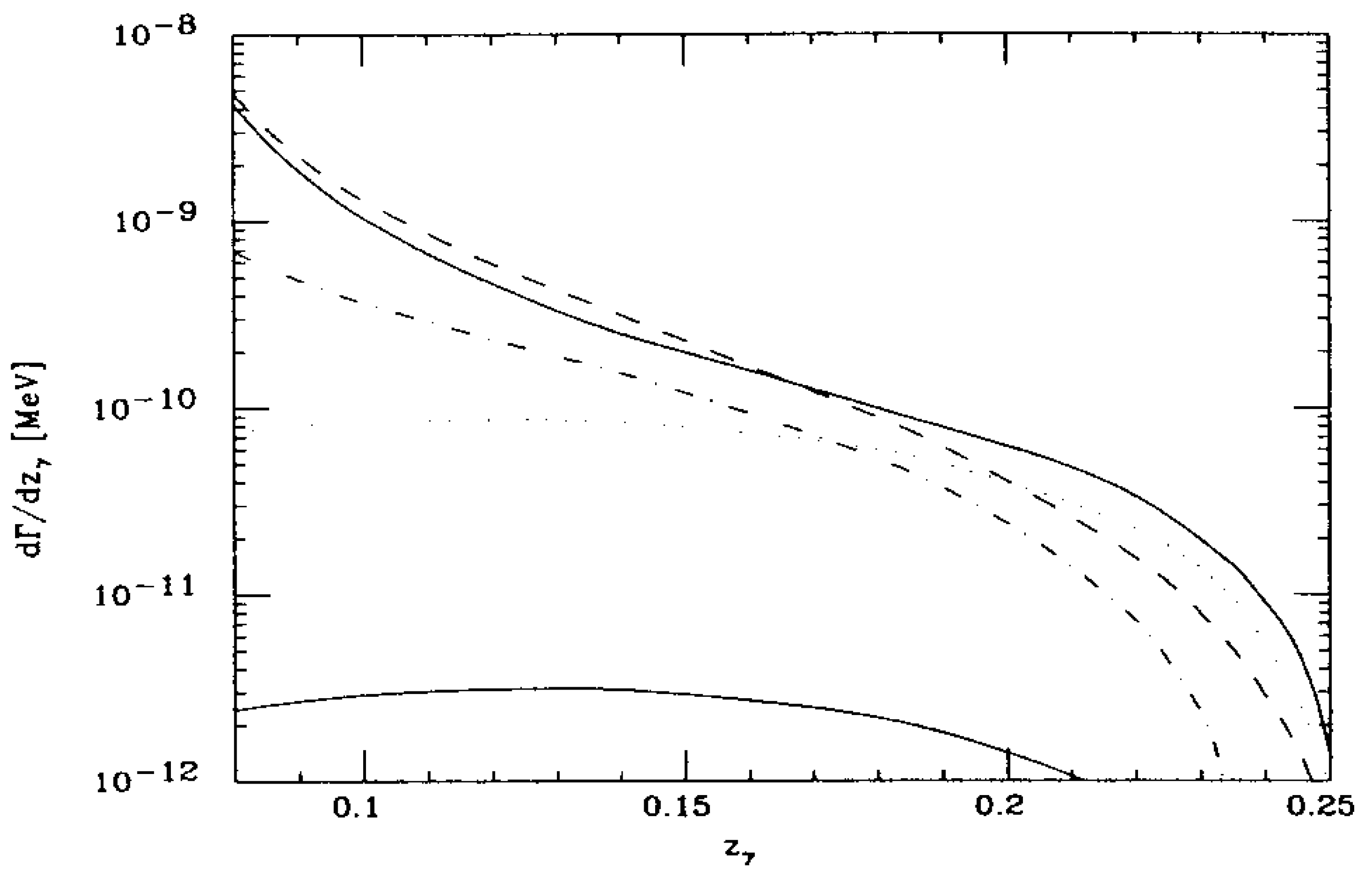

Fig. 2. Diphoton spectrum ($z_\gamma = s_{\gamma\gamma}/m_\eta^2$) for the decay $\eta \to \pi^0\pi^0\gamma\gamma$. The upper full line is the total contribution. The dashed line is the resonant contribution ($|A_R^{phys}|^2$), the dotted line is the one-loop non-resonant contribution ($|A_{NR}^{1PI}|^2$) and the dash-dotted line is the asbolute value of their interference ($\rho = 2$, $\alpha_0 = 0.18$). The lower full line is the tree-level non-resonant contribution ($|A_{NR}^{(4)}|^2$).

