LABORATORI NAZIONALI DI FRASCATI
SIS – Pubblicazioni

LNF–96/047 (R)
10 Settembre 1996

Hysteretic and AC Losses of High T_c Superconductors by numerical Solution of the Nonlinear Magnetic Diffusion Equation

D. Di Gioacchino, P. Tripodi, F. Celani
INFN–Laboratori Nazionali di Frascati, P.O. Box 13, I–00044 Frascati (Rome), Italy
A.M.Testa
ICMAT–CNR, Area della Ricerca di Roma, P.B. 10 Monterotondo Stazione (Rome), Italy
S. Pace
Phys. Dept., University of Salerno, I–84100 Baronissi (Sa), Italy

Abstract- **The complex AC susceptibility (χ) of high T_c superconducting materials has been described in terms of hysteretic and AC losses. By including both flux creep and flux flow resistivities in the expression of the flux diffusivity, induction profiles have been numerically calculated from the non linear flux diffusion equation. The imaginary part of χ has been evaluated as function of temperature, frequency and amplitude of the applied magnetic field. The role of pinning and AC losses occuring in high T_c materials is discussed.**

I. INTRODUCTION

The critical state model for hard superconductors assumes that, in absence of thermally activated phenomena, supercurrents can flow in macroscopic regions, with density equal to a critical value, $J_c(B)$, which is a function of the local induction B. However, thermal activation is very effective for high T_c materials and it produces non linear resistive effects [1]. Thus, in AC susceptibility measurements both hysteretic and frequency dependent losses are present [2,3] and the critical state model is not accurate for calculating losses.

In order to take into account both kind of losses, a useful approach is the solution of the non linear diffusion equation [4] for the magnetic induction (B) in the sample. In this case the non-linear I-V characteristic of the material governs the spatial induction profile and its temporal evolution, determining therefore the frequency and amplitude dependence of losses [5].

In this paper we report the results of a numerical evaluation of the complex AC susceptibility ($\chi = \chi' + i\chi''$) as a function of temperature, frequency and amplitude of the applied magnetic field. In such a way, it is possible to probe the various regimes of flux dynamics (i.e. flux flow, flux creep) and to investigate the contribution of different pinning mechanisms.

II. NUMERICAL METHOD

We considered a infinitely long cylindrical sample, placed in a parallel field; in this geometry, the diffusion equation for the induction B(r,t) can be written as:

$$\partial B/\partial t = r^{-1}\partial/\partial r\,[r\,(\rho(B,J)/\mu_0)\,\partial B/\partial r] \qquad 1)$$

where r is the radial coordinate, $J(r,t) = (1/\mu_0)\partial B/\partial r$ is the current and $\rho(B,J)=E(B,J)/J$ the resistivity.

In a conventional approach [6,7], the total time for fluxon motion can be considered as the sum of the creep and flow time ; therefore, the resulting resistivity is the parallel of the creep (ρ_{cr}) and of the flux flow (ρ_{ff}) resistivities:

$$1/\rho = 1/\rho_{cr} + 1/\rho_{ff}$$

Equation (1), with the boundary conditions $\partial B/\partial r=0$

Manuscript received August 26, 1996

Proceedings of "1996 Applied Superconductivity Conference", Pittsburgh, August 25–30, 1996, MBB-7

Submitted to *IEEE Transaction Magnetics*

at $r=0$ and $B(r=R) = B_0 \sin\omega t$, has been numerically solved by means of the NAG Library routines. The algorithm computes the time evolution of the flux profile for a fixed number of spatial meshes. The periodic steady magnetization loops have been calculated starting from the difference between the volume average $\langle B(r,t)\rangle$ of the profile $B(r,t)$ and the istantaneous value of the applied field. The fundamental susceptibility $\chi=\chi'+i\chi''$ was then calculated as the first Fourier coefficients of the magnetization cycle.

