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Radiative four-meson amplitudes in chiral perturbation theory *

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Abstract

We present a general discussion of radiative four-meson processes to $O(p^4)$ in chiral perturbation theory. We propose a definition of "generalized bremsstrahlung" that takes full advantage of experimental information on the corresponding non-radiative process. We also derive general formulae for one-loop amplitudes which can be applied, for instance, to $\eta \to 3\pi\gamma$, $\pi\pi \to \pi\pi\gamma$ and $K \to 3\pi\gamma$.

1. Chiral perturbation theory (CHPT) [1–3] incorporates electromagnetic gauge invariance. To lowest order in the derivative expansion, $O(p^2)$ in the meson sector, amplitudes for radiative transitions are completely determined by the corresponding non-radiative amplitudes. Direct emission, carrying genuinely new information, appears only at $O(p^4)$. Nevertheless, even in higher orders of the chiral expansion part of the amplitude is related to the respective non-radiative process. In order to isolate the direct emission amplitude, an operational definition of "generalized bremsstrahlung" is needed.

In this letter, we investigate in a general manner radiative transitions involving four pseudoscalar mesons and one real photon. Possible applications to be discussed elsewhere include $\eta \to 3\pi\gamma$ and $\pi\pi \to \pi\pi\gamma$ in the strong sector and the nonleptonic weak decays $K \to 3\pi\gamma$. Our purpose is twofold:

i. We extend Low's theorem [4] by terms of $O(k)$ (k is the photon momentum) to define generalized bremsstrahlung. This part will include in particular *all* local terms of $O(p^4)$ that contribute also to the non-radiative four-meson transition.

ii. We give a compact expression for the loop amplitude of a general four-meson process with a real photon. The resulting formula is immediately applicable to both strong and nonleptonic weak processes. We also consider the limiting case of a radiative three-meson amplitude to recover known results for $K \to 2\pi\gamma$ decays [5–8].

2. The amplitude for a four-meson transition with a single photon can be decomposed into an electric and a magnetic part:

$$A(\varphi_a\varphi_b\varphi_c\varphi_d\gamma) = e\varepsilon^{\mu}(k)(E_{\mu} + \varepsilon_{\mu\nu\rho\sigma}M^{\nu\rho\sigma}) , \quad (1)$$

with

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$$k^\mu E_\mu = 0, \quad \varepsilon_{\mu\nu\rho\sigma} k^\mu M^{\nu\rho\sigma} = 0 .$$

The magnetic amplitude $M^{\nu\rho\sigma}$ can only occur in nonleptonic weak processes (for a review, see Ref. [9]). It appears first at $O(p^4)$ as a tree-level contribution and will not concern us further. Here, we are only interested in the electric amplitude E_μ that is in particular sensitive to bremsstrahlung. The kinematics of the process is specified by five scalar variables which we choose as

$$s = (p_1 + p_2)^2, \quad \nu = p_3(p_1 - p_2) ,$$
$$t_i = k \cdot p_i \quad (i = 1, \ldots, 4) , \tag{2}$$

with

$$\sum_{i=1}^{4} p_i + k = 0 , \quad \sum_{i=1}^{4} t_i = 0 .$$

Any three of the t_i together with s and ν form a set of independent variables.

The non-radiative transition is characterized by the two Dalitz variables s and ν. Denoting the non-radiative amplitude by $A(s,\nu)$, Low's theorem [4] amounts to the following expansion in the photon momentum k:

$$E^\mu = A(s,\nu)\Sigma^\mu + 2\frac{\partial A(s,\nu)}{\partial s}\Lambda_{12}^\mu + \frac{\partial A(s,\nu)}{\partial \nu}(\Lambda_{13}^\mu - \Lambda_{23}^\mu) + O(k) , \tag{3}$$

with (the meson charges in units of e are denoted q_i)

$$\Sigma^\mu = \sum_{i=1}^{4} \frac{q_i p_i^\mu}{t_i} ,$$
$$\Lambda_{ij}^\mu = \Lambda_{ji}^\mu = (q_i t_j - q_j t_i) D_{ij}^\mu ,$$
$$D_{ij}^\mu = -D_{ji}^\mu = \frac{p_i^\mu}{t_i} - \frac{p_j^\mu}{t_j} . \tag{4}$$

The explicit terms in (3) are often called "internal bremsstrahlung". It is straightforward to show that there are no terms of $O(k)$ at lowest order in the chiral expansion. Thus, for radiative four-meson processes the leading chiral amplitude of $O(p^2)$ is completely determined by the non-radiative amplitude $A(s,\nu)$ as expressed by Eq. (3).

