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Lower Scaling Dimensions of Quarks and Gluons and new Energy Scales

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Abstract We consider the possibility that quarks and gluons, due to confinement, have lower scaling dimensions. In such a case there appear naturally new energy scales below which the standard theory is recovered. Arguments are given whereby for dimension $1/2$ of the quarks the theory is unitary also above these energy scales.

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1-It is generally assumed that local gauge invariance, renormalizability and unitarity completely determine a lagrangian apart from mass parameters. This is certainly true for the leptons, for which unitarity requires scaling dimension $3/2$, gauge invariance determines the form of the interaction and renormalizability limits the possible terms. But for the quarks the situation might be different. Since unitarity is needed only in the space of physical states, the scaling dimension $3/2$ can be neither sufficient nor necessary. Of course it is not sufficient in a perturbative framework, and unitarity of QCD is not really proven, even though there are convincing indications of confinement. But just because of confinement it is not obvious that it is necessary, and one can wonder whether the quarks can have lower dimensions. The same argument applies to gluons. It is the purpose of this paper to point out that if the scaling dimensions are lower

- i) there appear naturally new energy scales below which the Standard Model is recovered
- ii) it is plausible that the dimension $1/2$ for the quarks provides the proper scaling dimensions of hadrons also at higher energies
- iii) the dimension zero for the gluons can give a linear quark-quark potential already at the classical level.

With lower dimensions an action renormalizable by power counting will include terms up to dimension 4, and it will therefore contain composite fields with their kinetic and interaction terms. Such a model reduces, at low energy, to the Standard Model with the addition of nonrenormalizable interactions suppressed by inverse powers of the new energy scales. In order to investigate the actual dimensions of quarks and gluons one should then examine the effects of these interactions, which might be interesting to do in relation to the possible discrepancies between some recent experimental results [1] and the Standard Model, especially in view of the fact that such discrepancies occur only in the quark sector. But the internal consistency of a model with lower scaling dimensions has a wider interest. First, if different dimensions are theoretically possible, the ones which are not realized must be forbidden by some mechanism. The exclusion of lower dimensions for the quarks could be guaranteed, for instance, by a quark-lepton symmetry and would therefore be in strong support of a Grand Unification. Note that quarks and leptons are in any case related by the requirement of anomaly cancellation, but this does not affect the power counting of the interactions [2].

Second, the problem we are considering can be formulated in a different way, relevant to theories where there occur higher dimension terms . Given a Lagrangian with fermionic contact interactions, is it possible to make it renormalizable and unitary? If the answer is affirmative, we can incorporate the first corrections in the above mentioned models in the standard framework. As far as renormalizability by power counting is concerned, we can achieve it by lowering the dimensions of the fundamental fermions by the addition of higher derivative terms. The real difficulty is with unitarity. We will argue that this difficulty can be overcome if the elementary fermions are confined. We will in fact show that, under certain conditions, if the quarks have scaling dimensions 1/2, the hadrons have the appropriate scaling behaviour to lowest order in a perturbative scheme we are investigating.

2-Let us first define the model and show how it behaves at low energies, starting from pure QCD. The lagrangian density of quarks of dimension 1/2 including terms of dimension not greater than 4 is

$$\begin{aligned}
 \mathcal{L}_Q = & \sum_f -\bar{\lambda}_f \left[\frac{1}{2} \{D_{1f}, \mathcal{P}\} + \Lambda_{1f} D_{2f} + \Lambda_f^2 (\mathcal{P} + m_f) \right] \lambda_f \\
 & + \sum_{\text{colorless composite fields}} \alpha_M \phi (-\square + m_M^2) \phi + \alpha_B \bar{\psi} (\not{\partial} + m_B) \psi + \alpha_Y \bar{\psi} \psi \phi \dots \\
 & + \sum_{\text{colored composite fields}} \dots
 \end{aligned} \tag{1}$$

where

$$\begin{aligned}
 \mathcal{D}_\mu &= \partial_\mu - igG_\mu, \\
 D_{if} &= \mathcal{D}_\mu \mathcal{D}_\mu + \frac{1}{2} \gamma_{if} [\mathcal{D}_\mu, \mathcal{D}_\nu] \sigma_{\mu\nu}, \quad i = 1, 2.
 \end{aligned} \tag{2}$$

