

LNF-96/051(P)

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Physics Letters B, 402, 122-129, (1997)



ELSEVIER

5 June 1997

PHYSICS LETTERS B

Physics Letters B 402 (1997) 122–129

Anatomy of the Higgs mass spectrum

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Received 7 October 1996; revised manuscript received 28 February 1997

Editor: R. Gatto

Abstract

We analyze the implications of a Higgs discovery on possible “new-physics” scenarios, for m_H up to ~ 700 GeV. For this purpose we critically review lower and upper limits on the Higgs mass in the SM and in the MSSM, respectively. Furthermore, we discuss the general features of possible “heavy” ($m_H \gtrsim 2m_Z$) Higgs scenarios by means of a simple heavy-fermion condensate model. © 1997 Published by Elsevier Science B.V.

1. Introduction

The discovery of the Higgs particle is of utmost importance in particle physics. Over the years, various theoretical bounds have been made [1–8], and most recently an experimental *lower* bound of 65 GeV was set [9]. But the Higgs boson still remains elusive. Its nature – mass and couplings – would reveal the most fundamental aspects of the kind of mechanism that governs the spontaneous symmetry breakdown of the standard model (SM). In particular, one would like to know whether or not such a discovery, if and when it will be made, will be accompanied by “new physics” at some energy scale Λ . Of equal importance is the following question: at roughly what mass scale will the Higgs boson be considered elementary or composite? Can one make some meaningful statement concerning its nature once it is discovered? These are the issues we would like to explore in this paper.

A first step in this direction has been recently achieved by detailed analyses of the Higgs potential [6–8]. Indeed, with the discovery of the top quark

with mass $m_t = 175 \pm 9$ GeV [10], the Higgs mass (m_H) is severely constrained by the requirement of vacuum stability. In particular, two interesting conclusions have been drawn:

- i. If a Higgs will be discovered at LEP200, i.e. with $m_H \leq m_Z$, then some new physics must appear at very low scales: $\Lambda \lesssim 10$ TeV [7,8,11].
- ii. The standard model with an high cut-off (without new particles below 10^{15} GeV) requires $m_H \gtrsim 130$ GeV and is incompatible with the minimal supersymmetric standard model (MSSM), where the mass of the lightest Higgs boson is expected below 130 GeV [8].

We shall re-analyze the above statements, trying to clarify the stability of the physical conclusions with respect to the theoretical errors, and we shall extend the discussion studying the implications of a Higgs discovery up to approximately 700 GeV.

The plan of the paper is as follows. In the next section we shall briefly review what is known about the Higgs sector of the SM and of the MSSM. We then divide our analysis into three separate mass regions:

the region below m_Z , between m_Z and $2m_Z$, above $2m_Z$.

2. The Higgs boson in the SM and in the MSSM

The symmetry breaking sector of the SM is highly unstable and make sense only in presence of a cut-off scale Λ . The instability of the scalar sector implies that upper and lower limits on m_H , imposed by the requirement of no Landau pole and vacuum stability, both below Λ , tend to shrink together as the cut-off increases [1,2].

The instability of the scalar potential is generated by the quantum loop corrections to the classical expression

$$V^{\text{tree}}(\Phi) = -m^2\Phi^\dagger\Phi + \frac{\lambda}{6}(\Phi^\dagger\Phi)^2, \quad \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad (1)$$

where $v^2 = 2\langle\phi^0\rangle^2 = 1/\sqrt{2}G_F \simeq (246 \text{ GeV})^2$ and $\phi = \sqrt{2}\text{Re}\phi^0 - v$ is the physical Higgs field. As already noticed in Ref. [1], and successively confirmed by detailed analysis of the renormalization group (RG) improved potential [3,12], the issue of vacuum stability for $\phi \sim \Lambda \gg m_Z$ practically coincides with the requirement that the running coupling $\lambda(\Lambda)$ never becomes negative. On the other hand, the requirement that no Landau pole appears before Λ is equivalent to the condition that $\lambda(\Lambda)$ always remains in the perturbative region.