3. At $O(p^4)$, there are as usual both one-loop and tree-level contributions with a single vertex from the strong Lagrangian $\mathcal{L}_4$ [2,3] or the nonleptonic weak Lagrangian $\mathcal{L}_4^{\Delta S=1}$ [10,11]. Let us first consider the tree-level amplitude. The different terms in either $\mathcal{L}_4$ or $\mathcal{L}_4^{\Delta S=1}$ that can contribute to the processes under consideration can be grouped in four classes:

A. Terms of $O(m_q^2)$ without derivatives: fully covered by internal bremsstrahlung (3).
B. Terms of $O(m_q)$ with two (covariant) derivatives: again included in (3).
C. Four-derivative terms: in general not fully covered by (3).
D. Terms with two derivatives and one field strength tensor containing the electromagnetic field: contribute only to the radiative transition and thus are never included in (3).

Obviously, groups A, B correspond to internal bremsstrahlung while the contributions of type D belong to direct emission. Class C falls in between if we adopt Eq. (3) as the definition of bremsstrahlung. On the other hand, it would have both conceptual and practical advantages to include *all* terms under the heading "bremsstrahlung" that contribute to both radiative and non-radiative transitions. One practical advantage arises for $K \to 3\pi\gamma$ decays where the low-energy constants of the four-derivative terms are only partly known [12,11]. If those terms could be included in bremsstrahlung, we may use experimental data for $K \to 3\pi$ decays directly without having to worry about the values of the aforementioned coupling constants [13].

In order to incorporate class C in what we shall call generalized bremsstrahlung, we must add explicit terms of $O(k)$ to Low's formula (3). The clue for the solution is the observation that both $\mathcal{L}_4$ and $\mathcal{L}_4^{\Delta S=1}$ give rise to at most three independent four-derivative couplings at the mesonic level:

$$D_\mu\varphi_a D^\mu\varphi_b D_\nu\varphi_c D^\nu\varphi_d ,$$
$$D_\mu\varphi_a D^\mu\varphi_c D_\nu\varphi_b D^\nu\varphi_d ,$$
$$D_\mu\varphi_a D^\mu\varphi_d D_\nu\varphi_b D^\nu\varphi_c ,$$
$$D_\mu\varphi_a = (\partial_\mu + iq_a e A_\mu)\varphi_a . \tag{5}$$

At the same time, we have three independent second derivatives of the non-radiative amplitude $A(s,\nu)$. Therefore, the following extension of (3)

solves the problem (GB stands for "generalized bremsstrahlung"):

$$E^\mu = E^\mu_{\rm GB} + O(k)\,,$$

$$E^\mu_{\rm GB} = A(s,\nu)\Sigma^\mu + 2\frac{\partial A(s,\nu)}{\partial s}\Lambda^\mu_{12} + \frac{\partial A(s,\nu)}{\partial \nu}(\Lambda^\mu_{13} - \Lambda^\mu_{23}) + 2\frac{\partial^2 A(s,\nu)}{\partial s^2}(t_1+t_2)\Lambda^\mu_{12} + \frac{1}{2}\frac{\partial^2 A(s,\nu)}{\partial \nu^2}[(t_1-t_2)(\Lambda^\mu_{13}-\Lambda^\mu_{23}) - t_3 t_4 \Sigma^\mu] + 2\frac{\partial^2 A(s,\nu)}{\partial s \partial \nu}[t_2\Lambda^\mu_{13} - t_1\Lambda^\mu_{23}]\,. \quad (6)$$