An explicit expression for the resistivity has been reported by Brandt [8], where the creep electric field is:

$$E_{cr}(J) = 2\rho_c \exp(-U_p(T)/K_B T) * \sinh[J U_p(T)/(J_c(T) K_B T)] \qquad 2)$$

where $U_p(T)$ is the pinning potential, and we assume $\rho_c=\rho_{ff}$. For the flux flow electrical field the Bardeen-Stephen model [9] has been assumed:

$$E_{ff}(J)=J\rho_n B/B_{c2}(T) \qquad 3)$$

where the temperature dependence of the upper critical field is: $B_{c2}(t)=B_{c2}(0)\,(1-t^2)/(1+t^2)$, with $t=T/T_c$.

The material parameters refer to an YBCO sample [8,10,11,12] of radius $R=1$cm, $T_c=92.28$K, $B_{c2}(0)=112$T, $U_0(0)/K=2*10^4$K, $J_c(0)=10^{10}$A/m^2.

Neglecting fluctuations around T_c, the normal state resistivity is: $\rho_n(T) = 1.1*10^{-8}\ (\Omega m/K)*T + 2*10^{-6}\ \Omega m$. Moreover, as $U_p(T)$ and $J_c(T)$ vanish as $T\to T_c$, in this limit the superconducting state resistivity tends to the normal state value. Therefore, the results of the simulation are not very accurate near the transition temperature.

III. RESULTS AND DISCUSSIONS

The analysis has been restricted to the Bean critical-state model, neglecting the local magnetic field dependence of the pinning parameters J_c and U_p. The main reasons of such choice are the following:

a) analytical results for the susceptibility are available [13] in different geometries, making easier the comparison to the numerical solutions;

b) the Bean picture is effective for the comprehension of the effects related to thermally activated processes.

In the Bean model, the temperature dependence of the susceptibility is regulated by the temperature dependence of J_c, which in turn depends upon the assumed pinning model. In particular, two pinning models have been considered .

In the first one, denoted as CP[14], vortices may be supposed to be collectively pinned by randomly distributed weak pinning centers related to local variations of the mean free path. Such model has been reported to describe the behaviour of stoichiometric yttrium - based thin films [14]. The temperature dependences of U_p and J_c have the form:

$$U_p(B,t)=U_0\,\mathcal{U}_{CP}(t)=U_0(1-t^4) \qquad 4a)$$

$$J_c(B,t)=J_0\,\mathcal{J}_{CP}(t)=J_0\,(1-t^2)^{5/2}(1+t^2)^{-1} \qquad 4b)$$

In the second model, introduced by Tinkham and Malozemoff [15] (denoted as TM), the elementary pinning force is given by $f_p=U_p/\lambda$, where λ is the penetration depth, U_p is estimated by taking the condensation energy times the volume ξa_0^2, where $a_0^2=\phi_0/B$ [15] and ξ is the coherence length. Then the macroscopic force F_p results from a direct summation procedure of elementary forces f_p [8]. In this case temperature dependences are:

$$U_p(t)=U_0\,\mathcal{U}_{TM}(t)=U_0(1-t^2)^{3/2}(1+t^2)^{-1/2} \qquad 5a)$$

$$J_c(t)=J_0\,\mathcal{J}_{TM}(t)=J_0(1-t^2)(1+t^2) \qquad 5b)$$

As a general fact it should be remarked that for $t>0.8$ $\mathcal{J}_{CP}(t) \ll \mathcal{J}_{TM}(t)$, whereas $\mathcal{U}_{CP}(t) \gg \mathcal{U}_{TM}(t)$ for $t>0.9$.

For the CP model, plots of χ'' vs temperature at differents frequencies are reported in fig.1 for H=2.20mT; the same quantities are shown in fig.2 for TM model.

The Bean predictions are also reported in both figures. As the normalized current density $\mathcal{J}_{TM}(t)$ is greater than

$J_{CP}(t)$ for t>0.8, the corresponding peaks shown in figs.2 are sharper than those in figs.1; thus for the TM model the peak temperatures (T_p) are placed practically at T_c for both field amplitudes.