The evolution of λ as a function of Λ is ruled by a set of coupled differential equations

$$\begin{aligned} d\lambda(t)/dt &= \beta_\lambda(\lambda, g_i), \\ dg_i^2(t)/dt &= \beta_i(\lambda, g_i), \end{aligned} \quad t = \ln(\Lambda/\mu) \quad (2)$$

with the corresponding set of initial conditions which relate $\lambda(\mu)$ and $g_i^2(\mu)$ to physical observables (g_3, g_2 and g_1 denote $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge couplings and g_t the top-quark Yukawa coupling, all couplings are understood in the $\overline{\text{MS}}$ scheme). The β -functions of Eq. (2) are known in perturbation theory up to two loops (see Ref. [12,5] for the complete expressions), i.e. up to the third order in the expansion around zero in terms of λ and g_i^2 , whereas the finite parts of the initial conditions around $\mu = m_Z$ (threshold corrections) are known up to one-loop accuracy [13–15]. This knowledge enables us to re-sum

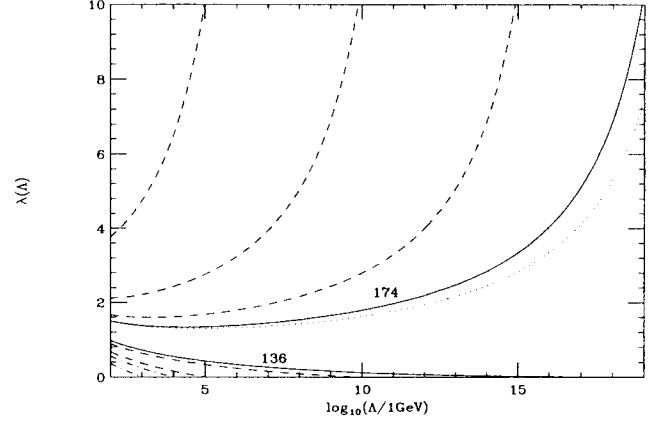


Fig. 1. $\lambda(\Lambda)$ for $m_t = 175 \text{ GeV}$ (pole mass) and $\alpha_S(m_Z) = 0.118$. Full and dashed lines have been obtained by integrating two-loop beta functions and using one-loop matching conditions, whereas the dotted one has been obtained by integrating one-loop beta functions. The physical values of the Higgs mass corresponding to the full lines are $m_H = 136$ and 174 GeV .

all the next-to-leading logs in the evolution of the coupling constants and thus to calculate them with high accuracy in the perturbative region. Nevertheless, the unstable character of $\lambda(t)$ can be simply read-off by the one-loop expression

$$\beta_\lambda = \frac{1}{16\pi^2} [4\lambda^2 + 12\lambda g_t^2 - 36g_t^4 + O(g_1^2, g_2^2)], \quad (3)$$

together with the tree-level relations

$$\lambda(m_H) = \frac{3m_H^2}{v^2} \quad \text{and} \quad g_t^2(m_t) = \frac{2m_t^2}{v^2}. \quad (4)$$

For small values of m_H the g_t^4 term in Eq. (3) drives λ to negative values, whereas if m_H is large enough the Higgs self-interaction dominates and eventually λ “blows up”.

The situation is summarized in Fig. 1 where we plot the evolution of λ as obtained by integrating two-loop beta functions. For $m_t = 175$ (pole mass) and $\alpha_S(m_Z) = 0.118$, if we impose the condition

$$0 < \lambda(\Lambda) < 10 \quad (5)$$

at the Planck scale, then m_H is confined in a very narrow range (full lines in Fig. 1)¹

$$136 \text{ GeV} \leq m_H \leq 174 \text{ GeV}. \quad (6)$$

¹ With respect to Ref. [7] we have removed a small error in the threshold correction of g_t (the correct expression is given in Ref. [15]) obtaining a $\sim 1 \text{ GeV}$ decrease of the lower limit.

The lower limit on m_H depends strongly on the values of m_t and $\alpha_S(m_Z)$ [6–8] whereas the upper one is more or less independent from them, both are weakened if the condition (5) is imposed at scales Λ below the Planck mass (dashed lines in Fig. 1). The consequences of a Higgs mass outside the range (6) will be discussed in the following sections.