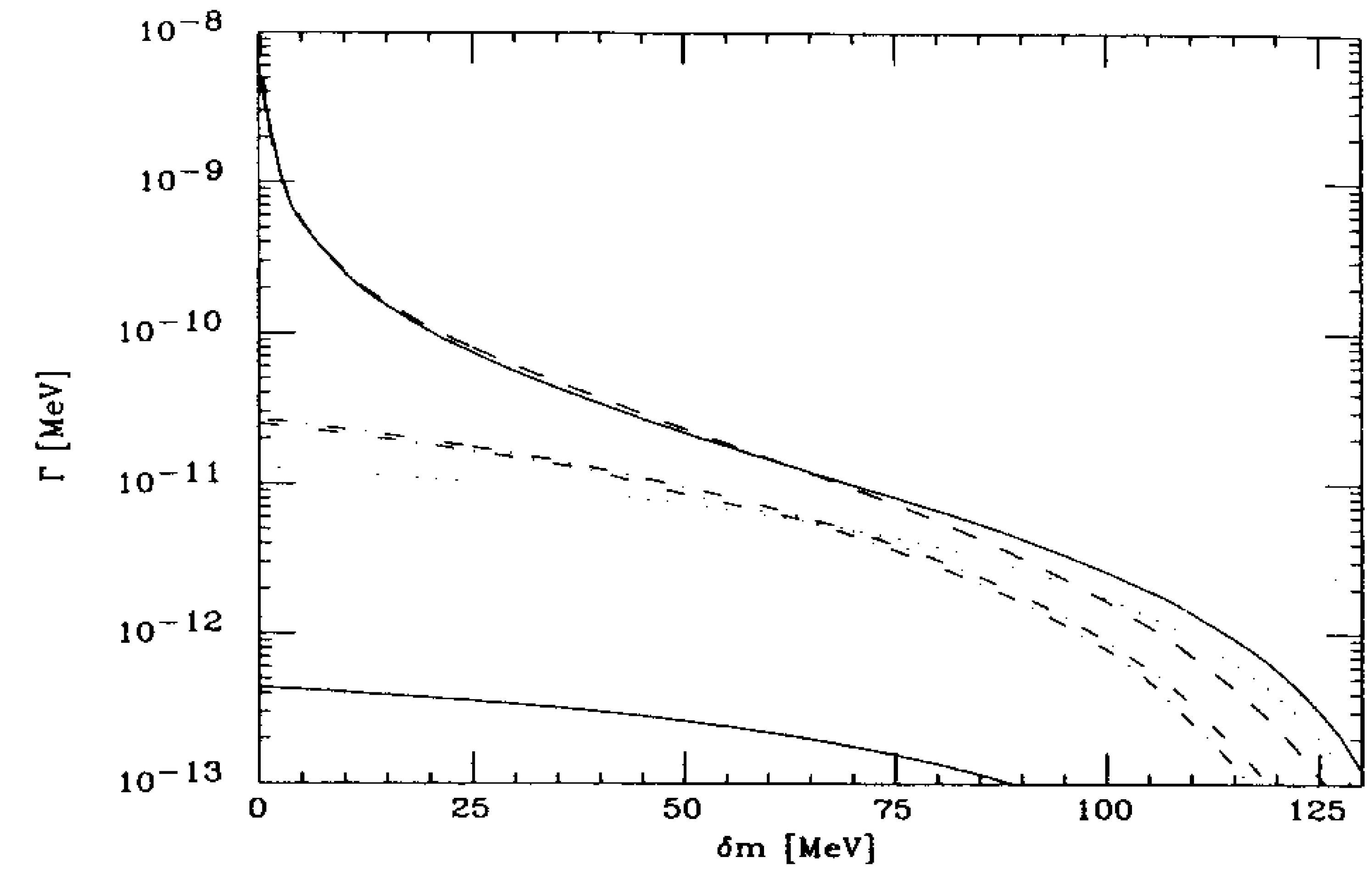

Fig. 3. Partial decay rate of $\eta \to \pi^0\pi^0\gamma\gamma$ as a function of the energy cut $|s_{\gamma\gamma}^{1/2} - m_{\pi^0}| < \delta m$. Full, dashed and dotted curves as in Fig. 2. The two dash-dotted lines, denoting the absolute value of the interference between A_R^{phys} and A_{NR}^{1PI}, have been obtained for $\alpha_0 = 0.16$ (upper line) and $\alpha_0 = 0.20$ (lower line).