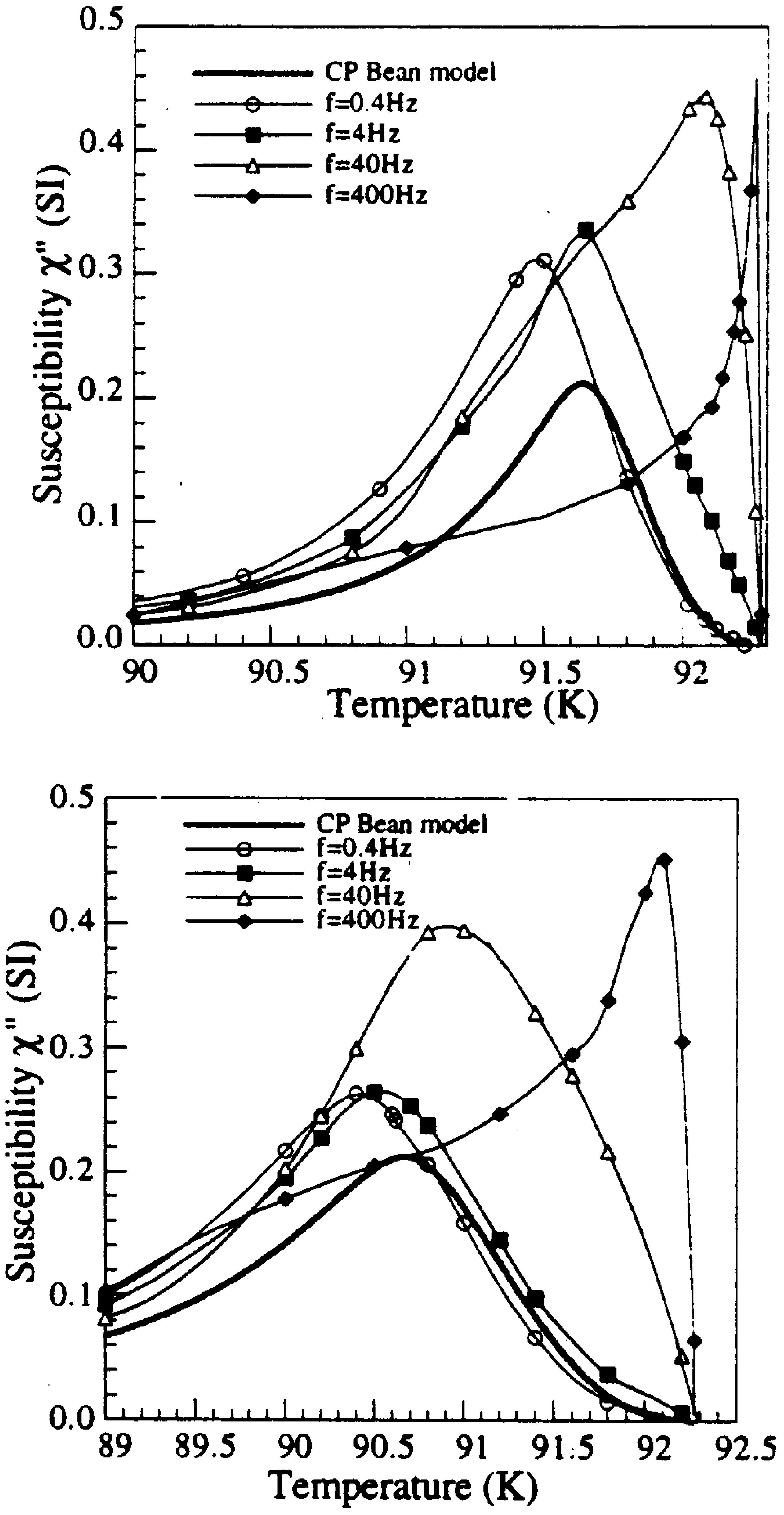

Fig.1. χ'' as a function of temperature at different frequencies for the CP model. Upper panel : B_0=2mT; lower panel B_0=20mT

For both models, as the frequency increases, curves of χ'' obtained by the diffusion equation show the following features :

a) an increase of T_p towards T_c ;

b) a progressive increase (up to a factor ≈2) of the peak amplitude respect to the Bean prediction.