The Higgs sector of the MSSM (see Refs. [17] for excellent reviews) contains two Higgs doublets, one is responsible for charged-lepton and down-type-quark masses (H_1), the other for up-type-quark masses (H_2). Of the eight degrees of freedom, two charged, one CP -odd and two CP -even neutral scalars correspond to physical particles after the $SU(2)_L \times U(1)_Y$ breaking. The tree-level potential for the two neutral components H_1^0 and H_2^0 is given by

$$V^{\text{tree}}(H_1^0, H_2^0) = \frac{g_1^2 + g_2^2}{8} (|H_1^0|^2 - |H_2^0|^2)^2 + m_1^2 |H_1^0|^2 + m_2^2 |H_2^0|^2 + [m_{12}^2 H_1^0 H_2^0 + \text{h.c.}]. \quad (7)$$

The sum of the two vacuum expectation values squared is fixed by the gauge boson masses: $v_1^2 + v_2^2 = v^2$ ($v_{1,2} = \sqrt{2}\langle H_{1,2}^0 \rangle$), while the ratio $\tan(\beta) = v_2/v_1$ is a free parameter. The remarkable feature of the potential (7) is that the coefficient of the dimension-four operator is completely fixed in terms of the gauge couplings g_1 and g_2 . This property leads to the tree-level relation $m_{H,H'}^2 = [m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta}] / 2$ ($m_{H,H'}$ are the two CP -even Higgs boson masses and m_A is the CP -odd one) that implies a strict upper bound

$$m_H \leq m_Z \cos 2\beta \leq m_Z \quad (8)$$

on the lightest Higgs boson mass.

As is well-known [18,19,8], the bound (8) receives large radiative corrections if SUSY particles, and in particular the \tilde{t} squark, are heavy. This can be easily understood by means of the SM evolution of λ previously discussed. Indeed, if all SUSY particles (including additional Higgs bosons) have a mass of the order of M_S ($M_S^2 \gg m_Z^2$), the lightest Higgs boson decouples below M_S and mimics the SM Higgs. Then, the evolution of the scalar self-coupling $\lambda(\Lambda)$ is dictated by SM beta functions up to $\Lambda = M_S$, where SUSY is restored and, according to the potential (7), the following relation must hold

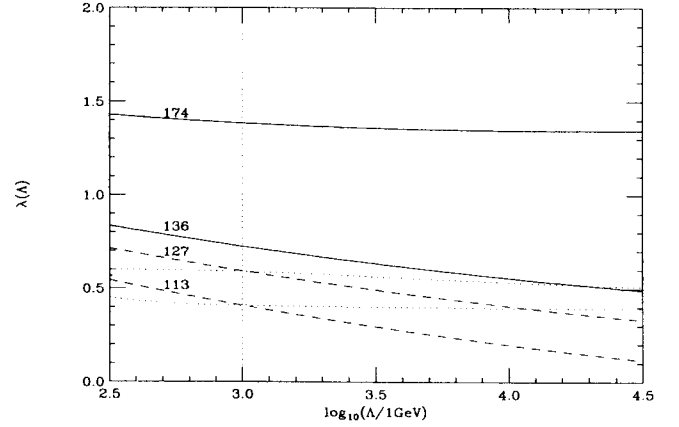


Fig. 2. $\lambda(\Lambda)$ in the small Λ region. The horizontal dotted lines indicate the MSSM upper limit imposed by Eq. (10) for $M_S = \Lambda$ and $X_t = 0$ (lower curve) or $X_t = X_t(M_S)$ chosen to maximize the threshold effect (upper curve). Full and dashed lines indicate the SM evolution of λ (two-loop beta functions and one-loop matching conditions with $m_t = 175$ GeV and $\alpha_S(m_Z) = 0.118$) for different values of m_H (as indicated above each line).

$$\lambda(M_S) = \frac{3}{4} [g_1(M_S)^2 + g_2(M_S)^2] \cos^2 2\beta. \quad (9)$$