The last two terms in Eq. (4.1) are the CHPT results given in Eqs. (3.2) and (3.4), whereas A_R^{phys} denotes a phenomenological expression for the resonant amplitude:

$$A_R^{phys} = A_R^{(4)} \rho e^{i\alpha_0} . \tag{4.2}$$

The factor $\rho e^{i\alpha_0}$ in the above equation takes into account the corrections to the tree-level amplitude of $\eta \to 3\pi^0$, which are known to be large [11]. Assuming a flat Dalitz Plot for this decay – not withstanding experimental constraints – and using the relation [11]

$$B_0(m_d - m_u) = m_{K^0}^2 - m_{K^+}^2 - m_{\pi^0}^2 + m_{\pi^+}^2 , \tag{4.3}$$

from the experimental data on $\Gamma(\eta \to 3\pi^0)$ [9] we find $\rho = 2.0 \pm 0.1$.

Contrary to ρ, the phase α_0 cannot be extracted from the $\eta \to 3\pi^0$ data. Similarly to the $K \to 3\pi$ analysis of [12], in order to evaluate α_0, we expand the one-loop $\eta \to 3\pi^0$ amplitude [11] around the center of the Dalitz Plot. Hence we obtain

$$\alpha_0 = \frac{1}{32\pi F_\pi^2} \left(1 - \frac{4m_\pi^2}{s_0}\right)^{1/2} (2s_0 + m_\pi^2) \simeq 0.18 , \tag{4.4}$$

where $s_0 = (m_\eta^2 + 3m_\pi^2)/3$.

As anticipated, the two figures show clearly that in the non-resonant amplitude the one-loop result dominates over the tree-level one in the whole phase space. Moreover, for $s_{\gamma\gamma} \gtrsim 0.15 m_\eta^2$ the non-resonant amplitude becomes non-negligible with respect to the resonant one. For $s_{\gamma\gamma} \gtrsim 0.20 m_\eta^2$ the dominant contribution to the decay is provided by A_{NR}.

All the distributions have been obtained using $\rho = 2$ in A_R^{phys} and considering the dominant π–π loops only. We have explicitly checked by means of Eq. (3.4) that the kaon loops give a very small contribution, confirming the statement of the previous Section that the π–π loops dominate the non-resonant amplitude in the whole phase space. The dependence of our result on α_0 is quite small, as shown in Fig. 3, while a precise determination of ρ is important, in order to determine with a good accuracy the region where the non-resonant amplitude dominates over the resonant one. Our analysis could be improved if high precision data on $\eta \to 3\pi^0$ were available. They would allow, in principle, to include the (small) D-wave contribution we neglected. However, even with the present uncertainty on the normalization factor ρ, we can state that the loop effect is in principle observable. Indeed, for $s_{\gamma\gamma} > 0.23 m_\eta^2$, we find that the non-resonant contribution is larger than the resonant one by more than a factor of 2.

Notice that our distributions differ from those in [1] – apart from the one-loop corrections to A_{NR} – by an overall normalization factor. Since we agree with the analytic expressions reported in [1], this discrepancy can be traced to a problem in the program used to produce their plots.

5. Discussion and outlook

In this paper we have explicitly calculated the dominant one-loop corrections in CHPT to the decay $\eta \to \pi^0\pi^0\gamma\gamma$, going beyond the lowest order treatment of [1] – the latter essentially corresponds to a simple current algebra calculation. The phenomenological interest of this process is due to the experimental facilities which can effectively act, in the next few years, as η-factories. A similar physical motivation led recently many authors to calculate leading corrections to the lowest-order CHPT prediction in both decays, such as $\eta \to \pi^0\gamma\gamma$ [13], and scattering processes, such as $\gamma\gamma \to \pi^+\pi^-$ [14] and $\gamma\gamma \to \pi^0\pi^0$ [15]. Earlier work on the CHPT predictions of pion polarizabilities can be found in [16], and the comparison with forward-angle dispersion sum-rules is discussed in [17]. Recently the charged-pion polarizabilities have been computed to two loops [18].