It is important to realize that $E^\mu_{\rm GB}$ in (6) does not contain *all* terms of at most $O(k)$. In fact, it is impossible in general to relate all terms of $O(k)$ to derivatives of $A(s,\nu)$. On the other hand, the definition of generalized bremsstrahlung in (6) guarantees that all local (counter)terms that contribute to both radiative and non-radiative processes (classes A, B, C) are included in $E^\mu_{\rm GB}$. The difference $E^\mu - E^\mu_{\rm GB}$ is at least $O(k)$ and will be referred to as the direct emission amplitude (of the electric type).

4. We now turn to the loop amplitude. Most of the renormalization procedure can trivially be carried over from the non-radiative to the radiative amplitude because all diagrams (tadpoles) relevant for mass, charge and wave function renormalization contribute only to internal bremsstrahlung. Thus, this part is completely taken care of by the non-radiative amplitude. Moreover, for a real photon there are no diagrams of the form factor type where the photon emerges from a mesonic bubble.

Restricting attention to transitions of at most first order in the Fermi coupling constant, the only non-trivial diagram is of the type shown in Fig. 1 where the photon can hook on to any charged meson line and to any vertex with at least two charged fields. The two vertices are either both from the lowest-order chiral Lagrangian $\mathcal{L}_2$ [2,3] (strong transition) or one from $\mathcal{L}_2$ and one from $\mathcal{L}_2^{\Delta S=1}$ (cf., e.g., Ref. [9]) for a non-leptonic weak transition. Despite the comparative simplicity, diagrams of the type displayed in Fig. 1 with a photon in all possible places generate a considerable number of terms due to the derivative structure of vertices. Moreover, there are usually several permutations $(1234) \to (abcd)$ that have to be added for a given process.

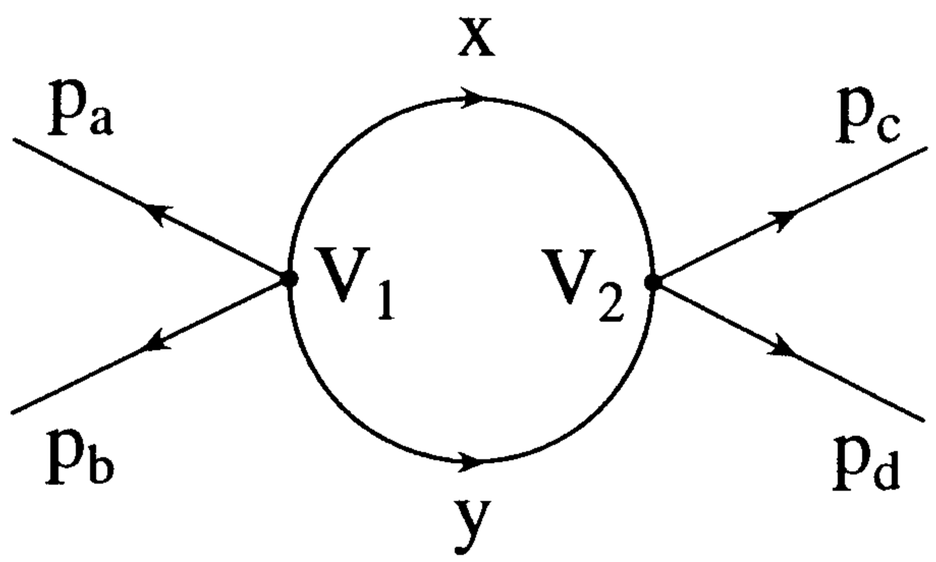

Fig. 1. One-loop diagram for the four-meson transition. For the radiative amplitude, the photon must be appended to every charged meson line and to every vertex with at least two charged fields. The vertices V_1, V_2 are defined in Eq. (7).

We have therefore found it useful, both for our own work and for possible future applications, to present the loop amplitude in a compact form suitable for any strong or nonleptonic weak transition. For this purpose, we first calculate the loop contribution to the non-radiative amplitude $A(s,\nu)$.