Moreover, regardless to the frequency, as the field amplitude increases, T_p decreases due to the behaviour of the critical state contribution.

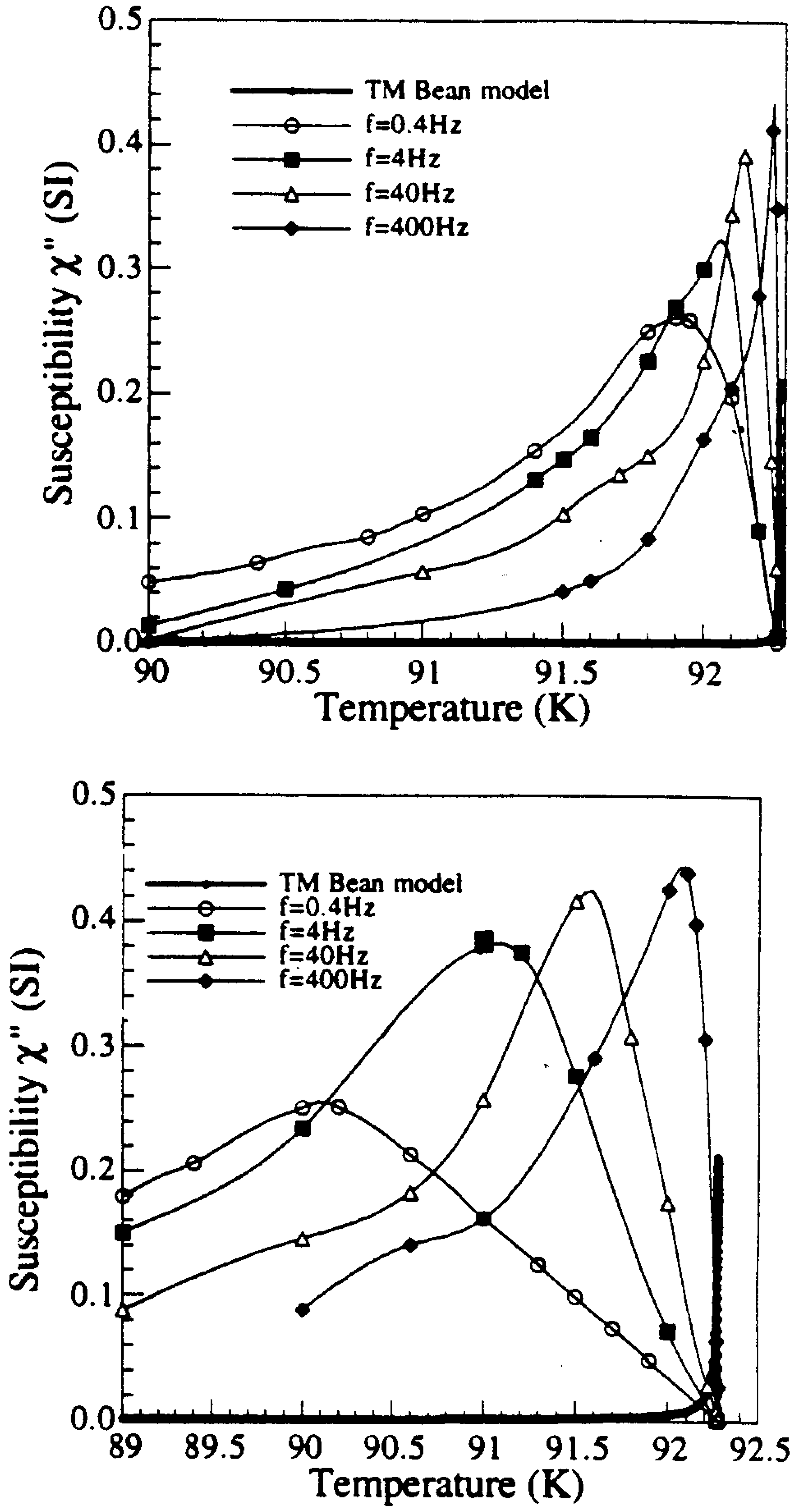

Fig.2. χ''as a function of temperature at different frequencies for the TM model. Upper panel : B_0=2mT; lower panel : B_0=20mT

The increase of the peak value respect to both the Bean prediction (χ''=0.21) and the metallic case (χ''=0.37) [3] is related to two combined effects: the creep rounding of the highly non linear I-V characteristics and the strong magnetic induction dependence of the flux flow resistivity.