Eq. (9) saturates the bound (8) for $M_S \sim m_Z$ but, due to the rapidly decreasing behaviour of $\lambda(\Lambda)$ (see Fig. 2), implies (20–30)% violations of the tree-level bound for $M_S \sim 1$ TeV [18]. Analogously to the tree-level relations (4), the boundary condition (9) is not differentiable with respect to the scale of λ : in order to calculate precise bounds on the Higgs mass is necessary to include threshold effects in both cases. The most important correction to Eq. (9) is the one generated by stop loops, that is proportional to g_t^4 . If we include this effect, Eq. (9) is modified in

$$\lambda(\Lambda) = \frac{3}{4} [g_1(\Lambda)^2 + g_2(\Lambda)^2] \cos^2 2\beta + \Delta\lambda(\Lambda), \quad (10)$$

where $d\Delta\lambda(\Lambda)/d\ln(\Lambda/\mu) = -36g_t^4/16\pi^2 + \dots$, by this way the leading terms in the derivatives of both sides of Eq. (10) are the same. The explicit expression of $\Delta\lambda(\Lambda)$, obtained by the one-loop stop contribution to the potential (7), is given by [8]

$$\Delta\lambda(\Lambda) = \frac{9g_t^4}{16\pi^2} \left\{ \frac{(m_t + X_t)^2}{\tilde{m}_+^2} \left[1 - \frac{(m_t + X_t)^2}{12\tilde{m}_+^2} \right] + \frac{(m_t - X_t)^2}{\tilde{m}_-^2} \left[1 - \frac{(m_t - X_t)^2}{12\tilde{m}_-^2} \right] + \ln \left(\frac{\tilde{m}_-^2 \tilde{m}_+^2}{\Lambda^4} \right) \right\}, \quad (11)$$

where $\tilde{m}_{\pm}^2 = M_S^2 + m_t^2 \pm m_t X_t$ are the eigenvalues of the stop mass matrix and X_t is the usual stop-mixing parameter [8,18]. As noticed in Ref. [8], $\Delta\lambda(M_S)$ has a maximum for $X_t^2 = 6M_S^2 + O(m_t^2)$. Imposing the boundary condition (11) at $\Lambda \sim M_S$, using two-loop SM beta functions to evolve down at $\mu \sim m_Z$ and finally using SM one-loop matching conditions to relate m_H and m_t to λ and g_t , we find (masses are in units of GeV)

$$M_H^{\text{MSSM}} < 127 + 0.9 [m_t - 175] - 0.8 \left[\frac{\alpha_S(m_Z) - 0.118}{0.006} \right] + 7 \log_{10} \left(\frac{M_S}{10^3} \right) \pm 4 \quad (12)$$

in good agreement with the analysis of Ref. [8]². The error in Eq. (12) has been estimated by varying low and high energy matching scales in the following intervals: $\Lambda \in [M_S, 2M_S]$ and $\mu \in [m_Z, 2m_t]$. Obviously the upper limit is very sensitive to M_S , defined as the soft stop mass, and is valid for M_S near 1 TeV; on the other hand, the dependence on other SUSY masses is within the quoted error. As can be noticed from Fig. 2, for $m_t = 175$ GeV and $\alpha_S = 0.118$, the SM with $\Lambda = M_{\text{Plank}}$ is compatible with the MSSM only for unnatural large values of M_S .

3. Physics of the “low” mass Higgs boson: $m_H \leq m_Z$ GeV

As we have discussed in the previous section, the SM vacuum becomes unstable when $m_H \leq 136$ GeV. In this case the usual wisdom asserted that new degrees of freedom must show up at or before the scale Λ where $\lambda(\Lambda)$ becomes negative. If the Higgs mass is below the Z mass this implies new physics in the TeV region [7,8,11].

Recently, it has been pointed out [21] that for small values of Λ the lower limit on m_H imposed by the condition

$$\left. \frac{dV^{1\text{-RG}}(\phi)}{d\phi} \right|_{\phi=\Lambda} > 0, \quad (13)$$

where $V^{1\text{-RG}}(\phi)$ denotes the one-loop RG-improved potential, do not coincide with the one imposed by

² Lower bounds are obtained with more stringent assumptions, like the naturalness of various MSSM parameters [20].