We characterize the vertices V_1, V_2 in momentum space by real constants a_i, b_i:

$$V_1 = a_0 + a_1 p_a\cdot p_b + a_2 p_a\cdot x + a_3(x^2 - M_x^2) + a_4(y^2 - M_y^2) + a_5(p_a^2 - M_a^2) + a_6(p_b^2 - M_b^2)\,,$$
$$V_2 = b_0 + b_1 p_c\cdot p_d + b_2 p_c\cdot x + b_3(x^2 - M_x^2) + b_4(y^2 - M_y^2) + b_5(p_c^2 - M_c^2) + b_6(p_d^2 - M_d^2)\,. \quad (7)$$

In calculating the loop amplitude for the non-radiative process, we do not associate the various scalar products with the Dalitz variables s, ν that will depend on the specific assignment $(abcd) \to (1234)$. Instead, we write the loop amplitude as a function of the four-momentum

$$P = p_c + p_d\,. \quad (8)$$

The non-photonic loop amplitude of Fig. 1 can be represented in the following form that will turn out to be useful (all external lines are on-shell):

$$
\begin{aligned}
F(P) &= A(M_x)[a_1 b_4 p_a\cdot p_b + a_4 b_1 p_c\cdot p_d \\
&\quad + a_4 b_4(P^2 + M_x^2 - M_y^2) + a_0 b_4 + a_4 b_0] \\
&+ A(M_y)[a_1 b_3 p_a\cdot p_b + a_2 b_3 p_a\cdot P + a_3 b_1 p_c\cdot p_d \\
&\quad + a_3 b_2 p_c\cdot P + a_3 b_3(P^2 - M_x^2 + M_y^2) \\
&\quad + a_0 b_3 + a_3 b_0] \\
&+ B(P^2, M_x, M_y)[a_0 b_0 + a_0 b_1 p_c\cdot p_d \\
&\quad + a_1 b_0 p_a\cdot p_b + a_1 b_1 p_a\cdot p_b p_c\cdot p_d] \\
&+ B_1(P^2, M_x, M_y)[a_0 b_2 p_c\cdot P + a_2 b_0 p_a\cdot P \\
&\quad + a_1 b_2 p_a\cdot p_b p_c\cdot P + a_2 b_1 p_c\cdot p_d p_a\cdot P] \\
&+ a_2 b_2[p_a\cdot p_c B_{20}(P^2, M_x, M_y) \\
&\quad + p_a\cdot P p_c\cdot P B_{22}(P^2, M_x, M_y)]\,. \qquad (9)
\end{aligned}
$$

The various functions in (9) are as defined conventionally (in d dimensions):

$$
\begin{aligned}
&A(M) = \frac{1}{i}\int \frac{d^d x}{(2\pi)^d}\frac{1}{x^2 - M^2}\,, \\
&(B, B_1 P_\mu, g_{\mu\nu} B_{20} + P_\mu P_\nu B_{22}) \\
&\quad = \frac{1}{i}\int \frac{d^d x}{(2\pi)^d}\frac{(1, x_\mu, x_\mu x_\nu)}{(x^2 - M_x^2)[(x-P)^2 - M_y^2]}\,. \qquad (10)
\end{aligned}
$$

Several comments are in order at this point.

i. In order to limit the number of terms generated, it is preferable to express the functions B_1, B_{20} and B_{22} via the usual recursion relations in terms of A, B only at the very end.

ii. The analytically non-trivial part of (9), involving the various B functions, contains only the on-shell couplings $a_0, a_1, a_2, b_0, b_1, b_2$. The off-shell couplings a_3, a_4, b_3, b_4 appear only together with the divergent constants $A(M)$. Since these terms are polynomials in the momenta of at most degree two, they will enter in the radiative amplitude only through internal bremsstrahlung. All the divergences in (9) will be absorbed by counterterms belonging to classes A,B,C of the previous classification. As emphasized before, the generalized bremsstrahlung (6) contains all these divergences plus the corresponding counterterms.

iii. For a reason soon to become evident, we have chosen to express $F(P)$ in terms of the scalar products

$$
\begin{aligned}
&p_a\cdot p_b, \quad p_c\cdot p_d, \quad P^2, \quad p_a\cdot P, \quad p_c\cdot P, \\
&p_a\cdot p_c\,. \qquad (11)
\end{aligned}
$$

In other words, we have not used kinematical relations to write $F(P)$ in terms of only two independent scalar variables [like s, ν defined in Eq. (2)].