For both models near the transition temperature, the presence of these combined effects clearly appears in the magnetization curves computed by the diffusion equation. As shown in fig. 3 for the CP model in a Bean approach, the cycle shapes would suggest an apparent Kim like decreasing of the critical current density with the local induction B. It should be noted that at the same frequencies such feature completely disappears at lower temperatures.

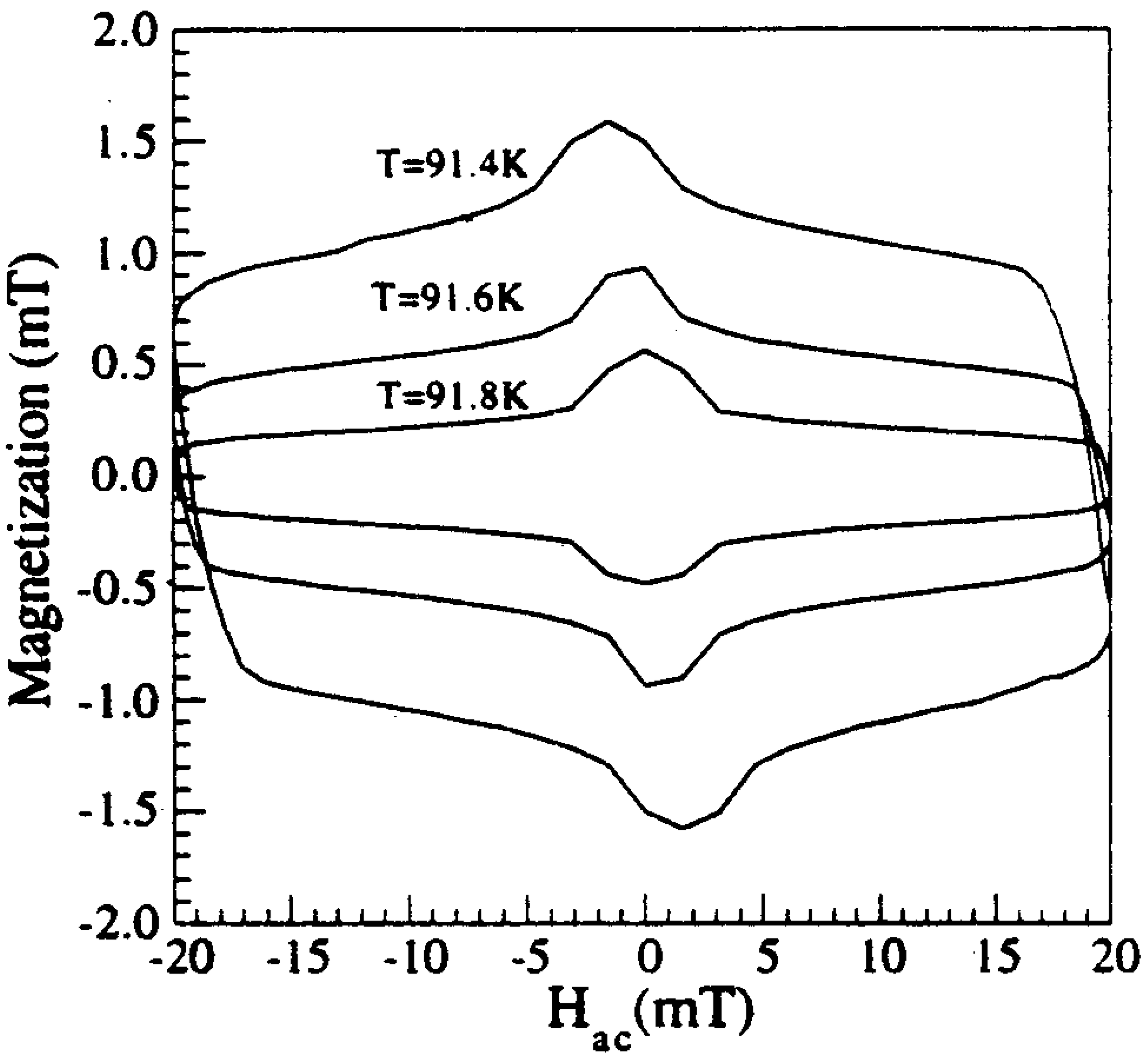

Fig.3 Magnetization curves at f=0.4Hz for the collective pinning model.

As far as the temperature dependence of χ'' is concerned, in the CP model the peak temperatures calculated at the lowest frequencies are lower than the Bean prediction, whereas at larger frequencies the reverse is found. As the applied field increases, T_p moves at lower temperatures, so that the freezing of the thermal activation determines a similarity between the low frequency diffusion results and the critical state behavior.

In the TM model, the calculated curves differ remarkably from the Bean result; indeed, thermally activated processes play a relevant role since for t>0.9 the pinning potential $U_{TM}(t)$ is much lower than $U_{CP}(t)$. As a consequence, the χ'' peak temperature lies well below the critical state value for each frequency and field amplitude value used in the numerical simulations .

CONCLUSIONS

We have investigated the frequency and temperature dependences of the imaginary part of the complex AC susceptibility by the numerical solution of the non linear diffusion equation for the magnetic induction, taking into account both hysteretic and AC losses. From a comparison with the results of the pure critical state model, it turns out that the determination of the critical current density