In the above equation G_μ is the gluon field, the m 's are mass parameters, the α 's and γ 's dimensionless constants, the Λ 's energy scales which appear naturally due to the dimension of the quark fields λ_f (of flavor f), and ψ and ϕ colorless composite fields with the quantum numbers of baryons and mesons respectively. Examples of these fields, which we will need later, are [3]

$$\begin{aligned}
 \psi_{ps} &= \epsilon_{abc}(u^a \sigma_2 \sigma_h u^b)(\sigma_h d^c)_s, \quad \text{for the proton} \\
 \psi_{ns} &= \psi_{pn}(u \leftrightarrow d), \quad \text{for the neutron} \\
 \phi^+ &= d^* \bar{u}^* - \bar{d}u, \quad \text{for the } \pi^+,
 \end{aligned}
 \tag{3}$$

where we have adopted the convention of summation over repeated indices, a,b,c are color indices, u and \bar{u}^* are the upper and lower components of λ_1

$$\lambda_{1,s}^a = u_s^a, \quad \lambda_{1,s+2}^a = (\bar{u}_s^a)^*, \quad s = 1, 2,
 \tag{4}$$

and similarly for the other quarks. In addition there are in the lagrangian terms with colored composite fields and their covariant derivatives. We will comment later on this proliferation of couplings.

The free quark propagator in such a model

$$\frac{1}{p^2 \not{p} + \Lambda_{1f} p^2 + \Lambda_f^2 (\not{p} + m_f)}
 \tag{5}$$

becomes the usual one for $p^2 \ll \Lambda_f^2, \Lambda_{1f} \leq m_f$. We can then consider the standard theory as a low energy approximation of the present model. Below the energy scales Λ_f , we can introduce the dimension 3/2 quark field q_f

$$q_f(x) = \Lambda_f \lambda_f(x)
 \tag{6}$$

and rescale the composites according to

$$\psi' = \Lambda^3 \beta_\psi^{-\frac{1}{2}} \psi, \quad \phi' = \Lambda^2 \beta_\phi^{-\frac{1}{2}} \phi,
 \tag{7}$$

where Λ is the smallest of the Λ_f 's and the β 's are ratios of the different Λ_f , depending on the flavor content of the hadrons. We can thus rewrite the quark lagrangian in the form

$$\begin{aligned}
 \mathcal{L}'_Q &= \sum_f \bar{q}_f [(\not{P} + m_f) + \frac{\Lambda_{1f}}{\Lambda_f^2} D_{2f} + \frac{1}{\Lambda_f^2} \frac{1}{2} \{D_{1f}, \not{P}\}] q_f \\
 &+ \sum_{\text{colorless composites fields}} \frac{\alpha_M \beta_\phi}{\Lambda^4} \phi' (-\square + m_M) \phi' + \frac{\alpha_B \beta_\psi}{\Lambda^6} \bar{\psi}' (\not{P} + m_B) \psi' + \frac{\alpha_Y \beta_\psi \beta_\phi^{\frac{1}{2}}}{\Lambda^8} \bar{\psi}' \psi' \phi' \dots \\
 &+ \sum_{\text{colored composites fields}} \dots
 \end{aligned}
 \tag{8}$$

It appears as the standard lagrangian with a partial (because of the remaining one loop divergencies) regularization by higher derivatives. But we want to keep Λ finite, so that \mathcal{L}'_Q is a non renormalizable lagrangian with cutoff Λ . If the Λ_f 's are all of the same order of magnitude, the dominant corrections come from the terms quadratic in the quark fields.