5. We now have all the ingredients for calculating the radiative loop amplitude $E^\mu_{\rm loop}$ corresponding to the diagrams of Fig. 1 with a photon in all possible places. For the general vertices V_1, V_2 given in (7), this loop amplitude contains several hundred terms even before reducing the various B functions via recursion relations. A compact representation will therefore be of great use for avoiding tedious repetitions of the same procedure.

We find it useful to decompose the radiative loop amplitude into two parts:

$$
E^\mu_{\rm loop} = G^\mu + H^\mu. \qquad (12)
$$

The more tedious part of the calculation is contained in the amplitude G^μ that can be expressed through various derivatives of the non-radiative loop amplitude F in (9) with respect to the scalar products (11). In some of the following terms, the momentum P has to be replaced by $P + k$, leaving all scalar products unchanged that do not contain P explicitly:

$$
\begin{aligned}
G^\mu &= F(P)\Sigma^\mu + \frac{F(P+k) - F(P)}{k\cdot P}\Lambda^\mu_{cd} \\
&\quad + \frac{\partial F}{\partial(p_a\cdot p_b)}(P)\Lambda^\mu_{ab} + \frac{\partial F}{\partial(p_a\cdot P)}(P)\Lambda^\mu_{aP} \\
&\quad + \frac{\partial F}{\partial(p_c\cdot p_d)}(P+k)\Lambda^\mu_{cd} + \frac{\partial F}{\partial(p_c\cdot P)}(P+k)\Lambda^\mu_{cP} \\
&\quad + \left[q_a t_c \frac{\partial F}{\partial(p_a\cdot p_c)}(P)\right. \\
&\qquad \left. - q_c t_a \frac{\partial F}{\partial(p_a\cdot p_c)}(P+k)\right] D^\mu_{ac} \\
&\quad - \frac{1}{2}(q_c + q_d) t_a t_c \left[\frac{\partial^2 F}{\partial(p_a\cdot P)\partial(p_c\cdot P)}(P) D^\mu_{aP}\right. \\
&\qquad \left. - \frac{\partial^2 F}{\partial(p_a\cdot P)\partial(p_c\cdot P)}(P+k) D^\mu_{cP}\right]. \qquad (13)
\end{aligned}
$$

We have used the definitions (4). When P appears as an index (e.g., in Λ^μ_{aP} or D^μ_{cP}), the corresponding

momentum and charge in (4) are P and $q_c + q_d$, respectively. For better understanding of the notation in (13), we give two explicit examples:

$$\frac{\partial F}{\partial(p_c\cdot P)}(P+k) = a_3 b_2 A(M_y) + (a_0 b_2 + a_1 b_2 p_a\cdot p_b) B_1((P+k)^2, M_x, M_y) + a_2 b_2 p_a\cdot(P+k) B_{22}((P+k)^2, M_x, M_y)\,,$$

$$\frac{\partial^2 F}{\partial(p_a\cdot P)\partial(p_c\cdot P)}(P) = a_2 b_2 B_{22}(P^2, M_x, M_y)\,. \tag{14}$$