$\lambda(\Lambda) > 0$. We agree with the above statement, however it must be stressed that the two conditions lead to equivalent results up to a small re-definition of Λ [21]. As an example, the lower limit on m_H imposed by Eq. (5) with $\Lambda = 1$ TeV, namely $m_H > 72$ GeV, is equivalent to the one imposed by Eq. (5) with $\Lambda \simeq 3.4$ TeV. On the other hand, the two conditions coincide for large values of the cut-off, where the corresponding $\lambda(\Lambda)$ curves are almost flat (Fig. 1). Since the exact relation between Λ , understood as the scale where the evolution of λ is no more ruled by standard model beta functions, and the masses of hypothetical new particles depends on the details of the new-physics model [11], in our opinion is meaningless to fix Λ with great accuracy. In other words, for a given value of m_H , the scale Λ where Eq. (5) or Eq. (13) are no more satisfied can give only an indication of the order of magnitude below which new physics must appear, and within this interpretation the two conditions are completely equivalent and consistent with the statement (i) of Section 1.

To stabilize the SM vacuum, one has to add more scalar degrees of freedom which couple to the SM Higgs, a well-known fact from studies of the effective potential or from studies of the RG equations. The most natural new-physics candidate in this case is the MSSM. There is a plethora of scalar fields: the supersymmetric partners of quarks and leptons, and the additional Higgses. However, as we have seen in the previous section, the “stabilizing scalar” is the stop which cancels the g_t dependence in the evolution of λ . The lighter the Higgs is, the lighter the stop must be.

What happens if the Higgs mass is very light, say 70 GeV, and the stop is not found in the TeV region? Either the MSSM is not correct and a more complicated version is needed or something other than SUSY enters the picture. In Ref. [11] this question has been studied using a toy model with electroweak singlet scalars, with multiplicity N and with a coupling δ to the standard Higgs field. It was found that the mass of the new singlet scalars could be as high as ten times the scale Λ where $\lambda(\Lambda)$ becomes negative.

In the above discussion, there was never any need for the Higgs boson to be composite. In fact, it appears to be more natural for the Higgs boson to be *elementary* in this case. Although there are models for a “light” Higgs boson where an elementary Higgs field is mixed with a top condensate [22], it does not

appear to be possible to construct a model where the Higgs boson is entirely composite. It is in this sense that we conclude that the Higgs boson is elementary if its mass is m_Z or below.

4. Physics of the Higgs boson with $m_Z \leq m_H \leq 2m_Z$

This is a region where it will be extremely hard to detect the Higgs boson [4]. Theoretically, this is a region where one can still presume that the Higgs boson is an elementary particle. Indeed, a look at Fig. 1 will convince us that λ blows up below the Planck scale only when $m_H \gtrsim 2m_Z$. Furthermore, there is no known mechanism which can give rise to a composite Higgs boson that light (without additional scalars).

As we have seen in Section 2, if $m_H \lesssim 130$ GeV the most natural candidate is still the MSSM. On the other hand, for $m_H \gtrsim 130$ GeV the MSSM it is unnatural because the SUSY scale is too high. Above 130 GeV natural candidates are SUSY extensions of the SM with a non-minimal scalar sector [23]. In this region also the SM itself can be considered a good candidate. Indeed, a part from the problem of quadratic divergences, new physics can be pushed up to the Planck scale if $m_H \gtrsim 130$. In this framework, an interesting scenario is the one proposed in Ref. [24].