Let us now consider the quark sector of the Standard Model. Since, as we have seen, terms involving the composite fields are suppressed at low energy by higher inverse powers of the energy scales, we consider here only the terms quadratic in the left and right quark fields

$$\mathcal{L}_Q = \bar{\lambda}_{L\alpha}[\mathcal{P} + \frac{1}{\Lambda_{L\alpha}^2} \frac{1}{2}\{\mathcal{P}, D_{L\alpha}\}]\lambda_{L\alpha} + \bar{\lambda}_{R\alpha}[\mathcal{P} + \frac{1}{\Lambda_{R\alpha}^2} \frac{1}{2}\{\mathcal{P}, D_{R\alpha}\}]\lambda_{R\alpha} \quad (9)$$

where α is the family index and, in standard notation

$$\begin{aligned} \mathcal{D}_\mu &= \partial_\mu - \frac{i}{2}g\tau W_\mu - \frac{i}{2}g'B_\mu, \\ D_{L,R\alpha} &= \mathcal{D}_\mu \mathcal{D}_\mu + \gamma_{L,R\alpha}[\mathcal{D}_\mu, \mathcal{D}_\nu]\sigma_{\mu\nu}. \end{aligned} \quad (10)$$

In a process involving one intermediate vector boson, for instance, we have corrections to the SM given by terms of the type

$$\frac{1}{\Lambda_L^2} \bar{\lambda}_L \frac{1}{2}\{\square, \mathcal{P}\}\lambda_L. \quad (11)$$

These are analogous to the corrections arising from the "leptophobic" vector boson introduced by Altarelli et al. [4]. If we want this correction to be smaller than 1%, we must take $\Lambda > 1TeV$.

We conclude by briefly considering the possibility of lower dimensions for the gluons. In this case the scaling dimension zero is natural in the sense that it can provide a linear potential among quarks already at the classical level [5]. The lagrangian density including terms of dimension not greater than 4 is

$$\mathcal{L}_G = \mathcal{D}_\lambda F_{\mu\nu} \mathcal{D}^\lambda F^{\mu\nu} + \alpha_{G1}[F_{\mu\nu} F^{\mu\nu}]^2 + \alpha_{G2}\square F_{\mu\nu} F_{\mu\nu} + \Lambda_G^2 F_{\mu\nu} F^{\mu\nu}. \quad (12)$$

In the above equation the α_G 's are dimensionless constants, Λ_G is an energy scale, $F_{\mu\nu}$ the stress tensor and \mathcal{D}_μ the covariant derivative in the adjoint representation. Again

one can rescale the gauge fields and the gauge coupling (which has here the dimension of an energy) to check that at low energy the model reduces to the standard one plus nonrenormalizable interactions.

4-We now show why it is plausible that the scaling dimension $1/2$ for the quarks is compatible with the right scaling dimensions for the hadrons and therefore with unitarity (from a perturbative point of view) in the hadron sector. We shall use an approach proposed [6] for the standard theory.

Consider first a barion-barion correlation function

$$\langle \psi_1(x_1)\psi_2(x_2) \rangle = \frac{1}{Z_0} \int [d\bar{\lambda}d\lambda] \psi_1(x_1)\psi_2(x_2) \exp[- \int d^4x \mathcal{L}_Q]. \quad (13)$$

We want to show that if we retain only the kinetic term for the composite field ψ in \mathcal{L}_Q , under certain conditions we get the right free propagator. The other terms must then be treated as a perturbation in the way outlined below.

Our argument is based on the use of the composite fields as independent variables. This requires a definition of the integral of a function of the ψ 's such that its value be equal to that obtained by expressing the ψ 's in terms of the λ 's and performing the Berezin integral over the latter [6]

$$\int [d\psi] f(\psi) = \int [d\lambda] f[\psi(\lambda)]. \quad (14)$$

Since the most general function of the ψ 's is a polynomial, it is sufficient to give this definition for monomials. We say that a monomial Θ is fundamental if it is a product of the ψ 's with coefficient unity and such that

$$\Theta_m = \psi_1^{m_1} \psi_2^{m_2} \dots = w_m \prod \lambda, \quad w_m \neq 0. \quad (15)$$