The second part H^μ of the loop amplitude (12) cannot be expressed in terms of F or derivatives thereof. Since the dominant contributions to $E^\mu_{\rm loop}$ are usually due to pion loops (if they contribute at all), we give the explicit expression for H^μ only for the case of equal loop masses ($M_x = M_y =: M$). In this special case, H^μ takes on the following compact form:

$$\begin{aligned} H^\mu &= a_2(t_b p_a^\mu - t_a p_b^\mu)\\ &\times\{(q_x-q_y)(2b_0+2b_1 p_c\cdot p_d + b_2 p_c\cdot P)\\ &\quad\times\widetilde{C_{20}}(P^2,-k\cdot P)\\ &\quad + b_2(q_x+q_y)[-2p_c\cdot P\widetilde{C_{31}}(P^2,-k\cdot P)\\ &\quad + 2t_c\widetilde{C_{32}}(P^2,-k\cdot P) - p_c\cdot P\widetilde{C_{20}}(P^2,-k\cdot P)]\}\\ &+ b_2(t_d p_c^\mu - t_c p_d^\mu)\\ &\times\{(q_x-q_y)[2a_0+2a_1 p_a\cdot p_b + a_2(p_a\cdot P + t_a)]\\ &\quad\times\widetilde{C_{20}}((P+k)^2, k\cdot P)\\ &\quad + a_2(q_x+q_y)[-2(p_a\cdot P+t_a)\\ &\quad\times\widetilde{C_{31}}((P+k)^2,k\cdot P) - 2t_a\widetilde{C_{32}}((P+k)^2,k\cdot P)\\ &\quad - (p_a\cdot P+t_a)\widetilde{C_{20}}((P+k)^2,k\cdot P)]\}\,. \end{aligned} \tag{15}$$

The functions $\widetilde{C_{ij}}$ are defined as

$$\widetilde{C_{ij}}(u,v) = \frac{C_{ij}(u,v) - C_{ij}(u,0)}{v} \tag{16}$$

in terms of the three-propagator one-loop functions $C_{ij}(p^2, k\cdot p)$ for $k^2 = 0$:

$$\frac{1}{i}\int\frac{d^dx}{(2\pi)^d} \times \frac{\{x_\mu x_\nu, x_\mu x_\nu x_\rho\}}{(x^2-M^2)[(x+p)^2-M^2][(x+k)^2-M^2]} = \{C_{20}(p^2,k\cdot p)g_{\mu\nu} + \ldots, C_{31}(p^2,k\cdot p)(p_\mu g_{\nu\rho} + p_\nu g_{\mu\rho} + p_\rho g_{\mu\nu}) + C_{32}(p^2,k\cdot p)(k_\mu g_{\nu\rho} + k_\nu g_{\mu\rho} + k_\rho g_{\mu\nu}) + \ldots\}. \tag{17}$$

As in the case of $F(P)$ in (9), it is advisable not to use the standard recursion relations for the functions C_{20}, C_{31} and C_{32} in (15) until the actual numerical analysis. At the expense of introducing the functions B, B_{20}, B_{22} defined in (10), one may express the functions $\widetilde{C_{ij}}((P+k)^2, k\cdot P)$ in terms of the $\widetilde{C_{ij}}(P^2, -k\cdot P)$ or vice versa.

The following comments (valid also in the case of different loop masses) explain the motivation for splitting the loop amplitude $E^\mu_{\rm loop}$ in (12) into two parts.

i. The amplitudes G^μ in (13) and H^μ in (15) are separately gauge invariant.

ii. The amplitude H^μ is finite and at least of $O(k)$ as is evident from Eqs. (15), (16) and (17). Moreover, it only contains the on-shell couplings $a_0, a_1, a_2, b_0, b_1, b_2$ defined in (7) and the charges q_x, q_y of the particles in the loop. Of course, we have

$$q_x + q_y = -q_a - q_b = q_c + q_d\,. \tag{18}$$

iii. The amplitude G^μ contains the generalized bremsstrahlung (6) for the non-radiative loop amplitude (9). If we denote by $E^\mu_{\rm GB}({\rm loop})$ the result obtained by inserting for $A(s,\nu)$ the on-shell loop amplitude (9) in Eq. (6), then the difference

$$\Delta^\mu = G^\mu - E^\mu_{\rm GB}({\rm loop}) \tag{19}$$

is at least of $O(k)$. Moreover, by construction of $E^\mu_{\rm GB}$ the divergences in Δ^μ are renormalized by counterterms of class D only, i.e. by counterterms with an explicit field strength tensor. In the strong sector, the relevant couplings of $O(p^4)$ are L_9 for chiral $SU(3)$ [ℓ_6 for $SU(2)$] and $N_{14}, N_{15}, N_{16}, N_{17}$ for the octet part of the nonleptonic weak Lagrangian [11]. Finally, it

can be shown that if $a_2 b_2 = 0$ then Δ_μ is finite and at least of $O(k^2)$ for $s = \sum_{i=1}^4 M_i^2/3$, $\nu = 0$ and arbitrary t_i.