5. Physics of the Higgs boson with $m_H \geq 2m_Z$

We finally come to the question of which kind of new physics is expected if m_H is found above ~ 180 GeV, i.e. in the region where $\lambda(\Lambda)$ develops a Landau singularity below the Planck mass. If one believes that the Landau pole is not an artifact of perturbation theory but a non-perturbative feature of the model, as suggested by lattice simulations (see, e.g. Refs. [16,5]), then it is natural to think that some new physics must show up at or below Λ_L . If that is so, this kind of new physics must be very different from the one needed to stabilize the vacuum, since one is now dealing with a strong coupling domain. One is then tempted to attribute this behaviour (strong coupling) to the nature of the Higgs boson. In particular, one might think that the Higgs boson is a *composite* particle which acts like an elementary field

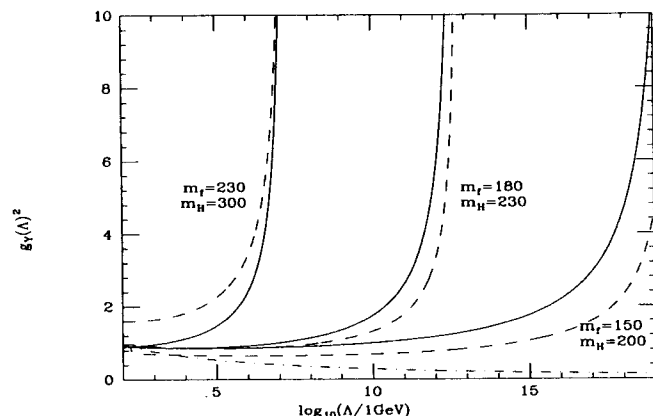


Fig. 3. RG evolution of the Yukawa couplings g_t^2 (full and dash-dotted lines) and g_q^2 (dashed lines). The top mass is always fixed to be 175 GeV and the heavy-lepton mass is assumed to be 90 GeV. The dash-dotted line is the evolution of g_t^2 without extra fermions. Near each dashed line is indicated the value of m_H and the corresponding value of m_q obtained by the requirement $\Lambda_L = \Lambda_q$ (the error on both m_H and m_q is about ± 10 GeV).

below the scale Λ_L . This is the scenario we would like to explore below.

The physics below the compositeness scale can be described in terms of an effective field theory whose couplings are constrained by the boundary conditions at the compositeness scale. In this framework, a class of models which is particularly attractive, relevant to the present discussion and quite general, is the class of the top-condensate models [25,26]. There the relevant boundary conditions are [26,27]

$$\lambda(\mu), g_t(\mu) \xrightarrow{\mu \rightarrow \Lambda_c} \infty, \quad (14)$$

$$\lambda(\mu)/g_t^2(\mu) \xrightarrow{\mu \rightarrow \Lambda_c} \text{const.} \quad (15)$$

Thus, the Landau singularity of the Higgs self-couplings naturally fits into this scheme. The only problem is the requirement of a pole also in the evolution of the top Yukawa coupling. As can be noticed in Fig. 3, the top Yukawa coupling itself is not large enough to develop a singularity since its evolution is “softened” by QCD. However, as we will show in the following, if we include additional heavy fermions with a mass m_f above a critical value, both g_t and g_f can “blow up” at a scale Λ_f .

To analyze better the model, let us consider the Lagrangian of a single degenerate quark doublet $q = (u, d)$ coupled to the Higgs field. If we re-scale the Higgs field in the following way

$$\Phi \longrightarrow \Phi_0/g_f, \quad (16)$$

the Lagrangian becomes

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{kinetic}}(u, d) + Z_\phi D_\mu \Phi_0^\dagger D^\mu \Phi_0 + \tilde{m}^2 \Phi_0^\dagger \Phi_0 \\ & - \frac{\tilde{\lambda}}{6} (\Phi_0^\dagger \Phi_0)^2 + \bar{q}_L \Phi_0 d_R + \bar{q}_L \Phi_0^C u_R + \text{h.c.}, \end{aligned} \quad (17)$$

where

$$\begin{aligned} \Phi_0^C = & i\sigma_2 \Phi_0^*, \quad Z_\phi = 1/g_f^2, \quad \tilde{m}^2 = Z_\phi m^2 \\ & \text{and } \tilde{\lambda} = Z_\phi^2 \lambda. \end{aligned} \quad (18)$$