from the peak temperature of χ'' should be exploited very carefully. Indeed, in a collective pinning picture, at larger frequencies. AC losses can be comparable or larger than the hysteretic ones. In the TM scheme, the giant creep phenomenon strongly depresses the critical current and mainly determines the χ'' versus T curves. Finally, in both approaches the amplitude of χ'' peaks is found to be much higher than both critical and the normal state predictions.

REFERENCES

[1] J.R.Clem *Magnetic Susceptibility of Superconductors and other Spin System*, p.177 Plenum press, New York (1991).
[2] Q.Y.Chen *Magnetic Susceptibility of Superconductors and other Spin System*, p.81 Plenum press, New York (1991).
[3]S.Samarappuli,A.Schilling,M.A.Chernikov, H.R.Ott,Th.Wolf,Phys. C vol 201, 159-165.(1992)
[4] C.J. van der Beek et al . Physica C vol 197, 320-336.(1992)
[5] A. Gurevich Int.Jour.Mod.Phys.vol B9 , 1045.(1995)
[6]S.Gjolmesli,K.Fossheim,T.Laegreid,*Transport Properties of Superconductors*,p .135 vol.25,(1990).
[7] R.Griessen Phys.Rev.Lett. vol. 64, 1674-1677 (1990).
[8]E.H.Brandt *Transport Properties of Superconductors*, p.97 vol.25,(1990).
[9] M. Tinkham *Introduction of Superconductivity* McGraw-Hill (1975)
[10]Mark W.Coffey and J.R.Clem Phys.Rev.B vol.45,p.9872-9881(1992).
[11] M. Tinkham, Helvetica Physica Acta vol. 61, 443 (1988).
[12] R. Civale, *Magnetic Susceptibility of Superconductors and other Spin System*, p.313 Plenum press, New York (1991).
[13] R.B.Goldfarb et al. *Magnetic Susceptibility of Superconductors and other Spin System*, p.49 Plenum press, New York (1991).
[14] R.Griessen,et al Phys. Rev. Let.t. vol.72, p.1910-1913 (1994).
[15] M.Tinkham ,Phys. Rev.Lett vol. 61,1658 (1988); Y. Yeshurun, A.P. Malozemoff, Phys. Rev. Lett. vol.60, 2202 (1988).