iv. The apparent asymmetry of G^μ and H^μ under interchanges $a \leftrightarrow b$ or $c \leftrightarrow d$ is due to the asymmetric definition of vertices (7). For the same reason, G^μ and H^μ are in general not invariant under interchanges of the loop particles x and y.

For the realistic case with experimental information on the non-radiative amplitude $A(s,\nu)$, the complete electric amplitude to $O(p^4)$ accuracy can be written as

$$E^\mu = E^\mu_{\rm GB}(\exp) + E^\mu_{\rm counter} + \sum_{\rm loops}(\Delta^\mu + H^\mu)\,, \qquad (20)$$

where several loop diagrams may have to be added for a given physical transition. Only counterterms with an explicit field strength tensor must be included in $E^\mu_{\rm counter}$. Consequently, only the renormalized coupling constants $L_9^r(\ell_6^r)$ and/or $N_{14}^r, \ldots, N_{17}^r$ appear in (20). Of course, the amplitude is finite and scale independent by construction. All other counterterms of $O(p^4)$ are hidden in $E^\mu_{\rm GB}(\exp)$. An alternative approach is to use instead of the experimental amplitude $E^\mu_{\rm GB}(\exp)$ in (20) the theoretical prediction $E^\mu_{\rm GB}(\text{theory})$ in terms of the amplitude $A(s,\nu)$ calculated in CHPT to $O(p^4)$ accuracy. Both approaches are equivalent to $O(p^4)$. The difference between them gives an indication of the size of effects of $O(p^6)$ and higher.

6. As a final application, we consider the limit $q_a \to 0$, $p_a \to 0$ to connect with known results for $K \to 2\pi\gamma$ decays. In this case, the vertex V_1 is necessarily of the weak nonleptonic type and the coupling constants a_1, a_2, a_5 disappear. A straightforward calculation shows that G^μ becomes in this limit

$$G^\mu = F(P+k)\Sigma^\mu\,, \qquad (21)$$

where $F(P+k) = F(-p_b)$ is now the on-shell loop amplitude for the decay $b \to c + d$. Eq. (21) is nothing but the familiar bremsstrahlung amplitude for a radiative three-meson transition and as a consequence

$$\Delta^\mu = 0\,. \qquad (22)$$

Likewise, the amplitude H^μ in (15) reduces in the three-meson limit to the single term [6–8]

$$H^\mu = 2a_0 b_2 (q_x - q_y)\widetilde{C_{20}}(M_b^2, -k\cdot p_b)(t_d p_c^\mu - t_c p_d^\mu)\,. \qquad (23)$$

Thus, the loop contribution to direct emission is finite and proportional to a_0, the on-shell tree-level amplitude for the nonleptonic weak transition $b \to c + d$.

7. To summarize, we have presented a general discussion of radiative four-meson processes to $O(p^4)$ in CHPT. We have proposed a definition of generalized bremsstrahlung in Eq. (6) that has the advantage of including all counterterms of $O(p^4)$ that contribute to both radiative and non-radiative amplitudes. For general vertices of $O(p^2)$ that encompass all strong and nonleptonic weak transitions of interest, we have calculated the non-trivial loop amplitudes in terms of two gauge invariant parts. The amplitude G^μ given in (13) is expressed in terms of the non-radiative loop amplitude F in (9). The divergences are all contained in G^μ. The remainder H^μ given in Eq. (15) is finite and at least of $O(k)$.

Applications of these general results to $K \to 3\pi\gamma$ decays and other radiative four-meson processes will be presented elsewhere [13].

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