If λ and g_f^2 develop a singularity at the same scale $\Lambda_f = \Lambda_L = \Lambda_C$, so that the boundary conditions (14)–(15) are satisfied, then with an appropriate tuning of the quadratic divergences we can have [27]

$$\tilde{m}^2/\Lambda_C^2 \xrightarrow{\mu \rightarrow \Lambda_C} \text{const} < 0. \quad (19)$$

Thus at the compositeness scale the above Lagrangian becomes

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}}(u, d) + \bar{q}_L \Phi_0 d_R + \bar{q}_L \Phi_0^C u_R + \tilde{m}^2 \Phi_0^\dagger \Phi_0 + \text{h.c.} \quad (20)$$

and Φ_0 , which is now just an auxiliary field, can be integrated out to obtain a four-fermion Nambu–Jona-Lasinio Lagrangian [28]

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}}(u, d) + G_0 \bar{q}_L (u_R \bar{u}_R + d_R \bar{d}_R) q_L, \quad (21)$$

with $G_0 = -1/\tilde{m}^2$. Viewed in this way, the Higgs boson becomes a dynamical fermion-condensate below the scale Λ_C , in other words the Higgs boson becomes a composite particle.

The necessary conditions for the above view to hold are the constraints (14), (15). In order to understand if these conditions can be satisfied, it is useful to examine the RG equation of the ratio $x = \lambda/g_f^2$. At one loop, it is given by

$$16\pi^2 \frac{dx}{dt} = 4g_f^2(x - x_+)(x - x_-), \quad (22)$$

where $x_\pm = (-3 \pm 9)/2$, if both members of the quark doublet are degenerate in mass, or $x_\pm = \frac{3}{8}(-1 \pm \sqrt{65})$, if one member is much heavier than the other one (e.g. the 3rd generation case). The only possibility to have $\Lambda_f = \Lambda_L$ is that the initial value of x is one of the two fixed points. Since x_- is always negative,

the solution $x = x_-$ is ruled out by vacuum stability. Thus the boundary conditions can be satisfied only if $x = x_+$ and this implies a precise relation between m_H and m_f . Using the tree-level relations (4) we find

$$m_H^2 = \frac{2}{3} m_f^2 x_+. \quad (23)$$

Since x_+ is always greater than 3/2, the fixed point scenario implies

$$m_H > m_f. \quad (24)$$

It is easy to see that, in the large N_c limit, the fixed point takes on the value $x = 6$, giving $m_H = 2m_f$, a familiar result found in the Nambu–Jona-Lasinio model. For finite N_c one finds in general

$$m_f < m_H < 2m_f. \quad (25)$$

As we have mentioned earlier, the top quark is not heavy enough to solely fit into this scenario. This is because the growth of the top Yukawa coupling is dampened by QCD. The minimum top mass for which there will be a Landau singularity at the Planck mass is $m_t \approx 216$ GeV, a value which is way outside the experimental range. Let us then assume that there exist some extra heavy fermions. To avoid the anomaly problem we will add a complete family of heavy quarks and leptons. For simplicity we will give a Dirac-mass to the neutrino and, in order to satisfy the constraints of electroweak precision measurements [29], we will assume that both quarks and leptons are degenerate $SU(2)_L$ doublets. As we shall see below, the addition of these extra fermions changes dramatically the behaviour of the couplings at high energy. To see this let us write the RG equations for λ , g_t (the top Yukawa coupling), g_q and g_l (the Yukawa couplings of the heavy quark and lepton doublets)

$$\begin{aligned} 16\pi^2 \frac{d\lambda}{dt} = & 4\lambda^2 + 4\lambda(3g_t^2 + 6g_q^2 + 2g_l^2) \\ & - 12(3g_t^4 + 6g_q^4 + 2g_l^4) + O(g_1^2, g_2^2), \end{aligned} \quad (26)$$

$$16\pi^2 \frac{dg_t^2}{dt} = g_t^2 [9g_t^2 + 12g_q^2 + 4g_l^2 - 16g_3^2] + O(g_1^2, g_2^2), \quad (27)$$

$$16\pi^2 \frac{dg_q^2}{dt} = g_q^2 [6g_t^2 + 12g_q^2 + 4g_l^2 - 16g_3^2] + O(g_1^2, g_2^2), \quad (28)$$

$$16\pi^2 \frac{dg_l^2}{dt} = g_l^2 [6g_t^2 + 12g_q^2 + 4g_l^2] + O(g_1^2, g_2^2). \quad (29)$$