Recent results on $\gamma\gamma \to \pi^0\pi^0\pi^0$ [6] have inspired us, since there the lowest-order amplitude is suppressed and the corrections due to chiral loops dominate the cross-section. We found a similar result to hold for the non-resonant contribution to the decay $\eta \to \pi^0\pi^0\gamma\gamma$.

Despite the enhancement due to the one-loop corrections, the non-resonant amplitude is shadowed from the resonant (i.e. the π^0-exchange) contribution, over a large portion of the diphoton spectrum. We have shown, however, that for large $s_{\gamma\gamma}$ the one-loop corrections to the non-resonant amplitude dominate also over the resonant contribution (see Figs. 2 and 3). A measurement of the partial width of $\eta \to \pi^0\pi^0\gamma\gamma$ in this kinematical region – within the reach of the future facilities – would represent a new interesting test of CHPT at $O(p^6)$.

We wish to conclude this Letter with a few comments concerning the possibility of future developments. In particular, one might ask why we concentrated on the neutral pion channel, rather than calculating the amplitude of $\eta \to \pi^+\pi^-\gamma\gamma$, which is statistically favored. The reason is that the decay $\eta \to \pi^+\pi^-\gamma\gamma$ is dominated by the bremsstrahlung of $\eta \to \pi^+\pi^-\gamma$ [1]. Since the latter is not suppressed already at the tree level, we expect that the one-loop corrections not related to the $\eta \to \pi^+\pi^-\gamma$ amplitude will be hardly detectable. [4] From this point of view, when considering the outlook for a future calculation and a potentially related measurement, we expect that the scatterings $\gamma\gamma \to \pi^+\pi^-\eta$ and $\gamma\gamma \to \pi^0\pi^0\eta$ will provide more interesting tools for the study of chiral-loop effects.

Added note

After submitting this Letter, we learned about a similar calculation which included also the η–η' mixing effect and an estimate of $O(p^8)$ contributions in the resonant amplitude [19]. This result confirms that the effect we calculated is the dominant one in the non-resonant amplitude.

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Appendix A

The general definition of $\widetilde{C_{20}}(x,y)$ in terms of the three-denominator one-loop scalar functions can be found in [10]. In the π–π case and for x, $x-2y > 4m_\pi^2$ the explicit expression is given by:

$$(4\pi)^2 \,\mathrm{Re}\,\widetilde{C_{20}}(x,y) = \frac{x}{8y^2}\left\{\left(1-2\frac{y}{x}\right)\right.$$
$$\times\left[\beta\log\left(\frac{1+\beta}{1-\beta}\right) - \beta_0\log\left(\frac{1+\beta_0}{1-\beta_0}\right)\right]$$
$$\left.+\frac{m_\pi^2}{x}\left[\log^2\left(\frac{1+\beta_0}{1-\beta_0}\right) - \log^2\left(\frac{1+\beta}{1-\beta}\right)\right] + 2\frac{y}{x}\right\}, \tag{A.1}$$

[4] From the theoretical point of view, in order to isolate these effects it is necessary to implement an appropriate definition of 'generalized bremsstrahlung', similarly to what has been done in [10] for the radiative-four-meson amplitudes.

$$(16\pi)\,\mathrm{Im}\,\widetilde{C_{20}}(x,y) = -\frac{x}{8y^2}\left\{\left(1-2\frac{y}{x}\right)[\beta-\beta_0] + \frac{2m_\pi^2}{x}\left[\log\left(\frac{1+\beta_0}{1-\beta_0}\right) - \log\left(\frac{1+\beta}{1-\beta}\right)\right] + 2\frac{y}{x}\right\}, \tag{A.2}$$

where

$$\beta_0 = \sqrt{1-\frac{4m_\pi^2}{x}} \quad \beta = \sqrt{1-\frac{4m_\pi^2}{(x-2y)}}. \tag{A.3}$$

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