In the above equation $\prod \lambda$ is the product of all the quark field components in a given space-time point, m is a vector index with components $m_i = 0, 1$ and w_m is the weight of Θ_m . The definition we are looking for is

$$\int [d\psi] \Theta_m = w_m. \quad (16)$$

The important property of this definition is that if the fields ψ are chosen in such a way that there is only one fundamental monomial (this is one of the conditions referred to above), the integral defined by Eq.(16) is identical, after a rescaling of the ψ 's to get rid of the weight, to the Berezin integral. It follows that if we assume as free action for these ψ -fields the Dirac action, their propagator is the canonical propagator of a Dirac particle. Let us report, as an example, that if we confine ourselves to the quarks u and d the condition of a unique fundamental monomial is naturally realized by assuming as barion variables the nucleon ones. In this case in fact the only monomial with nonvanishing weight is [6]

$$\Theta = \psi_{p1}\psi_{p2}\psi_{n1}\psi_{n2} = 2^7 \cdot 3^2 \cdot 5P(u_1)P(u_2)P(d_1)P(d_2), \quad (17)$$

where the ψ_{pk}, ψ_{nk} are given by Eq.(3) and

$$P(u_k) = u_k^1 u_k^2 u_k^3. \quad (18)$$

Let us now consider the case of scalars. Again we can define fundamental monomials and their integrals, but now the exponents of the bilinear composites in these monomials can be higher than 1, and even when they are one the evaluation of the propagators is much more complicated, and cannot in general be performed analytically. It has however been shown [6] that the propagator of a composite complex scalar like ϕ of Eq.(3) is equal to the selfavoiding random walk in any number of space-time dimensions where this is a generalized free theory. As it is well known this happens in dimension greater than 4 and it is conjectured and numerically confirmed [7] to be true also in dimension 4. Now a generalized free field is characterized by the fact that its n -point functions factorize into products of the 2-point function, but we do not know anything about the Lehmann representation of this 2-point function, and at this stage we must assume that it is not too different from that of a Klein-Gordon particle. The validity of this assumption is another of the conditions referred to at the beginning, which makes our result for scalar composites weaker than for the spinor ones.

Processes involving at the same time nucleons and mesons can be treated for instance by a regularization on an interpenetrating hypercubic body centered lattice, by performing

alternatively the transformation to trilinear and bilinear fields, and for different barions and mesons we can proceed similarly.

The above results can be substantiated only if one can show in this framework that the interactions do not violate the unitarity of the free approximation. To do this we are studying a perturbative scheme along the following lines. Selected the physical fields relevant in a given process, we assume their kinetic term as the free action and treat all the other terms, including the ones quadratic in the quark fields, as a perturbation. We then expand wr to this perturbation and, at each order, we express everything in terms of the fundamental monomials and perform the integration according to the above rules assuming the relevant composites as independent variables.

The fact that one can construct generalized free fields by polynomials of free fields has been known for a long time [8], but we think we have gone a little bit further. First, we have shown that barions are canonical fields, which is encouraging in view of their behaviour as the constituents of matter. Second, mesons are to some extent characterized and third, the way these results are obtained opens the perspective of actual perturbative or numerical computations.

A last comment about the proliferation of couplings. The evaluation of composite correlation functions in the standard theory requires in general additional normalization conditions. The additional parameters appearing with lower dimensions might be, to some extent, the counterpart of these additional conditions. The hadronic masses, however, in the standard theory can be calculated in terms of the fundamental parameters, while they appear as free parameters in the present model. To asses whether this is really so, one should understund how the bound state problem can be formulated in the present context.

If instead too many of the parameteres appearing in the actions (1),(9) turn out to be independent and the hadronic masses cannot be calculated, even if internally consistent the model is really unsatisfactory and it will not be realized in Nature. The point we want to raise is that even so, its internal consistency would have interesting consequences, pointing toward the mechanism which prevents its occurrence.

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