In absence of extra fermions one can easily see from the above equations that the growth of g_t is dampened by the gauge couplings (mainly by g_3). On the other hand, in the presence of the extra fermions g_t is no longer dampened provided g_q or g_l exceed some critical values³. In addition, all the Yukawa couplings tend to “drag” each other. Assuming for simplicity that the leptons are relatively light ($m_l \sim M_Z$), i.e. assuming that for small t the major role in dragging g_t is played by g_q , then a numerical integration of the above equations shows that for $m_q \gtrsim 150$ GeV all the g_Y ($Y = t, q, l$) develop a singularity around the Planck scale (see Fig. 3). Imposing the condition that λ develops a singularity at the same scale as the g_Y , we can then determine the Higgs mass (e.g. we find $m_H \simeq 200$ GeV if the singularity is around the Planck scale). As m_q increases, the compositeness scale Λ_C decreases and the relation between m_H and m_q approaches the fixed point prediction (23) with $x_+ \simeq 3$. If one relaxes the assumption that the leptons are light then a new parameter enters the game: the relation between the Higgs mass and the heaviest fermion receives small corrections but the corresponding compositeness scale can be substantially smaller.

The above scenario cannot be considered as a realistic model. Indeed, if the scale Λ_C is high there is a “fine tuning” problem related to the large disparity between Λ_C and the electroweak scale. However, it is beyond the scope of this paper to try to construct an underlying theory around Λ_C and thus we will ignore it. Our purpose is just to show some general features of a wide class of models. In particular, if there are no new bosons (scalars or gauge bosons) below the compositeness scale, the following features hold independently of the multiplicity of the new fermions:

- i. The compositeness scale Λ_C , the heavy-fermion mass m_f , and the effective Higgs mass m_H , are tied together so that m_H and m_f increase as Λ_C decreases.
- ii. As shown in Eq. (24), one typically finds $m_H > m_f$. Thus if m_H is not found below $2m_Z$ it should be “easier” to search for new fermions instead of searching for the Higgs boson itself.

³ The $O(g_1^2, g_2^2)$ terms in Eqs. (28), (29), which tend to split the evolution of U and D Yukawa couplings, cannot be neglected if we are interested in a precise determination of the critical values of g_q and/or g_l .

- iii. For $\Lambda_C \approx 1$ TeV both m_H and m_f are $O(\Lambda_C)$ and the Higgs effective theory becomes meaningless. In this sense we agree with the more precise and well-defined lattice bound: $m_H \lesssim 700$ GeV [16].

6. Conclusions

In this paper we have analyzed the consequences of a Higgs discovery up to approximately 700 GeV, dividing the mass region into three parts: the region below m_Z , between m_Z and $2m_Z$, above $2m_Z$.

Regarding the first two regions we have confirmed and refined the results stated in the introduction, namely the SM lower bound due to vacuum stability and the MSSM upper bound.

Regarding the last region ($m_H \gtrsim 2m_Z$) we have shown, by means of a simple heavy-fermion condensate model, how the Landau pole of the Higgs self-coupling can be related to the compositeness of the Higgs particle. We have analyzed the general features of such scenarios. In particular, we have shown that there exists a precise relationship between the effective Higgs mass, the new-fermion mass and the compositeness scale, which should hold in a wide class of models.

Acknowledgements

G.I. wishes to thank G. Altarelli and L. Maiani for interesting discussions, and the warm hospitality of the University of Virginia where part of this work was done. P.Q.H. wishes to thank the warm hospitality of the Theory Groups at the University of Rome “La Sapienza” and at Fermilab where part of this work was done. P.Q.H. is supported in parts by the US Department of Energy under grant No. DE-A505-89ER40518.

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